This Code is revised by supplement on an annual basis. The official Code includes this document plus the accumulated revisions

9th PRINTING, SEPTEMBER 1977

Editorial Corrections made as of August 1975

ANSI
A89.1-1972

ACI Standard

Building Code Requirements for Reinforced Concrete (ACI 318-71)*

Reported by ACI Committee 318

EDWARD COHEN
Chairman

GEORGE F. LEYH
Secretary

W. C. E. BECKER
W. BURR BENNETT, JR.
DELMAR L. BLOEM†
FRANK B. BROWN
T. Z. CHASTAIN
WILLIAM D. CROMARTIE
OWEN L. DELEVANTE
JAMES N. DE SERIO
FRANK G. ERSKINE
NOEL J. EVERARD
PHIL M. FERGUSON
ASHBY T. GIBBONS, JR.
WILLIAM A. HEITMANN
EIVIND HOGNESTAD
EUGENE P. HOLLAND
FRITZ KRAMRISCH
T. Y. LIN
MICHAEL A. LOMBARD
ROBERT F. MAST
WILLIAM V. MERKEL
ROBERT B. B. MOORMAN
KEITH O. O'DONNELL
DOUGLAS E. PARSONS
EDWARD O. PFRANG
W. G. PLEWES
RAYMOND C. REESE
THEODORE O. REYHNER
PAUL F. RICE
FRANCISCO ROBLES
PAUL ROGERS
JOHN A. SBAROUNIS
MORRIS SCHUPACK
CHESTER P. SIESS
I. J. SPEYER
JOHN P. THOMPSON
M. P. VAN BUREN
A. CARL WEBER
GEORGE WINTER
ALFRED ZWEIG

This Code covers the proper design and construction of buildings of reinforced concrete. It is written in such form that it may be incorporated verbatim or adopted by reference in a general building code, and earlier editions have been widely used in this manner.

Among the subjects covered are: permits and drawings; inspection; specifications; materials; concrete quality; mixing and placing; formwork, embedded pipes, and construction joints; reinforcement details; analysis and design; strength and serviceability; flexural and axial loads; shear and torsion; development of reinforcement; slab systems; walls; footings; precast concrete; prestressed concrete; shells and folded plate members; strength evaluation of existing structures; and special provisions for seismic design.

The quality and testing of materials used in the construction are covered by reference to the appropriate ASTM standard specifications. Welding of reinforcement is covered by reference to the appropriate AWS standard.

Keywords: admixtures; aggregates; anchorage (structural); beam-column frame; beams (supports); building codes; cements; cold weather construction; columns (supports); combined stress; composite construction (concrete to concrete); composite construction (concrete and steel); compressive strength; concrete construction; concretes; concrete slabs; construction joints; continuity (structural); cover; curing; deep beams; deflections; drawings; earthquake resistant structures; embedded service ducts; flexural strength; floors; folded plates; footings; formwork (construction); frames; hot weather construction; inspection; joists; lightweight concretes; loads (forces); load tests (structural); materials; mixing; mix proportioning; modulus of elasticity; moments; pipe columns; pipes (tubes); placing; precast concrete; prestressed concrete; prestressing steels; quality control; reinforced concrete; reinforcing steels; roofs; serviceability; shear strength; shear walls; shells (structural forms); spans; specifications; splicing; strength; strength analysis; structural analysis; structural design; T-beams; torsion; walls; water; welded wire fabric.

*Adopted as a standard of the American Concrete Institute at its 1970 Fall Convention, St. Louis, Mo., Nov. 5, 1970, as amended; ratified by letter ballot Feb. 9, 1971. ACI 318-71 supersedes ACI 318-63.

†Deceased

First printing, Nov. 1971.

CONTENTS

PART 5—STRUCTURAL SYSTEMS OR ELEMENTS

PART 6—SPECIAL CONSIDERATIONS

APPENDICES

PART 1—GENERAL

CHAPTER 1—GENERAL REQUIREMENTS

1.1—Scope

1.1.1—This Code provides minimum requirements for the design and construction of reinforced concrete structural elements of any structure erected under the requirements of the general building code of which this Code forms a part.

1.1.2—This Code supplements the general building code and shall govern in all matters pertaining to design and construction wherever it is in conflict with the requirements in the general building code.

1.1.3—For special structures, such as arches, tanks, reservoirs, grain elevators, blast-resistant structures, and chimneys, the provisions of this Code shall govern where applicable.

1.2—Permits and drawings

1.2.1—Copies of structural drawings, typical details and specifications for all reinforced concrete construction shall bear the seal of a licensed engineer or architect and shall be filed with the building department as a permanent record before a permit to construct such work will be issued. These drawings, details, and specifications shall show: the size and position of all structural elements and reinforcing steel; provision for dimensional changes resulting from creep, shrinkage, and temperature; the specified strength of the concrete at stated ages or stages of construction; the specified strength or grade of reinforcing steel; the magnitude and location of prestressing forces; and the live load and other loads used in the design.

1.2.2—Calculations pertinent to the design shall be filed with the drawings when required by the Building Official. When automatic data processing is used, design assumptions and identified input and output data may be submitted in lieu of calculations. Calculations may be supplemented by model analysis.

1.2.3—Building Official means the officer or other designated authority charged with the administration and enforcement of this Code, or his duly authorized representative.

1.3—Inspection

1.3.1—Concrete construction shall be inspected throughout the various work stages by a competent engineer or architect, or by a competent representative responsible to him. The inspector shall require compliance with design drawings and specifications and keep a record which shall cover: quality and proportions of concrete materials; mixing, placing, and curing of concrete; placing of reinforcement; tensioning of prestressed reinforcement; form placement and removal; reshoring; sequence of erection and connection of precast members; any significant construction loadings on completed floors, members or walls; and general progress of the work.

1.3.2—When the ambient temperature falls below 40 F or rises above 95 F, a complete record of concrete temperatures and of the protection given to the concrete during placement and curing shall be kept.

1.3.3—The records of inspection required in Sections 1.3.1 and 1.3.2 shall be kept available to the Building Official during the progress of the work and for 2 years thereafter and shall be preserved by the inspecting engineer or architect for that purpose.

1.4—Approval of special systems of design or construction

The sponsors of any system of design or construction within the scope of this Code, the adequacy of which has been shown by successful use or by analysis or test, but which does not conform to or is not covered by this Code, shall have the right to present the data on which their design is based to a board of examiners appointed by the Building Official. This board shall be composed of competent engineers and shall have the authority to investigate the data so submitted, to require tests, and to formulate rules governing the design and construction of such systems to meet the intent of this Code. These rules when approved by the Building Official and promulgated shall be of the same force and effect as the provisions of this Code.

CHAPTER 2—DEFINITIONS

2.1—General

The following terms are defined for general use in this Code. Specialized definitions appear in individual chapters.

Admixture—A material other than portland cement, aggregate, or water added to concrete to modify its properties.

Aggregate—Inert material which is mixed with portland cement and water to produce concrete.

Aggregate, lightweight—Aggregate having a dry, loose weight of 70 lb per cu ft or less.

Anchorage—See Chapter 12. Also, the means by which the prestress force is permanently transferred to the concrete.

Bonded tendons—Tendons which are bonded to the concrete either directly or through grouting.

Building Official—See Section 1.2.3.

Column—An element used primarily to support axial compressive loads and with a height at least three times its least lateral dimension.

Composite concrete flexural member—See Chapter 17.

Compressive strength of concrete (f_c')—Specified compressive strength of concrete in pounds per square inch (psi) (see Section 4.3). Wherever this quantity is under a radical sign, the square root of the numerical value only is intended, and the resultant is in pounds per square inch (psi).

Concrete—A mixture of portland cement, fine aggregate, coarse aggregate and water.

Concrete, structural lightweight—A concrete containing lightweight aggregate which conforms to Section 3.3 and having an air-dry unit weight as determined by "Method of Test for Unit Weight of Structural Lightweight Concrete" (ASTM C 567), not exceeding 115 pcf. In this Code, a lightweight concrete without natural sand is termed "all-lightweight concrete" and lightweight concrete in which all fine aggregate consists of normal weight sand is termed "sand-lightweight concrete."

Curvature friction—Friction resulting from bends or curves in the specified profile of post-tensioned tendons.

Deformed reinforcement—Reinforcing bars, deformed wire, welded wire fabric, and welded deformed wire fabric conforming to Sections 3.5.1, 3.5.7, 3.5.6 or 3.5.8, respectively.

Development length—The length of embedded reinforcement required to develop the design strength of the reinforcement at a critical section. (See Section 9.2.2.)

Effective area of concrete—The area of a section which lies between the centroid of the tension reinforcement and the compression face of a flexural member.

Effective area of reinforcement—The area obtained by multiplying the right cross-sectional area of the reinforcement by the cosine of the angle between its direction and the direction for which the effectiveness is to be determined.

