Tree-CNN: A Hierarchical Deep Convolutional Neural Network for Incremental Learning

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Abstract

Over the past decade, Deep Convolutional Neural Networks (DCNNs) have shown remarkable performance in most computer vision tasks. These tasks traditionally use a fixed dataset, and the model, once trained, is deployed as is. Adding new information to such a model presents a challenge due to complex training issues, such as "catastrophic forgetting", and sensitivity to hyperparameter tuning. However, in this modern world, data is constantly evolving, and our deep learning models are required to adapt to these changes. In this paper, we propose an adaptive hierarchical network structure composed of DCNNs that can grow and learn as new data becomes available. The network grows in a tree-like fashion to accommodate new classes of data, while preserving the ability to distinguish the previously trained classes. The network organizes the incrementally available data into feature-driven superclasses and improves upon existing hierarchical CNN models by adding the capability of self-growth. The proposed hierarchical model, when compared against fine-tuning a deep network, achieves significant reduction of training effort, while maintaining competitive accuracy on CIFAR-10 and CIFAR-100.

Keywords: Convolutional Neural Networks, Deep Learning, Incremental Learning, Transfer Learning

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1. Introduction

In recent years Deep Convolutional Neural Networks (DCNNs) have emerged as the leading architecture for large scale image classification [1]. In 2012, AlexNet [2], an 8 layer Deep CNN, won the ImageNet Large Scale Visual Recognition Challenge (ISLVRC) and catapulted DCNNs into the spotlight. Since then, they have dominated ISLVRC and have performed extremely well on popular image datasets such as MNIST [3, 4], CIFAR-10/100 [5], and ImageNet [6].

Today, with increased access to large amounts of labeled data (eg. ImageNet [6] contains 1.2 million images with 1000 categories), supervised learning has become the leading paradigm in training DCNNs for image recognition. Traditionally, a DCNN is trained on a dataset containing a large number of labeled images. The network learns to extract relevant features and classify these images. This trained model is then used on real world unlabeled images to classify them. In such training, all the training data is presented to the network during the same training process. However, in real world, we hardly have all the information at once, and data is, instead, gathered incrementally over time. This creates the need for models that can learn new information as it becomes available. In this work, we try to address the challenge of learning on such incrementally available data in the domain of image recognition using deep networks.

A DCNN embeds feature extraction and classification in one coherent architecture within the same model. Modifying one part of the parameter space immediately affects the model globally [7]. Another problem of incrementally training a DCNN is the issue of "catastrophic forgetting" [8]. When a trained DCNN is retrained exclusively over new data, it results in the destruction of existing features learned from earlier data. This mandates using previous data when retraining on new data.

To avoid catastrophic forgetting, and to leverage the features learned in previous task, this work proposes a network made of CNNs that grows hierarchically as new classes are introduced. The network adds the new classes like new leaves to the hierarchical structure. The branching is based on the similarity of features between new and old classes. The initial nodes of the *Tree-CNN* assign the input into coarse super-classes, and as we approach the leaves of the network, finer classification is done. Such a model allows us to leverage the convolution layers learned previously to be used in the new bigger network.

The rest of the paper is organized as follows. The related work on incremental learning in deep neural networks is discussed in Section 2. In Section 3 we present our proposed network architecture and incremental learning method. In Section 4, the two experiments using CIFAR-10 and CIFAR-100 datasets are described. It is followed by a detailed analysis of the performance of the network and its comparison with transfer learning and fine tuning in Section 5. Finally, Section 6 discusses the merits and limitations of our network, our findings, and possible opportunities for future work.

2. Related Work

The modern world of digitized data produces new information every second [9], thus fueling the need for systems that can learn as new data arrives. Traditional deep neural networks are static in that respect, and several new approaches to incremental learning are currently being explored. "One-shot learning" [10] is a Bayesian transfer learning technique, that uses very few training samples to learn new classes. Fast R-CNN [11], a popular framework for object detection, also suffers from "catastrophic forgetting". One way to mitigate this issue is to use a frozen copy of the original network compute and balance the loss when new classes are introduced in the network [12]. "Learning without Forgetting" [13] is another method that uses only new task data to train the network while preserving the original capabilities. The original network is trained on an extensive dataset, such as ImageNet [6], and the new task data is a much smaller dataset. "Expert Gate" [14] adds networks (or experts) trained on new tasks sequentially to the system and uses a set of gating autoencoders to select the right network ("expert") for the given input. Progressive Neural Networks [15] learn to solve complex sequences of task by leveraging prior knowledge with lateral connections. Another recent work on incremental learning in neural networks is "iCaRL" [16], where they built an incremental classifier that can potentially learn incrementally over an indefinitely long time period.

It has been observed that initial layers of a CNN learn very generic features [17] that has been exploited for transfer learning [18], [19]. Common features, that are shared between images, have been used previously to build hierarchical classifiers. These features can be grouped semantically, such as in [20], or be feature-driven, such as "FALCON" [21]. Similar to the progression of complexity of convolutional layers in a DCNN, the upper nodes of a hierarchical CNN classify the images into coarse super-classes using

basic features, like grouping green-colored objects together, or humans faces together. Then deeper nodes perform finer discrimination, such as "boy" v/s "girl", "apples" v/s "oranges", etc. Such hierarchical CNN models have been shown to perform at par or even better than standard DCNNs [22]. "Discriminative Transfer Learning" [23] is one of the earliest works where classes are categorized hierarchically to improve network performance. Deep Neural Decision Forests [24] unified decision trees and deep CNN's to build a hierarchical classifier. "HD-CNN" [22], is a hierarchical CNN model that is built by exploiting the common feature sharing aspect of images. However, in these works, the dataset is fixed from the beginning, and prior knowledge of all the classes and their properties is used to build a hierarchical model.

In our work, *Tree-CNN* starts out as a single root node and generates new hierarchies to accommodate the new classes. Images belonging to the older dataset are required during retraining, but by localizing the change to a small section of the whole network, our method tries to reduce the training effort and complexity. In [7], a similar approach is applied, where the new classes are added to the old classes, and divided into two super-classes, by using an error-based model. The initial network is cloned to form two new networks which are fine tuned over the two new super-classes. While their motivation was a "divide-and-conquer" approach for large datasets, our work tries to incrementally grow with new data over multiple learning stages. In the next section, we lay out in detail our design principle, network topology and the algorithm used to grow the network.

3. Incremental Learning Model

3.1. Network Architecture

Inspired from hierarchical classifiers, our proposed model, *Tree-CNN* is composed of multiple nodes connected in a tree-like manner. Each node (except leaf nodes) has a DCNN which is trained to classify the input to the node into one of it's children. The root node is the highest node of the tree, where the first classification happens. The image is then passed on to its child node, as per the classification label. This node further classifies the image, until we reach a leaf node, the last step of classification. Branch nodes are intermediary nodes, each having a parent and two or more children. The leaf node is the last level of the tree. Each leaf node is uniquely associated to a class and no two leaf nodes have the same class. Fig. 1 shows the root node and branch nodes for a two-stage classification network. Each output

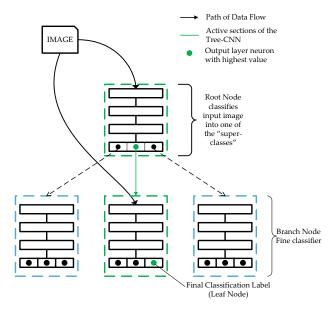


Figure 1: A generic model of 2-level Tree-CNN: The output of the root node is used to select the branch node at the next level.

of the second level branch node is a leaf node, which is the output node of the branch CNN. The inference methodology of such a network is given by Algorithm 1.

