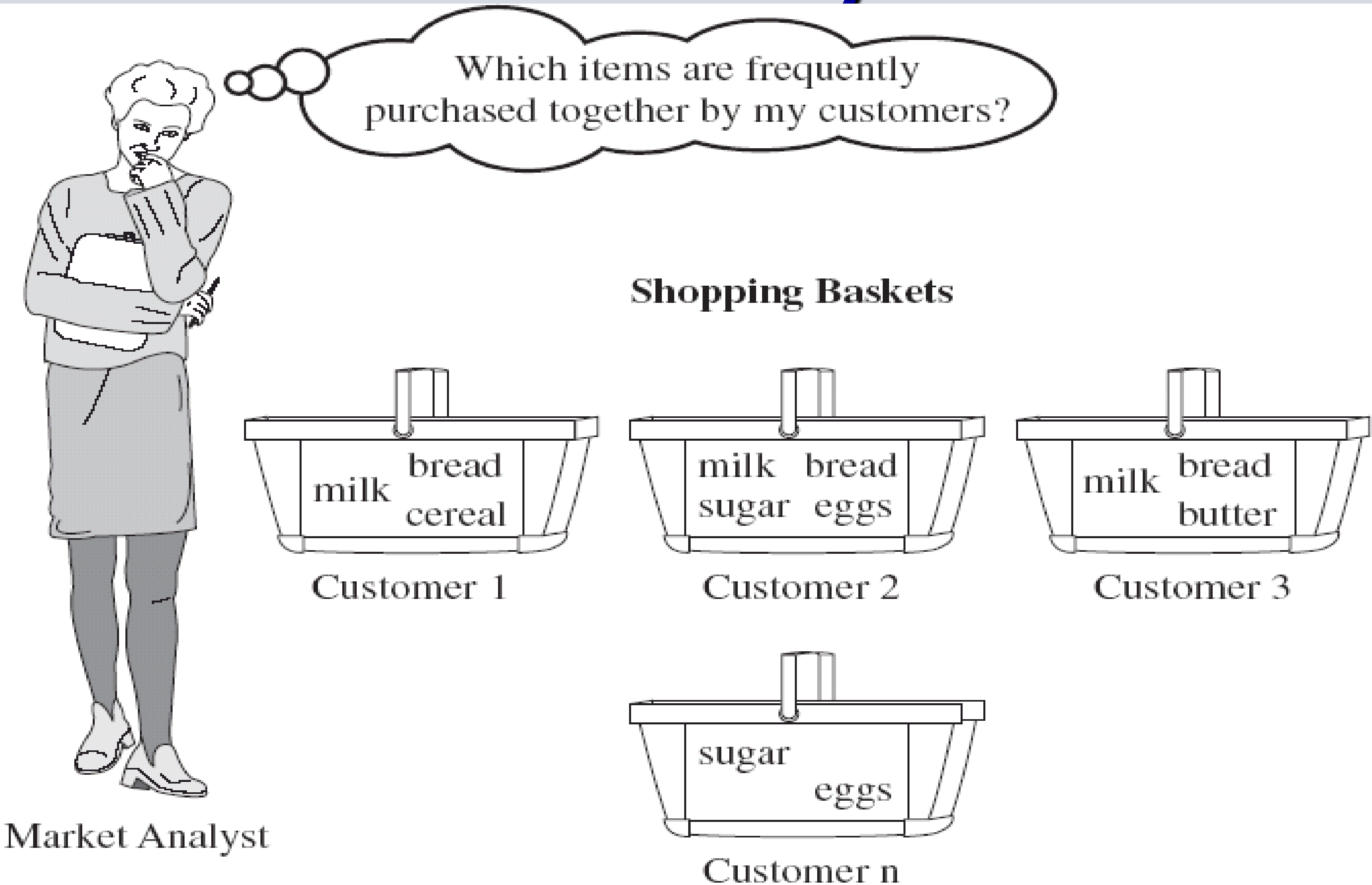
Association Rules

Association Rules Outline

Goal: Provide an overview of basic Association Rule mining techniques

- Association Rules Problem Overview
  - Market Basket Analysis
  - Association Rules Definition
- Association Rules Algorithms
  - Apriori

