Response to the Two Reviewers

We would like to thank the associate editor for the time and efforts in putting our manuscript through the review process. We would also like to thank the two anonymous referees for their careful reviews and constructive comments. The concerns of the reviewers have been addressed point by point. We are resubmitting the revised manuscript for review.

Let us summarize the major changes in this new revision:

- We corrected all of the typos based on the first reviewer's comments.
- In response to the second reviewer's criticisms, we emphasized on the new algorithms and new applications in our article and, in addition, added a new primal-dual coordinate-update algorithm in Section 4.4. We provided the convergence proof for the added algorithm in Appendix D.
- We updated a numerical experiment in Section 6.2 to demonstrate the superiority of coordinate friendly methods over start-of-the-art methods.
- We edited the introduction to reflect the changes made, as well as the other contributions of our article.

1. Response to reviewer 1

"This paper proposes a high-level abstraction for coordinate update method in optimization. The concept of "coordinate friendly operator" is introduced and the proposed framework unifies many previously studied coordinate update type methods in the literature."

"In general, the paper is well written and well organized. But this reviewer still finds some typos and awkward statements. In the following, I will list some of the typos that I found. The authors are suggested to do more thorough and careful checking to make sure that there is no more typos."

Response: Thanks for your careful review. We have read through the paper multiple times and improved then awkward statements. We have also corrected the typos, one by one, in the new manuscript. Thanks again.

2. Response to reviewer 2

"This paper discusses coordinate update methods, which are useful for solving large-scale problems that admit a fixed-point formulation, including various coordinate descent methods in optimization, solution of linear systems

of equations, and many operator splitting algorithms. In particular, the authors introduce several notions of Coordinate Friendly (CF) operators, discuss composite and combination of CF operators, use them to characterize several classical and recent operator splitting schemes, and also obtain some new coordinate update methods. These methods are illustrated by examples from machine learning, imaging, finance, and distributed computing, among others."

Response: Thanks for the summary.

Before we respond to other comments from the reviewer, we would like to share with the reviewer the motivation for our work.

Coordinate update methods have had very exciting progress recently. Nonetheless, most recent papers focus on a few classes of important applications such as regularized empirical risk minimization. It would be a pity if coordinate update methods are not applied to more problems and its strong performance is not shown. Clearly, the strong performance of coordinate update methods relies on solving simple subproblems. However, it was not always easy to get simple subproblems and implement their solvers properly. Even for domain experts, when they face problems combining constraints, objective functions, both separable and non-separable terms, and/or both convex and nonconvex parts, those questions are nontrivial to answer.

Clearly, not all problem structures and algorithms are amenable to coordinate update. Therefore, we need a qualifier, a simple condition that needs to be satisfied for an efficient coordinate update. That is the coordinate-friendly property we propose in our article.

The techniques to obtain simple subproblems, develop their solvers, and qualify them for coordinate updates are crucial to the successful applications of coordinate update methods, which are the focus of our article.

Comment 1: The focus of this paper is on summarizing the components and composition of efficient coordinate-update methods, and largely ignore convergence guarantees.

Response: We agree that the convergence guarantees were ignored in our article on purpose. In the new revision, we added the convergence guarantee for one of the novel algorithms in the paper. It is an algorithm performing an async-parallel primal-dual coordinate update, capable of solving problems such as second-order cone programs by multiple computers simultaneously performing coordinate updates. To avoid a sharp increase in the technicality level, we have added the proof to the Appendix. Due to the focus and the page limit, we have to ignore the convergence guarantees of other new algorithms.

Comment 2: On one hand, this seems to be a timely topic given the recent flurry of research activities on coordinate update methods and their applications, and the materials presented may help the understanding of the common theme of these methods, especially for practitioners.

Response: We agree with the reviewer that the target audiences of our paper are the practitioners.

We would like to emphasize that those coordinate-update algorithms obtained after the Douglas-Rachford splitting or primal-dual splitting are new and uncommon. Those algorithms embrace two different sets of techniques, coordinate update and operator splitting, for challenging applications.

Comment 3: But on the other hand, I feel there is not enough innovative ideas other than summarizing several well-known structure of coordinate update methods, and the technical depth of this paper does not reach the high-quality of a first-class journal on mathematical sciences.

Response: Our exposition includes both well–known and *new* structures. Several composite structures are new, and their integration with operator splitting such as the proposed primal-dual coordinate-update algorithm is also new. We have systematically reasoned that certain quantities must be cached and maintained in memory, or coordinate updates would not be efficient. These expositions lead to new coordinate-update algorithms for second-order cone programming, nonnegative matrix factorization, CT image processing, and portfolio optimization, which were not previously treated with similar coordinate updates. We also get new coordinate-update algorithms for overlapping group LASSO.

Considering the audience and goal of this article, we intentionally keep its technical depth at a low–medium level so that nondomain experts can understand this article. Operator splitting and coordinate descent convergence can go very sophisticated, fending off potential users. Hence, we decide to leave them out in order to promote applications.

Regarding whether our paper will fit the journal "Annals of Mathematical Sciences and Applications", we would note that our paper is written to emphasize on *applications*, so a low technical depth might well be preferred by the journal and its readers. Of course, it will take time for us to see which emphases the journal "Annals of Mathematical Sciences and Applications" will eventually settle.

Comment 4: From a more application oriented view, there is no sufficient justification (either theoretical or empirical) for the efficiency of the derived algorithms, nor comparison with state of the arts for the particular applications discussed. Thus it is not clear how useful they will become, other than illustrating some general ideas.

Response: We respectfully disagree with the reviewer for the following reasons.

The original submission of this article does include the numerical results for three different coordinate-update algorithms, which are applied to three different problems. We had compared the performance of our coordinate-update algorithms with the corresponding full-update algorithms, which are themselves state-of-the-art methods. The numerical experiments had shown the advantage of coordinate-update algorithms. In the third numerical experiment, we had compared an *async*-parallel coordinate-update algorithm with the corresponding *sync*-parallel one and observed significant speedups.

On the other hand, we do agree that the original submission ignored theoretical justification (as explained in our last response).

In this revision, we updated our second numerical example to a new instance of CT image. The coordinate-update algorithm continues to show its superiority in efficiency.

We also added the convergence analysis for one of our primal-dual coordinate-update algorithms (the most sophisticated one, under the async-parallel setting).

Hopefully, these numerical and theoretical results will convince the reviewer and the readers that coordinate update algorithms are worth consideration.

The reviewer comments have helped us understand the potential concerns from the reader. Therefore, we made also changes throughout the paper to clarify our goals, as well as the contributions of this paper.