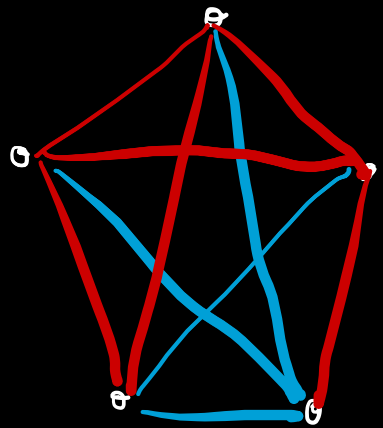
Multicolour Ramsey numbers of short odd cycles

Wouter Cames van Batenburg, January 2026

Joint work with
Maria Axenovich, Oliver Janzer, Lukas Michel & Mathieu Rundström

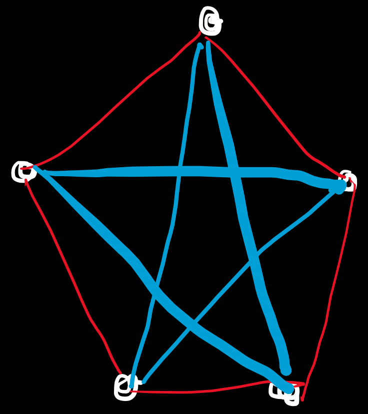
What is smallest n such that every red/blue colouring of the edges of K_n contains a monochromatic triangle, i.e. a $\triangle$ or $\triangle$?

What is smallest n such that <u>every</u> red/blue colouring of the edges of K_n contains a monochromatic triangle, i.e. a △ or △ ?



has △, but ...

What is smallest n such that every red/**blue** colouring of the edges of K_n contains a monochromatic triangle, i.e. a $\triangle$ or $\triangle$?



has no $\triangle$ or $\triangle$,

so $n > 5$.

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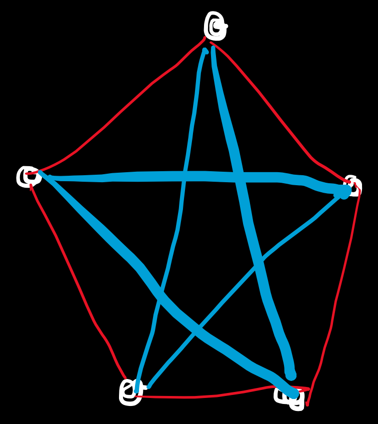

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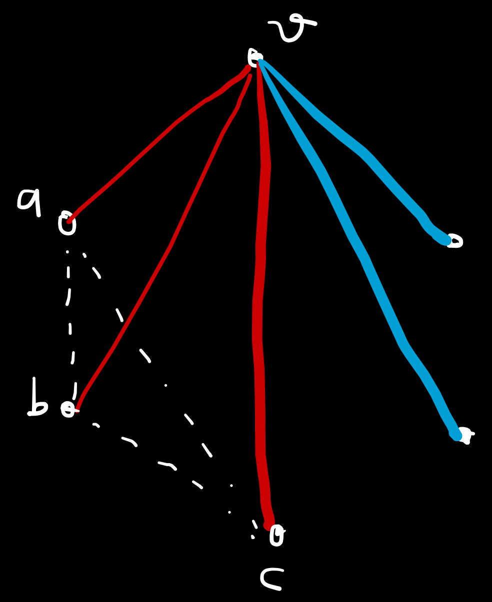

wlog red neighborhood of v has size ≥ 3.
$\rightsquigarrow$ $\triangle$ or $\triangle$ on $\{v, a, b, c\}$
$\rightsquigarrow$ $n \leq 6$

What is smallest n such that every red/blue colouring of the edges of K_n contains a monochromatic triangle, i.e. a $\triangle$ or $\triangle$?

Answer : $n = 6$.

What is smallest n such that every red/blue/yellow colouring of the edges of K_n contains a monochromatic triangle, i.e. a $\triangle$ or $\triangle$ or $\triangle$?

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Answer : $n = 17$

What is smallest n such that <u>every</u> red/blue/yellow/green colouring of the edges of K_n contains a monochromatic triangle, i.e. a $\triangle$ or $\triangle$ or $\triangle$ or $\triangle$?

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Unknown!

$$51 \leq n \leq 62$$

Def Given graphs $H_1, H_2, \dots, H_k$, the <u>Ramsey number</u>

$\boxed{R(H_1, H_2, \dots, H_k)}$ is the smallest integer n

such that every $\{1, 2, \dots, k\}$-edge colouring of K_n

contains an i-coloured H_i, for some i.

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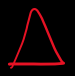
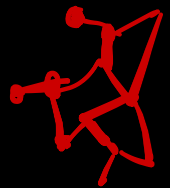
Exp. $R(C_3, C_3) = 6$  or 