Market basket analysis



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Example: Market Basket Data

- Items frequently purchased together:

  Bread $\Rightarrow$PeanutButter

- Uses:
  - Placement
  - Advertising
  - Sales
  - Coupons
- Objective: increase sales and reduce costs

Association Rule Definitions

- ***Set of items:*** $I=\{I_1,I_2,\ldots,I_m\}$
- ***Transactions:*** $D=\{t_1,t_2, \ldots, t_n\}, t_j \subseteq I$
- ***Itemset:*** $\{I_{i1},I_{i2}, \ldots, I_{ik}\} \subseteq I$
- ***Support of an itemset:*** Percentage of transactions which contain that itemset.
- ***Large (Frequent) itemset:*** Itemset whose number of occurrences is above a threshold.

Association Rule Definitions

- ***Association Rule (AR):*** implication $X \Rightarrow Y$ where $X,Y \subseteq I$ and $X \cap Y = ;$
- ***Support of AR (s) X $\Rightarrow$ Y***: Percentage of transactions that contain $X \cup Y$.
  - $P(X,Y) = \sigma\ (X \cup Y)\ /\ |T|$
- ***Confidence of AR (α) X $\Rightarrow$ Y:*** Ratio of number of transactions that contain $X \cup Y$ to the number that contain X.
  - $P(Y|X) = \sigma\ (X \cup Y)\ /\ \sigma\ (X)$

Transaction	Items
t_1	Bread,Jelly,PeanutButter
t_2	Bread,PeanutButter
t_3	Bread,Milk,PeanutButter
t_4	Coffee ,Bread
t_5	Coffee ,Milk

$X \Rightarrow Y$	s	α
Bread $\Rightarrow$ PeanutButter		
PeanutButter $\Rightarrow$ Bread		
Coffee $\Rightarrow$ Bread		
PeanutButter $\Rightarrow$ Jelly		
Jelly $\Rightarrow$ PeanutButter		
Jelly $\Rightarrow$ Milk		

Dunham

Transaction 1	🍎 🍺 🍚 🍗
Transaction 2	🍎 🍺 🍚
Transaction 3	🍎 🍺
Transaction 4	🍎 🍐
Transaction 5	🍼 🍺 🍚 🍗
Transaction 6	🍼 🍺 🍚
Transaction 7	🍼 🍺
Transaction 8	🍼 🍐

Support {🍎} = $\frac{4}{8}$

Confidence {🍎 → 🍺} = Support {🍎, 🍺} / Support {🍎} $\quad \frac{3}{4}$

Lift {🍎 → 🍺} = Support {🍎, 🍺} / (Support {🍎} x Support {🍺}) $\quad \frac{3}{24}$

https://www.kdnuggets.com/2016/04/association-rules-apriori-algorithm-tutorial.html