Effective prestress—The stress remaining in the tendons after all losses have occurred, excluding the effects of dead load and superimposed loads.

Embedment length—The length of embedded reinforcement provided beyond a critical section.

Embedment length, equivalent (l_e) — The length of embedded reinforcement which can develop the same stress as that which can be developed by a hook or mechanical anchorage.

End anchorage—Length of reinforcement, or a mechanical anchor, or a hook, or combination thereof, beyond the point of nominal zero stress in the reinforcement.

Jacking force—In prestressed concrete, the temporary force exerted by the device which introduces the tension into the tendons.

Load, dead—The dead weight supported by a member, as defined by the general building code of which this Code forms a part (without load factors).

Load, design—Load, multiplied by appropriate load factor, used to proportion members. (See Sections 8.1 and 9.3.)

Load, live—The live load specified by the general building code of which this Code forms a part (without load factors).

Load, service—Live and dead loads (without load factors).

Modulus of elasticity—See Section 8.3.

Pedestal—An upright compression member having a ratio of unsupported height to average least lateral dimension of 3 or less.

Plain concrete—Concrete that does not conform to the definition for reinforced concrete.

Plain reinforcement—Reinforcement that does not conform to the definition of deformed reinforcement.

Post-tensioning—A method of prestressing in which the tendons are tensioned after the concrete has hardened.

Precast concrete—A plain or reinforced concrete element cast in other than its final position in the structure.

Prestressed concrete—Reinforced concrete in which there have been introduced internal stresses of such magnitude and distribution that the stresses resulting from loads are counteracted to a desired degree.

Pretensioning—A method of prestressing in which the tendons are tensioned before the concrete is placed.

Reinforced concrete—Concrete containing reinforcement, including prestressing steel, and designed on the assumption that the two materials act together in resisting forces.

Reinforcement—Material that conforms to Section 3.5, excluding prestressing steel unless specifically included.

Segmental member—A structural member made up of individual elements prestressed together to act as a monolithic unit under service loads.

Span length—See Section 8.5.2.

Spiral—Continuously wound reinforcement in the form of a cylindrical helix.

Splitting tensile strength (f_{ct})—The tensile strength of concrete determined by splitting test made in accordance with "Specifications for Lightweight Aggregates for Structural Concrete" (ASTM C 330). See Section 4.2.9.

Stirrups or ties—Lateral reinforcement formed of individual units, open or closed (see Section 7.12.7), or of continuously wound reinforcement. The term "stirrups" is usually applied to lateral reinforcement in horizontal members and the term "ties" to those in vertical members.

Stress—Intensity of force per unit area.

Surface water—Water carried by an aggregate except that held by absorption within the aggregate particles themselves.

Tendon—A tensioned steel element used to impart prestress to the concrete.

Ties—see Stirrups

Transfer—In prestressed concrete, the operation of transferring the tendon force to the concrete.

Wall—A vertical element used primarily to enclose or separate spaces.

Wobble friction—In prestressed concrete, the friction caused by the unintended deviation of the prestressing tendon from its specified profile.

Yield strength or yield point (f_y)—Specified minimum yield strength or yield point of reinforcement in pounds per square inch. Yield strength or yield point shall be determined in tension according to applicable ASTM specifications or Section 3.5.

PART 2—SPECIFICATIONS FOR TESTS AND MATERIALS

CHAPTER 3—MATERIALS

3.0—Notation

f_{pu} = ultimate strength of prestressing steel, psi

f_y = specified yield strength of nonprestressed reinforcement, psi

3.1—Tests of materials

3.1.1—The Building Official shall have the right to order the testing of any materials used in concrete construction to determine if they are of the quality specified.

3.1.2—Tests of materials and of concrete shall be made in accordance with standards of the American Society for Testing and Materials, listed in Section 3.8.1. The complete records of such tests shall be available for inspection during the progress of the work and for 2 years thereafter, and shall be preserved by the inspecting engineer or architect for that purpose.

3.2—Cements

3.2.1—Cement shall conform to one of the specifications for portland cement listed below:

(a) *Portland cement*—"Specification for Portland Cement" (ASTM C 150).

(b) *Air-entraining portland cement*—"Specification for Air-Entraining Portland Cement" (ASTM C 175).

(c) *Portland blast-furnace-slag cement or portland-pozzolan cement*—"Specification for Blended Hydraulic Cements" (ASTM C 595), excluding Types S and SA which are not intended as the principal cementing constituent of structural concrete.

3.2.2—The cement used in the work shall correspond to that on which the selection of concrete proportions was based (see Section 4.2).

3.3—Aggregates

3.3.1—Concrete aggregates shall conform to "Specifications for Concrete Aggregates" (ASTM C 33) or to "Specifications for Lightweight Aggregates for Structural Concrete" (ASTM C 330), except that aggregates failing to meet these specifications but which have been shown by special test or actual service to produce concrete of adequate strength and durability may be used where authorized by the Building Official.

3.3.2—The nominal maximum size of the aggregate shall not be larger than one-fifth of the narrowest dimension between sides of forms, one-third of the depth of slabs, nor three-fourths of the minimum clear spacing between individual reinforcing bars or bundles of bars or pretensioning tendons or post-tensioning ducts. These limitations may be waived if, in the judgment of the engineer, workability and methods of consolidation are such that the concrete can be placed without honeycomb or void.

3.4—Water

3.4.1—Water used in mixing concrete shall be clean and free from injurious amounts of oils, acids, alkalis, salts, organic materials, or other substances that may be deleterious to concrete or steel. In addition, the mixing water for prestressed concrete or for concrete which will contain aluminum embedments, including that portion of the mixing water contributed in the form of free moisture on the aggregates, shall not contain deleterious amounts of chloride ion.

3.4.2—If nonpotable water is proposed for use, the selection of proportions shall be based on concrete mixes using water from the same source. Mortar test cubes made with nonpotable mixing water shall have 7-day and 28-day strengths equal to at least 90 percent of the strengths of similar specimens made with potable water. The strength test comparison shall be made on mortars, identical except for the mixing water, prepared and tested in accordance with "Method of Test for Compressive Strength of Hydraulic Cement Mortars (Using 2-inch Cube Specimens)" (ASTM C 109).

3.5—Metal reinforcement

3.5.1—Reinforcing bars shall conform to one of the following specifications, except that yield strength shall correspond to that determined by tests on full-size bars and for reinforcing bars with a specified yield strength, f_y, exceeding 60,000 psi, f_y shall be the stress corresponding to a strain of 0.35 percent.

(a) "Specification for Deformed Billet-Steel Bars for Concrete Reinforcement" (ASTM A 615). If #14 or #18 bars meeting these specifications are to be bent, they shall also be capable of being bent, 90 deg, at a minimum temperature of 60 F, around a ten-bar-diameter pin without cracking transverse to the axis of the bar.

(b) "Specification for Rail-Steel Deformed Bars for Concrete Reinforcement" (ASTM A 616). If bars meeting these specifications are to be bent, they shall also meet the bending requirements of ASTM A 615 for Grade 60.

(c) "Specification for Axle-Steel Deformed Bars for Concrete Reinforcement" (ASTM A 617).

3.5.2—Plain bars for spiral reinforcement shall conform to the strength requirements and minimum elongation of the appropriate specification prescribed in Section 3.5.1.

3.5.3—Reinforcement to be welded shall be indicated on the drawings and the welding procedure to be used shall be specified. The ASTM specifications shall be supplemented by requirements assuring satisfactory weldability by this procedure in conformity with "Recommended Practices for Welding Reinforcing Steel, Metal Inserts, and Connections in Reinforced Concrete Construction" (AWS D12.1). The supplementary specification requirements shall be designated in the order, and conformance with these requirements shall be confirmed by the supplier at the time of delivery.

3.5.4—Bar and rod mats for concrete reinforcement shall be the clipped type conforming to "Specifications for Fabricated Steel Bar or Rod Mats for Concrete Reinforcement" (ASTM A 184).

3.5.5—Plain wire for spiral reinforcement shall conform to "Specifications for Cold-Drawn Steel Wire for Concrete Reinforcement" (ASTM A 82), except that f_y shall be the stress corresponding to a strain of 0.35 percent if the yield strength specified in the design exceeds 60,000 psi.

3.5.6—Welded plain wire fabric for concrete reinforcement shall conform to "Specifications for Welded Steel Wire Fabric for Concrete Reinforcement" (ASTM A 185) and to the stipulation of Section 3.5.5 regarding measurement of f_y, except that welded intersections shall be spaced not farther apart than 12 in. in the direction of the principal reinforcement.

3.5.7—Deformed wire for concrete reinforcement shall conform to "Specification for Deformed Steel Wire for Concrete Reinforcement" (ASTM A 496), except that wire shall not be smaller than size D-4 and that f_y shall be the stress corresponding to a strain of 0.35 percent if the yield strength specified in the design exceeds 60,000 psi.