$R(C_3, C_3, C_3) = 17$  or  or 

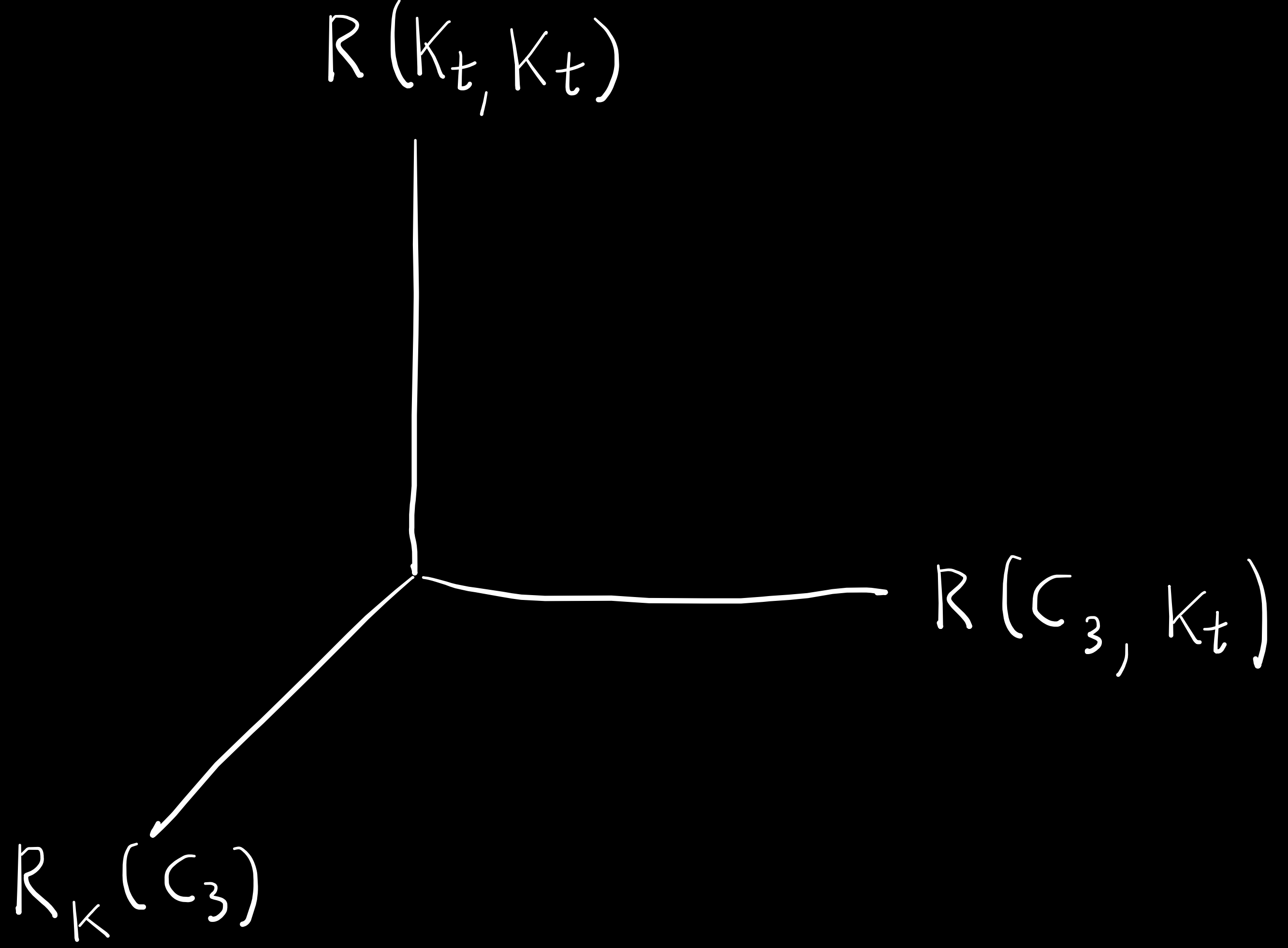
$R(H, K_2) = \# V(H)$, for every graph H.  or 

Def Given graphs $H_1, H_2, \dots, H_k$, the <u>Ramsey number</u> $\boxed{R(H_1, H_2, \dots, H_k)}$ is the smallest integer n such that every $\{1, 2, \dots, k\}$-edge colouring of K_n contains an i-coloured H_i, for some i.

Notation $\boxed{R_k(H)} := R(\underbrace{H, H, \dots, H}_{k \text{ times}})$.

Exp. $R_2(C_3) = 6$, $R_3(C_3) = 17$.

Three „orthogonal" main challenges in Ramsey theory



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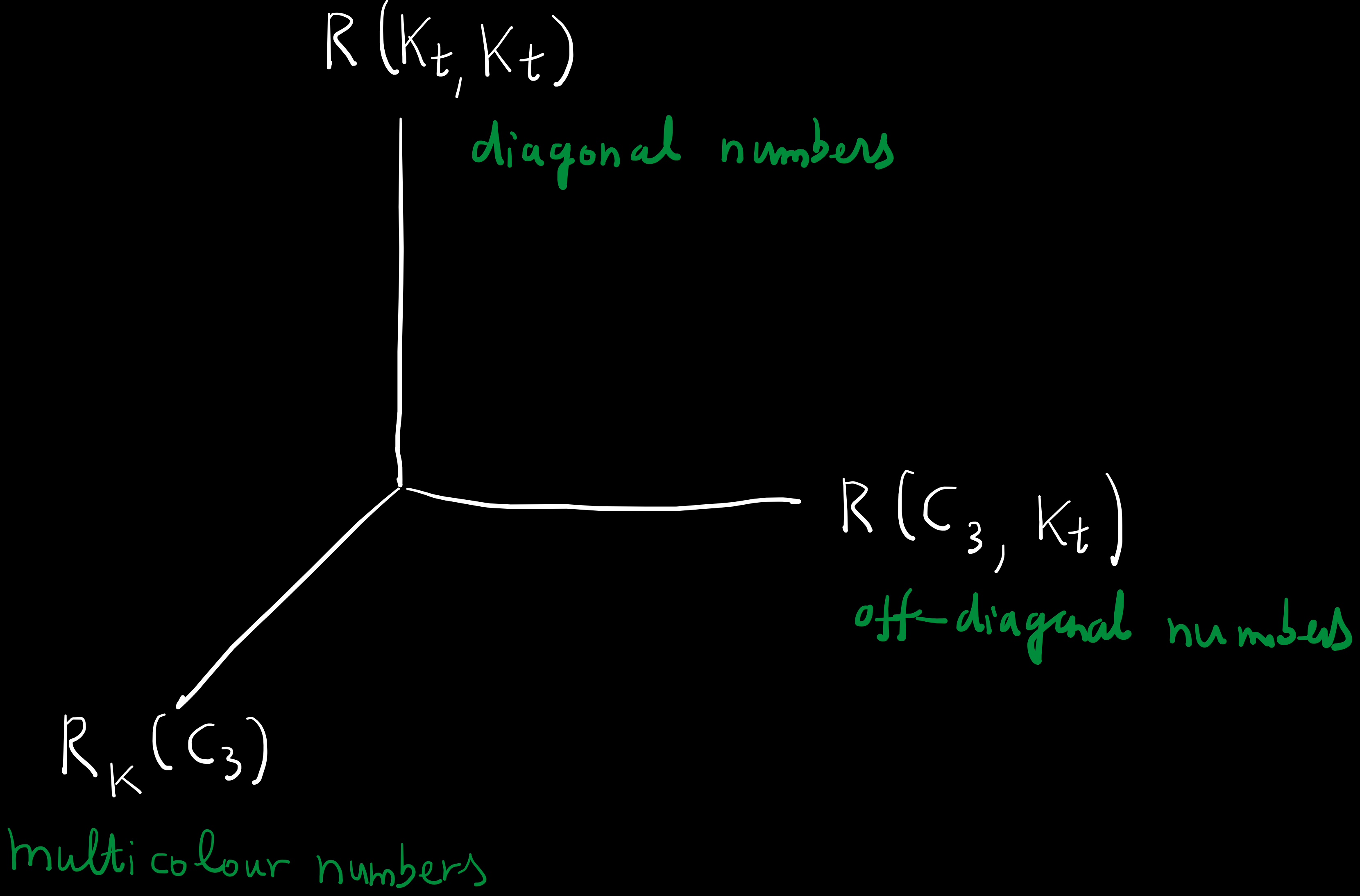
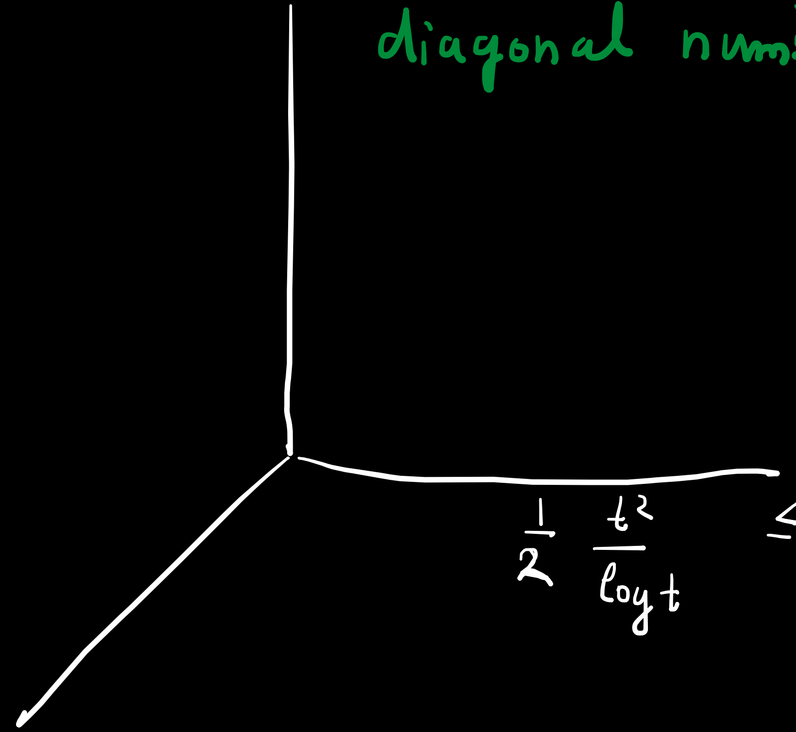


