1 Conclusion

The paper contains interesting observations worth publishing, but it requires a lot of restructuring and cleaning up before it could be considered for publication. The present form suffers from confusing and non-standard terminology, long proofs that reproduce standard arguments, sloppy formulations, and many typos.

Below are some of the main issues, a longer list is in the last section.

- Problems with terminology and notation:
  - The author uses three different names for 'finite limit preserving functor' interchangeably: *finite limit preserving functor* (e.g. Def. 3), *finitely continuous functor* (e.g. abstract), and *left exact functor* (e.g. Def. 13).
  - The symbol $\simeq$ is correctly used for equivalence of categories, but also incorrectly for isomorphism of functors (e.g. first page bottom) ($\cong$ should be used for isomorphism).
  - What the author calls 'Stone' in Definition 32 should really be called *sober*, in accordance with Taylor's [6].
- Missing hypotheses in definitions and lemmas. E.g. definition 5, lemma 12, definition 29 (see comments below)
- Inaccurate language and formulations.
  - Theorem 18 refers to the strict concept 'right adjoint right inverse', but really means the non-strict 'fully faithful right adjoint'.
  - Definition 49 states that "A tripos is a presheaf [of sets] ... with the following properties". This is incorrect since the order on the fibers is not a property but a structure.
  - Definition 56 defines a notion of 'isomorphism' of functors which is clearly not invertible. Consequently, the groupoid mentioned in theorem 57 is not well defined.
  - The author refers to MacLane [3] as a reference for Beck's theorem. However, this source contains a version of the theorem that characterizes categories of algebras up to *isomorphism*, and what is needed in this article is a characterization up to equivalence.
- Error in Example 11.
- Several lemmas rephrase known results with non-standard terminology – see comments on Lemma 36 and Section 4.4 below.

In the next section I give a summary of what I consider the central concepts and insights of the article, and I give some suggestions how these could be presented more clearly and efficiently.

In the last section I list issues that I came across while reading the article. I emphasize that this list is non-exhaustive, and addressing these issues one by one will not necessarily result into a publishable article.

2 Summary

2.1 Resolvent functors

A resolvent functor is a functor $F : \mathcal{C} \to \mathcal{D}$ such that for every $D \in \mathcal{D}$ there exists a $C \in \mathcal{C}$ and a regular epi $FC \to D$ which is weakly terminal in F/D.

The following results are shown on resolvent functors:

- A fully faithful finite limit preserving functor $F : \mathcal{C} \to \mathcal{C}'$ between finite limit categories is resolvent iff for every exact category $\mathcal{E}$, the precomposition functor $Lex(\mathcal{D}, \mathcal{E}) \to Lex(\mathcal{C}, \mathcal{E})$ has a fully faithful left adjoint, i.e. certain Kan extensions exist.

- The embedding $\mathcal{D} \to \mathcal{D}_{ex/reg}$ of a regular category into its ex/reg completion is resolvent iff the canonical exact functor $\mathcal{D}_{ex/lex} \to \mathcal{D}_{ex/reg}$ has a fully faithful right adjoint (this generalizes and strengthens a result of Menni [4]).

- A regular category $\mathcal{D}$ has weak dependent products and a generic mono iff $\mathcal{D}_{ex/reg}$ is a topos and $\mathcal{D} \to \mathcal{D}_{ex/reg}$ is resolvent.

- Constant objects functors $\mathcal{B} \to \mathcal{B}[P]$ (P is a tripos on a finite product category $\mathcal{B}$) are characterized up to equivalence as finite product preserving resolvent functors $\mathcal{B} \to \mathcal{E}$ into toposes.

Most of this (except the third item, on which I comment in the next subsection) can be presented much more concisely. The author should also take care to clearly state which hypotheses on categories and functors he is using where, which is not always clear in the present form.

On size issues: local smallness and well-poweredness are mentioned in a few places, but the author might consider just ignoring these issues, which are mainly relevant when relative size-differences come into play, such as e.g. small products in a large category. As the constructions discussed in the paper do not rely on such relative size differences, one can just assume that everything is small w.r.t. an appropriate universe.

The result mentioned in the last item should be compared to a similar result in in Pitts' thesis (see comments in last section).