3.5.8—Welded deformed wire fabric for concrete reinforcement shall conform to "Specification for Welded Deformed Steel Wire Fabric for Concrete Reinforcement" (ASTM A 497) and to the stipulation of Section 3.5.7 regarding measurement of f_y, except that welded intersections shall be spaced not farther apart than 16 in. in the direction of the principal reinforcement.

3.5.9—Wire and strands for tendons in prestressed concrete shall conform to "Specifications for Uncoated Seven-Wire Stress-Relieved Strand for Prestressed Concrete" (ASTM A 416) or "Specifications for Uncoated Stress-Relieved Wire for Prestressed Concrete" (ASTM A 421). Strands or wire not specifically itemized in ASTM A 416 or A 421 may be used provided they conform to the minimum requirements of these specifications and have no properties which make them less satisfactory than those listed in ASTM A 416 or A 421.

3.5.10—High strength alloy steel bars for post-tensioning tendons shall be proof-stressed during manufacture to 85 percent of the minimum guaranteed tensile strength. After proof-stressing, bars shall be subjected to a stress-relieving heat treatment to produce the prescribed physical properties. After processing, the physical properties of the bars when tested on full sections, shall conform to the following minimum properties:

Yield strength (0.2 percent offset):	$0.85f_{pu}$
Elongation at rupture in 20 diameters:	4 percent
Reduction of area at rupture:	20 percent

3.5.11—Steel pipe or tubing for a composite compression member composed of a steel encased concrete core meeting requirements of Section 10.15.4 shall conform to one of the following specifications:

(a) Grade B of "Specifications for Welded and Seamless Steel Pipe" (ASTM A 53).

(b) "Specifications for Cold-Formed Welded and Seamless Carbon Steel Structural Tubing in Rounds and Shapes" (ASTM A 500).

(c) "Specifications for Hot-Formed Welded and Seamless Carbon Steel Structural Tubing" (ASTM A 501).

3.5.12—Structural steel used in conjunction with reinforcing steel in a composite compression member meeting the requirements of Section 10.15.5 or 10.15.6 shall conform to one of the following specifications:

(a) "Specification for Structural Steel" (ASTM A 36)

(b) "Specifications for High Strength Low Alloy Structural Steel" (ASTM A 242)

(c) "Specifications for High-Strength Structural Steel" (ASTM A 440)

(d) "Specifications for High-Strength Low-Alloy Structural Manganese Vanadium Steel" (ASTM A 441)

(e) "Specifications for High-Strength Low-Alloy Columbium-Vanadium Steels of Structural Quality" (ASTM A 572)

(f) "Specification for High-Strength Low-Alloy Structural Steel with 50,000 psi Minimum Yield Point to 4 in. Thick" (ASTM A 588).

3.6—Admixtures

3.6.1—Admixtures to be used in concrete shall be subject to prior approval by the Engineer. The admixture shall be shown capable of maintaining essentially the same composition and performance throughout the work as the product used in establishing concrete proportions in accordance with Section 4.2. Admixtures containing chloride ions shall not be used in prestressed concrete or in concrete containing aluminum embedments if their use will produce a deleterious concentration of chloride ion in the mixing water.

3.6.2—Air-entraining admixtures shall conform to "Specification for Air-Entraining Admixtures for Concrete" (ASTM C 260).

3.6.3—Water-reducing admixtures, retarding admixtures, accelerating admixtures, water-reducing and retarding admixtures, and water-reducing and accelerating admixtures shall conform to "Specification for Chemical Admixtures for Concrete" (ASTM C 494).

3.6.4—Fly ash or other pozzolans used as admixtures shall conform to "Specifications for Fly Ash and Raw or Calcined Natural Pozzolans for Use in Portland Cement Concrete" (ASTM C 618).

3.7—Storage of materials

Cement and aggregates shall be stored in such a manner as to prevent their deterioration or the intrusion of foreign matter. Any material which has deteriorated or which has been contaminated shall not be used for concrete.

3.8—Specifications cited in this Code

3.8.1—The specifications of the American Society for Testing and Materials referred to in this Code are listed below with their serial designations, including the year of adoption or revision, and are declared to be a part of this Code the same as if fully set forth elsewhere herein:

A 36-70	Standard Specification for Structural Steel
A 53-69a	Standard Specifications for Welded and Seamless Steel Pipe
A 82-70	Standard Specifications for Cold-Drawn Steel Wire for Concrete Reinforcement
A 184-65	Standard Specifications for Fabricated Steel Bar or Rod Mats for Concrete Reinforcement
A 185-70	Standard Specifications for Welded Steel Wire Fabric for Concrete Reinforcement
A 242-70	Standard Specifications for High-Strength Low-Alloy Structural Steel
A 370-68	Standard Methods and Definitions for Mechanical Testing of Steel Products
A 416-68	Standard Specifications for Uncoated Seven-Wire Stress-Relieved Strand for Prestressed Concrete
A 421-65	Standard Specifications for Uncoated Stress-Relieved Wire for Prestressed Concrete

A 440-70 Standard Specifications for High-Strength Structural Steel
A 441-70 Standard Specifications for High-Strength Low-Alloy Structural Manganese Vanadium Steel
A 496-70 Standard Specification for Deformed Steel Wire for Concrete Reinforcement
A 497-70 Standard Specification for Welded Deformed Steel Wire Fabric for Concrete Reinforcement
A 500-68 Standard Specifications for Cold-Formed Welded and Seamless Carbon Steel Structural Tubing in Rounds and Shapes
A 501-68a Standard Specifications for Hot-Formed Welded and Seamless Carbon Steel Structural Tubing
A 572-70 Standard Specification for High-Strength, Low-Alloy, Columbium Vanadium Steels of Structural Quality
A 588-70 Standard Specifications for High-Strength Low-Alloy Structural Steel with 50,000 psi Minimum Yield Point to 4 in. Thick
A 615-68 Standard Specification for Deformed Billet-Steel Bars for Concrete Reinforcement
A 616-68 Standard Specification for Rail-Steel Deformed Bars for Concrete Reinforcement
A 617-68 Standard Specification for Axle-Steel Deformed Bars for Concrete Reinforcement
C 31-69 Standard Method of Making and Curing Concrete Compressive and Flexural Strength Test Specimens in the Field
C 33-67 Standard Specifications for Concrete Aggregates
C 39-66 Standard Method of Test for Compressive Strength of Molded Concrete Cylinders
C 42-68 Standard Method of Obtaining and Testing Drilled Cores and Sawed Beams of Concrete
C 94-69 Standard Specification for Ready-Mixed Concrete
C 109-64 Standard Method of Test for Compressive Strength of Hydraulic Cement Mortars (Using 2-inch Cube Specimens)
C 144-70 Standard Specifications for Aggregate for Masonry Mortar
C 150-69a Standard Specification for Portland Cement
C 172-68 Standard Method of Sampling Fresh Concrete
C 175-69 Standard Specification for Air-Entraining Portland Cement
C 192-69 Standard Method of Making and Curing Concrete Test Specimens in the Laboratory
C 260-69 Standard Specification for Air-Entraining Admixtures for Concrete
C 330-69 Standard Specifications for Lightweight Aggregates for Structural Concrete
C 494-68 Standard Specification for Chemical Admixtures for Concrete
C 496-69 Standard Method of Test for Splitting Tensile Strength of Molded Concrete Cylinders
C 567-69 Standard Method of Test for Unit Weight of Structural Lightweight Concrete
C 595-68 Standard Specification for Blended Hydraulic Cements
C 618-68T Tentative Specifications for Fly Ash and Raw or Calcined Natural Pozzolans for Use in Portland Cement Concrete
E 6-66 Standard Definitions of Terms Relating to Methods of Mechanical Testing

3.8.2—The specifications of the American Welding Society "Recommended Practices for Welding Reinforcing Steel, Metal Inserts and Connections in Reinforced Concrete Construction" (AWS D12.1-61) is declared to be a part of this Code the same as if fully set forth herein.

PART 3—CONSTRUCTION REQUIREMENTS

CHAPTER 4—CONCRETE QUALITY

4.0—Notation

f_c' = specified compressive strength of concrete, psi

f_{ct} = average splitting tensile strength of lightweight aggregate concrete, psi

4.1—General

4.1.1—Concrete shall be proportioned and produced to provide an average compressive strength sufficiently high to minimize the frequency of strength tests below the value of the specified compressive strength of the concrete, f_c'. See Section 4.2.2.1.

4.1.2—Plans submitted for approval or used for any project shall clearly show the compressive strength of concrete, f_c', for which each part of the structure is designed.

4.1.3—Requirements for f_c' shall be based on tests of cylinders made and tested in accordance with ASTM methods as prescribed in this chapter.

4.1.4—Unless otherwise specified, f_c' shall be based on 28-day tests. For high-early-strength concrete, the test age for f_c' shall be as indicated in the plans or specifications.