3.2. The Learning Algorithm

We start with the assumption that we have a model that is already trained to recognize a certain number of objects. The model could be hierarchical with multiple CNNs or could be just a single CNN acting as a root node with multiple leaf nodes. A new task is defined as learning to identify images belonging to M new classes. We start at the root node of our given model, and we provide a small sample of images ($\sim 10\%$) from the new training set as input to this node.

We obtain a 3 dimensional matrix from the output layer, $O^{K\times M\times I}$, where, K is the number of children of the root node, M is the number of new classes, and I is the number of sample images per class. O(k,m,i) denotes the output of the k^{th} neuron for the i^{th} image belonging to the m^{th} class where $k\in[1,K]$, $m\in[1,M]$, and $i\in[1,I]$. $O_{avg}^{K\times M}$ is the average of the outputs over I images. Softmax likelihood is computed over O_{avg} (eq. 1) to obtain the likelihood

Algorithm 1 Tree-CNN: At Inference

```
1: I = \text{Input Image}, node = \text{Root Node of the Tree}
2: procedure CLASSPREDICT(I, node)
       count = # of children of node
       if count = 0 then
4:
          label = class label of the node
5:
          return label
6:
 7:
       else
          nextNode = EvaluateNode(I, node)
8:
          returns the address of the child node of highest output neuron
9:
          return ClassPredict(I, nextNode)
10:
       end if
11:
12: end procedure
```

matrix $L^{K\times M}$ (eq. 2).

$$O_{avg}(k,m) = \sum_{i=1}^{I} \frac{O(k,m,i)}{I}$$
 (1)

$$L(k,m) = \frac{e^{O_{avg}(k,m)}}{\sum_{k=1}^{K} e^{O_{avg}(k,m)}}$$
(2)

(3)

We generate an ordered list S from $L^{K\times M}$, having the following properties

- ullet The list S has M objects. Each object corresponds uniquely to one of the new M classes.
- Each object S[i] has the following attributes:
 - -S[i].label = label of the new class
 - $S[i].value = [v_1, v_2, v_3]$, top 3 average softmax (o_{avg}) output values for that class in descending order, $v_1 \ge v_2 \ge v_3$
 - $S[i].nodes = [n_1, n_2, n_3]$, output nodes corresponding to the softmax outputs v_1, v_2, v_3
- S is ordered in the decreasing value of S[i].value[1]

The ordering is done to ensure that new classes with high likelihood values are added first to *Tree-CNN*. Softmax likelihood is used instead of number of images that get classified as each of the child nodes because it translates the

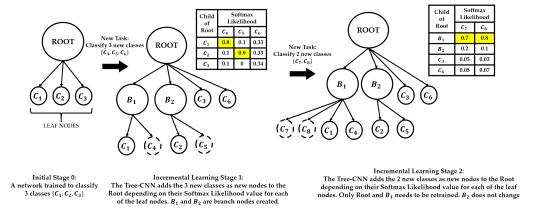


Figure 2: An example illustrating multiple incremental learning stages of the Tree-CNN. The network starts as a single root node, and expands as new classes are added.

output layer's response to the images into an exponential scale and helps us better identify how similar an image is to one of the already existing labels. After constructing S, we look at its first element, S[1], and take one of the 3 actions.

- i. Add the new class to an existing child node: If v_1 is greater than the next value (v_2) by a threshold, α (a design specification), that class indicates a strong resemblance/association with a particular child node. The new class is added to corresponding child node n_1 .
- ii. Merge two child nodes to form a new child node and add the new class to this node: If there are more than 1 child nodes that the new class has a strong likelihood for, we can combine them to form a new child node. It happens when $v_1 v_2 < \alpha$, and $v_2 v_3 > \beta$ (another threshold, defined by the user). For example, if the top 3 likelihood values were $v_1 = 0.48$, $v_2 = 0.45$, and $v_3 = 0.05$. Then, provided n_2 is a leaf node, we merge n_2 into n_1 , and add the new class to n_1 .
- iii. Add the new class as a new child node: If the new class doesn't have a likelihood value that is greater than other values by a good margin $(v_1 v_2 < \alpha, v_2 v_3 < \beta)$, or all child nodes are full, the network expands horizontally by adding the new class as a new child node. This node will be a leaf node.

As the root node keeps adding new branches and sub-branches, the branch nodes with more children tend to get heavier. Incoming new classes tend to have a higher softmax likelihood for branch nodes with greater number of children. To prevent the *Tree-CNN* from becoming lop-sided, one can set the maximum number of children a branch node can have.

When calculating L(k, m), we substitute $e^{O_{avg}(k,m)}$ with 0 for those k branches that are 'full', i.e have reached the limit for number of children per branch. We assign S[1].label a location in the Tree-CNN depending on its value. After that, we remove the column corresponding to that class from L(k, m), we check for "full" branch nodes, and modify L(k, m) for those output nodes. Finally we generate the ordered list S, and again apply our conditions on the new S[1] to determine where it is added to the root node. This is done iteratively till all new classes are assigned a location under the root node.

The pseudo-code is outlined in Algorithm 2. We also illustrate a toy example of incremental learning in Tree-CNN with Fig. 2. The network starts as a single CNN that can classify 3 classes, C_1 , C_2 , C_3 . We want to increase the network capability by adding 3 new classes. In the first incremental learning stage, the softmax likelihood table L is generated, as shown in the figure. C_4 and C_5 are added to the leaf nodes containing C_1 and C_2 respectively, converting them into branch nodes B_1 and B_2 , as per condition (i). For C_6 , the 3 likelihood values are $v_1 = 0.34$, $v_2 = 0.33$, $v_3 = 0.33$. It satisfies neither condition (i) nor condition (ii), thus it is added as a new node to the root, as per condition (iii). Again, as new information is available, we want the Tree-CNN to be able to recognize 2 new image classes, C_7 , and C_8 . Both the new classes satisfy $v_1 - v_2 > \alpha (= 0.1)$. Thus, both the classes are added to B_1 . While this example is for a two level Tree-CNN, the algorithm can potentially be extended to deeper Tree-CNN models.

To create deeper Tree-CNN models, once the "Grow-Tree" algorithm is completed for the M classes at the root node, one can move to the next level of the tree. The same process is applicable on the child nodes that now have new classes to be added to them. The decision on how to grow the tree is semi-supervised: the algorithm itself decides how to grow the tree, given the constraints by the user. We can limit parameters such as maximum children for a node, maximum depth for the tree, etc. as per our system requirements.