2.2 Exact completions of regular wlccc's with generic mono

In connection with the third item in the above list, the author studies two subcategories, $\mathcal{S}, \mathcal{T}$ of the ex/lex completion $\mathcal{D}_{ex/lex}$ of a regular wlccc $\mathcal{D}$ with generic mono. These subcategories fit into a sequence

$$\mathcal{D} \to \mathcal{S} \to \mathcal{T} \to \mathcal{D}_{ex/lex}$$

of full inclusions, and the author gives the following alternative characterizations of $\mathcal{S}$ and $\mathcal{T}$:

- In terms of Taylor's *abstract Stone duality*. There is a dominance in $\mathcal{D}_{ex/lex}$ classifying the monos that are right orthogonal to (the images of) regular epis in $\mathcal{D}$ (which are not necessarily regular epic in $\mathcal{D}_{ex/lex}$). Using Taylor's terminology [5, 6], $\mathcal{S}$ is the category of *sober* objects, and $\mathcal{T}$ is the category of *discrete* objects relative to this dominance. The author then uses a theorem of Taylor [5, Thm. 11.12] to deduce that $\mathcal{S}$ is a topos.

- In terms of Menni's [4]. Menni defines the *canonical topology* to be the largest universal closure operator on $\mathcal{D}_{ex/lex}$ such that all objects in the image of $\mathcal{D} \to \mathcal{D}_{ex/lex}$ are sheaves. The category $\mathcal{S}$ is the category of *sheaves* w.r.t. this topology, and $\mathcal{T}$ is the category of *separated objects*.

- In terms of Frey's [1, 2]. As the author points out (but does not prove), $\mathcal{T}$ is a q-topos in Frey's terminology, and $\mathcal{S}$ is its subcategory of *coarse* objects.

The author also observes that $\mathcal{S}$ is equivalent to $\mathcal{D}_{ex/lex}$, which shows that the latter is a topos.

These observations are interesting, but they require a clearer presentation and discussion:

- On Taylor. The term 'abstract Stone duality' should be mentioned to explain the terminology of 'open mono', and Taylor's theorem should be stated in the form that it is used, together with auxiliary definitions. Beck's theorem should also be stated in the form that it is used.

- On Menni. His work is mentioned at several places, but instead of using it, the author reproves large parts using his own terminology. This makes things vastly more complicated and confusing than necessary.

- On Frey. The fact that $\mathcal{T}$ is a q-topos shouldn't be too hard to prove using properties of categories of separated objects, in particular that the regular monos in $\mathcal{T}$ are those that are closed w.r.t. the universal closure operator.

For what it's worth, here is a proof that $\mathcal{D}_{ex/reg}$ is a topos that I consider more direct:

- The subobject fibration of $\mathcal{D}$ is a tripos for $\mathcal{D}$ regular wlccc with generic mono: conjunction is given by pullback, implication and universal quantification can be constructed from weak dependent products followed by image factorization, and power objects are constructed from the generic mono and weak exponentials. Existential quantification and disjunction can be defined in terms of the other connectives.

- $\mathcal{D}_{ex/reg}$ is the category of partial equivalence relations and functional relations in the in the subobject fibration, which is also the tripos-to-topos construction.

3 Comments

1. Abstract and first paragraph:

    - could be merged, since the first paragraph only rephrases the beginning of the abstract, with "the effective topos" instead of "many ex/reg completions in realizability"

- give reference for things related to Eff and Asm, e.g. van Oosten's book
- make more clear in the abstract that resolvency is an original concept introduced in this paper (the second sentence of the abstract could be interpreted as saying that resolvency is an existing concept, that has been identified as reason for the existence of extensions)

2. Remark 2 : not your definition of 'having certain limits' is equivalent to AC, but the equivalence of this notion and existence of limiting cones.

3. page 3, lines 3,4 : maybe better 'counterexample' instead of 'example'?

4. last paragraph before "2. Preliminaries" (page 3) seems vague and could be left out. It is distracting from the main line of thought to refer to a potential reformulation of Giraud's theorem that is not carried out in the paper.

5. definition 3 : why are you using (2,1)-categories? usually, the universal property of ex/reg completions and ex/lex completions is formulated in terms of functor *categories*, e.g. $FC(C, E) \simeq EX(C_{ex/lex}, E)$ where FC(-,-) and Ex(-,-) are categories of functors, and not necessarily isomorphic natural transformations. If you have a good reason for using groupoids instead, you could consider to explain it.

6. lemma 4 :

   - You write "... Moreover, each regular epimorphism $X \to IY$ is split.", choosing not to introduce the technical concept of "projective object". However, you use "projective" in the proofs of Lemma 27, Lemma 28, and Theorem 31. In light of this, it would make sense to use the word also in Lemma 4 (which you are using in 27, 28, and 31 without saying), and possibly explain it.