4.2—Selection of concrete proportions

4.2.1—Proportions of ingredients for concrete shall be established on the basis of Sections 4.2.2 through 4.2.8 to provide:

(a) Conformance with the strength test requirements of Section 4.3

(b) Adequate workability and proper consistency to permit the concrete to be worked readily into the forms and around reinforcement under the conditions of placement to be employed, without excessive segregation or bleeding

(c) Resistance to freezing and thawing and other aggressive actions, where required

The criteria of Sections 4.2.2 through 4.2.4 are solely for the purpose of establishing required mixture proportions and do not constitute a basis for confirming the adequacy of concrete strength, which is covered in Section 4.3.

4.2.2—Except as permitted in Section 4.2.4 or required by Sections 4.2.5, 4.2.6, or 4.2.7, proportions, including water-cement ratio, shall be established on the basis either of laboratory trial batches or of field experience with the materials to be employed. The proportions shall be selected to produce an average strength at the designated test age exceeding f_c' by the amount indicated below, when both air content and slump are the maximums permitted by the specifications.

4.2.2.1 Where the concrete production facility has a record, based on at least 30 consecutive strength tests representing similar materials and conditions to those expected, the strength used as the basis for selecting proportions shall exceed the required f_c' by at least:

400 psi if the standard deviation is	less than 300 psi
550 psi if the standard deviation is	300 to 400 psi
700 psi if the standard deviation is	400 to 500 psi
900 psi if the standard deviation is	500 to 600 psi

Strength data for determining standard deviation shall be considered to comply with the foregoing stipulations if they represent either a group of at least 30 consecutive tests or the statistical average for two groups totaling 30 or more tests. The tests used to establish standard deviation shall represent concrete produced to meet a specified strength or strengths within 1000 psi of that specified for the proposed work. Changes in materials and proportions within the population of background tests shall not have been more closely restricted than they will be for the proposed work.

4.2.2.2 If the standard deviation exceeds 600 psi or if a suitable record of strength test performance is not available, proportions shall be selected to produce an average strength at least 1200 psi greater than the required f_c'.

Using the methods of "Recommended Practice for Evaluation of Compression Test Results of Field Concrete (ACI 214-65)," the amount by which the average strength must exceed f_c' may be reduced to an appropriate level below 1200 psi after sufficient test data become available from the job to indicate that, at the lower average strength, the probable frequency of tests more than 500 psi below f_c' will not exceed 1 in 100 and that the probable frequency of an average of three consecutive tests below f_c' will not exceed 1 in 100.

4.2.3—When laboratory trial batches are used as the basis for selecting concrete proportions, strength tests shall be made in accordance with "Method of Test for Compressive Strength of Molded Concrete Cylinders" (ASTM C 39) on specimens prepared in accordance with "Method of Making and Curing Test Specimens in the Laboratory" (ASTM C 192). A curve shall be established showing the relationship between water-cement ratio (or cement content) and compressive strength. The curve shall be based on at least three points representing batches which produce strengths above and below that

TABLE 4.2.4—MAXIMUM PERMISSIBLE WATER-CEMENT RATIOS FOR CONCRETE (WHEN STRENGTH DATA FROM TRIAL BATCHES OR FIELD EXPERIENCE ARE NOT AVAILABLE)

	Maximum permissible water-cement ratio			
	Non-air-entrained concrete		Air-entrained concrete	
Specified compressive strength f_c', psi*	Absolute ratio by weight	U.S. gal. per 94-lb bag of cement	Absolute ratio by weight	U.S. gal. per 94-lb bag of cement
2500	0.65	7.3	0.54	6.1
3000	0.58	6.6	0.46	5.2
3500	0.51	5.8	0.40	4.5
4000	0.44	5.0	0.35	4.0
4500	0.38	4.3	0.30	3.4
5000	0.31	3.5	†	†

*28-day strengths for cements meeting strength limits of ASTM C 150 Type I, IA, II, or IIA and 7-day strengths for Type III or IIIA; with most materials, the water-cement ratios shown will provide average strengths greater than indicated in Section 4.2.2 as being required.

†For strengths above 4500 psi with air-entrained concrete, proportions should be selected by the methods of Sections 4.2.2 or 4.2.3.

required. Each point shall represent the average of at least three specimens tested at 28 days or the earlier age designated.

The maximum permissible water-cement ratio (or minimum cement content) for the concrete to be used in the structure shall be that shown by the curve to produce the average strength indicated in Section 4.2.2 unless a lower water-cement ratio or higher strength is required by Sections 4.2.5, 4.2.6, or 4.2.7.

4.2.4—If suitable data from trial batches or field experience cannot be obtained, permission may be granted to base concrete proportions on the water-cement ratio limits shown in Table 4.2.4. This table shall be used only for concrete to be made with cements meeting the strength requirements for Type I, Type II, or Type III of "Specification for Portland Cement" (ASTM C 150) or, for air-entrained concrete only, Type IA, Type IIA, or Type IIIA of "Specification for Air-Entraining Portland Cement" (ASTM C 175), and shall not be applied to concrete containing lightweight aggregates or admixtures other than those for entraining air. Application of this method for estimating proportions does not remove the requirement to meet compressive strength test criteria of Section 4.3 and the water-cement ratio limits of Sections 4.2.5, 4.2.6, and 4.2.7.

4.2.5—Concrete that, after curing, will be subject to freezing temperatures while wet shall contain entrained air within the limts of Table 4.2.5. For such concrete made with normal weight aggregate, the water-cement ratio shall not exceed 0.53 by weight. When the concrete is made with lightweight aggregate, the specified compressive strength f_c' shall be at least 3000 psi.

TABLE 4.2.5—CONCRETE AIR CONTENT FOR VARIOUS SIZES OF COARSE AGGREGATE

Nominal maximum size of coarse aggregate, in.	Total air content, percent by volume
3/8	6 to 10
1/2	5 to 9
3/4	4 to 8
1	3.5 to 6.5
1 1/2	3 to 6
2	2.5 to 5.5
3	1.5 to 4.5

4.2.6—When made with normal weight aggregate, concrete that is intended to be watertight shall have a maximum water-cement ratio of 0.48 for exposure to fresh water and 0.44 for exposure to sea water. With lightweight aggregate, the specified compressive strength f_c' shall be at least 3750 psi for exposure to fresh water and 4000 psi for exposure to seawater.

4.2.7—Concrete that will be exposed to injurious concentrations of sulfate-containing solutions shall conform to Section 4.2.6 and be made with sulfate-resisting cement.

4.2.8—Where different materials are to be used for different portions of the work, each combination shall be evaluated separately.

4.2.9—Where design criteria in Sections 9.5.2.2, 11.3 and 12.5 (c) provide for the use of a splitting tensile strength value of concrete as a modifier, laboratory tests shall be made in accordance with "Specifications for Lightweight Aggregates for Structural Concrete" (ASTM C 330) to establish the value of f_{ct} corresponding to the specified value of f_c'.

4.2.9.1 Tensile splitting tests of field concrete shall not be used as a basis for acceptance.

4.3—Evaluation and acceptance of concrete

4.3.1—Samples for strength tests of each class of concrete shall be taken not less than once a day nor less than once for each 150 cu yd of concrete or for each 5000 sq ft of surface area placed. The samples for strength tests shall be taken in accordance with "Method of Sampling Fresh Concrete" (ASTM C 172). Cylinders for acceptance tests shall be molded and laboratory-cured in accordance with "Method of Making and Curing Concrete Compressive and Flexural Strength Test Specimens in the Field" (ASTM C 31) and tested in accordance with "Method of Test for Compressive Strength of Molded Concrete Cylinders" (ASTM C 39). Each strength test result shall be the average of two cylinders from the same sample tested at 28 days or the specified earlier age.

4.3.2—When the frequency of testing of Section 4.3.1 will provide less than five tests for a given class of concrete, tests shall be made from at least five randomly selected batches or from each batch if fewer than five are used. When the total quantity of a given class of concrete is less than 50 cu yd, the strength tests may be waived by the Building Official if, in his judgment, adequate evidence of satisfactory strength is provided.

4.3.3—The strength level of the concrete will be considered satisfactory if the averages of all sets of three consecutive strength test results equal or exceed the required f_c' and no individual strength test result falls below the required f_c' by more than 500 psi.

4.3.4—Strength tests of specimens cured under field conditions in accordance with Section 7.4 of "Method of Making and Curing Concrete Compressive and Flexural Strength Test Specimens in the Field" (ASTM C 31) may be required by the Building Official to check the adequacy of curing and protection of the concrete in the structure. Such specimens shall be molded at the same time and from the same samples as the laboratory-cured acceptance test specimens. Procedures for protecting and curing the concrete shall be improved when the strength of field-cured cylinders at the test age designated for measuring f_c' is less than 85 percent of that of the companion laboratory-cured cylinders. When the laboratory-cured cylinder strengths are appreciably higher than f_c' the field-cured cylinder strengths need not exceed f_c' by more than 500 psi even though the 85 percent criterion is not met.