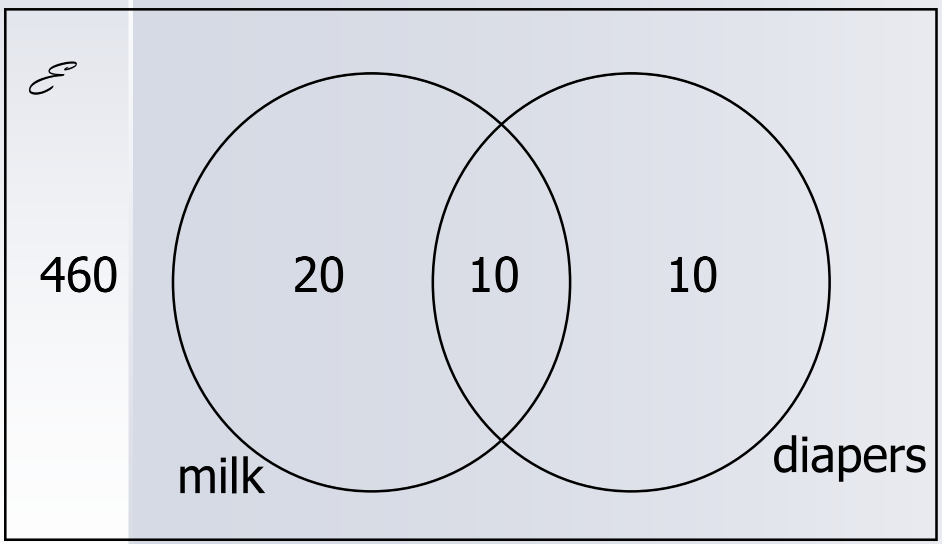
Example

500,000 transactions

20,000 transactions contains diapers

30,000 transactions contains milk

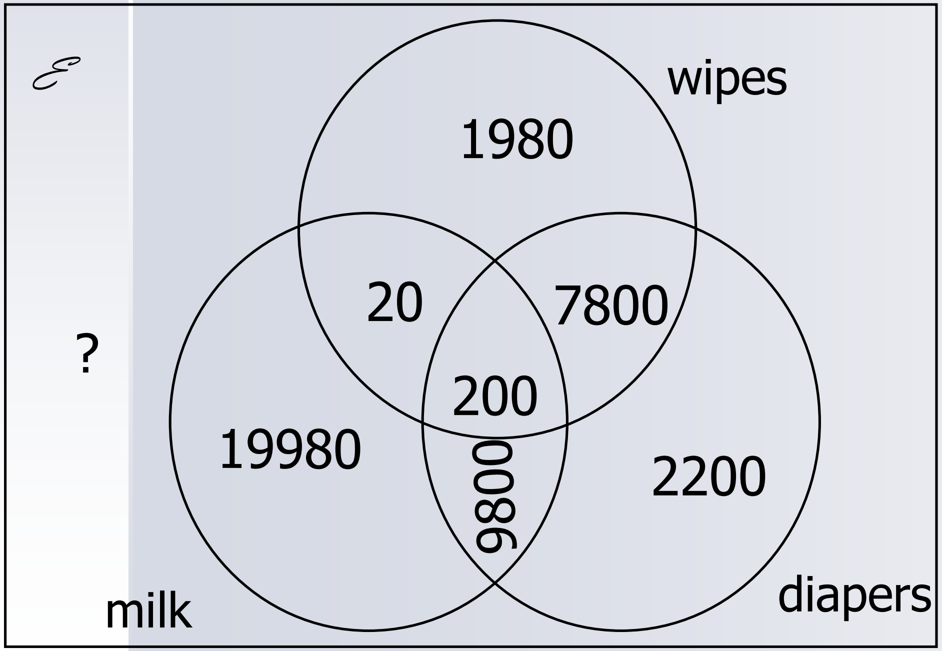
10,000 transactions contains both diapers & milk



	s	c
Milk ⇒ Diapers	0.02	0.33
Diapers ⇒ Milk	0.02	0.50

Example

10,000 transactions contains wipes
8,000 transactions contains wipes & diapers
220 transactions contains wipes & milk
200 transactions contains wipes & diapers & milk



	s,%	c,%
Milk ⇒ Diapers	2	33.33
Milk ⇒ Wipes	0.04	0.73
Diapers ⇒ Milk	2	50
Diapers ⇒ Wipes	1.6	40
…		
Wipes & Milk ⇒ Diapers	0.04	90.91

Association Rule Techniques

1. Find Large Itemsets.
2. Generate rules from frequent itemsets.

Association Rule Mining

- Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction

Market-Basket transactions

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Cocoa, Eggs**
3	**Milk, Diaper, Cocoa, Coke**
4	**Bread, Milk, Diaper, Cocoa**
5	**Bread, Milk, Diaper, Coke**

Example of Association Rules

{Diaper} $\rightarrow$ {Cocoa},
{Milk, Bread} $\rightarrow$ {Eggs,Coke},
{Cocoa, Bread} $\rightarrow$ {Milk},

Implication means co-occurrence, not causality!