Once the new classes are allotted locations in the tree, supervised gradient descent based training is performed on the modified/new nodes. This saves us from modifying the whole network, and only affected portions of the network require retraining/fine-tuning. At every incremental learning stage, the root node is trained on all the available data as it needs to learn to classify all the objects into the new branches. During inference, a branch

Algorithm 2 Grow Tree-CNN

```
1: L = \text{Likelihood Matrix}
2: maxChildren = max. number of children per branch node
3: RootNode = Root Node of the Tree-CNN
4: procedure GrowTree(L, Node)
       S = GenenerateS(L, Node, maxChildren)
5:
       while S is not Empty do
6:
           ▶ Get attributes of the first object
 7:
           [label, value, node] = GetAttributes(S[1])
8:
           if value[1] - value[2] > \alpha then
9:
               ▶ The new class has a strong preference for n_1
10:

ightharpoonup Adds label to node[1]
11:
               RootNode = AddClasstoNode(RootNode, label, node[1])
12:
           else
13:
              if value[2] - value[3] > \beta then
14:
                  \blacktriangleright The new class has similar strong preference n_1 and n_2
15:
16:
                  Merge = Check for Merge(Node, node[1], node[2])
                  \blacktriangleright Merge is True only if node[2] is a leaf node, and,
17:
                  ▶ the # of children of node[1] less than maxChildren - 1
18:
                  if Merge then
19:
                      \blacktriangleright Merge node[2] into node[1]
20:
                      RootNode = MergeNode(RootNode, node[1], node[2])
21:
                      RootNode = AddClasstoNode(RootNode, label, node[1])
22:
23:
                  else
24:
                      ► Add new class to the smaller output node
                      sNode = Node with lesser children (node[1], node[2])
25:
                      RootNode = AddClasstoNode(RootNode, label, sNode)
26:
                  end if
27:
               else
28:
29:
                  ▶ Add new class as a new Leaf node to Root Node
                  RootNode = AddNewNode(RootNode, label)
30:
               end if
31:
           end if
32:
           \blacktriangleright Remove the columns of the added class from L
33:
           \blacktriangleright Remove the rows of "full" nodes from L
34:
35:
           \triangleright Regenerate S
36:
           S = GenenerateS(L, Node, maxChildren)
37:
       end while
38: end procedure
```

node is activated only when the root node classifies the input to that branch node. If an incorrect classification happens at Root Node, for example it classifies an image of a car into the "Animal Node" (CIFAR-10 example, Sec 4.1), irrespective of what the branch node classifies it as, it would still be an incorrect classification. Hence we only train the branch node with the classes it has been assigned to. If there is no change in the branch node's look up table at an incremental learning stage, it is left as is.

Handling input labels inside the Tree-CNN

The dataset available to the user will have unique labels assigned to each of it's object classes. However, the root and branch nodes of the Tree-CNN tend to group/merge/split these classes as required by the algorithm. To ensure label consistency, each node of the Tree-CNN maintains it's own "LabelsTransform" lookup table. For example, when a new class is added to one of the pre-existing output nodes of a root node, the lookup table is updated with new class being assigned to that output node. Similarly when a new class is added as a new node, the class label and the new output node is added as a new entry to the lookup table. Every class is finally associated with a unique leaf node, hence leaf nodes do not require a look up table. Whenever two nodes are merged, the node with lower average softmax value (say, node A) gets integrated with the node with the higher average softmax value (say, node B) for the new class in consideration. If the two softmax values are equal, it is chosen at random. At the root node level, the lookup table is modified as follows: The class labels that were assigned to node A, will now be assigned to node B. The look up table of merged node B will add these class labels from node A as new entries and assign them to new leaf nodes.

4. The Experimental Setup

4.1. Adding Multiple New Classes (CIFAR-10)

4.1.1. Dataset

CIFAR-10 dataset [5], having 10 mutually exclusive classes, was used for this experiment. The network is first trained on 6 classes, and then learns the remaining 4 classes in the next learning stage.

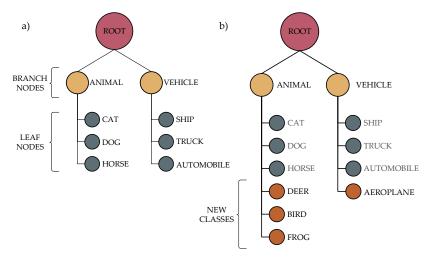


Figure 3: Graphical representation of Tree-CNN for CIFAR-10 a) before incremental learning, b) after incremental learning

4.1.2. The Network Initialization

The *Tree-CNN* for CIFAR-10 starts out as a two level network with a root node with two branch nodes as shown in Fig 3. The six initial classes of CIFAR-10 are grouped into "Vehicles" and "Animals" and the CNN (Table 1) at the root is trained to classify input images into these two categories. Each of the two branch nodes has a CNN (Table 2) that does finer classification into leaf nodes. Fig. 3a) represents the initial model of *Tree-CNN A*. This experiment illustrates, that given provided a 2-level *Tree-CNN*, how the learning model can add new classes. The root node achieves a testing accuracy of 98.73%, while the branch nodes, "Animals" and "Vehicles", achieve 86% and 94.43% testing accuracy respectively. Overall, the network achieves a testing accuracy of 89.10%.

4.1.3. Incremental Learning

The remaining four classes are now introduced as the new learning task. 50 images per class (10% of the training set) are selected at random, and shown to the root node. We obtain the L matrix, which is a 2×4 matrix with each element $l_{ij} \in (0, 1)$. The 1^{st} row of the matrix indicates the softmax likelihood of each of the 4 classes as being classified as "Vehicles", while the second row presents the same information for "Animals". In this experiment, α is set at 0 (Algorithm 2), and the network is bound to take only one action:

Table 1: Root Node Tree-CNN (CIFAR-10)

Input 32×32×3
CONV-1
64 5×5 ReLU
[2 2] Max Pooling
CONV-2
128 3×3 ReLU
Dropout 0.5
128 3×3 ReLU
[2 2] Max Pooling
FC
8192×512 ReLU
Dropout 0.5
512×128 ReLU
Dropout 0.5
128×2 ReLU
Softmax Layer

Table 2: Branch Node Tree-CNN (CIFAR-10)

Input 32×32×3
CONV-1
32.5×5 ReLU
[2 2] Max Pooling
Dropout 0.25
CONV-2
64 5×5 ReLU
[2 2] Max Pooling
Dropout 0.25
CONV-3
64 3×3 ReLU
[2 2] Avg Pooling
Dropout 0.25
FC
$1024\times128 \text{ ReLU}$
Dropout 0.5
128×N ReLU
(N = # of Classes)
Softmax Layer

Table 3: Network B

add the new class to one of the two child nodes. The branch node with higher likelihood value adds the new class to itself. The $\mathit{Tree-CNN}$ before and after addition of these 4 classes is shown in Fig. 3 .

Once the new classes have been assigned locations in the Tree-CNN, we begin the re-training of the network. The root node is re-trained using all 10 classes, divided into to subclasses. The branch node "animal" is retrained

using training data from 6 classes, 3 old and 3 new added to it. Similarly, branch node "vehicles" is retrained with training data from 4 classes, 3 old, 1 new.