4.3.5—If individual tests of laboratory-cured specimens produce strengths more than 500 psi below f_c' or if tests of field-cured cylinders indicate deficiencies in protection and curing, steps shall be taken to assure that load-carrying capacity of the structure is not jeopardized. If the likelihood of low-strength concrete is confirmed and computations indicate that the load carrying capacity may have been significantly reduced, tests of cores drilled from the area in question may be required in accordance with "Method of Obtaining and Testing Drilled Cores and Sawed Beams of Concrete" (ASTM C 42). Three cores shall be taken for each case of a cylinder test more than 500 psi below f_c'. If the concrete in the structure will be dry under service conditions, the cores shall be air dried (temperature 60 to 80 F, relative humidity less than 60 percent) for 7 days before test and shall be tested dry. If the concrete in the structure will be more than superficially wet under service conditions, the cores shall be immersed in water for at least 48 hr and tested wet.

4.3.5.1 Concrete in the area represented by the core tests will be considered structurally adequate if the average of the three cores is equal to at least 85 percent of f_c' and if no single core is less than 75 percent of f_c'. To check testing accuracy, locations represented by erratic core strengths may be retested. If these strength acceptance criteria are not met by the core tests, and if structural adequacy remains in doubt, the responsible authority may order load tests as outlined in Chapter 20 for the questionable portion of the structure, or take other action appropriate to the circumstances.

CHAPTER 5—MIXING AND PLACING CONCRETE

5.1—Preparation of equipment and place of deposit

5.1.1—Before concrete is placed, all equipment for mixing and transporting the concrete shall be clean, all debris and ice shall be removed from the spaces to be occupied by the concrete, forms shall be properly coated, masonry filler units that will be in contact with concrete shall be well drenched, and the reinforcement shall be thoroughly clean of ice or other deleterious coatings.

5.1.2—Water shall be removed from the place of deposit before concrete is placed unless a tremie is to be used or unless otherwise permitted by the Building Official.

5.1.3—All laitance and other unsound material shall be removed from hardened concrete before additional concrete is placed.

5.2—Mixing of concrete

5.2.1—All concrete shall be mixed until there is a uniform distribution of the materials and shall be discharged completely before the mixer is recharged.

5.2.2—For job-mixed concrete, mixing shall be done in a batch mixer of approved type. The mixer shall be rotated at a speed recommended by the manufacturer and mixing shall be continued for at least 1½ min after all materials are in the drum, unless a shorter time is shown to be satisfactory by the criteria of "Specification for Ready-Mixed Concrete" (ASTM C 94) for central mixers.

5.2.3—Ready-mixed concrete shall be mixed and delivered in accordance with the requirements set forth in "Specification for Ready-Mixed Concrete" (ASTM C 94).

5.3—Conveying

5.3.1—Concrete shall be conveyed from the mixer to the place of final deposit by methods which will prevent the separation or loss of materials.

5.3.2—Conveying equipment shall be capable of providing a supply of concrete at the site of placement without separation of ingredients and without interruptions sufficient to permit loss of plasticity between successive increments.

5.4—Depositing

5.4.1—Concrete shall be deposited as nearly as practicable in its final position to avoid segregation due to rehandling or flowing. The concreting shall be carried on at such a rate that the concrete is at all times plastic and flows readily into the spaces between the bars. No concrete that has partially hardened or been contaminated by foreign materials shall be deposited in the structure, nor shall retempered concrete or concrete which has been remixed after initial set be used unless approved by the Engineer.

5.4.2—After concreting is started, it shall be carried on as a continuous operation until the placing of the panel or section is completed except as permitted or prohibited by Section 6.4. The top surfaces of vertically formed lifts shall be generally level. When construction joints are necessary, they shall be made in accordance with Section 6.4.

5.4.3—All concrete shall be thoroughly consolidated by suitable means during placement, and shall be thoroughly worked around the reinforcement and embedded fixtures and into the corners of the forms.

5.4.4—Where conditions make consolidation difficult, or where reinforcement is congested, batches of mortar containing the same proportions of cement, sand, and water as used in the concrete, shall first be deposited in the forms to a depth of at least 1 in.

5.5—Curing

5.5.1—Unless cured in accordance with Section 5.5.2, concrete shall be maintained above 50 F and in a moist condition for at least the first 7 days after placing, except that high-early-strength concrete shall be so maintained for at least the first 3 days. Supplementary strength tests in accordance with Section 4.3.4 may be required to assure that curing is satisfactory.

5.5.2—Curing by high pressure steam, steam at atmospheric pressure, heat and moisture, or other accepted processes, may be employed to accelerate strength gain and reduce the time of curing. Accelerated curing shall provide the compressive strength of the concrete at the load stage considered at least equal to the design strength required at that load stage. The curing process shall produce concrete with a durability at least equivalent to the curing method of Section 5.5.1.

5.6—Cold weather requirements

Adequate equipment shall be provided for heating the concrete materials and protecting the concrete during freezing or near-freezing weather. All concrete materials and all reinforcement, forms, fillers, and ground with which the concrete is to come in contact shall be free from frost. No frozen materials or materials containing ice shall be used.

5.7—Hot weather requirements

During hot weather, proper attention shall be given to ingredients, production methods, handling, placing, protection, and curing to prevent excessive concrete temperatures or water evaporation which will impair the required strength or serviceability of the member or structure.

CHAPTER 6—FORMWORK, EMBEDDED PIPES, AND CONSTRUCTION JOINTS

6.1—Design of formwork

6.1.1—Forms shall result in a final structure which conforms to the shape, lines, and dimensions of the members as required by the plans and specifications, and shall be substantial and sufficiently tight to prevent leakage of mortar. They shall be properly braced or tied together so as to maintain position and shape. Forms and their supports shall be designed so that previously placed structure will not be damaged.

6.1.2—Design of formwork shall include consideration of the following factors:

(a) Rate and method of placing concrete

(b) Construction loads, including vertical, horizontal, and impact loads

(c) Special form requirements necessary for the construction of shells, folded plates, domes, architectural concrete, or similar types of elements

6.1.3—Forms for prestressed members shall be constructed to permit movement of the member without damage during application of the prestressing force.

6.2—Removal of forms and shores

6.2.1—No construction loads exceeding the dead load plus live load shall be supported on any unshored portion of the structure under construction. No construction loads shall be supported on, nor any shoring removed from, any part of the structure under construction except when that portion of the structure in combination with the remaining forming and shoring system has sufficient strength to support safely its weight and the loads placed thereon. This strength may be demonstrated by job-cured test specimens and by a structural analysis considering the proposed loads in relation to these test strengths and the strength of the forming and shoring system. Such analysis and test data shall be furnished by the contractor to the Building Official when so required.

6.2.2—Forms shall be removed in such manner as to insure the complete safety of the structure. Where the structure as a whole is adequately supported on shores, the removable floor forms, beam and girder sides, column forms, and similar vertical forms may be removed after 24 hr, provided the concrete is sufficiently strong not to be injured thereby.

6.2.3—Form supports of prestressed members may be removed when sufficient prestressing has been applied to enable them to carry their dead loads and anticipated construction loads.

6.3—Conduits and pipes embedded in concrete

6.3.1—Electric conduits and other pipes whose embedment is allowed shall not, with their fittings, displace more than 4 percent of the area of the cross section of a column on which stress is calculated or which is required for fire protection. Sleeves, conduits, or other pipes passing through floors, walls, or beams shall be of such size and in such location as not to impair significantly the strength of the construction. Such sleeves, conduits, or pipes may be considered as replacing structurally in compression the displaced concrete, provided they are not exposed to rusting or other deterioration, are of uncoated or galvanized iron or steel not thinner than standard Schedule 40 steel pipe, have a nominal inside diameter not over 2 in., and are spaced not less than three diameters on centers. Except when plans of conduits and pipes are reviewed by the structural engineer, embedded pipes or conduits, other than those merely passing through, shall be not larger in outside dimension than one-third the thickness of the slab, wall, or beam in which they are embedded, nor shall they be spaced closer than three diameters or widths on center, nor so located as to impair significantly the strength of the construction. Sleeves, pipes, or conduits of any material not harmful to concrete and within the limitations of this section may be embedded in the concrete with the approval of the Engineer, provided they are not considered to replace the displaced concrete. Sleeves, pipes, or conduits of aluminum shall not be embedded in structural concrete unless effectively coated or covered to prevent aluminum-concrete reaction or electrolytic action between aluminum and steel.

6.3.2—Pipes which will contain liquid, gas, or vapor may be embedded in structural concrete under the following additional conditions:

6.3.2.1 Pipes and fittings shall be designed to resist the effects of the material, pressure, and temperature to which they will be subjected.