Definition: Frequent Itemset

- **Itemset**
  - A collection of one or more items
    - Example: {Milk, Bread, Diaper}
  - k-itemset
    - An itemset that contains k items
- **Support count (σ)**
  - Frequency of occurrence of an itemset
  - E.g. σ({Milk, Bread,Diaper}) = 2
- **Support**
  - Fraction of transactions that contain an itemset
  - E.g. s({Milk, Bread, Diaper}) = 2/5

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Cocoa, Eggs
3	Milk, Diaper, Cocoa, Coke
4	Bread, Milk, Diaper, Cocoa
5	Bread, Milk, Diaper, Coke

Frequent Itemset

An itemset whose support is greater than or equal to a *minsup* threshold

Definition: Association Rule

- **Association Rule**
  - An implication expression of the form X → Y, where X and Y are itemsets
  - Example:
    {Milk, Diaper} → {Cocoa}

- **Rule Evaluation Metrics**
  - Support (s)
    - Fraction of transactions that contain both X and Y
  - Confidence (c)
    - Measures how often items in Y appear in transactions that contain X

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Cocoa, Eggs**
3	**Milk, Diaper, Cocoa, Coke**
4	**Bread, Milk, Diaper, Cocoa**
5	**Bread, Milk, Diaper, Coke**

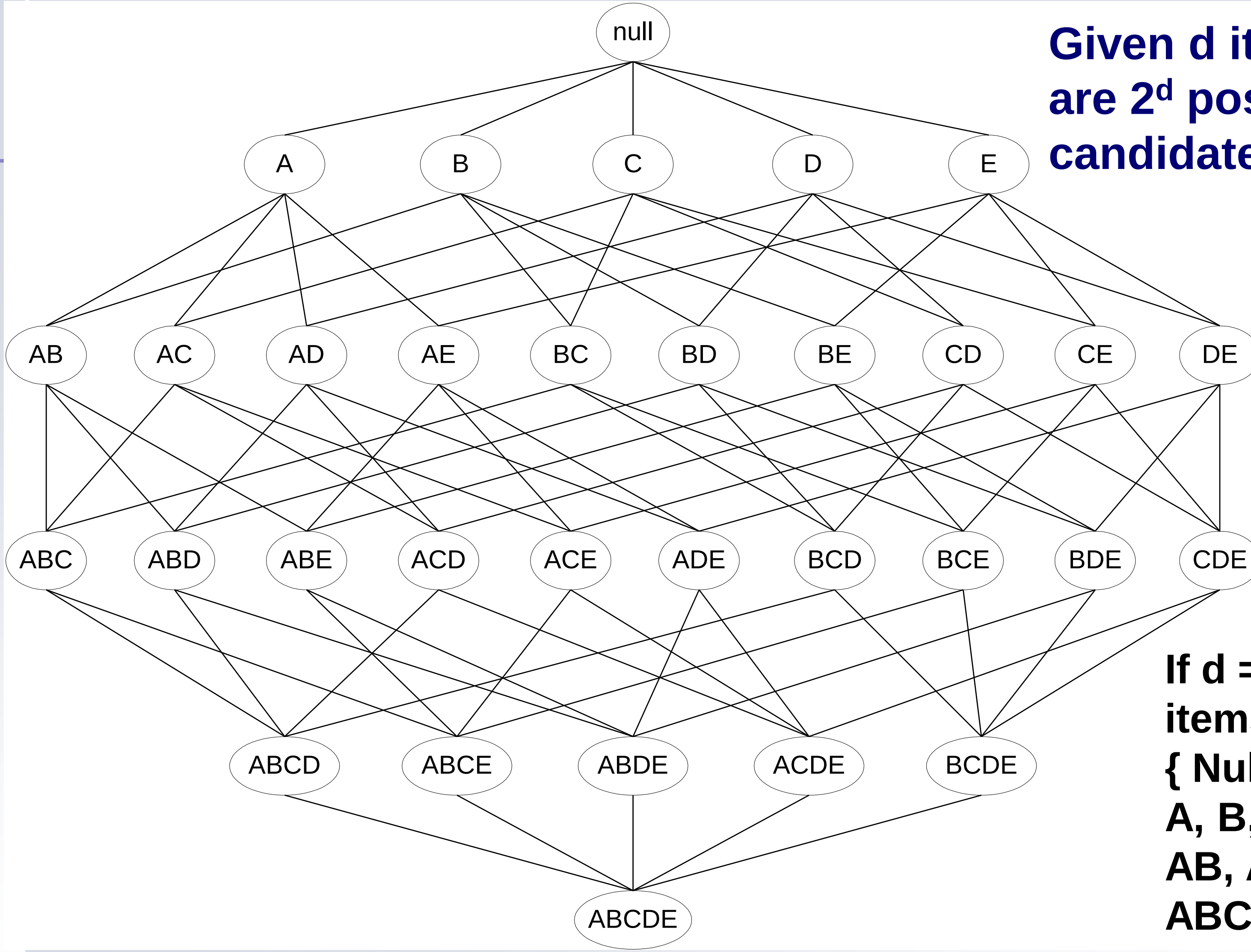
Example:

$$\{\text{Milk}, \text{Diaper}\} \Rightarrow \text{Cocoa}$$

$$s = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Cocoa})}{|\text{T}|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Cocoa})}{\sigma(\text{Milk}, \text{Diaper})} = \frac{2}{3} = 0.67$$

Frequent Itemset Generation



Given d items, there are 2^d possible candidate itemsets

If d = 3, possible itemsets is 8:
{ Null
A, B, C
AB, AC, BC
ABC }

Apriori

- ***Large Itemset Property:***

  Any subset of a large itemset is large.
- Contrapositive:

  If an itemset is not large,

  none of its supersets are large.

Illustrating Apriori Principle 1

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Cocoa, Eggs**
3	**Milk, Diaper, Cocoa, Coke**
4	**Bread, Milk, Diaper, Cocoa**
5	**Bread, Milk, Diaper, Coke**

Minimum Support = 3

- **Step 1: Generating 1-itemset frequent itemset**

Item	Count
Bread	**4**
Coke	**2**
Milk	**4**
Cocoa	**3**
Diaper	**4**
Eggs	**1**

Eliminate itemset which is less than the minimum support

Item	Count
Bread	**4**
~~**Coke**~~	~~**2**~~
Milk	**4**
Cocoa	**3**
Diaper	**4**
~~**Eggs**~~	~~**1**~~

Tan,Steinbach, Kumar

Illustrating Apriori Principle 2

- **Step 2: Generating 2-itemset frequent itemset**

Item	Count
Bread, Milk	**3**
Bread, Cocoa	**2**
Bread, Diaper	**3**
Milk, Cocoa	**2**
Milk, Diaper	**3**
Cocoa, Diaper	**3**

Eliminate itemset which is less than the minimum support

Item	Count
Bread, Milk	**3**
~~**Bread, Cocoa**~~	~~**2**~~
Bread, Diaper	**3**
~~**Milk, Cocoa**~~	~~**2**~~
Milk, Diaper	**3**
Cocoa, Diaper	**3**

Illustrating Apriori Principle 3

- Step 3: Generating 3-itemset frequent itemset

Item	Count
Bread, Milk, Diaper	**2**
Milk, Cocoa, Diaper	**3**

Eliminate itemset which is less than the minimum support

Item	Count
~~**Bread, Milk, Diaper**~~	~~**2**~~
Milk, Cocoa, Diaper	**3**

Illustrating Apriori Principle 4

- **Step 4: Generating association rules from frequent itemsets: Milk, Cocoa, Diaper**