Three „orthogonal" main challenges in Ramsey theory

$$\sqrt{2}^{k} \lesssim R(K_t, K_t) \lesssim (4-\varepsilon)^{k}$$

diagonal numbers



$$\frac{1}{2}\,\frac{t^2}{\log t} \le R(C_3, K_t) \lesssim \frac{t^2}{\log t}$$

off-diagonal numbers

$$R_k(C_3)$$

multicolour numbers

Three "orthogonal" main challenges in Ramsey theory

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diagonal numbers

battling for constants

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$$\boxed{3{,}28^{k} \lesssim R_k(C_3) \lesssim k!}$$

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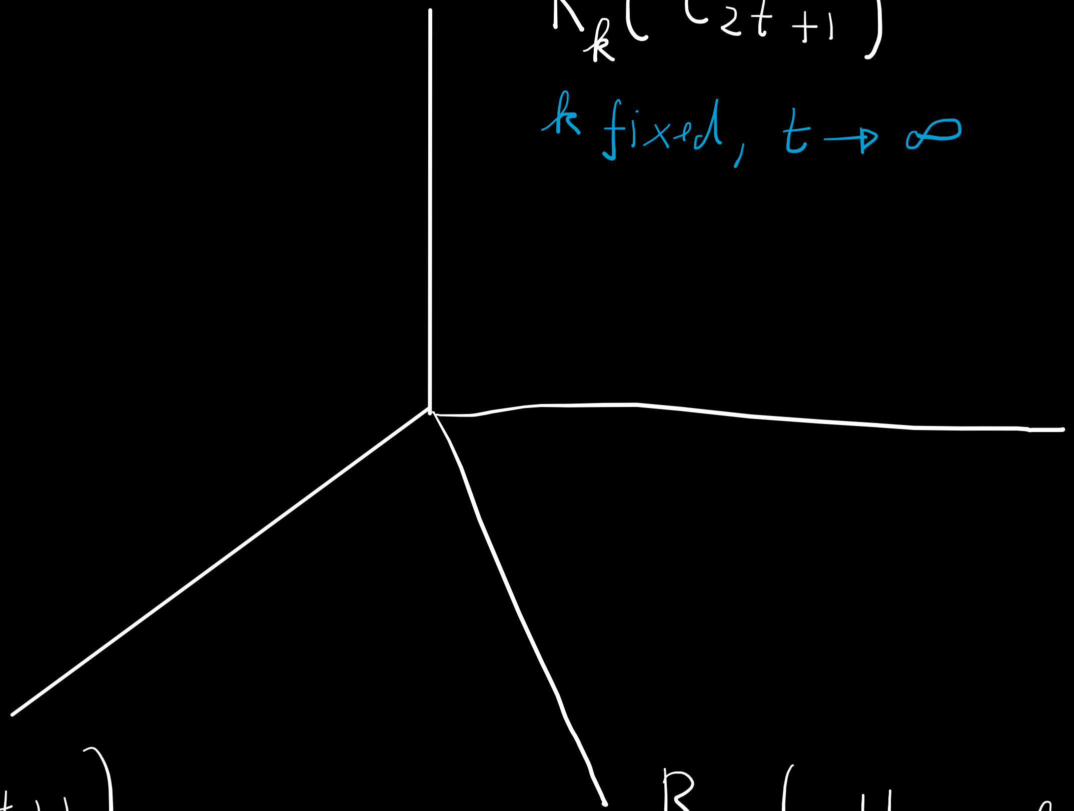
multicolour numbers

← Truth exponential or superexponential in k?

Essentially no improvement since Schur (1916)

Relaxations of $R_k(C_3)$



$R_k(C_{2t+1})$

k fixed, $t \to \infty$

$R_k(\text{odd cycles} \le L(k)) = 2^k + 1$

find smallest $L(k)$, as $k \to \infty$

$R_k(C_{2t+1})$

t fixed, $k \to \infty$

$R_k(\text{odd cycles} \le L(k, b)) \le b^k$

find smallest $L(k, b)$, for every $b > 2$, as $k \to \infty$

Relaxations of $R_k(C_3)$

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k fixed, $t \to \infty$

Asked by Erdős

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t fixed, $k \to \infty$

Erdős: $\lim_{k \to \infty} \dfrac{R_k(C_{2t+1})}{R_k(C_3)} = 0$ if $t \ge 2$?