7. definition 5 :

   - i don't see the point in bringing up the term "trivial bigroupoid" for the well established "pseudo-equivalence relation", especially since it's never used again in the text
   - line before last: "simply" can be omitted without loss

8. beginning of section 3.2 (page 5) : probably "should" has to be replaced by "shows"

9. definition 7 :

   - you write that F is an arbitrary functor and don't impose conditions on the domain and codomain categories, in condition 1 you ask the morphism to be the coequalizer of "its kernel pair", implicitly assuming that the latter exists.
   - D_0 should be in $\mathcal{C}$, not in $\mathcal{D}$

10. Example 11 (page 6) :

- $Fam(\mathcal{C})$ is *not* regular for regular $\mathcal{C}$ in general. To show this, I show that regularity of $Fam(\mathcal{C})$ implies that $\mathcal{C}$ has small joins of subobjects, which is not true for in general for regular categories. Given a family $(m_i : U_i \to A)$ of subobjects, take the image factorization of the induced map of type $(U_i)_{i\in I} \to (A)_1$. This image factorization is of the form $(U_i)_{i\in I} \xrightarrow{e} (V_j)_{j\in J} \xrightarrow{m} (A)_1$, which is mapped to $I \xrightarrow{f} J \xrightarrow{g} 1$ by $P : Fam(\mathcal{C}) \to Set$. Since P preserves finite limits, g has to be monic, and J subterminal. If I is nonempty, then $J = 1$. Hence m is of the form $(V)_1 \to (A)_1$, and we can deduce that the map $V \to A$ is monic as well, since that's how monos in $Fam(\mathcal{C})$ are. Thus, V is a subobject of A containing all U_i, and the universal property of the image factorization implies that it is the least such.
- If you assume $\mathcal{C}$ to be a geometric category (in the sense of the elephant or the nLab – it's just a regular category with pullback stable small joins of subobjects, and we just saw that the small joins are necessary to make $Fam(\mathcal{C})$ regular), then i think that indeed $Fam(\mathcal{C})$ is regular and $Fam(\mathcal{C})_{ex/reg}$ is a topos. However, it is not the topos of sheaves for the regular topology, but for the *canonical* topology.

11. Lemma 12:
    - before this, you only refer to concrete functors as embeddings, which is acceptable practice, but here you refer to the "concept" of embedding. I assume you mean full and faithful functor, but the meaning of embedding is ambiguous in the literature, as the discussion on the wikipedia page http://en.wikipedia.org/wiki/Subcategory shows (MacLane CwM means only "faithful" by embedding). Thus, "embedding" should be avoided or at least explained.
    - what do you mean by "...induce every morphism"? If you are referring to Lemma 6 – I see a problem since the equivalence relations live in $\mathcal{C}$, and the morphisms to be induced live in D. I think for this to make sense, equivalence relations in $\mathcal{C}$ have to get mapped to equivalence relations in $\mathcal{D}$, and for this, F has to preserve finite limits. Thus, you should add this to your assumptions.
12. Definition 13 (page 7) wouldn't it make sense to put this before Lemma 12? Here you also mention the preservation of finite limits, that I think is missing in 12.
13. First line of Section 3.3 (page 7) : *right*, not rights
14. Proof of Lemma 14 (page 7):
    - In the construction of $F_!(G)(X)$ you can not just take any pseudoequivalence relation representing X. You actually have to take a resolution. Otherwise the next step, which states that morphisms $X \to Y$ are represented by moprhisms of pseudoequivalence relations is not valid.
    - regarding the invertibility of η you could point out that this is a consequence of the Kan extension being "pointwise" – see "categories

for the working mathematician", Section X.3, Corollary 3 (actually MacLane proves invertibility in presence of additional assumptions, but the proof applies to arbitrary pointwise extensions, see also Corollary 4)

- 'By generalization ...' : This phrase is used thirteen times in the paper, and I find it unusual. I have the impression that the author uses it as a kind of ∀-introduction, but I haven't seen it anywhere else, so maybe it could just be left out, or replaced by an adverb?