Rule	Support	Confidence
Milk → Cocoa, Diaper	0.4	0.5
Milk, Cocoa → Diaper	0.4	1.0
Milk, Diaper → Cocoa	0.4	0.67
Cocoa → Milk, Diaper	0.4	0.67
Cocoa, Diaper → Milk	0.4	0.67
Diaper → Milk, Cocoa	0.4	0.5

Apriori – example 2

Database TDB

$Sup_{min} = 2$

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1st scan

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2nd scan

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

L_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

3rd scan

L_3

Itemset	sup
{B, C, E}	2

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Apriori – example 3

Minimum Support = 60%

Transaction ID	Items Bought
T1	{Mango, Onion, Nintendo, Key-chain, Eggs, Yo-yo}
T2	{Doll, Onion, Nintendo, Key-chain, Eggs, Yo-yo}
T3	{Mango, Apple, Key-chain, Eggs}
T4	{Mango, Umbrella, Corn, Key-chain, Yo-yo}
T5	{Corn, Onion, Onion, Key-chain, Ice-cream, Eggs}

Transaction ID	Items Bought
T1	{M, O, N, K, E, Y }
T2	{D, O, N, K, E, Y }
T3	{M, A, K, E}
T4	{M, U, C, K, Y }
T5	{C, O, O, K, I, E}

http://nikhilvithlani.blogspot.my/2012/03/apriori-algorithm-for-data-mining-made.html

Apriori – example 3 (cont.)

Item	No of transactions
M	3
O	3
N	2
K	5
E	4
Y	3
D	1
A	1
U	1
C	2
I	1

Item	Number of transactions
M	3
O	3
K	5
E	4
Y	3

Eliminate itemset which is less than the minimum support

Apriori – example 3 (cont.)

Item Pairs	Number of transactions
MO	1
MK	3
ME	2
MY	2
OK	3
OE	3
OY	2
KE	4
KY	3
EY	2

Item Pairs	Number of transactions
MK	3
OK	3
OE	3
KE	4
KY	3

Eliminate itemset which is less than the minimum support

Apriori – example 3 (cont.)

Item Set	Number of transcations
MOK	
MOE	
MOY	
MEK	
MEY	
KYM	
KOE	
KOY	
KEY	
...	...

Item Set	Number of transactions
OKE	3
KEY	2

Support? Confidence?

- Problems with support and confidence; a rule may have high support & high confidence because the rule is an obvious rule... *someone who buys potato chips is highly likely to buy soft drink -> not surprising*
- Confidence totally ignore $P(B)$
  - $P(A,B) = \sigma(A \cup B) / |T|$
  - $P(B|A) = \sigma(A \cup B) / \sigma(A)$

Interestingness Measure: Correlations (Lift)

- Lift (Wipes, Milk $\Rightarrow$ Diapers) is 22.73
- People who purchased Wipes & Milk are 22.73 times more likely to also purchase Diapers than people who do not purchase Wipes & Milk .
- Measure of dependent/correlated events: lift

$$lift(A \Rightarrow B) = \frac{P(A \cup B)}{P(A)P(B)}$$

>1 positively correlated
= 1 independent
< 1 negatively correlated

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Chi-square, χ^2

$$\chi^2 = \sum \frac{(observed - expected)^2}{expected}$$

$$\chi^2 = \sum_{i=1}^{c} \sum_{j=1}^{r} \frac{(o_{ij} - e_{ij})^2}{e_{ij}}$$

$$e_{ij} = \frac{count(A = a_i) \times count(B = b_j)}{N}$$

$$DOF = (r - 1) \times (c - 1)$$

A has c distinct values (a_1, a_2, a_3, …)
B has r distinct values (b_1, b_2, b_3, …)

Example

Observed

Significance level is 0.01

	Coke	Non-Coke	TOTAL
Osteoporosis	22	16	38
No osteoporosis	10	28	38
TOTAL	32	44	76