$R_k(\text{odd cycles} \le L(k,b)) \le b^k$

find smallest $L(k,b)$, for every $b > 2$

our intermediate question

Relaxations of $R_k(C_3)$

$R_k(C_{2t+1}) = t \cdot 2^k + 1$ (Bondy & Erdős '73) (Jenssen & Skokan '21)

k fixed, $t \to \infty$

$R_k(C_{2t+1})$

t fixed, $k \to \infty$

$\lesssim \sqrt{k!}$ (Lin & Chen '19) if $t \geq 2$.

$\lesssim (k!)^{1/t}$ (ACJMR '25)

$R_k(\text{odd cycles} \leq L(k)) = 2^k + 1$

find smallest $L(k)$

$L(k) \lesssim \frac{1}{k} 2^k$ (Girão & Hunter '24)

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$L(k) \gtrsim 2^{c\sqrt{\log k}}$ (Day & Johnson '17)

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$L(k,b) \lesssim k/\sqrt{b-2}$ (Girão & Hunter, Janzer & Yip, '24/'25)

$L(k,b) \lesssim \log_{b/2}(k)$ (ACJMR '25)

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- Solves conjecture of Fox

[Li 2009] gave a conditional proof.

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A naive lemma for multicolour Ramsey

Lemma Consider a k-colouring of K_n. Suppose that for every colour i, the subgraph G_i formed by the i-coloured edges has chromatic number $\leq \chi$.

Then $\boxed{n \leq \chi^k}$

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Then $\boxed{n \leq \chi^k}$

Proof Base case $k=1$: then $G_1 \cong K_n$ has chromatic number n so $n \leq \chi$ ✓.

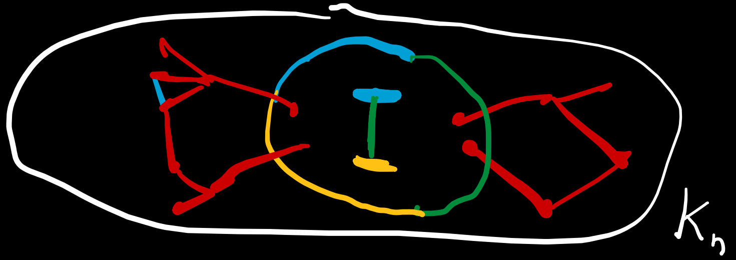
A naive lemma for multicolour Ramsey

Lemma Consider a k-colouring of K_n. Suppose that for every colour i, the subgraph G_i formed by the i-coloured edges has chromatic number $\le \chi$. Then $\boxed{n \le \chi^k}$.

Proof Induction step: Let I be a maximum independent set of G_k.

Then $$\boxed{\frac{n}{\chi}} \le \frac{n}{\chi(G_k)} \le \# I \underset{\text{by induction}}{\le} \boxed{\chi^{k-1}}.$$

Since the induced subgraph $K_n[I]$ does not use colour k. $\square$



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Corollary Let $\mathcal{H}$ be a set of graphs s.t. every $\mathcal{H}$-free graph has chromatic number $\leq \chi$.

Then $R_k(\mathcal{H}) \leq \chi^k + 1$.

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(**Proof** Suppose that K_n admits a k-colouring without monochromatic $H \in \mathcal{H}$. Then by Lemma, $n \leq \chi^k$. $\square$)

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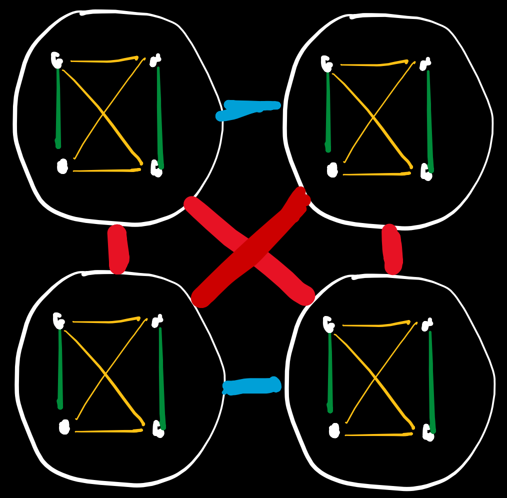
Sharp example $\mathcal{H} = \{\text{all odd cycles}\}$.

Then ALL $\mathcal{H}$-free graphs have chromatic number ≤ 2.

$$\Rightarrow \boxed{R_k(\mathcal{H}) \leq 2^k + 1},$$

Conversely,

K_{2^k} can be partitioned into k bipartite (i.e. $\mathcal{H}$-free) graphs:



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What about $\mathcal{H} = \{\text{all odd cycles of length} \leq t\}$?

$\mathcal{H} = \{C_{2t+1}\}$?

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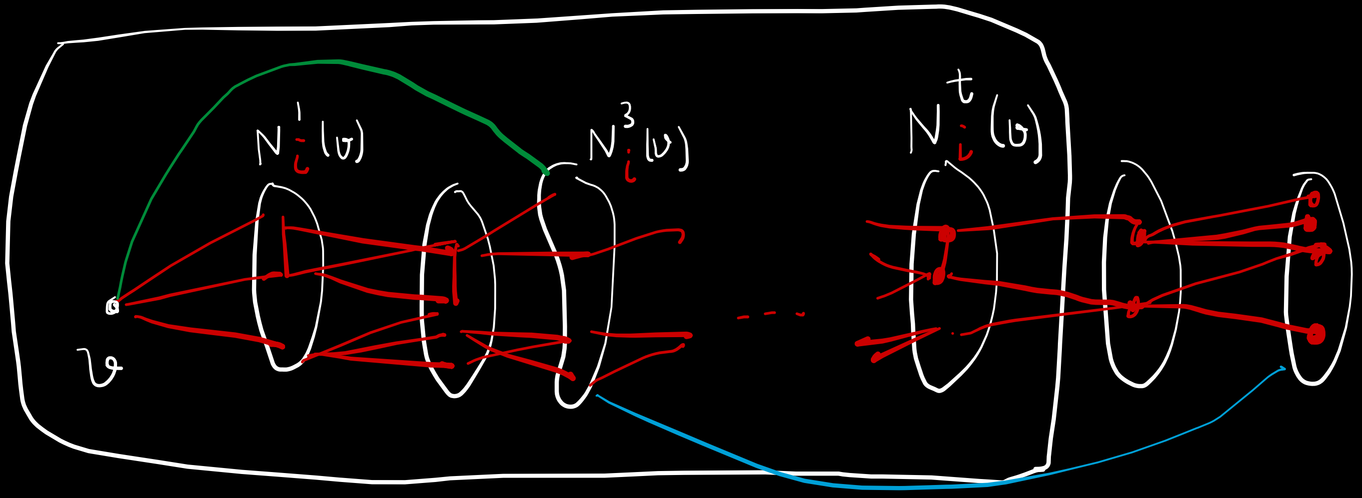
:(

The naive Lemma / Corollary is useless,
since $\exists$ graphs with arbitrarily large girth & χ.

