Response to the Two Reviewers

We would like to thank the associate editor for the time and efforts in putting our manuscript through the review process. We would also like to thank the two anonymous referees for their carful reviews and constructive comments. The concerns of the reviewers have been addressed point by point. We are resubmitting the revised manuscript for review.

Let us summarize the big changes in this new version:

- We corrected all of the typos based on the first reviewer's comments.
- We added new async-parallel primal-dual coordinate-update algorithms in Section 4.4 and their convergence proof in Appendix D.
- We added a new numerical experiment in Section 6.2 to demonstrate the superior of coordinate friendly methods over the start-of-the-art method.
- We edited the introduction for it to be consistent with our revision and better reflect our contribution.

1. Response to reviewer 1

This paper proposes a high-level abstraction for coordinate update method in optimization. The concept of "coordinate friendly operator" is introduced and the proposed framework unifies many previously studied coordinate undate type methods in the literature.

Comment 1: In general, the paper is well written and well organized. But this reviewer still finds some typos and awkward statements. In the following, I will list some of the typos that I found. The authors are suggested to do more thorough and careful checking to make sure that there is no more typos.

Response: Thanks for careful reading of our manuscript. We have read through the paper multiple times and corrected the awkward statements. According to the reviewer's comments, we have also corrected the typos and addressed the comments in the new manuscript.

2. Response to reviewer 2

This paper discusses coordinate update methods, which are useful for solving large-scale problems that admit a fixed-point formulation, including various

coordinate descent methods in optimization, solution of linear systems of equations, and many operator splitting algorithms. In particular, the authors introduce several notions of Coordinate Friendly (CF) operators, discuss composite and combination of CF operators, use them to characterize several classical and recent operator splitting schemes, and also obtain some new coordinate update methods. These methods are illustrated by examples from machine learning, imaging, finance, and distributed computing, among others.

Comment 1: The focus of this paper is on summarizing the components and composition of efficient coordinate-update methods, and largely ignore convergence guarantees.

Response: We agree with the reviewer that we do ignore the convergence guarantees for the initial submission, however, we do not agree that this paper is about summarizing the existing algorithms. In fact, the goal of this paper is to identify the favorable structures of an operator which make coordinate-update computationally worthy. As for convergence guarantees, in this revision, we proved the convergence of our primal-dual coordinate-update schemes under the async-parallel algorithmic framework (which includes the stochastic coordinate selection rule as a special case). The convergence of other coordinate-update schemes is omitted due to page limit, but they all converge under the async-parallel algorithmic framework according to the previous work [1].

Comment 2: On one hand, this seems to be a timely topic given the recent flurry of research activities on coordinate update methods and their applications, and the materials presented may help the understanding of the common theme of these methods, especially for practitioners.

Response: We agree with the reviewer that the target audiences for this paper are the practitioners. For many of the applications, coordinate-update algorithms were never proposed to solve them, yet our numerical experiments show that they have strong performance. One of the aims of our paper is to bridge the gap between coordinate-update theory and the applications.

Comment 3: But on the other hand, I feel there is not enough innovative ideas other than summarizing several well-known structure of coordinate update methods, and the technical depth of this paper does not reach the high-quality of a first-class journal on mathematical sciences.

Response: There are many innovative ideas in this paper. First, we identified and classified coordinate friendly structures. Formal CF analysis, especially that for composite operators, has never appeared in the literature, although it appears to be simple to understand. Second, we apply our new

coordinate-update algorithms to problems that were never treated with coordinate update methods before, for example, second-order cone programming, nonnegative matrix factorization, image processing, portfolio optimization, etc. Third, we also provide new coordinate-update approaches for problems which were treated before with coordinate-update methods, for instance, support vector machines and group Lasso. Forth, our overlapping-block coordinate update scheme for primal-dual algorithms has never appeared in the literature.

For the first submission, we intentionally reduced the technical depth of this paper for the following reason. We realize the big gap between coordinate-update theory and the applications in the current literature. Most of the existing coordinate-update papers only consider the several limited well known problems which have separable structures. Many other applications can actually be solved with the more efficient coordinate update methods as demonstrated in this paper.

Based on our understanding, the aim and scope of this first class journal is not just about mathematical science, but also applications. In order to explain how to apply coordinate method to more complicated applications and expand their influences, we put less emphasis on the technical depth.

Comment 4: From a more application oriented view, there is no sufficient justification (either theoretical or empirical) for the efficiency of the derived algorithms, nor comparison with state of the arts for the particular applications discussed. Thus it is not clear how useful they will become, other than illustrating some general ideas.

Response: We do not agree with the reviewer to this point for the following reasons.

Firstly, we ignored the convergence proof for the reasons stated in our response to Comment 3. In this revision, we proved the convergence of our primal-dual coordinate-update schemes (the most complicated coordinate update algorithm in this paper) under the async-parallel algorithmic framework.

Secondly, in the original version, we already included numerical results for 3 different coordinate-update algorithms applied to problems arising from 3 different areas. We compared the performance of our coordinate-update algorithms with the corresponding full-update algorithms, which are themselves state-of-the-art methods. The numerical experiments have shown the advantage of coordinate-update algorithms. In our third numerical experiment, we compare the async-parallel algorithmic framework with the sync-parallel algorithmic framework and achieve significant speedup through the async-parallel framework. In this revision, we changed our second example

to a new instance of CT image, and the coordinate-update algorithm still showed its efficiency.

References

[1] Z. Peng, Y. Xu, M. Yan, and W. Yin. ARock: an Algorithmic Framework for Asynchronous Parallel Coordinate Updates. *ArXiv e-prints*, June 2015.