Drinking Coke and Getting Osteoporosis are not related

Steps:

- State null hypothesis: A & B are not correlated
- Calculate expected values
- Calculate chi-square value
- Compare calculated & tabulated values
- Justify hypothesis

Example

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Expected

	Coke	Non-Coke	TOTAL
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Example

Observed

Significance level is 0.01

	Coke	Non-Coke	TOTAL
Osteoporosis	22	16	38
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TOTAL	32	44	76

$$\chi^2 = \sum \frac{(observed - expected)^2}{expected}$$

Steps:

- State null hypothesis: A & B are not correlated
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- Calculate chi-square value
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Observed

Significance level is 0.01

	Coke	Non-Coke	TOTAL
Osteoporosis	22	16	38
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TOTAL	32	44	76

$$\chi^2 = \frac{(22-16)^2}{16} + \frac{(16-22)^2}{22} + \frac{(10-16)^2}{16} + \frac{(28-22)^2}{22} = 7.77$$

Expected

	Coke	Non-Coke	TOTAL
Osteoporosis	16	22	38
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Example

Observed

Significance level is 0.01

	Coke	Non-Coke	TOTAL
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TOTAL	32	44	76

$$DOF = (r-1)\times(c-1)$$

$$DOF = (2-1)\times(2-1) = 1$$

Steps:

- State null hypothesis: A & B are not correlated
- Calculate expected values
- Calculate chi-square value
- Compare calculated & tabulated values
- Justify hypothesis

Chi-square distribution table

TABLE F X^2 distribution critical values

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	[illegible]	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	[illegible]	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	[illegible]	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

Example

Observed

Significance level is 0.01

	Coke	Non-Coke	TOTAL
Osteoporosis	22	16	38
No osteoporosis	10	28	38
TOTAL	32	44	76

Steps:

- State null hypothesis: A & B are not correlated
- Calculate expected values
- Calculate chi-square value

$7.77 > 6.63$

- Compare calculated & tabulated values
- Justify hypothesis

Example

Observed

Significance level is 0.01

	Coke	Non-Coke	TOTAL
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TOTAL	32	44	76

Steps:

- State null hypothesis: A & B are not correlated
- Calculate expected values
- Calculate chi-square value
- Compare calculated & tabulated values
- Justify hypothesis

$7.77 > 6.63$

Reject null hypothesis

Example

- Null hypothesis: A & B are not correlated
- Calculate expected values
- Calculate chi-square value
- Compare calculated & tabulated values
- Justify hypothesis

$$\chi^2 = \frac{(22-16)^2}{16} + \frac{(16-22)^2}{22} + \frac{(10-16)^2}{16} + \frac{(28-22)^2}{22} = 7.77$$

- DOF (df) = (2-1)(2-1) = 1
- significant level, a = 0.01
- χ^2 = 6.63 (tabulated)

Further example: Chi Square

	Male	Female	TOTAL
Like Sci. Fic.	250	200	450
Dislike Sci.Fic.	50	1000	1050
TOTAL	300	1200	1500

Expected

Significance level is 0.001

	Male	Female	TOTAL
Like Sci. Fic.	90	360	450
Dislike Sci.Fic.	210	840	1050
TOTAL	300	1200	1500

Which Measures Should Be Used?