4.2. Sequentially Adding Multiple Classes (CIFAR-100)

Table 4: Root Node Tree-CNN (CIFAR-100) Table 5: Branch Node Tree-CNN (CIFAR-100)

Input 32×32×3
CONV-1
64 5×5 ReLU
[2 2] Max Pooling
CONV-2
128 3×3 ReLU
Dropout 0.5
128 3×3 ReLU
[2 2] Max Pooling
CONV-3
256 3×3 ReLU
Dropout 0.5
256 3×3 ReLU
[2 2] Avg Pooling
FC
4096×1024 ReLU
Dropout 0.5
1024×1024 ReLU
Dropout 0.5
1024×N
(N = # of Children)
· · ·

Input $32 \times 32 \times 3$
CONV-1
$32 5 \times 5 \text{ ReLU}$
[2 2] Max Pooling
Dropout 0.25
CONV-2
$64.5 \times 5 \text{ ReLU}$
[2 2] Max Pooling
Dropout 0.25
CONV-3
64.3×3 ReLU
Dropout 0.5
$64 3 \times 3 \text{ ReLU}$
[2 2] Avg Pooling
FC
$1024 \times 512 \text{ ReLU}$
Dropout 0.5
512×128 ReLU
Dropout 0.5
$128\times N$
(N = # of Children)

4.2.1. Dataset

The dataset, CIFAR-100 [5], has 100 classes, 500 training and 100 testing images per class. The 100 classes are randomly divided into 10 groups of 10 classes each and organized in a fixed order (Appendix A). These groups of classes are introduced to the network incrementally.

4.2.2. The Network Initialization

We initialize the *Tree-CNN* as a root node with 10 leaf nodes. The root node, thus comprises of a CNN (Table 4), with 10 output nodes. Initially this CNN is trained to classify the 10 classes belonging to group 0 of the incremental CIFAR-100 dataset (Appendix A). In subsequent learning stages, as new classes get grouped together under same output nodes, the network adds branch nodes. The DCNN model used in these branch nodes is given in Table 5. The branch node has a higher chance of over-fitting than the root node as the dataset per node shrinks in size as we move deeper into the tree. Hence we introduce more dropout layers to the CNNs at these nodes to enhance regularization.

4.2.3. Incremental Learning

The remaining 9 groups, each containing 10 classes is incrementally introduced to the network in 9 learning stages. At each stage, 50 images belonging to each class are shown to the root node and a likelihood matrix L is generated. The columns of the matrix are used to form an ordered set S, as described in section 3.2 . For this experiment, we applied the following constraints to the Algorithm 2:

- Maximum depth of the tree is 2.
- We set $\alpha = 0.1$ and $\beta = 0.1$.
- Maximum number of child nodes for a branch node is set at 5, 10, 20 for the three test cases: *Tree-CNN-5*, *Tree-CNN-10*, and *Tree-CNN-20* respectively.

At every learning stage, once the new classes have been assigned the location in the Tree-CNN, we update the corresponding branch and root CNNs by retraining them on the combined dataset of old and new classes added to them. The branch nodes to which new children have not been added are left untouched.

4.3. Benchmarking

There is an absence of standardized benchmark protocol for incremental learning, which led us to use a benchmarking protocol similar to one used in iCaRL [16]. The classes of the dataset are grouped and arranged in a fixed random order. At each learning stage, a selected set of classes would be introduced to the network. Once training is completed for a particular learning stage, the network would be evaluated on all the classes it has learned so far and the accuracy is reported.

4.3.1. Baseline Network

To compare against the proposed Tree-CNN, we defined a baseline network (Network B) with a complexity level similar to two stage Tree-CNN. The network is has a VGG-net [25] like structure with 11 layers. It has 4 convolutional blocks, each block having 2 sets of 3×3 convolutional kernels (Table 3).

4.3.2. Fine-tuning the baseline network using old + new data

The baseline network is trained in incremental stages using fine-tuning. The new classes are added as new output nodes of the final layer and 5 different fine tuning strategies have been used. Each method retrains/fine-tunes certain layers of the network. While fine tuning, all of the available dataset is used, both old data and new data. It is assumed that the system has access to all the data that has been introduced so far. As listed below, we set 5 different depths of back-propagation when retraining with the incremental data and the old data.

- B:I [FC]
- B:II [FC + CONV-1]
- B:III [FC + CONV-1 + CONV-2]
- B:IV [FC + CONV-1 + CONV-2 + CONV-3]
- B:V [FC + CONV-1 + CONV-2 + CONV-3 + CONV-4] (equivalent to training a new network with all the classes)

4.3.3. Evaluation Metrics

We compare Tree-CNN against retraining Network B on two metrics: Testing Accuracy, and Training Effort , which is defined as Training Effort = \sum_{nets} (total number of weights \times total number of training samples)

Training Effort attempts to capture the number of weight updates that happen per training epoch. As batch size and number of training epochs is kept the same, the product of the number of weights and the number of training samples used can provide us with the measure of the computation cost of a learning stage. For Tree-CNN the training effort of each of the nodes (or nets) is summed together. Whereas, for network B, it is just one node/neural network, and for each of the cases (B:I-B:V), we simply sum the number of weights in the layers that are being retrained and multiply it with total number of training samples available at a learning stage to calculate the Training Effort.

4.4. The Training Framework

We used MatConvNet [26], an open-source Deep Learning toolbox for MATLAB [27], for training the networks. During training, data augmentation was done by flipping the training images horizontally at random with a probability of 0.5 [28]. All images were whitened and contrast normalized [28]. The activation used in all the networks is rectified linear activation ReLU, $\sigma(x) = max(x, 0)$. The networks are trained using mini-batch stochastic gradient descent with fixed momentum of 0.9. Dropout [29] is used between the final fully connected layers, and between pooling layers to regularize the network. We also employed batch-normalization [30] at the output of every convolutional layer. Additionally, a weight decay $\lambda = 0.001$ was set to regularize each model. The weight decay helps against overfitting of our model. The final layer performs softmax operation on the output of the nodes to generate class probabilities. All CNNs are trained for 300 epochs. The learning rate is kept at 0.1 for first 200 epochs, then reduced by a factor of 10 every 50 epochs.

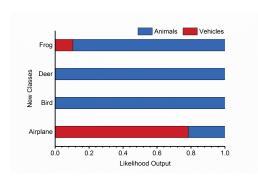
5. Results

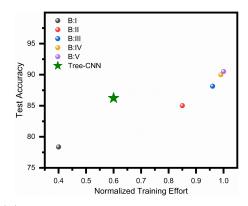
5.1. Adding multiple new classes (CIFAR-10)

Table 6: Training Effort and Test Accuracy comparison of Tree-CNN against Network B for CIFAR-10 $\,$

	B:I	B:II	B:III	B:IV	B:V	Tree-CNN
Testing Accuracy	78.37	85.02	88.15	90.00	90.51	86.24
Normalized Training Effort	0.40	0.85	0.96	0.99	1	0.60

We initialized a Tree-CNN that can classify six classes (Fig. 3a). It had a root node and two branch nodes. The sample images from the 4 new classes generated the softmax likelihood output at root node as shown in Fig. 4a. Accordingly, the new classes are added to the two nodes, and the new Tree-CNN is shown in Fig. 3b. In Table 6, we report the test accuracy and the training effort for the 5 cases of fine-tuning network B against our Tree-CNN for CIFAR-10. We observe that retraining only the FC layers of baseline network (B:I) requires the least training effort, however, it gives us the lowest accuracy of 78.37%. And as more classes are introduced, this method causes significant loss in accuracy, as shown with CIFAR-100 (Fig.