Solution :

Instead of bounding $\chi(\mathcal{G}_i)$,

bound chromatic number of small-radius balls in $\mathcal{G}_i$



$\mathcal{G}_i[N_i^{\leq t}(v)]$:= ball of radius t in the red graph, centered at v.

Given: a k-colouring of K_n.

Naive Lemma

If for every colour i, the subgraph G_i formed by the i-coloured edges has chromatic number $\leq \chi$, then $\boxed{n \leq \chi^k}$.

New Lemma (ACJMR 2025+)

If for every colour i and every vertex v of K_n, the induced subgraph $G_i[N_i^{\leq t}(v)]$ has chromatic number $\leq \chi$, then $\boxed{n \leq \chi^k \cdot k^{k/t}}$.

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Note: New $\overset{t\to\infty}{\Longrightarrow}$ Naive

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Corollary Let $b > 2$. If $n > b^k$, then every k-edge colouring of K_n contains a monochromatic odd cycle of length $\le 2 \cdot \lceil \log_{b/2}(k) \rceil + 1$.

Proof Write $t := \lceil \log_{b/2}(k) \rceil$. Suppose the contrary.

Then there is no i-coloured odd cycle of length $\le 2t+1$,

so $G_i[N_i^{\le t}(v)]$ is bipartite for all v and i.

Therefore $n \le 2^k \cdot k^{k/t} \le b^k$. $\lightning$ $\square$

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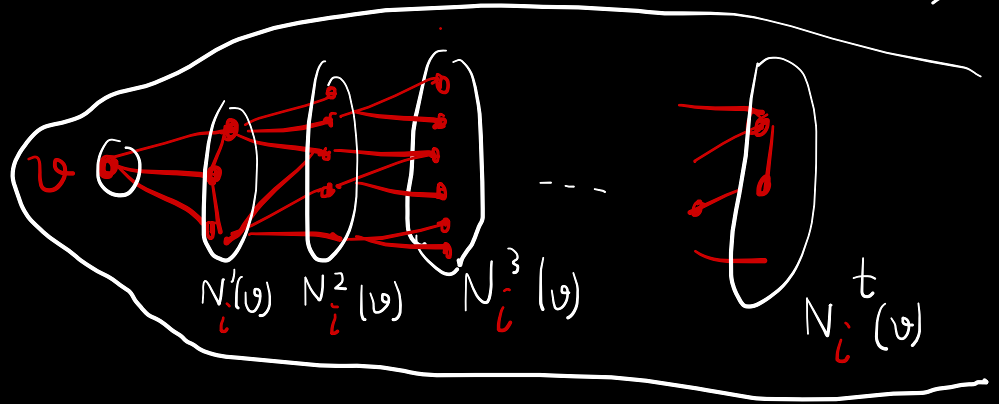

Corollary $\boxed{R_k(C_{2t+1}) \leq (4t-2)^k \cdot k^{k/t} + 1} := n+1$

Proof Suppose a k-colouring of K_n has no monochromatic C_{2t+1}.
Then $G_i[N_i^{\leq t}(v)]$ is a C_{2t+1}-free graph with radius $\leq t$ and hence $(4t+2)$-colourable $\square$

(Erdős, Faudree, Rousseau, Schelp '78)

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Proof Idea: it's easier to find a monochromatic structure if there are few colours.

$\Rightarrow$ punish vertices that have many distinct incident colours

$\Rightarrow$ define vertex weight $w(v) = C^{-d(v)}$ where $d(v) :=$ # distinct colours incident to v, & $C := \chi \cdot k^{1/t}$ is the smallest constant that will work.

$$w(v) := \left(\chi \cdot k^{1/t}\right)^{-d(v)}$$

& For each vertex subset $U \subseteq V(K_n)$, $w(U) = \sum_{v \in U} w(v)$ = weight of U

$w(U)$ small $<\approx>$ many distinct colours incident to U, on average $<\approx>$ monochromatic structure less likely

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<u>Obs</u> Suffices to prove $w(V_n) \leq 1$.

<u>Proof</u> Then $1 \geq w(V_n) = \sum_{v \in V_n} \frac{1}{\left(\chi \cdot k^{1/t}\right)^{d(v)}} \geq \frac{n}{\left(\chi \cdot k^{1/t}\right)^{k}}$ $\square$

So our task is to show that $\omega(V_n) \leq 1$

So our task is to show that $w(V_n) \leq 1$

Observation

By induction on n, $w(V_n - U) \leq 1$ for all $U \subseteq V_n$.

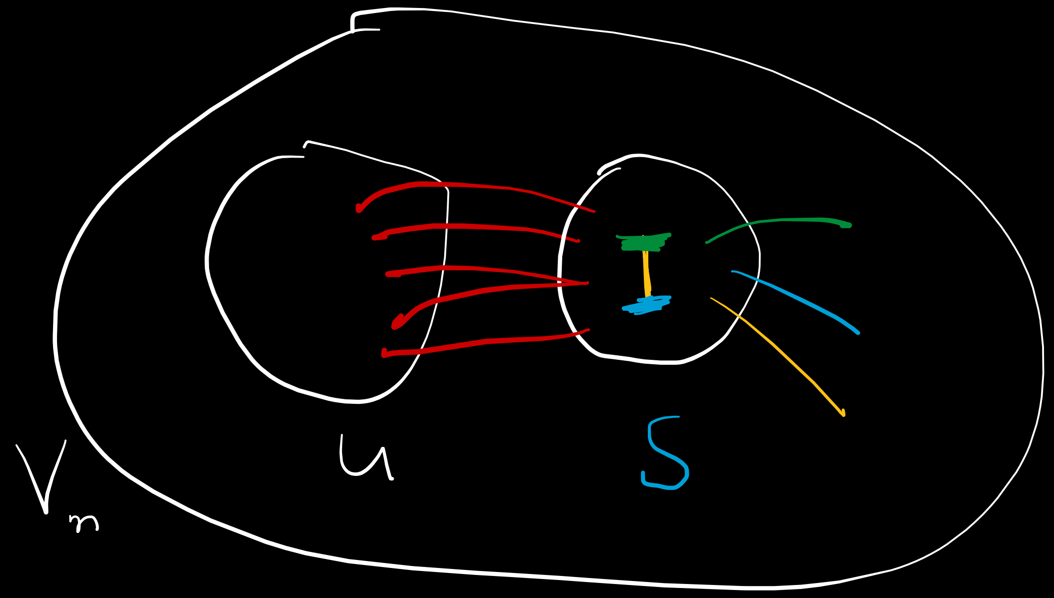
It's sufficient to find $U \subseteq V_n$ s.t. $\boxed{w(V_n) \leq w(V_n - U)}$.