15. proof of Lemma 17 (page 9)
    - $id_{\mathcal{D}}$ instead of $id_{\mathcal{C}_{ex/reg}}$ at several places
    - F instead of J at several places

16. Theorem 18:
    - 'right adjoint right inverse' means a reflection whose counit is the *identity*, which doesn't play a role here. Write 'fully faithful right adjoint'.
    - Claim 3. : $\mathcal{E}$ instead of $\mathcal{D}$
    - Proof:
      - K : $\mathcal{C}_{ex/lex} \to \mathcal{D}$ (not $\mathcal{C}_{ex/lex} \to \mathcal{C}_{ex/lex}$)
      - $FY = KIY \to X$ (not $KIY \to X$)

17. section 4, first paragraph : "if and only if", not "in and only if"

18. proof of prop 21: "...inherits local cartesian closure" – mention that this depends on the reflector preserving finite limits

19. Example 22:
    - you might want to write somewhere explicitly what this is an example of (I assume of a category $\mathcal{C}$ s.th. $\mathcal{C} \to \mathcal{C}_{ex/reg}$ is not resolvent)
    - $\mathcal{C}$ is closed under finite products and arbitrary subobjects as a subcategory of **Set**, hence it is a geometric category (see comments on Ex.11). Thus, $Fam(C)$ is indeed regular, and $Fam(C)_{ex/reg}$ a topos
    - more precisely $Fam(C)_{ex/reg}$ is the topos of sheaves for the canonical topology on $\mathcal{C}$ – and thus equivalent to **Set**, since we can recover any Grothendieck topos from a generating set by taking sheaves for the canonical topology on the full subcategory spanned by the generating set.

20. definition 26: explain why you call them 'open monos', i.e. mention ASD.

21. proof of lemma 27:
    - 4th line: replace "Since pullback" by "As a pullback"
    - 5th line after 1st diagram: "p_y is a regular epimorphism" (not e)

22. definition 29:
    - I think you have to assume somewhere that C is weakly lccc

- does "congruence" have a technical meaning here?
- "pullbacks of $I\gamma$ along f are pullbacks of $I\gamma$ along g" – wouldn't it be easier to write "$f^*(I\gamma) \cong g^*(I\gamma)$"? Or do you mean something different?
- "Because $C_{ex/lex}$ is lccc and exact, there exists a congruence ..." : it is not at all clear how this follows. You could define $x \equiv y$ as $I(\gamma)(x) \Leftrightarrow I(\gamma)(y)$ in the internal language of the posetal reflection of the fundamental fibration (where the existence of implication follows from the lccc structure), but if you don't want to use the logical language you might have spell this out by hand somehow

23. beginning of 4.3: the *contravariant* power object functor $P : E^{\mathrm{op}} \to E$ is monadic

24. definition 32 (page 14) : what you call "Stone" is called "sober" by Taylor ("Sober spaces and continuations", Def. 4.7). Indeed it is precisely the sober spaces that can be reconstructed from their opens – not the Stone spaces as you write. Those can be reconstructed from their boolean algebra or *clopens*

25. remark 34: This all should be explained in much more detail. Cf. summary section above.

26. lemma 36: this is proposition 4.9 in taylor's "sober spaces and continuations". I think the passage to coequalizers at the end of the proof is misplaced

27. lemma 37:

    - statement of lemma: If you want to show that $\Omega^{(-)}$ creates split coequalizers, then you in particular have to show that f,g have an equalizer whenever Ω^f, Ω^g have a split coequalizer. Or are you claiming that $\mathcal{S}$ has all equalizers anyway? This would have to be proven. Is it relevant that you write "a coequalizer" but "the equalizer"?
    - 1st sentence of proof: you are using that subobjects of discrete objects are discrete. could you maybe state and prove that explicitly as a further clause in lemma 35?
    - " ... and ... *have* a split coequalizer" not "has"
    - "... composition with h on the left is a section of Ω^h": I don't see how this typechecks

28. theorem 38:

    - CWM is not a good choice of reference for the monadicity theorem, since it characterizes categories of algebras up to *isomorphism* of categories, and I think in your situation monadicity can only be expected to hold up to equivalence. Better choices of references are Barr/Wells or Borceux. In any case it might be helpful to include an explicit statement of the monadicity theorem in the form that you are using.
    - $\mathcal{S}^{\mathrm{op}} \to \mathcal{S}$ (not $\mathcal{S} \to \mathcal{S}$)

- replace “en” by “and”
- you should mention Taylor earlier and include an explicit statement of his result, in the form that you use. you should also say something about “compact”, or alternatively phrase your statement of his result in a way that avoids this term.