(a) Likelihood values at the root node for the new classes

(b) Testing Accuracy vs Normalized Training Effort (CIFAR-10)

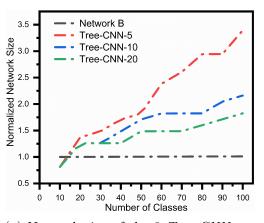
Figure 4: Incrementally learning CIFAR-10: 4 New classes are added to Network B and Tree-CNN. Networks B:I to B:V represent 5 increasing depths of retraining for Network B. (a) The softmax likelihood output at the root node for the two branches. (b) Testing Accuracy vs Normalized Training Effort for Tree-CNN and networks B:I to B:V

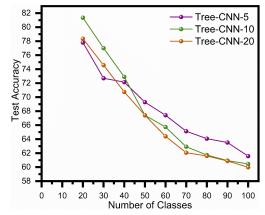
6b). The Tree-CNN has the second lowest normalized training effort, $\sim 40\%$ less than B:V, and $\sim 30\%$ less than B:II. At the same time, Tree-CNN had comparable accuracy to B:II and B:III, while just being less than the ideal case B:V by a margin of 3.76%. This accuracy vs training effort trade-off is presented in Fig. 4b, where it is clearly visible that Tree-CNN provided the most optimal solution for adding the 4 new classes.

5.2. Sequentially adding new classes (CIFAR-100)

We initialized a root node that can classify 10 classes, i.e. has 10 leaf nodes. Then, we incrementally grew the *Tree-CNN* for 3 different values of maximum children per branch node (*maxChildren*), namely 5, 10, and 20. We label these 3 models as *Tree-CNN-5*, *Tree-CNN-10* and *Tree-CNN-20* respectively. At the end of 9 incremental learning stages, the root node of *Tree-CNN-5* had 23 branch nodes and 3 leaf nodes. Whereas, the root node of *Tree-CNN-10* has 12 branch nodes and 5 leaf nodes. As expected, the root node of *Tree-CNN-20* had least number of child nodes, 9 branch nodes and 3 leaf nodes. The final hierarchical structure of the *Tree-CNNs* can be found in Appendix B, Fig. B.9-B.11.

We observe that as new classes are added, the *Tree-CNNs* grow in size by adding more branches (Fig. 5a). The size of Network B remains relatively unchanged, as only additional output nodes are added, which translates to a





(a) Network size of the 3 Tree-CNNs as new classes are added to the models, normalized with respect to Network B (CIFAR-100)

(b) Test Accuracy of the 3 *Tree-CNNs* as as new classes are added to the models (CIFAR-100)

Figure 5: Tree-CNN: Effect of varying the maximum number of children per branch node (maxChildren) as new classes are added to the models (CIFAR-100)

small fraction of new weights in the final layer. Tree-CNN-5 almost grows $3.4\times$ the size of Network B, while Tree-CNN-10 and Tree-CNN-20 reach $2.2\times$ and $1.8\times$ the baseline size, respectively. The training effort for the 3 Tree-CNNs was almost identical, within 1e-2 margin of each other (Fig. 6a), over the 9 incremental learning stages. As maxChildren is reduced, the test accuracy improves, as observed in Fig. 5b. If maxChildren is set to 1, we obtain a situation similar to test case B:V, where every new class is just a new output node.

We compare the training effort needed for the Tree-CNNs against the 5 different fine-tuning cases of Network B over the 9 incremental learning stages in Fig. 6a. We normalized the training effort by dividing all the values with the highest training effort. i.e. B:V. For all the models, the training effort required at a particular learning stage was greater than the effort required by the previous stage. This is because we had to show images belonging to old classes to avoid "catastrophic forgetting". The Tree-CNNs exhibit a lower training effort than 4 fine-tuning test cases, B:II - B:V. the test case B:II has a significantly lower training effort than all the other cases, as it only retrains the final fully connected layer. However, it suffers the worst accuracy degradation over the 9 learning stages (Fig. 6b). This shows that

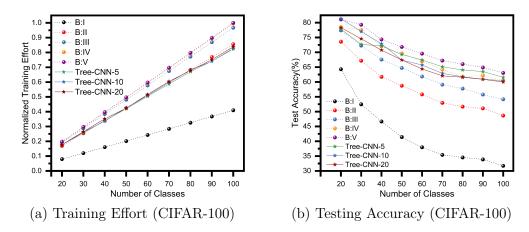


Figure 6: CIFAR-100: New classes are added to Network B and *Tree-CNNs* in batches of 10. Networks B:I to B:V represent 5 increasing depths of retraining for Network B (a) Training effort for every learning stage (Table C.8) (b) Testing Accuracy at the end of each learning stage (Table C.9)

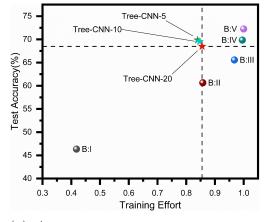
only retraining the final linear classifier, i.e. the fully connected layer, is not sufficient. We need to train the feature extractors, i.e. convolutional blocks, as well on the new data.

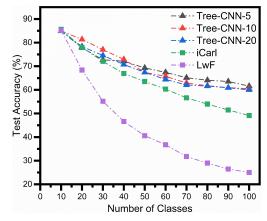
While B:I is the worst performer in terms of accuracy, Fig. 6b shows that all the networks suffer from some accuracy degradation with increasing number of classes. B:V provides the baseline accuracy at each stage, as it represents a network fully trained on all the available data at that stage. The three Tree-CNNs perform almost at par with B:IV, and outperform all other variants of network B. From Fig. 6, we can conclude that Tree-CNNs offer the most optimal trade-off between training effort and testing accuracy. This is further illustrated in Fig. 7a, where we plot the average test accuracy and average training effort over all the learning stages.

We compare our model against two works on incremental learning, 'iCaRL'[16] and 'Learning without Forgetting' [13] as shown in Fig. 7b. We use the accuracy reported in [16] for CIFAR-100, and compare it against our method. For 'LwF', a ResNet-32 [32] is retrained exclusively on new data at every stage. Hence it suffers the most accuracy degradation. In 'iCarl' [16], a ResNet-32 is retrained with new data and only 2000 samples of old data (called 'exemplars') at every stage. It is able to recover a good amount performance, compared to 'LwF' but still falls short of state-of-the-art by $\approx 18\%$. Tree-CNNs yield 10% higher accuracy than 'iCaRL' and over 50% higher accuracy than

Table 7: Test Accuracy over all 100 classes of CIFAR-100

Model	Final Test	Average Test
Model	Accuracy (%)	Accuracy(%)
B:V	63.05	72.23
Tree-CNN-5	61.57	69.85
Tree-CNN-10	60.46	69.53
Tree-CNN-20	59.99	68.49
iCarl (Rebuffi, et al. 2017) [16]	49.11	64.10
LwF (Li, et al. 2017) [16, 13]	25.00	44.49
HD-CNN (Yan, et al. 2015) [22]	67.38	N/A
Hertel, et al. 2015 [31]	67.68	N/A





- (a) Average test accuracy vs average training effort over all the learning stages of Tree-CNNs and Network B:I B:V (CIFAR-100)
- (b) Accuracy over incremental learning stages of *Tree-CNNs*, iCaRL[16] and Learning without Forgetting(LwF) [13] where new classes are added in batches of 10 (CIFAR-100)

Figure 7: The performance of Tree-CNN compared with a) Fine-tuning Network B b) Other incremental learning methods [16, 13]

'Learning without Forgetting' (LwF). This shows that our learning method using the hierarchical structure is more resistant to catastrophic forgetting as new classes are added. *Tree-CNNs* are able to achieve near state-of-the-art accuracy for CIFAR-100 as illustrated in Table 7. While the second column reports the final accuracy, the third column reports the average accuracy of

the incremental learning methods where new classes are added in batches of 10.