⇝ It's sufficient to find $U \subseteq V_n$ s.t. $\boxed{w(V_n) \leq w(V_n - U)}$.

⇝ need to find U s.t.

weight of $V_n - U$ increases to compensate loss of $w(U)$.

Goal may be achieved by finding a large $S \subseteq V_n - U$ s.t. every $v \in S$ has some red neighbour in U, but <u>not</u> in $V_n - U$.



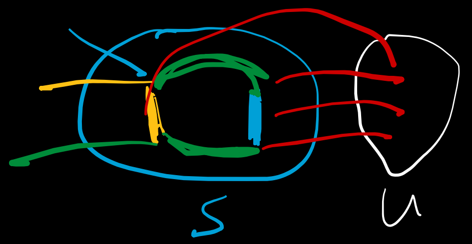
⇝ Removing U decreases $d(v)$ for all $v \in S$

⇝ increases weight of S by factor $\geq x \cdot k^{1/t}$.

⇝ suffices that

$$\boxed{\underbrace{w(U)}_{\text{weight loss}} \leq \underbrace{(x \cdot k^{1/t} - 1) \cdot w(S)}_{\text{weight gain}}}$$

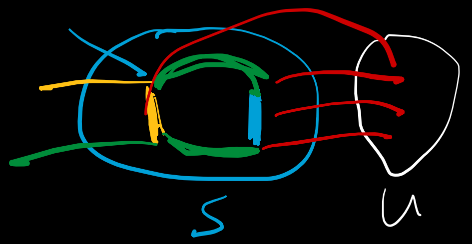
$\rightsquigarrow$ Suffices to find $U \subseteq V_n$ & $S \subseteq V_n - U$ s.t.



* Every vertex of S has some red neighbour in U, but not in $V_n - U$.

* $w(U) \leq (\chi \cdot k^{1/t} - 1) \cdot w(S)$.

$\leadsto$ Suffices to find $U \subseteq V_n$ & $S \subseteq V_n - U$ s.t.



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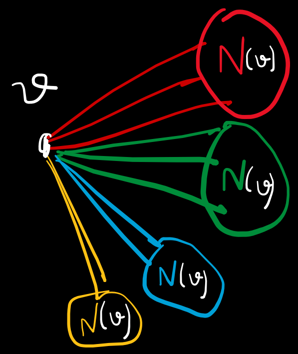
Finding U and S :

Pick arbitrary vertex v. It has $\leq k$ incident colours.



Pigeon hole $\leadsto$ $\exists$ colour (wlog red) with $w(N(v)) \geq \dfrac{w(V_n \setminus \{v\})}{k}$

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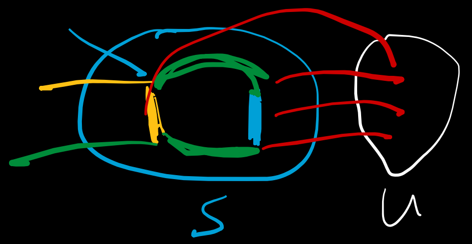

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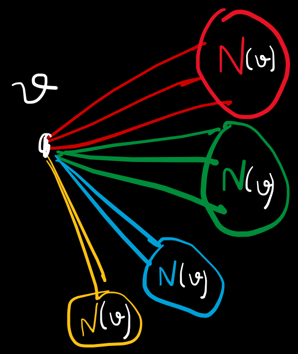

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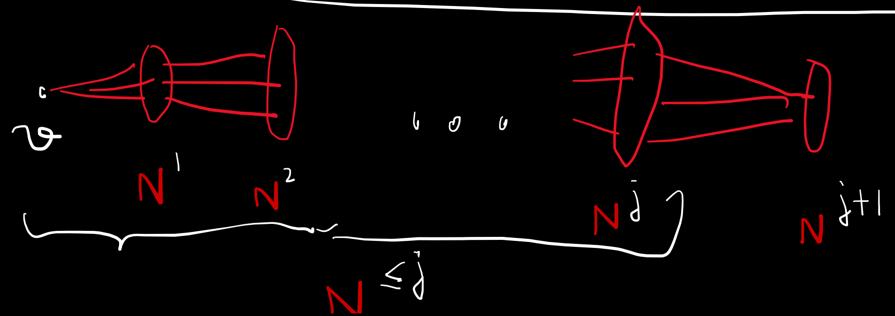
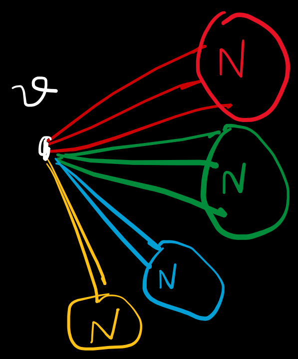
Claim $\exists$ $j \in \{1, 2, \dots, t\}$ s.t. $\boxed{w(N^{j+1}(v)) \leq (k^{1/t} - 1) \cdot w(N^{\leq j}(v))}$

ie.: $\exists$ index $j+1$ where the red graph does not grow too much





Pick arbitrary vertex v. It has $\leq k$ incident colours.