29. section 4.4: you seem to be rephrasing large parts of Menni [11] while changing terminology

    - Definition 40 (local isomorphisms): this seems to be a special case of ‘dense mono’ for universal closure operator induced by the ‘canonical topology’ on $\mathcal{C}$ (special case because the codomain is required to be in the image of I)
    - Lemma 41: stability under pullback always holds for dense maps of a universal closure operator, and orthogonality to open monos follows since open monos are precisely the *closed* (sic!) ones in the sense of the universal closure operator
    - Definition 42 (local object) you already mention that they are equivalent to Menni’s sheaves, so why not just adopt his definition and terminology?
    - Lemma 43: for the statement that I is regular to make sense, shouldn’t you also show that $\mathcal{L}$ is a regular category? Menni does that in 4.15
    - Corollary 44: this is (a weakening of) Menni’s Corollary 4.16.
    - Lemma 45: this is easier to understand and proved when phrased in terms of familiar concepts: “X is a sheaf iff it is a *closed* subobject of some Ω^Y”. Then one only has to prove that Ω is a sheaf, since (1) sheaves form an exponential ideal (2) subobjects of sheaves are sheaves iff they are closed (see Lemmas A4.3.8, A4.4.3 in the Elephant). To see that Omega (the classifier of closed monos) is a sheaf, note that for any dense mono $m : A \to B$ we have $\mathcal{C}(A,\Omega) = subc(A) = subc(B) = \mathcal{C}(B,\Omega)$, where subc(-) is the presheaf of closed subobjects, and the second bijection is Lemma A4.3.7 in the elephant. This reduces two pages of proof to one paragraph.

30. theorem 48:

    - you mention theorem 31 in the proof, but what is it good for? I don’t see how it helps to prove that $\mathcal{C}$ has a generic morphism, but this can be deduced directly from resolvency of J

31. section 5, 1st paragraph: thankS to

32. definition 49 (tripos):

    - being a tripos is *not* a property of a presheaf of sets, but additional structure (it is a property of a presheaf of posets/preorders). this also makes the $T : \mathcal{B}^{\mathrm{op}} \to \mathbf{Set}$ on page 32, line 2 problematic
    - definition 49-2 : if a sentence ends with a displayed equation, i would put a fullstop at the end of the equation

- is it really necessary to mention "internal heyting algebras"? this point of view doesn't seem to play a role in the following, and people who don't already know that presheaves of certain first order structures can be understood as internal such structures might only be confused by it being brought up in a half-sentence.
- definition 49, clause 3: you write ϵ_X as subscript later, which i think looks nicer than simply juxtaposing like here
- definition 49, clause 3: "generic predicate" is only used for the power object of 1 in the literature. what you mean is called "membership predicate" e.g. by van Oosten

33. generally on triposes: you sometimes use functorial notation (like $phi = \exists_\pi \psi$, e.g. pg. 23, first display) and sometimes "internal language" notation with variables (like $\phi(x) = \exists y \psi(x, y)$, e.g. pg. 24, first display). I think you should comment on that and explain your notational conventions regarding the internal language, e.g. by referring to a textbook (Jacobs, Johnstone, van Oosten). The same applies to defining morphisms using variables, like $pi_{02}(x, y, z) = (x, z)$.

34. definition 50: say what you are talking about, e.g. "Given a tripos $P : \mathcal{B}^{\mathrm{op}} \to \mathbf{Poset}, \ldots$"

35. Lemma 51: insert "…in $\mathcal{B}$, *let* $\nabla(f) = \ldots$" for better readability

36. Lemma 53:
    - display equation: $\exists x \in X\, \forall y \in X$ (not Y)
    - Let $\Sigma(X, e) = (\pi X, \exists_{\delta_{\pi X}} \ldots)$ (not $\exists_{\delta_X}$)
    - …defines a monomorphism $\cdots \to \Delta \pi X$ (not ΔX)
    - end of proof: what do you mean with "by further generalization"? I think what is missing is essentially the observation that $\epsilon_X : \Sigma(X, e) \to (X, e)$ is epic (and thus the coequalizer of its kernel pair, since we are in a topos)

37. Page 24, middle: "If $\mathcal{E}$ is well powered, then there is a functor $Sub : \mathcal{E}^{\mathrm{op}} \to$ **Set** which assigns a poset of subobjects to each object". If the functor assigns a poset to each object, then it's a functor of type $\mathcal{E}^{\mathrm{op}} \to$ **Poset**! This seems to be a similar misunderstanding as in the definition of triposes.