6.3.2.2 The temperature of the liquid, gas, or vapor shall not exceed 150 F.

6.3.2.3 The maximum pressure to which any piping or fittings shall be subjected shall be 200 psi above atmospheric pressure.

6.3.2.4 All piping and fittings except as noted in Section 6.3.2.5 shall be tested as a unit for leaks immediately prior to concreting. The testing pressure above atmospheric pressure shall be 50 percent in excess of the pressure to which the piping and fittings may be subjected, but the minimum testing pressure shall be not less than 150 psi above atmospheric pressure. The pressure test shall be held for 4 hr with no drop in pressure except that which may be caused by air temperature.

6.3.2.5 Drain pipes and other piping designed for pressures of not more than 1 psi above atmospheric pressure need not be tested as required in Section 6.3.2.4.

6.3.2.6 Pipes carrying liquid, gas, or vapor which is explosive or injurious to health shall again be tested as specified in Section 6.3.2.4 after the concrete has hardened.

6.3.2.7 No liquid, gas, or vapor, except water not exceeding 90 F nor 50 psi pressure, is to be placed in the pipes until the concrete has attained its design strength.

6.3.2.8 In solid slabs the piping, unless it is for radiant heating or snow melting, shall be placed between the top and bottom reinforcement.

6.3.2.9 The concrete covering of the pipes and fittings shall be not less than 1½ in. for concrete surfaces exposed to the weather or in contact with the ground, nor ¾ in. for concrete surface not exposed directly to the ground or weather.

6.3.2.10 Reinforcement with an area equal to at least 0.2 percent of the area of the concrete section shall be provided normal to the piping.

6.3.2.11 The piping and fittings shall be assembled by welding, brazing, solder-sweating, or other equally satisfactory method. Screw connections shall be prohibited. The piping shall be so fabricated and installed that it will not require any cutting, bending, or displacement of the reinforcement from its proper location.

6.4—Construction joints

6.4.1—Joints not indicated on the plans shall be so made and located as not to impair significantly the strength of the structure. Where a joint is to be made, the surface of the concrete shall be thoroughly cleaned and all laitance and standing water removed. Vertical joints also shall be thoroughly wetted and coated with neat cement grout immediately before placing of new concrete.

6.4.2—A delay at least until the concrete in columns and walls is no longer plastic must occur before casting or erecting beams, girders, or slabs supported thereon. Beams, girders, brackets, column capitals, and haunches shall be considered as part of the floor system and shall be placed monolithically therewith.

6.4.3—Construction joints in floors shall be located near the middle of the spans of slabs, beams, or girders, unless a beam intersects a girder at this point, in which case the joints in the girders shall be offset a distance equal to twice the width of the beam. Provision shall be made for transfer of shear and other forces through the construction joints.

CHAPTER 7—DETAILS OF REINFORCEMENT

7.0—Notation

A_w = area of a deformed wire, sq in.

d = distance from extreme compression fiber to centroid of tension reinforcement, in.

d_b = nominal diameter of bar, wire, or prestressing strand, in.

f_c' = specified compressive strength of concrete, psi

$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi

f_y = specified yield strength of nonprestressed reinforcement, psi

h = overall thickness of member, in.

l_d = development length, in. See Chapter 12

l_w = total lengths of wire extending beyond outermost cross wires, for each pair of spliced wires, in.

n = number of pairs of cross wires in splice

s = tie spacing, in.

s_w = spacing of deformed wires, in.

7.1—Hooks and bends

7.1.1 *Hooks*—The term "standard hook" as used herein shall mean either:

7.1.1.1 A semicircular turn plus an extension of at least four bar diameters but not less than 2½ in. at the free end of the bar, or

7.1.1.2 A 90-deg turn plus an extension of at least 12 bar diameters at the free end of the bar, or

7.1.1.3 For stirrup and tie anchorage only, either a 90-deg or a 135-deg turn plus an extension of at least six bar diameters but not less than 2½ in. at the free end of the bar.

7.1.2 *Minimum bend diameter*—The diameter of bend measured on the inside of the bar for standard hooks, other than stirrup and tie hooks, shall not be less than the values of Table 7.1.2, except that for sizes #3 to #11, inclusive, in Grade 40 bars with 180 deg hooks only, the minimum diameter shall be five bar diameters.

TABLE 7.1.2—MINIMUM DIAMETERS OF BEND

Bar size	Minimum diameter
#3 through #8	6 bar diameters
#9, #10, and #11	8 bar diameters
#14 and #18	10 bar diameters

7.1.3 *Stirrup and tie hooks and bends other than standard hooks*

7.1.3.1 Inside diameter of bends for stirrups and ties shall not be less than 1½ in. for #3, 2 in. for #4, and 2½ in. for #5.

7.1.3.2 Bends for all other bars shall have diameters on the inside of the bar not less than allowed by Section 7.1.2.

7.1.3.3 Inside diameter of bends in welded wire fabric, plain or deformed, for stirrups and ties shall not be less than four wire diameters for deformed wire larger than D6 and two wire diameters for all other wires. Bends with inside diameter of less than eight wire diameters shall not be less than four wire diameters from the nearest welded intersection.

7.1.4 *Bending*—All bars shall be bent cold, unless otherwise permitted by the Engineer. No bars partially embedded in concrete shall be field bent, except as shown on the plans or permitted by the Engineer.

7.2—Surface conditions of reinforcement

7.2.1—Metal reinforcement at the time concrete is placed shall be free from mud, oil, or other nonmetallic coatings that adversely affect bonding capacity.

7.2.2—Metal reinforcement, except prestressing steel, with rust, mill scale, or a combination of both shall be considered as satisfactory, provided the minimum dimensions, including height of deformations, and weight of a hand wire brushed test specimen are not less than the applicable ASTM specification requirements.

7.2.3—Prestressing steel shall be clean and free of excessive rust, oil, dirt, scale, and pitting. A light oxide is permissible.

7.3—Placing reinforcement

7.3.1 *Supports*—Reinforcement, prestressing steel, and ducts, shall be accurately placed and adequately supported before concrete is placed, and shall be secured against displacement within permitted tolerances. Welding of crossing bars shall not be permitted for assembly of reinforcement unless authorized by the Engineer.

7.3.2 *Tolerances*—Unless otherwise specified by the engineer, reinforcement, prestressing steel, and prestressing steel ducts shall be placed within the following tolerances:

7.3.2.1 For clear concrete protection and for depth, d in flexural members, walls, and compression members where d is

8 in. or less:	±¼ in.
More than 8 in. but less than 24 in.:	±⅜ in.
24 in. or more:	±½ in.

but the cover shall not be reduced by more than one-third of the specified cover.

7.3.2.2 For longitudinal location of bends and ends of bars: ±2 in. except at discontinuous ends of members where tolerance shall be ±½ in.

7.3.3 *Draped fabric*—When welded wire fabric with wire of ¼ in. diameter or less is used for slab reinforcement in slabs not exceeding 10 ft in span, the reinforcement may be curved from a point near the top of the slab over the support to a point near the bottom of the slab at midspan, provided such reinforcement is either continuous over, or securely anchored at, the support.

7.4—Spacing of reinforcement

7.4.1—The clear distance between parallel bars in a layer shall be not less than the nominal diameter of the bars, nor 1 in. See also Section 3.3.2. Where parallel reinforcement is placed in two or more layers, the bars in the upper layers shall be placed directly above those in the bottom layer with the clear distance between layers not less than 1 in.

7.4.2—Groups of parallel reinforcing bars bundled in contact, assumed to act as a unit, not more than four in any one bundle, may be used only when stirrups or ties enclose the bundle. Bars larger than #11 shall not be bundled in beams or girders. Individual bars in a bundle cut off within the span of flexural members shall terminate at different points with at least 40 bar diameters stagger. Where spacing limitations and minimum clear cover are based on bar size, a unit of bundled bars shall be treated as a single bar of a diameter derived from the equivalent total area.

7.4.3—In walls and slabs other than concrete joist construction, the principal reinforcement shall be spaced not farther apart than three times the wall or slab thickness, nor more than 18 in.

7.4.4—In spirally reinforced and tied compression members, the clear distance between longitudinal bars shall be not less than one and one-half times the nominal bar diameter, nor 1½ in. See also Section 3.3.2.

7.4.5—The clear distance limitation between bars shall also apply to the clear distance between a contact lap splice and adjacent splices or bars.

7.4.6—The clear distance between pretensioning steel at each end of the member shall be not less than four times the diameter of individual wires nor three times the diameter of strands. See also Section 3.3.2. Closer vertical spacing and bundling of strands may be permitted in the middle portion of the span.

7.4.7—Ducts for post-tensioning steel may be bundled if it can be shown that the concrete can be satisfactorily placed and when provision is made to prevent the steel, when tensioned, from breaking through the duct.

