Programming 1

Lesson 03 – Binary, Octal, Decimal, and Hexadecimal Conversion

Binary, Octal, Decimal, and Hexadecimal Conversion

Different Data Representations

- We have learned that the computer can only understand machine code (0 and 1).
- In this case, if you have a number 39, it cannot be stored directly as it is in the computer (there is neither ‘3’ nor ‘9’ in machine code).
- Actually, there are more than one data representations. What we use in our everyday life is called the “Decimal” number.
- Beyond Decimal (D), there are also
  - Binary (B)
  - Octal (O)
  - Hexadecimal (H)

Different Data Representations

- In Decimal representation, we have 10 basic symbols: 0, 1, 2, …, 9
- In Binary representation, we have 2 basic symbols: 0 and 1
- In Octal representation, we have 8 basic symbols: 0, 1, 2, …, 7
- In Hexadecimal representation, we have 16 basic symbols: 0, 1, 2, …, 9, plus a, b, c, d, e, f

Different Data Representations

Representation	Number of symbols	Symbols
Binary	2	0 1
Octal	8	0 1 2 3 4 5 6 7
Decimal	10	0 1 2 3 4 5 6 7 8 9
Hexadecimal	16	0 1 2 3 4 5 6 7 8 9 a b c d e f

Decimal Representation

- First we have to take a close look of Decimal numbers.
- Starting from the smallest decimal number 0, if we increase the number one by one, then you will get 2, 3, 4, …. , 9.
- At this moment we reach the biggest symbol (9) in the Decimal number system. If we continuously increase the number, we will have to increase the number in the next position by one, and reset the current position back to the smallest symbol(0), after that we will get 10.
- This will happen when we reach 99, and it will reset the two digits back to 00, and the next position will add one, and we will get 100.

Binary Representation

- The binary representation will follow the same strategy, but with two symbols only.
- Let’s start with 0, and increase the number one by one again. After one time, we will get 1, which is already the maximum symbol in the Binary Representation.
- So the next time when we want to increase by one, it will increase one in the next position and reset the current position back into 0, and we get 10
- From here we can see that 10 in Binary represents 2 in Decimal.

Octal and Hexadecimal Representations

- You can use the same way to represent the numbers in Octal (8 symbols) and Hexadecimal (16 symbols) representations.

Decimal, Binary, Octal and Hexadecimal Representations

Decimal	Binary	Octal	Hexadecimal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F


Questions?

Fast Conversion

- The next question is how to convert between these representations.
- First let’s take a look at conversion from Other representations (Binary, Octal, and Hexadecimal) to Decimal

Binary to Decimal Fast Conversion

- First let's take a look at conversion from Binary to Decimal
- These Binary numbers on the right have 4 digits. Let's count from right to the left,
  - the 1st digit of Binary has a weight of $2^0 = 1$.
  - the 2nd digit of Binary has a weight of $2^1 = 2$.
  - the 3rd digit of Binary has a weight of $2^2 = 4$.
  - the 4th digit of Binary has a weight of $2^3 = 8$.
- If you time each digit with its weight and sum them together, you will get the decimal number. (0b1101 = 8 + 4 + 1 = 0d13)

Decimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

Octal to Decimal Fast Conversion

Decimal	Octal
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	10
9	11
10	12
11	13
12	14
13	15
14	16
15	17

- To convert Octal to Decimal, or Hexadecimal to Decimal follow the same rule:
  - the 1^{st} digit of Octal has a weight of $8^0 = 1$.
  - the 2^{nd} digit of Octal has a weight of $8^1 = 8$.
  - the 3^{rd} digit of Octal has a weight of $8^2 = 64$.
  - the 4^{th} digit of Octal has a weight of $8^3 = 512$.
- 0o17 = 8 + 7 = 0d15 0o34 = 8 * 3 + 4 = 0d28

Hexadecimal to Decimal Fast Conversion

Decimal	Hexadecimal
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	A
11	B
12	C
13	D
14	E
15	F

- To convert Hexadecimal to Decimal, or Hexadecimal to Decimal follow the same rule:
  - the 1st digit of Hexadecimal has a weight of $16^0 = 1$.
  - the 2nd digit of Hexadecimal has a weight of $16^1 = 256$.
  - the 3rd digit of Hexadecimal has a weight of $16^2 = 4096$.
  - the 4th digit of Hexadecimal has a weight of $16^3 = 65536$.
- 0h17 = 16 + 7 = 0d23
- 0h34 = 16 * 3 + 4 = 0d52


Questions?

