Viterbi Example

State space X_t, the state at time t

Initial prob. π

Transitions $P(X_t \mid X_{t-1})$

Observations E_t, the observation at time t

The most likely state seq $X_1 \dots X_t$

calculated using viterbi

$$V_{1,X_1} = P(E_1 \mid X_1) \cdot \pi$$

$$V_{t,X_t} = \max\left(P(E_t \mid X_t)\, P(X_t \mid X_{t-1})\, V_{t-1,X_{t-1}}\right), \quad \forall x_i \in X_t$$

Day 1:

$$V_{1,X_1} = P(E_1 \mid X_1) \cdot \pi$$

$X_1 = \text{healthy}$

$= P(\text{normal} \mid \text{healthy})\, P(\text{healthy})$

$= .5 \times .6 = .3$

$X_1 = \text{fever}$

$= P(\text{normal} \mid \text{fever})\, P(\text{fever})$

$= .1 \times .4 = .04$

$V_{1,X_1} = \langle .3, .04 \rangle$ (healthy, fever)

Day 2: observe cold

$$V_{2,X_2} = \max_{X_1, X_2}\left(P(E_2 \mid X_2)\, P(X_2 \mid X_1)\, V_{1,X_1}\right)$$

max over previous states X_1 to get to X_2

4 combinations, X_1 = healthy, fever; X_2 = healthy, fever

$= \max($

$P(\text{cold} \mid \text{healthy})\, P(\text{healthy} \mid \text{healthy}) \times .3$ (V_{1,X_1} = healthy), $\Big\}$ X_2 = healthy

$P(\text{cold} \mid \text{healthy})\, P(\text{healthy} \mid \text{fever}) \times .04$, $\Big\}$ X_2 = healthy

$P(\text{cold} \mid \text{fever})\, P(\text{fever} \mid \text{fever}) \times .04$, $\Big\}$ X_2 = fever

$P(\text{cold} \mid \text{fever})\, P(\text{fever} \mid \text{healthy}) \times .3$ $\Big\}$ X_2 = fever

$= \max($

$.4 \times .7 \times .3 = .084$, $.3 \times .6 \times .04 = .0072$

$.4 \times .4 \times .04 = .0064$, $.3 \times .3 \times .3 = .027$

$)$

Day 3: observe dizzy

$$V_{3,X_3} = \max_{X_2 \forall X_3} \left(P(E_3 | X_3) \times P(X_3 | X_2) \times V_{2,X_2} \right)$$

4 combinations

Day 3 observation: dizzy

X_3 = healthy, fever

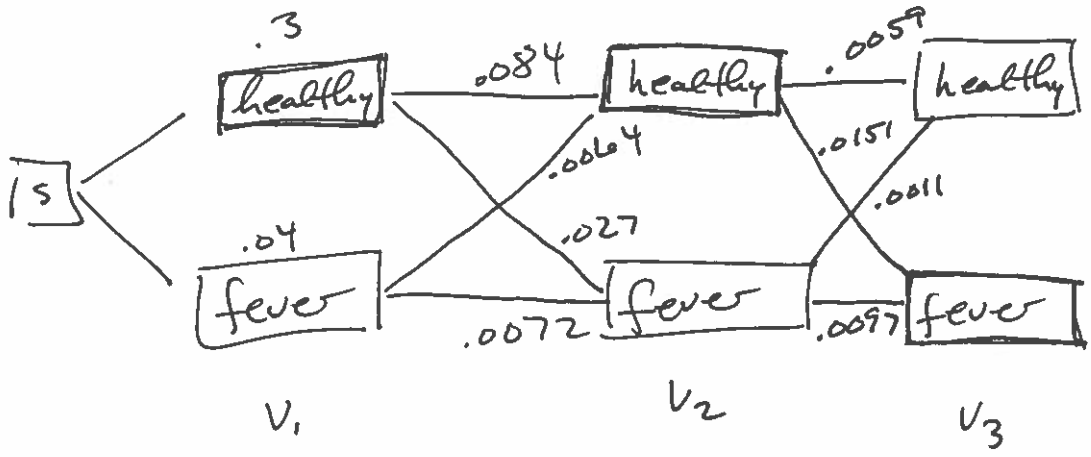
X_2 = healthy, fever

$$= \max_{X_2} \begin{cases} P(\text{dizzy} | \text{healthy}) P(\text{healthy} | \text{healthy}) \times .084, \\ P(\text{dizzy} | \text{healthy}) P(\text{healthy} | \text{fever}) \times .027, \end{cases} V_3 = \text{healthy}$$

$$\begin{cases} P(\text{dizzy} | \text{fever}) P(\text{fever} | \text{healthy}) \times .084 \\ P(\text{dizzy} | \text{fever}) P(\text{fever} | \text{fever}) \times .027 \end{cases} V_3 = \text{fever}$$

$$= \max_{x_2} \begin{cases} .1 \times .7 \times .084 & h \to h \\ .1 \times .4 \times .027 & f \to h \\ .6 \times .3 \times .084 & h \to f \\ .6 \times .6 \times .027 & f \to f \end{cases} \begin{matrix} V_3 = h \\ V_3 = f \end{matrix}$$

$$= \begin{matrix} .0058 \\ .0011 \\ .0151 \\ .0097 \end{matrix}$$



Max V_2, fever = .027
Back ptr to healthy at V_1

Max V_2, healthy = .084
Back ptr to healthy at V_1

Max V_3, fever = .0151
Back ptr to healthy at V_2

Max V_3, healthy = .0059
Back ptr to healthy at V_2