7.5—Splices in reinforcement—General

7.5.1—Splices of reinforcement shall be made only as required or permitted on the design drawings or in specifications, or as authorized by the engineer. Except as provided herein, all welding shall conform to "Recommended Practices for

Welding Reinforcing Steel, Metal Inserts, and Connections in Reinforced Concrete Construction" (AWS D12.1).

7.5.2—Lap splices shall not be used for bars larger than #11 except as provided in Section 15.6.8.

7.5.3—Lap splices of bundled bars shall be based on the lap splice length required for individual bars of the same size as the bars spliced and such individual splices within the bundle shall not overlap each other. The length of lap, as prescribed in Section 7.6.1 or 7.7.1 shall be increased 20 percent for a three-bar bundle and 33 percent for a four-bar bundle.

7.5.4—Bars spliced by noncontact lap splices in flexural members shall not be spaced transversely farther apart than one-fifth the required length of lap nor 6 in.

7.5.5—Welded splices or other positive connections may be used.

7.5.5.1 A full welded splice is one in which the bars are butted and welded to develop in tension at least 125 percent of the specified yield strength of the bar.

7.5.5.2 Full positive connections shall develop in tension or compression, as required, at least 125 percent of the specified yield strength of the bar.

7.5.5.3 Welded splices or positive connections not meeting the requirements of Section 7.5.5.1 or 7.5.5.2 may be used in regions of low computed stress in conformance with Section 7.6.3.2.

7.6—Splices in tension

7.6.1 *Classification of tension lap splices*—The minimum length of lap for tension lap splices covered in the Code shall be at least that given in this section. l_d is the tensile development length for the full f_y as given in Section 12.5.

7.6.1.1 Class A splices $1.0l_d$

7.6.1.2 Class B splices $1.3l_d$

7.6.1.3 Class C splices $1.7l_d$

7.6.1.4 Class D splices $2.0l_d$

The bars in a Class D splice shall be enclosed within a spiral meeting the requirements of Section 12.5(d) but no reduction in required development length shall be allowed for the effect of the spiral. The ends of bars larger than #4 shall be hooked 180 deg.

7.6.2 *Splices in tension tie members*—Where feasible, splices shall be staggered and made with full welded or full positive connections. If lap splices are used, they shall meet the requirements of a Class D splice as given in Section 7.6.1.4.

7.6.3 *Tension splices in other members*

7.6.3.1 In regions of maximum moment or high computed stress.

7.6.3.1.1 Splices in regions of maximum moment preferably shall be avoided. Where such splices must be used, they shall be lapped, welded, or otherwise anchored for their full f_y. If no more than one-half of the bars are lap spliced within a required lap length, splices shall meet the requirements for Class B splices (lap of $1.3l_d$). If more than one-half the bars are lap spliced within a required lap length, splices shall meet the requirements of Class C splices (lap of $1.7l_d$).

Welded splices or positive connections if used shall meet the requirements of Section 7.5.5.1 or 7.5.5.2.

7.6.3.1.2 Wherever the computed stress exceeds $0.5f_y$ splices must meet the requirements of Section 7.6.3.1.1.

7.6.3.2 In regions of low computed stress. Splices in regions where the maximum computed stress in the bar is always less than $0.5f_y$ shall meet the following requirements:

7.6.3.2.1 If no more than three-quarters of the bars are lap spliced within a required lap length, splices shall meet the requirements for Class A splices (lap of $1.0l_d$).

7.6.3.2.2 If more than three-quarters of the bars are lap spliced within a required lap length, splices shall meet the requirements for Class B splices (lap of $1.3l_d$).

7.6.3.2.3 The requirements of Sections 7.5.5.1 and 7.5.5.2 for welded splices or positive connections may be waived if the splices are staggered at least 24 in. and in such a manner as to develop at every section at least twice the calculated tensile force at the section and in no case less than 20,000 psi on the total sectional area of all bars used. In computing the capacity developed at each section, spliced bars may be rated at the specified splice strength. Unspliced bars shall be rated at that fraction of f_y defined by the ratio of the shorter actual development length to the l_d required for f_y.

7.7—Splices in compression

7.7.1 *Lap splices in compression*

7.7.1.1 The minimum length of a lap splice in compression, shall be the development length in compression l_d, (Section 12.6) but not less in inches than $0.0005f_yd_b$ for f_y of 60,000 psi or less, nor $(0.0009f_y - 24)\ d_b$ for f_y greater than 60,000 psi, nor 12 in. When the specified concrete strengths are less than 3000 psi, the lap shall be increased by one-third.

7.7.1.2 In tied compression members where ties throughout the lap length have an effective area of at least $0.0015hs$, 0.83 of the lap length specified in Section 7.7.1.1 may be used, but the lap length shall be not less than 12 in. Tie legs perpendicular to dimension h shall be used in determining the effective area.

7.7.1.3 Within the spiral of spiral compression members, 0.75 of the lap length specified in Section 7.7.1.1 may be used, but the lap length shall not be less than 12 in.

7.7.2 *End bearing*—In bars required for compression only, the compressive stress may be transmitted by bearing of square cut ends held in concentric contact by a suitable device. Ends shall terminate in flat surfaces within 1½ deg of right angles to the axis of the bars and shall be fitted within 3 deg of full bearing after assembly. End bearing splices shall not be used except in members containing closed ties, closed stirrups, or spirals.

7.7.3 *Welded splices or positive connections*—Welded splices or positive connections used in compression shall meet the requirements of Section 7.5.5.

7.8—Splices of welded plain wire fabric

7.8.1—Lapped splices of wires carrying more than one-half of the permissible stress shall preferably be avoided. Where such splices must carry more than one-half of the permissible stress, they shall be so made that the overlap measured between outermost cross wires of each fabric sheet is not less than the spacing of cross wires plus 2 in.

7.8.2—Splices of wires stressed at not more than one-half the permissible stress shall be so made that the overlap measured between outermost cross wires is not less than 2 in.

7.9—Splices of deformed wire and welded deformed wire fabric

The minimum splice length for deformed wire reinforcement shall be $0.045 d_b f_y / \sqrt{f_c'}$ but in no case shall the splice be less than 12 in.

The minimum splice length for welded deformed wire fabric reinforcement shall be at least

$$0.045 d_b \frac{f_y - 20{,}000n}{\sqrt{f_c'}}$$

but the outermost cross wires in the splice shall be lapped at least

$$\frac{A_w}{s_w}\left(360 - \frac{8 l_w}{d_b}\right)$$

7.10—Special details for columns

7.10.1—Where longitudinal bars are offset, the slope of the inclined portion of the bar with the axis of the column shall not exceed 1 in 6, and the portions of the bar above and below the offset shall be parallel to the axis of the column. Adequate horizontal support at the offset bends shall be treated as a matter of design, and shall be provided by metal ties, spirals, or parts of the floor construction. Metal ties or spirals so designed shall be placed not more than 6 in. from the point of bend. The horizontal thrust to be resisted shall be assumed as one and one-half times the horizontal component of the nominal force in the inclined portion of the bar.

Offset bars shall be bent before they are placed in the forms. See Section 7.1.4. Bundled bars shall not be offset bent.

7.10.2—Where column faces are offset 3 in. or more, splices of vertical bars adjacent to the offset face shall be made by separate dowels lapped as required herein.

7.10.3—Where the design load stress in the longitudinal bars in a column, calculated for various loading conditions, varies from f_y in compression to $\frac{1}{2} f_y$ or less in tension, lap splices, butt welded splices, positive connections, or end bearing splices may be used. The total tensile capacity provided in each face of the column by the splices alone or by the splices in combination with continuing unspliced bars at specified yield stress shall be at least twice the calculated tension in that face of the column but not less than required by Section 7.10.5.

7.10.4—Where the design load stress in the longitudinal bars in a column calculated for any loading condition exceeds $\frac{1}{2} f_y$ in tension, lap splices designed for full yield stress in tension, or full welded splices or full positive connections shall be used.

7.10.5—At horizontal cross sections of columns where splices are located, a minimum tensile strength at each face equal to one-fourth the area of vertical reinforcement in that face multiplied by f_y shall be provided.

7.10.6—Metal cores in composite columns shall be accurately finished to bear at splices, and positive provision shall be made for alignment of one core above another. Bearing shall be considered effective to transfer 50 percent of the total compressive stress in the metal core. At the column base, provision shall be made to transfer the load to the footing, in accordance with Section 15.6.

The base of the metal section shall be designed to transfer the load from the entire composite column to the footing, or it may be designed to transfer the load from the metal section only, provided it is so placed as to leave ample section of concrete for the transfer of load from the reinforced concrete section of the column by means of bond on the vertical reinforcement and by direct compression on the concrete.

7.11—Connections

At connections of principal framing elements, such as beams and columns, enclosure shall be provided for splices of continuing reinforcement and for end anchorage of reinforcement terminating in such connections. Such enclosure may consist of external concrete or internal closed ties, spirals, or stirrups.