38. Page 24, next paragraph:
    - "…satisfies parts of the properties of a tripos trivially" – which parts?
    - "The poset $Sub(DX)$ is a Heyting algebra …" – so far you only have assumed that D : B —¿ E is a fp-preserving functor between fp-categories, so you won't get Heyting algebras in general
    - " The resolvency condition proved in lemma insures that $Sub(D-)$ also has generic predicates" – Which lemma are you referring to? Did the number get lost? Are you anticipating/announcing the following lemma? Maybe you made a copy-paste error somewhere? Also: *ensures*, not *insures*.

39. proof of Lemma 55 (pages 24, 25):

   - e : $X_0 \to \Omega^{DX}$ (not $\to DX$)
   - c : $D\pi X \to \Omega^{DX}$ (not $\to \Omega^X$)
   - determines *a* new resolution (not "an")
   - "the set of all pullbacks" — it took me some time to understand what you meant, until I realized that you probably refer to a subobject, viewed as equivalence class of monos (right?). Could you maybe rather write "the equivalence class of the pullback" or something like that? Also, on page 24 in the middle where you talk about well poweredness you mention that well poweredness allows to define posets of subobjects, but you never really say that the elements of the poset are equivalence classes (another possibility would be representatives, then "the set of all pullbacks" wouldn't make sense as far as I can see – in this case you could maybe write "the subobject given by the pullback")
   - "...determines a new resolution" — it would be easier to understand if you explicitly mention the corresponding object in the category of monos, which would be $id_{D\pi X}$ if I'm not mistaken

40. Definition 56 (page 25):

   - isomorphisms of (indexed) posets always preserve finite joins, so this condition is redundant
   - the "groupoid" of finite product preserving functors is in fact not a groupoid, for the simple reason that equivalences of categories are not isomorphisms, and don't have inverses on the nose. if you really want to phrase Thm. 57 as an equivalence of groupoids, then I think you have to use pairs (F, ϕ) of an equivalence and a natural iso as you write, but *modulo natural isomorphism*. Another possibility would be to phrase the result as a biequivalence of 2-categories, and then to conclude that the 2-category of functors into toposes is very simple (equivalence-relation-like) on the level of 2-cells, since it's biequivalent to a 1-category (even a groupoid). this would amount to the same thing, but would make clearer why it's justified to quotient by natural isos (since they are unique as soon as they exist)

41. Theorem 57 (page 25):

   - The *groupoid* (not groupoidS)
   - the equivalence of triposes you attribute to Frey can already be found in Pitts' thesis (Proposition 3.6)
   - Pitts gives a characterization of functors $\mathcal{B} \to \mathcal{B}[T]$ in Theorem 3.10 of his thesis. I think you might want to give a comparison to your theorem. Pitts' characterization is not in terms of resolvency (of course), but there are obvious similarities in the conditions.

42. page 25, line before last: "the full image". I think it would be good to define "full image", e.g. in a footnote. I don't remember encountering this terminology before reading the paper. The definition is easily found

on the nlab (if one recognizes it as an independent technical term, which already requires some careful reading), but there, no references are given.

43. page 26, "it is easy to see ..." : I don't think resolvency is necessary here – a regular functor $F : \mathcal{D} \to \mathcal{E}$ from a regular into an exact category is an ex/reg completion iff it is

    - full and faithful
    - covering (for every $D \in \mathcal{D}$ there exists a regular epi $F(C) \to D$)
    - full on subobjects

    van den Berg [7] attributes this to Carboni (some free constructions in realizability and proof theory), but I couldn't find the result in the latter.

References

[1] J. Frey. A 2-categorical analysis of the tripos-to-topos construction. *arXiv preprint arXiv:1104.2776*, 2011.

[2] J. Frey. Triposes, q-toposes and toposes. *Annals of pure and applied logic*, 166(2):232–259, 2015.

[3] S. Mac Lane. *Categories for the working mathematician*, volume 5 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, second edition, 1998.

[4] M. Menni. Closure operators in exact completions. *Theory and Applications of Categories*, 8(21):522–540, 2001.

[5] P. Taylor. Geometric and higher order logic in terms of abstract stone duality. *Theory and Applications of Categories*, 7(15):284–338, 2000.

[6] P. Taylor. Sober spaces and continuations. *Theory and Applications of Categories*, 10(12):248–300, 2002.

[7] B. van den Berg. Predicative toposes. *arXiv preprint arXiv:1207.0959*, 2012.