An interesting thing to note was similar looking classes, that were also semantically similar, were grouped under the same branches. At the end of the nine incremental learning stages, certain similar objects grouped together is shown in Fig. 8 for *Tree-CNN-10*. While there were some groups that had object sharing semantic similarity as well, there were odd groups as well, such as Node 13 as shown in Fig. 8. This opens up the possibility of using such a hierarchical structure for finding hidden similarity in the incoming data.

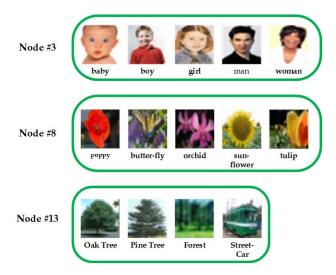


Figure 8: Examples of groups of classes formed when new classes were added incrementally to *Tree-CNN-10* in batches of 10 for CIFAR-100

6. Discussion

The motivation of this work stems from the idea that subsequent addition of new image classes to a network should be easier than retraining the whole network again with all the classes. We observed that each incremental learning stage required more effort than the previous, because images belonging to old classes needed to be shown to the CNNs. This is due to the inherent problem of "catastrophic forgetting" in deep neural networks. Our proposed method offers the best trade-off between accuracy and training effort when compared against fine-tuning layers of a deep network. It also achieves better accuracy, much closer to state-of-the-art on CIFAR-100, as compared to other works,

'iCarl' [16] and 'LwF' [13]. The hierarchical node-based learning model of the Tree-CNN attempts to confine the change in the model to a few nodes only. And, in this way, it limits the computation costs of retraining, while using all the previous data. Thus, it can learn with lower training effort than fine-tuning a deep network, while preserving much of the accuracy. However, the Tree-CNN continues to grow in size over time, and the implications of that on memory requirements needs to be investigated. During inference, a single node is evaluated at a time, thus the memory requirement per node inference is much lower than the size of the entire model. Tree-CNN grows in a manner such that images that share common features are grouped together. The correlation of the semantic similarity of the class labels and the featuresimilarity of the class images under a branch is another interesting area to explore. The Tree-CNN generates hierarchical grouping of initially unrelated classes, thereby generating a label relation graph out of these classes [33]. The final leaf nodes, and the distance between them can also be used as a measure of how similar any two images are. Such a method of training and classification can be used to hierarchically classify large datasets. Our proposed method, Tree-CNN, thus offers a better learning model that is based on hierarchical classifiers and transfer learning and can organically adapt to new information over time.

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Declaration of Interests

Declarations of interest: none

Appendix A. Incremental CIFAR-100 Dataset

The 100 classes of CIFAR-100 were randomly arranged and divided in 10 batches, each containing 10 classes. We randomly shuffled numbers 1 to 100 in 10 groups and then used that to group classes. We list the batches in the order they were added to the Tree-CNN for the incremental learning task below.

- 0 chair, bridge, girl, kangaroo, lawn mower, possum, otter, poppy, sweet pepper, bicycle
- 1 lion, man, palm tree, tank, willow tree, bowl, mountain, hamster, chimpanzee, cloud
- 2 plain, leopard, castle, bee, raccoon, bus, rabbit, train, worm, ray
- 3 table, aquarium fish, couch, caterpillar, whale, sunflower, trout, butterfly, shrew, house
- 4 bottle, orange, dinosaur, beaver, bed, snail, flatfish, shark, tractor, apple
- 5 woman, fox, lobster, skunk, can, turtle, cockroach, dolphin, bear, pickup truck
- 6 lizard, road, porcupine, mouse, seal, sea, tiger, telephone, rocket, tulip
- 7 baby, motorcycle, elephant, clock, maple tree, mushroom, pear, orchid, spider, oak tree
- 8 wardrobe, squirrel, crocodile, wolf, plate, skyscraper, keyboard, beetle, streetcar, crab
- 9 snake, lamp, camel, pine tree, cattle, boy, rose, forest, television, cup

Appendix B. Final Tree-CNN for max children 5, 10, 20 (CIFAR-100)

We trained the *Tree-CNN* with the incremental CIFAR-100 dataset, and we set the maximum number of children a branch node can have as 5, 10, and 20. The corresponding 3 *Tree-CNN*s were labeled - *Tree-CNN-5*, *Tree-CNN-10*, *Tree-CNN-20*. The 2-level hierarchical structure of these *Tree-CNNs* after 9 incremental learning stages is shown in Fig. B.9 - B.11. The nodes marked 'yellow' indicate completely filled branch nodes (B), the ones marked 'blue' indicate branch nodes (B) that are partially filled, while those marked 'green' refer to leaf nodes (L).

	В	APPLE	BOWL	SWEET PEPPER	TABLE	WORM
	В	AQ. FISH	RAY	SHREW	TROUT	WHALE
	В	BABY	LIZARD	LIZARD MUSHROOM		WOLF
	В	BEAR	BEAVER	MOUSE	PORCUPINE	SKUNK
	В	BED	BUS	COUCH	HOUSE	TRACTOR
	В	BEE	BUTTERFLY	CATERPILLAR	POPPY	SUNFLOWER
	В	BOTTLE	CAN	CLOCK	TELEPHONE	WARDROBE
	В	BRIDGE	CASTLE	PALM TREE	TANK	TRAIN
	В	CHIMPANZEE	CLOUD	MOUNTAIN	OTTER	WILLOW TREE
	В	CUP	KEYBOARD	LAMP	PLATE	TELEVISION
	В	DINOSAUR	ELEPHANT	FOX	KANGAROO	TIGER
	В	DOLPHIN	ROCKET	SEAL	SHARK	TURTLE
P -	ЬВ	FLATFISH	GIRL	HAMSTER	MAN	WOMAN
ROOT	В	LEOPARD	LION	POSSUM	RABBIT	RACCOON
	В	MAPLE TREE	OAK TREE	PLAIN	ROAD	SEA
	В	BOY	CAMEL	CATTLE	SQUIRREL	
	В	LOBSTER	ORCHID	ROSE	TULIP	
	В	BEETLE	COCKROACH	SNAIL		_
	В	LAWN MOWER	PICKUP TRUCK	STREETCAR		
	В	BICYCLE	MOTORCYCLE			
	В	CRAB	SNAKE			
	В	CROCODILE	FOREST			
	В	ORANGE	PEAR			
	L	CHAIR		•		
	L	PINE TREE				
	L	SKYSCRAPER				
			•			