symbol	measure	range	formula
ϕ	ϕ-coefficient	-1…1	$\frac{P(A,B)-P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$
Q	Yule's Q	-1 … 1	$\frac{P(A,B)P(\overline{A},\overline{B})-P(A,\overline{B})P(\overline{A},B)}{P(A,B)P(\overline{A},\overline{B})+P(A,\overline{B})P(\overline{A},B)}$
Y	Yule's Y	-1 … 1	$\frac{\sqrt{P(A,B)P(\overline{A},\overline{B})}-\sqrt{P(A,\overline{B})P(\overline{A},B)}}{\sqrt{P(A,B)P(\overline{A},\overline{B})}+\sqrt{P(A,\overline{B})P(\overline{A},B)}}$
k	Cohen's	-1 … 1	$\frac{P(A,B)+P(\overline{A},\overline{B})-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A)P(B)-P(\overline{A})P(\overline{B})}$
PS	Piatetsky-Shapiro's	-0.25 … 0.25	$P(A,B)-P(A)P(B)$
F	Certainty factor	-1 … 1	$\max(\frac{P(B\|A)-P(B)}{1-P(B)}, \frac{P(A\|B)-P(A)}{1-P(A)})$
AV	added value	-0.5 … 1	$\max(P(B\|A)-P(B), P(A\|B)-P(A))$
K	Klosgen's Q	-0.33 … 0.38	$\sqrt{P(A,B)}\max(P(B\|A)-P(B), P(A\|B)-P(A))$
g	Goodman-kruskal's	0 … 1	$\frac{\Sigma_j \max_k P(A_j,B_k)+\Sigma_k \max_j P(A_j,B_k)-\max_j P(A_j)-\max_k P(B_k)}{2-\max_j P(A_j)-max_k P(B_k)}$
M	Mutual Information	0 … 1	$\frac{\Sigma_i \Sigma_j P(A_i,B_j) \log \frac{P(A_i,B_j)}{P(A_i)P(B_J)}}{\min(-\Sigma_i P(A_i) \log P(A_i) \log P(A_i), -\Sigma_i P(B_i) \log P(B_i) \log P(B_i))}$
J	J-Measure	0 … 1	$\max(P(A,B)\log(\frac{P(B\|A)}{P(B)}) + P(A\overline{B})\log(\frac{P(\overline{B}\|A)}{P(\overline{B})}))$ $P(A,B)\log(\frac{P(A\|B)}{P(A)}) + P(\overline{A}B)\log(\frac{P(\overline{A}\|B)}{P(\overline{A})})$
G	Gini index	0 … 1	$\max(P(A)[P(B\|A)^2 + P(\overline{B}\|A)^2] + P(\overline{A}[P(B\|\overline{A})^2 + P(\overline{B}\|\overline{A})^2] - P(B)^2 - P(\overline{B})^2,$ $P(B)[P(A\|B)^2 + P(\overline{A}\|B)^2] + P(\overline{B}[P(A\|\overline{B})^2 + P(\overline{A}\|\overline{B})^2] - P(A)^2 - P(\overline{A})^2)$
s	support	0 … 1	$P(A,B)$
c	confidence	0 … 1	$max(P(B\|A), P(A\|B))$
L	Laplace	0 … 1	$\max(\frac{NP(A,B)+1}{NP(A)+2}, \frac{NP(A,B)+1}{NP(B)+2})$
IS	Cosine	0 … 1	$\frac{P(A,B)}{\sqrt{P(A)P(B)}}$
γ	coherence(Jaccard)	0 … 1	$\frac{P(A,B)}{P(A)+P(B)-P(A,B)}$
α	all_confidence	0 … 1	$\frac{P(A,B)}{\max(P(A),P(B))}$
o	odds ratio	0 … ∞	$\frac{P(A,B)P(\overline{A},\overline{B})}{P(\overline{A},B)P(A,\overline{B})}$
V	Conviction	0.5 … ∞	$\max(\frac{P(A)P(\overline{B})}{P(A\overline{B})}, \frac{P(B)P(\overline{A})}{P(B\overline{A})})$
λ	lift	0 … ∞	$\frac{P(A,B)}{P(A)P(B)}$
S	Collective strength	0 … ∞	$\frac{P(A,B)+P(\overline{AB})}{P(A)P(B)+P(\overline{A})P(\overline{B})} \times \frac{1-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A,B)-P(\overline{AB})}$
χ^2	χ^2	0 … ∞	$\Sigma_i \frac{(P(A_i)-E_i)^2}{E_i}$