Pigeon hole $\rightsquigarrow$ $\exists$ colour (wlog red) with $w(N(v)) \geq \dfrac{w(V_n \setminus \{v\})}{k}$

Claim $\exists$ $j \in \{1, 2, \dots, t\}$ s.t. $\boxed{w(N^{j+1}) \leq (k^{1/t} - 1) \cdot w(N^{\leq j})}$

Indeed, otherwise

$$\frac{w(N^{\leq t+1})}{w(N^{\leq 1})} = \prod_{j=1}^{t} \frac{w(N^{\leq j+1})}{w(N^{\leq j})} = \prod_{j=1}^{t} 1 + \frac{w(N^{j+1})}{w(N^{\leq j})} > \prod_{j=1}^{t} k^{1/t} = k$$

So that $w(N^{\leq t+1}) > k \cdot w(N^{\leq 1}) \geq w(V_n)$ $\lightning$

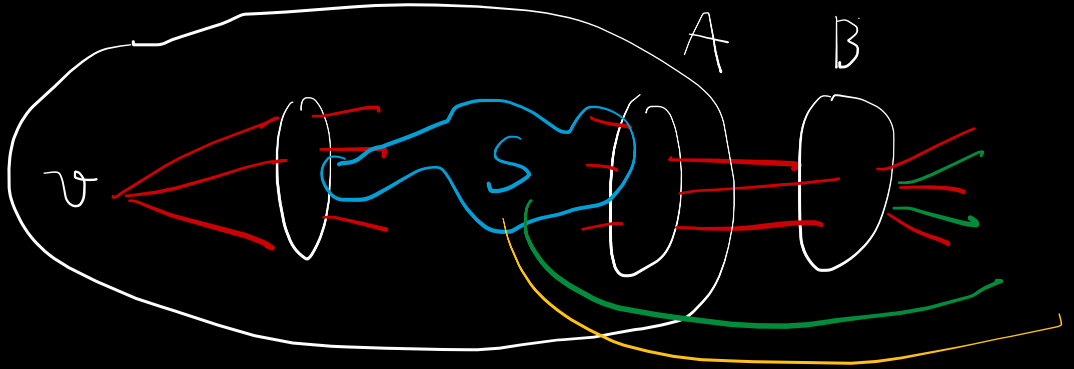
Claim $\exists$ $j \in \{1, 2, \ldots, t\}$ s.t. $\boxed{w\left(N^{j+1}(v)\right) \leq \left(k^{1/t} - 1\right) \cdot w\left(N^{\leq j}(v)\right)}$

Claim $\exists\ j \in \{1, 2, \dots, t\}$ s.t. $w(N^{j+1}(v)) \leq (k^{1/t} - 1) \cdot w(N^{\leq j}(v))$.

Let $A := N^{\leq j}(v)$ and $B := N^{j+1}(v)$. Then by claim, $\boxed{w(B) \leq (k^{1/t} - 1)\, w(A)}$.

By lemma assumption, $G_{red}[A]$ is χ-colourable, so has independent set S with $\boxed{w(S) \geq \frac{w(A)}{\chi}}$.

Now choose $\boxed{U = B \cup A \setminus S}$.



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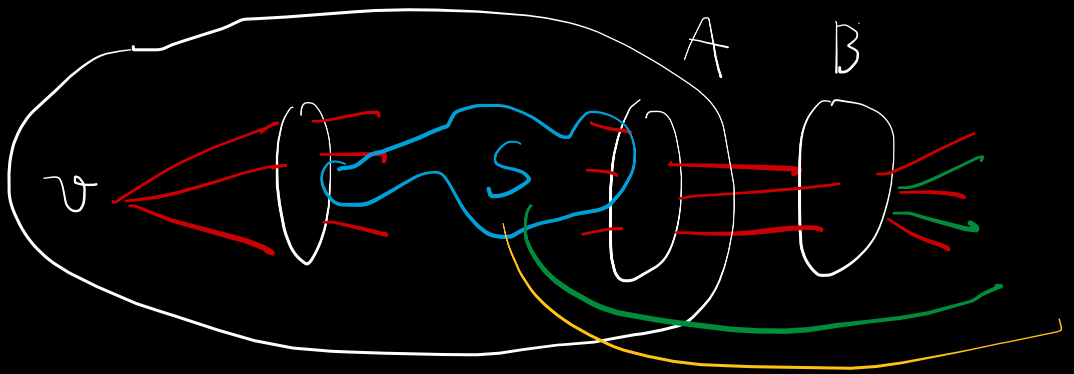
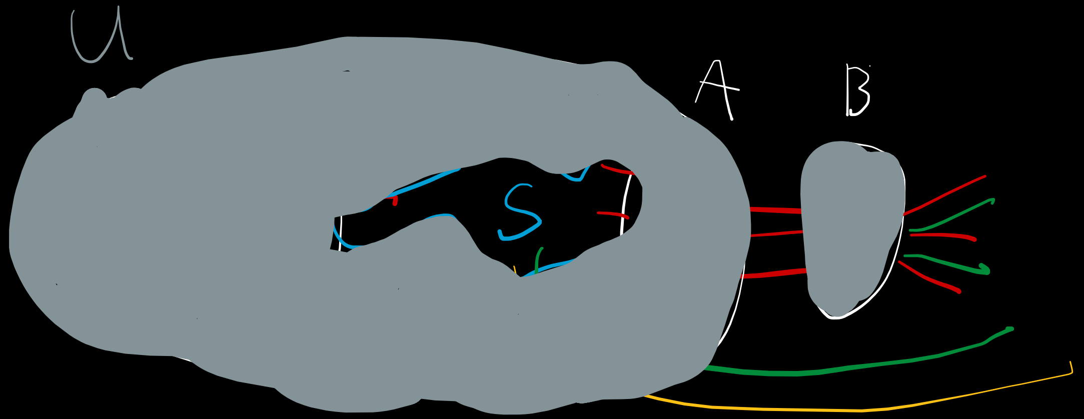




Every vertex of S has some red neighbour in U, but not in $V_n - U$.

Claim $\exists\ j \in \{1, 2, \dots, t\}$ s.t. $w(N^{j+1}(v)) \leq (k^{1/t} - 1) \cdot w(N^{\leq j}(v))$.

Let $A := N^{\leq j}(v)$ and $B := N^{j+1}(v)$. Then by claim, $\boxed{w(B) \leq (k^{1/t} - 1)\, w(A)}$.