Figure B.9: Tree-CNN-5: After 9 incremental learning stages

B AQ. FISH ORCHID POPPY SPIDER SUNFLOWER TULIP B BEAR BOTTLE CAN CATERPILLAR FOX TIGER PORCUPINE ROCKET SKUNK TELEPHONE TIGER B BEAVER DINOSAUR FLATFISH HAMSTER KANGAROO POSSUM SHARK SHREW SNAIL WHALE B BED BICYCLE CHAIR COUCH HOUSE MOTORCYCLE PICKUP TRUCK ROAD TABLE TRACTOR B BEE BOWL CHIMPANZEE LION OTTER RABBIT RACCOON RAY VLOUD WORM B BRIDGE BUS CASTLE LEOPARD MOUNTAIN WILLOW TREE PLAIN TANK TRAIN WILLOW TREE MUSHROOM SEA SEAL TROUT TURTLE B CLOCK CUP LAMP PLATE SNAKE B BABY BOY GIRL MAN WOMAN B APPLE ORANGE PEAR ROSE SWEET PEPPER B CAMEL CATTLE SQUIRREL L CROCODILE L LAWN MOWER L LOBSTER L SKYSCRAPER L WOLF							
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B PORCUPINE ROCKET SKUNK TELEPHONE TIGER B BEAVER DINOSAUR FLATFISH HAMSTER KANGAROO POSSUM SHARK SHREW SNAIL WHALE B BED BICYCLE CHAIR COUCH HOUSE MOTORCYCLE PICKUP TRUCK ROAD TABLE TRACTOR B BEE BOWL CHIMPANZEE LION OTTER RABBIT RACCOON RAY VLOUD WORM B BRIDGE BUS CASTLE LEOPARD MOUNTAIN PALM TREE PLAIN TANK TRAIN WILLOW TREE DOLPHIN ELEPHANT LIZARD MAPLE TREE MOUSE TELEVISION WARDROBE B BABY BOY GIRL MAN WOMAN B APPLE ORANGE PEAR ROSE B CAMEL CATTLE SQUIRREL L CROCODILE L LAWN MOWER L LOBSTER L SKYSCRAPER		D	ORCHID	POPPY	SPIDER	SUNFLOWER	TULIP
PORCUPINE ROCKET SKUNK TELEPHONE TIGER BEAVER DINOSAUR FLATFISH HAMSTER KANGAROO POSSUM SHARK SHREW SNAIL WHALE BED BICYCLE CHAIR COUCH HOUSE TRACTOR BEE BOWL CHIMPANZEE LION OTTER RABBIT RACCOON RAY VLOUD WORM BRIDGE BUS CASTLE LEOPARD MOUNTAIN PALM TREE PLAIN TANK TRAIN WILLOW TREE MUSHROOM SEA SEAL TROUT TURTLE BOLPHIN ELEPHANT LIZARD MAPLE TREE MOUSE TELEVISION WARDROBE B BABY BOY GIRL MAN WOMAN B APPLE ORANGE PEAR ROSE SWEET PEPPER B FOREST OAK TREE PINE TREE STREET CAR L CROCODILE L LAWN MOWER L LOBSTER L SKYSCRAPER		D	BEAR	BOTTLE	CAN	CATERPILLAR	FOX
B POSSUM SHARK SHREW SNAIL WHALE B BED BICYCLE CHAIR COUCH HOUSE MOTORCYCLE PICKUP TRUCK ROAD TABLE TRACTOR B BEE BOWL CHIMPANZEE LION OTTER RABBIT RACCOON RAY VLOUD WORM B BRIDGE BUS CASTLE LEOPARD MOUNTAIN PALM TREE PLAIN TANK TRAIN WILLOW TREE DOLPHIN ELEPHANT LIZARD MAPLE TREE MOUSE MUSHROOM SEA SEAL TROUT TURTLE CLOCK CUP LAMP PLATE SNAKE TELEVISION WARDROBE B BABY BOY GIRL MAN WOMAN B APPLE ORANGE PEAR ROSE SWEET PEPPER B FOREST OAK TREE PINE TREE STREET CAR CATTLE SQUIRREL L CROCODILE L LAWN MOWER L LOBSTER L SKYSCRAPER		В	PORCUPINE	ROCKET	SKUNK	TELEPHONE	TIGER
BED BICYCLE CHAIR COUCH HOUSE MOTORCYCLE PICKUP TRUCK ROAD TABLE TRACTOR BEE BOWL CHIMPANZEE LION OTTER RABBIT RACCOON RAY VLOUD WORM BRIDGE BUS CASTLE LEOPARD MOUNTAIN PALM TREE PLAIN TANK TRAIN WILLOW TREE DOLPHIN ELEPHANT LIZARD MAPLE TREE MOUSE MUSHROOM SEA SEAL TROUT TURTLE CLOCK CUP LAMP PLATE SNAKE TELEVISION WARDROBE B BABY BOY GIRL MAN WOMAN B APPLE ORANGE PEAR ROSE B CAMEL CATTLE SQUIRREL L CROCODILE L LAWN MOWER L LOBSTER L SKYSCRAPER		D	BEAVER	DINOSAUR	FLATFISH	HAMSTER	KANGAROO
B MOTORCYCLE PICKUP TRUCK ROAD TABLE TRACTOR B BEE BOWL CHIMPANZEE LION OTTER RABBIT RACCOON RAY VLOUD WORM B BRIDGE BUS CASTLE LEOPARD MOUNTAIN PALM TREE PLAIN TANK TRAIN WILLOW TREE DOLPHIN ELEPHANT LIZARD MAPLE TREE MOUSE MUSHROOM SEA SEAL TROUT TURTLE CLOCK CUP LAMP PLATE SNAKE TELEVISION WARDROBE B BABY BOY GIRL MAN WOMAN B APPLE ORANGE PEAR ROSE SWEET PEPPER B FOREST OAK TREE PINE TREE STREET CAR B CAMEL CATTLE SQUIRREL L CROCODILE L LAWN MOWER L LOBSTER L SKYSCRAPER		Ь	POSSUM	SHARK	SHREW	SNAIL	WHALE
BEE BOWL CHIMPANZEE LION OTTER RABBIT RACCOON RAY VLOUD WORM BRIDGE BUS CASTLE LEOPARD MOUNTAIN PALM TREE PLAIN TANK TRAIN WILLOW TREE BOLPHIN ELEPHANT LIZARD MAPLE TREE MOUSE MUSHROOM SEA SEAL TROUT TURTLE CLOCK CUP LAMP PLATE SNAKE TELEVISION WARDROBE B BABY BOY GIRL MAN WOMAN B APPLE ORANGE PEAR ROSE SWEET PEPPER B FOREST OAK TREE PINE TREE STREET CAR CROCODILE L LAWN MOWER L LOBSTER L SKYSCRAPER		R	BED	BICYCLE	CHAIR	COUCH	HOUSE
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RABBIT RACCOON RAY VLOUD WORM BRIDGE BUS CASTLE LEOPARD MOUNTAIN PALM TREE PLAIN TANK TRAIN WILLOW TREE B DOLPHIN ELEPHANT LIZARD MAPLE TREE MOUSE MUSHROOM SEA SEAL TROUT TURTLE CLOCK CUP LAMP PLATE SNAKE TELEVISION WARDROBE B BABY BOY GIRL MAN WOMAN B APPLE ORANGE PEAR ROSE SWEET PEPPER B FOREST OAK TREE PINE TREE STREET CAR B CAMEL CATTLE SQUIRREL L CROCODILE L LAWN MOWER L LOBSTER L SKYSCRAPER		R	BEE	BOWL	CHIMPANZEE	LION	OTTER
B PALM TREE PLAIN TANK TRAIN WILLOW TREE B DOLPHIN ELEPHANT LIZARD MAPLE TREE MOUSE MUSHROOM SEA SEAL TROUT TURTLE CLOCK CUP LAMP PLATE SNAKE B BABY BOY GIRL MAN WOMAN B APPLE ORANGE PEAR ROSE SWEET PEPPER B FOREST OAK TREE PINE TREE STREET CAR B CAMEL CATTLE SQUIRREL L CROCODILE L LAWN MOWER L LOBSTER L SKYSCRAPER			RABBIT	RACCOON	RAY	VLOUD	WORM
B DOLPHIN ELEPHANT LIZARD MAPLE TREE MOUSE MUSHROOM SEA SEAL TROUT TURTLE CLOCK CUP LAMP PLATE SNAKE TELEVISION WARDROBE B BABY BOY GIRL MAN WOMAN B APPLE ORANGE PEAR ROSE SWEET PEPPER B FOREST OAK TREE PINE TREE STREET CAR B CAMEL CATTLE SQUIRREL L CROCODILE L LAWN MOWER L LOBSTER L SKYSCRAPER		В					
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MUSHROOM SEA SEAL TROUT TURTLE CLOCK CUP LAMP PLATE SNAKE B CLOCK CUP LAMP PLATE SNAKE B BABY BOY GIRL MAN WOMAN B APPLE ORANGE PEAR ROSE SWEET PEPPER B FOREST OAK TREE PINE TREE STREET CAR B CAMEL CATTLE SQUIRREL L CROCODILE L LAWN MOWER L LOBSTER L SKYSCRAPER		В		ELEPHANT	LIZARD	MAPLE TREE	MOUSE
B BABY BOY GIRL MAN WOMAN B APPLE ORANGE PEAR ROSE SWEET PEPPER B FOREST OAK TREE PINE TREE STREET CAR B CAMEL CATTLE SQUIRREL L CROCODILE L LAWN MOWER L LOBSTER L SKYSCRAPER	9	ь					
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B APPLE ORANGE PEAR ROSE SWEET PEPPER B FOREST OAK TREE PINE TREE STREET CAR B CAMEL CATTLE SQUIRREL L CROCODILE L LAWN MOWER L LOBSTER L SKYSCRAPER			TELEVISION	WARDROBE			
B APPLE ORANGE PEAR ROSE PEPPER B FOREST OAK TREE PINE TREE STREET CAR B CAMEL CATTLE SQUIRREL L CROCODILE L LAWN MOWER L LOBSTER L SKYSCRAPER		В	BABY	BOY	GIRL	MAN	WOMAN
B FOREST OAK TREE PINE TREE STREET CAR B CAMEL CATTLE SQUIRREL L CROCODILE L LAWN MOWER L LOBSTER L SKYSCRAPER		В	APPLE	ORANGE	PEAR	ROSE	
B CAMEL CATTLE SQUIRREL L CROCODILE L LAWN MOWER L LOBSTER L SKYSCRAPER							PEPPER
L CROCODILE L LAWN MOWER L LOBSTER L SKYSCRAPER		В	FOREST	OAK TREE	PINE TREE	STREET CAR	
L LAWN MOWER L LOBSTER L SKYSCRAPER		В	CAMEL	CATTLE	SQUIRREL		
L LOBSTER L SKYSCRAPER		L	CROCODILE				
L SKYSCRAPER		L	LAWN MOWER				
		L	LOBSTER				
L WOLF		L	SKYSCRAPER				
		L	WOLF				