Fast Conversion

- Then let’s see how to convert Decimal to other representations

Decimal to Binary Fast Conversion

- First let’s take a look at conversion from Decimal to Binary
- We know that for binary numbers, each position has a special weight. 1, 2, 4, 8, 16, 32, ….
- And when converting from Binary to Decimal, we sum the weight of the position with 1 together.
- To convert from Decimal to Binary, we will do the opposite, we will write down all the weights that is smaller than the decimal numbers.

Fast Conversion

- For example: 0d314
  - First write down all the weight that are smaller than 314:

256	128	64	32	16	8	4	2	1

  - Then we will start from left and try to remove weight from the target number if the weight is smaller or equal than the target number, and then update the target number as the remaining value. If the weight is removed, we will set a 1 on that position in the binary number, else 0.
  - For example, 256 is smaller than 314, so we will remove it from 314, and set that position as 1, then the target is 314 – 256 = 58. Then 128 and 64 are bigger than 58, so we will skip them, and set two 00s at these two positions. After that, we have 32, that is smaller than 58, and we will update the target as 58 – 32 = 26, and set the position as 1. We will continue doing this, until the target get 0.

Fast Conversion

- For example: 0d314

256	128	64	32	16	8	4	2	1
1								

- Remaining 314 – 256 = 58

256	128	64	32	16	8	4	2	1
1	0	0	1					

- Remaining 58 – 32 = 26

Fast Conversion

- Remaining 58 – 32 = 26

256	128	64	32	16	8	4	2	1
1	0	0	1	1				

- Remaining 26 – 16 = 10

256	128	64	32	16	8	4	2	1
1	0	0	1	1	1			

- Remaining 10 – 8 = 2

Fast Conversion

- Remaining 2

256	128	64	32	16	8	4	2	1
1	0	0	1	1	1	0	1	0

- Remaining 2 – 2 = 0, conversion complete.

- 0d314 = 0b100111010


Questions?

Decimal to Octal and Hexadecimal Fast Conversion

- To convert Decimal to Octal and Hexadecimal, you can follow the same strategy, however, it will be more difficult to write the weights, and you might need to do two digits times two digits multiplication, which is not very convenient.

- Instead, I would recommend you to **first convert the Decimal number into Binary number, and then convert the Binary number into Octal or Hexadecimal number.**

Decimal to Octal and Hexadecimal Fast Conversion

- To convert the Binary to Octal, you can group each **3** digits **from right to the left**.
- For example: 0d314 = 0b100111010
- We can first group the Binary number as: 0b 100 111 010
- And then convert each three numbers into a Octal number. (3 Binary digits represents 0 – 7, which is the same as the Octal range)
- 0d314 = 0b100111010 = 0b 100 111 010 = 0o472

Decimal to Octal and Hexadecimal Fast Conversion

- To convert the Binary to Hexadecimal, you will need to group each **4** digits **from right to the left**.
- For example: 0d314 = 0b100111010
- We can first group the Binary number as: 0b 1 0011 1010
- And then convert each four numbers into a Hexadecimal number.
- 0d314 = 0b100111010 = 0b 1 0011 1010 = 0h13a


Questions?

Octal and Hexadecimal Fast Conversion

- The last is to convert between Octal and Hexadecimal.
- To convert between Octal and Hexadecimal, we will also need to use Binary as the bridge. **First Convert Octal (Hexadecimal) into Binary, and then convert the Binary into Hexadecimal (Octal).**

Fast Conversion

- For example: 0o217
- Each digit of a Octal number can be convert into 3 Binary digits
  - 0o217 = 0b 010 001 111
- Then we will need to regroup the Binary number (4 digits each group)
  - 0o217 = 0b 010 001 111 = 0b 0 1000 1111
- In the end we will convert the Binary number to Hexadecimal number
  - 0o217 = 0b 010 001 111 = 0b 0 1000 1111 = 0h08f

Fast Conversion

- For example: 0h3cf
- Each digit of a Hexadecimal number can be convert into 4 Binary digits
  - 0h3cf = 0b 0011 1100 1111
- Then we will need to regroup the Binary number (3 digits each group)
  - 0h3cf = 0b 0011 1100 1111 = 0b 001 111 001 111
- In the end we will convert the Binary number to Octal number
  - 0h3cf = 0b 0011 1100 1111 = 0b 001 111 001 111 = 0o 1717


Questions?

Hands on

- 0o345 to 0h
- 0d777 to 0b
- 0h2e to 0d
- 0b10010101110 to 0h
- 0o742651 to 0b
- 0d487 to 0h