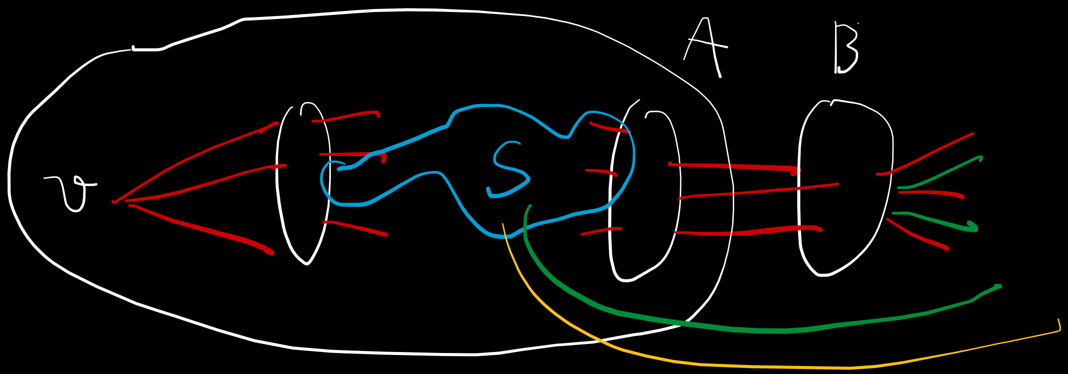
By lemma assumption, $G_{red}[A]$ is χ-colourable, so has independent set S with $\boxed{w(S) \geq \frac{w(A)}{\chi}}$.

Now choose $\boxed{U = B \cup A \setminus S}$.

Then $\boxed{w(U)} = w(B) + w(A) - w(S) \leq (k^{1/t} - 1)\, w(A) + w(A) - w(S)$

$$= k^{1/t}\, w(A) - w(S)$$

$$\leq \boxed{(\chi \cdot k^{1/t} - 1) \cdot w(S)}$$



$\square$

Summary

- growth rate of $R(C_3)$ wide open;
- we proved $R_k(C_{2t+1}) \lesssim (k!)^{1/t}$ and e.g.

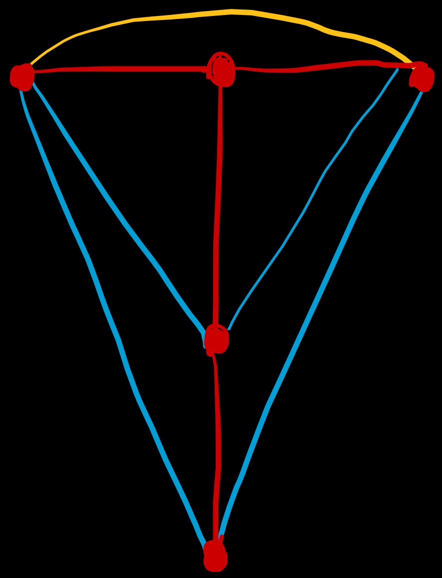
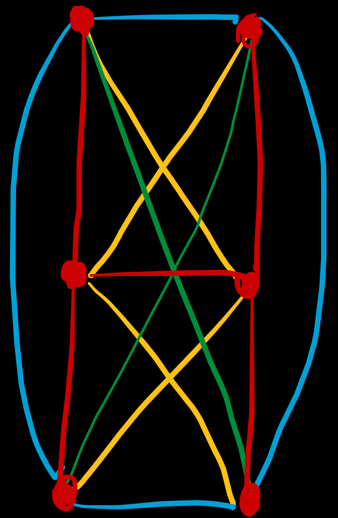
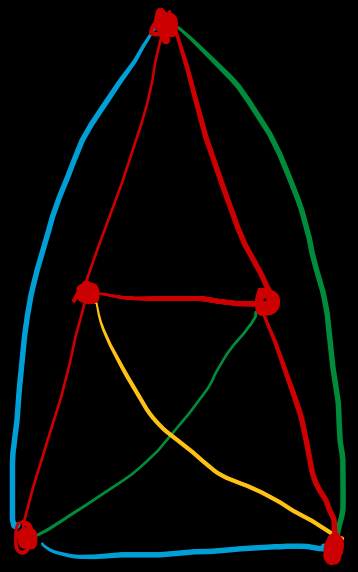
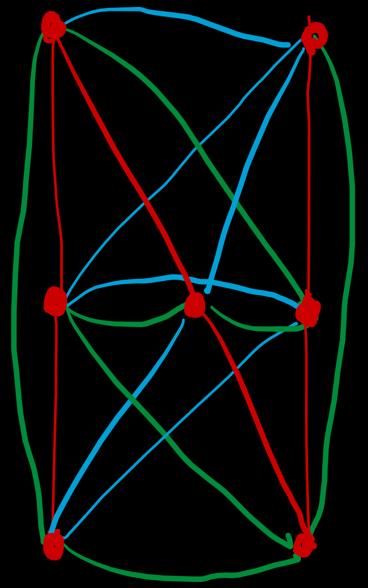
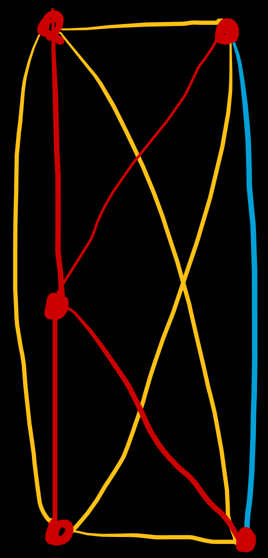
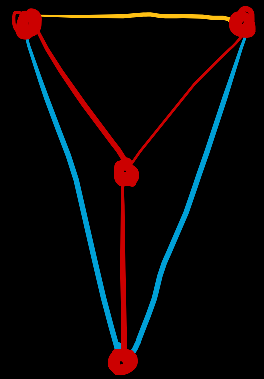
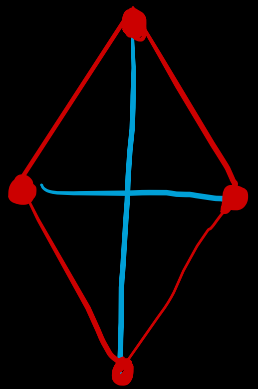
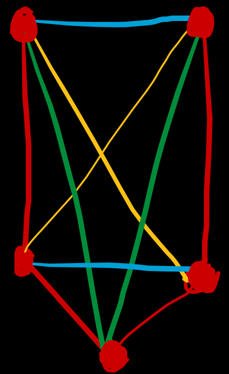
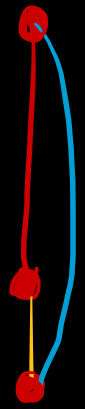
$$R_k(\text{odd cycles} \le \log_2 k) \le 4^k$$

- Proof via a Lemma that only needs input that small radius balls w/o C_{2t+1} have bounded χ.
- Proof of Lemma is via a weighted induction that favours vertices with few distinct adjacent colours.

Questions

* Does the new Lemma have further applications, for other multicolour Ramsey numbers?

* Can the locally weighted induction approach be adapted to 2-colour Ramsey numbers? e.g. ...
  Assign a low weight to a vertex v if $N^{\leq 2}(v)$ contains many unfavourable structures.

THANK
YOU!