Figure B.10: Tree-CNN-10: After 9 incremental learning stages

		APPLE	BED	BOTTLE	BRIDGE	BUS
	В	CAN	CASTLE	COUCH	DOLPHIN	HOUSE
	Ь	LOBSTER	PICKUP TRUCK	ROAD	ROCKET	SHARK
		TABLE	TANK	TELEPHONE	TRACTOR	TRAIN
		AQ. FISH	BEE	BOWL	BUTTERFLY	CATERPILLAR
	В	CHIMPANZEE	CLOUD	LEOPARD	MOUNTAIN	OTTER
	В	PALM TREE	PLAIN	RABBIT	RAY	SUNFLOWER
		SWEET PEPPER	TROUT	WHALE	WILLOW TREE	WORM
		BEAR	BEAVER	CLOCK	COCKROACH	DINOSAUR
	В	ELEPHANT	FLATFISH	FOX	LIZARD	MOUSE
	В	MUSHROOM	OAK TREE	PORCUPINE	SEAL	SHREW
Ι.		SKUNK	SNAIL	SPIDER	TIGER	TURTLE
00T	В	BABY	BOY	CHAIR	CUP	GIRL
&		HAMSTER	MAN	TELEVISION	WARDROBE	WOMAN
	В	CAMEL	CATTLE	CRAB	CROCODILE	LION
		POSSUM	RACCOON	SNAKE	SQUIRREL	WOLF
	В	ORANGE	ORCHID	PEAR	POPPY	TULIP
	ь	KEYBOARD	LAMP	PLATE	ROSE	
	В	BICYCLE	MOTORCYCLE	STREETCAR		
	В	FOREST	MAPLE TREE	PINE TREE		
	В	SEA	SKYSCRAPER		-	
	L	BEETLE				
	L	KANGAROO				
	L	LAWNMOWER				

Figure B.11: Tree-CNN-20: After 9 incremental learning stages

Appendix C. Full Simulation Results

Table C.8: Normalized Training Effort as classes are added incrementally in batches of 10 (CIFAR-100)

Number of Classes	B:I	B:II	B:III	B:IV	B:V	Tree- CNN-5	Tree- CNN-10	Tree- CNN-20
20	0.08	0.17	0.19	0.20	0.20	0.18	0.18	0.18
30	0.12	0.25	0.29	0.30	0.30	0.26	0.26	0.27
40	0.16	0.34	0.38	0.39	0.40	0.34	0.34	0.35
50	0.20	0.42	0.48	0.49	0.50	0.42	0.43	0.42
60	0.24	0.51	0.58	0.59	0.60	0.50	0.51	0.52
70	0.28	0.60	0.67	0.69	0.70	0.59	0.60	0.60
80	0.33	0.68	0.77	0.79	0.80	0.67	0.68	0.68
90	0.37	0.77	0.87	0.89	0.90	0.74	0.74	0.75
100	0.41	0.86	0.97	1.00	1.00	0.82	0.83	0.84

Table C.9: Test Accuracy as classes are added incrementally in batches of 10 (CIFAR-100)

Number of	D.I	B:II	B:III	B:IV	B:V	Tree-	Tree-	Tree-
Classes	B:I	D:11	D:111	D:1V	D: V	CNN-5	CNN-10	CNN-20
20	64.30	73.60	77.40	78.50	81.10	77.80	81.35	78.35
30	52.47	67.17	72.27	77.57	79.30	72.70	77.00	74.57
40	46.68	61.72	67.55	72.08	74.35	72.15	72.90	70.75
50	41.42	58.74	64.74	69.62	71.82	69.28	67.40	67.42
60	37.98	55.80	61.85	66.95	69.57	67.42	65.73	64.40
70	35.43	52.96	59.10	64.23	67.21	65.13	62.91	62.07
80	34.55	51.68	57.77	61.81	66.03	64.07	61.73	61.60
90	33.88	51.09	55.77	62.21	64.90	63.52	60.93	60.88
100	31.73	48.68	54.16	60.48	63.05	61.57	60.46	59.99

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