7.12—Lateral reinforcement

7.12.1—Lateral reinforcement shall meet the provisions of this section and, where shear or torsion reinforcement is required, shall comply with the provisions of Chapter 11.

7.12.2—Spiral reinforcement for compression members to conform to Section 10.9.2 shall consist of evenly spaced continuous spirals held firmly in place and true to line by vertical spacers. At least two spacers shall be used for spirals less than 20 in. in diameter, three for spirals 20 to 30 in. in diameter, and four for spirals more than 30 in. in diameter. When spiral wires or bars are ⅝ in. or larger, three spacers shall be used for spirals 24 in. or less in diameter and four for spirals more than 24 in. in diameter. The spirals shall be of such size and so assembled as to permit handling and placing without being distorted from the designed dimensions. For cast-in-place construction, the material used in spirals shall have a minimum diameter of ⅜ in. Anchorage of spiral reinforcement shall be provided by one and one-half extra turns of spiral bar or wire at each end of the spiral unit. Splices when necessary in spiral bars or wires shall be tension lap splices of 48 diameters minimum but not less than 12 in., or welds. The clear spacing between spirals shall not exceed 3 in. or be less than 1 in. See also Section 3.3.2. The reinforcing spiral shall extend from the floor level in any story or from the top of the footing to the level of the lowest horizontal reinforcement in the slab, drop panel, or beam above. Where beams or brackets are not present on all sides of the column, ties shall extend above the termination of the spiral to the bottom of the slab or drop panel. In a column with a capital, the spiral shall extend to a plane at which the diameter or width of the capital is twice that of the column.

7.12.3—All nonprestressed bars for tied columns shall be enclosed by lateral ties, at least #3 in size for longitudinal bars #10 or smaller, and at least #4 in size for #11, #14, #18, and bundled longitudinal bars. The spacing of the ties shall not exceed 16 longitudinal bar diameters, 48 tie bar diameters, or the least dimension of the column. The ties shall be so arranged that every corner and alternate longitudinal bar shall have lateral support provided by the corner of a tie having an included angle of not more than 135 deg and no bar shall be farther than 6 in. clear on either side from such a laterally supported bar. Ties shall be located vertically not more than half a tie spacing above the floor or footing, and shall be spaced as provided herein to not more than half a tie spacing below the lowest horizontal reinforcement in the slab or drop panel above, except that where beams or brackets provide enclosure on all sides of the column, the ties may be terminated not more than 3 in. below the lowest reinforcement in such beams or brackets. Welded wire fabric of equivalent area may be used. Where the bars are located around the periphery of a circle, a complete circular tie may be used. For lateral reinforcement with prestressed tendons in tied columns see Section 18.14.3. For lateral reinforcement with composite tied columns see Section 10.15.6.

7.12.4—All provisions of Sections 7.12.2, 7.12.3, 10.15.6, and 18.14.3 may be waived where tests and structural analysis show adequate strength and feasibility of construction.

7.12.5—Compression reinforcement in beams or girders shall be enclosed by ties or stirrups satisfying the size and spacing limitations in Section 7.12.3 or by welded wire fabric of equivalent area. Such stirrups or ties shall be used throughout the distance where the compression reinforcement is required.

7.12.6—Lateral reinforcement for flexural framing members subject to stress reversals or to torsion at supports shall consist of closed ties, closed stirrups, or spirals extending around main reinforcement.

7.12.7—Closed ties or stirrups may be formed in one piece by overlapping standard stirrup or tie end hooks around a longitudinal bar or in one or two pieces spliced in accordance with Section 7.6.1.3 or anchored in accordance with Section 12.13.

7.13—Shrinkage and temperature reinforcement

Reinforcement for shrinkage and temperature stresses normal to the principal reinforcement shall be provided in structural floor and roof slabs where the principal reinforcement extends in one direction only. At all sections where it is required, such reinforcement shall be developed for its specified yield strength in conformance with Section 7.6 or 12.1.1. Such reinforcement shall provide at least the following ratios of reinforcement area to gross concrete area, but not less than 0.0014 and in no case shall such reinforcement be placed farther apart than five times the slab thickness nor more than 18 in.

Slabs where Grade 40 or 50 deformed bars are used	0.0020
Slabs where Grade 60 deformed bars or welded wire fabric, deformed or plain, are used	0.0018
Slabs where reinforcement with yield strength exceeding 60,000 psi measured at a yield strain of 0.35 percent is used	$\frac{0.0018 \times 60{,}000}{f_y}$

7.14—Concrete protection for reinforcement

7.14.1—The following minimum concrete cover shall be provided for reinforcing bars, prestressing tendons, or ducts. For bar bundles, the minimum cover shall equal the equivalent diameter of the bundle but need not be more than 2 in. or the tabulated minimum, whichever is greater.

7.14.1.1 Cast-in-place concrete (nonprestressed)

	Minimum cover, in.
Cast against and permanently exposed to earth	3
Exposed to earth or weather:	
#6 through #18 bars	2
#5 bars, ⅝ in. wire, and smaller	1½
Not exposed to weather or in contact with the ground:	
Slabs, walls, joists:	
#14 and #18 bars	1½
#11 and smaller	¾
Beams, girders, columns:	
Principal reinforcement, ties, stirrups or spirals	1½
Shells and folded plate members:	
#6 bars and larger	¾
#5 bars, ⅝ in. wire, and smaller	½

7.14.1.2 Precast concrete (manufactured under plant control conditions)

	Minimum cover, in.
Exposed to earth or weather:	
Wall panels:	
#14 and #18 bars	1½
#11 and smaller	¾
Other members:	
#14 and #18 bars	2
#6 through #11	1½
#5 bars, ⅝ in. wire, and smaller	1¼
Not exposed to weather or in contact with the ground:	
Slabs, walls, joists:	
#14 and #18 bars	1¼
#11 and smaller	⅝
Beams, girders, columns:	
Principal reinforcement	d_b but not less than ⅝ and need not exceed 1½
Ties, stirrups, or spirals	⅜
Shells and folded plate members:	
#6 bars and larger	⅝
#5 bars, ⅝ in. wire, and smaller	⅜

7.14.1.3 Prestressed concrete members—Prestressed and nonprestressed reinforcement, ducts, and end fittings

Cast against and permanently exposed to earth	3
Exposed to earth or weather:	
Wall panels, slabs, and joists	1
Other members	1½
Not exposed to weather or in contact with the ground:	
Slabs, walls, joists	¾
Beams, girders, columns:	
Principal reinforcement	1½
Ties, stirrups, or spirals	1
Shells and folded plate members:	
Reinforcement ⅝ in. and smaller	⅜
Other reinforcement	d_b but not less than ¾

7.14.2—The cover for nonprestressed reinforcement in prestressed concrete members under plant control may be that given for precast members.

Cover specified in Section 7.14.1.3 is for prestressed members with stresses less than or equal to the limits of Section 18.4.2(b). When tensile stresses exceed this value for members exposed to weather, earth, or corrosive environment, cover shall be increased 50 percent.

7.14.3—In corrosive atmospheres or severe exposure conditions, the amount of concrete protection shall be suitably increased, and the denseness and nonporosity of the protecting concrete shall be considered, or other protection shall be provided.

7.14.4—Exposed reinforcing bars, inserts, and plates intended for bonding with future extensions shall be protected from corrosion.

7.14.5—When the general code, of which this Code forms a part, requires a fire-protective covering greater than the concrete protection specified in this section, such greater thicknesses shall be used.

PART 4—GENERAL REQUIREMENTS

CHAPTER 8—ANALYSIS AND DESIGN—GENERAL CONSIDERATIONS

8.0—Notation

A_s = area of nonprestressed tension reinforcement, sq in.
A_s' = area of compression reinforcement, sq in.
b = width of compression face of member
d = distance from extreme compression fiber to centroid of tension reinforcement, in.
E_c = modulus of elasticity of concrete, psi. See Section 8.3.1
E_s = modulus of elasticity of steel, psi. See Section 8.3.2
f_c' = specified compressive strength of concrete, psi
f_y = specified yield strength of nonprestressed reinforcement, psi
l_n = clear span for positive moment or shear and the average adjacent clear spans for negative moment
n = modular ratio = E_s/E_c
v_c = nominal permissible shear stress carried by concrete
w = design load per unit length of beam or per unit area of slab
w = weight of concrete, lb per cu ft
β_1 = a factor defined in Section 10.2.7
ρ = A_s/bd = ratio of nonprestressed tension reinforcement
ρ' = A_s'/bd
ρ_b = reinforcement ratio producing balanced conditions. See Section 10.3.3

8.1—Design methods

8.1.1—In the design of reinforced concrete structures, members shall be proportioned for adequate strength in accordance with the provisions of this Code, using the load factors and capacity reduction factors ϕ specified in Chapter 9.

8.1.2—Alternatively, for nonprestressed members the method provided in Section 8.10 may be used taking both the ϕ factors and load factors as one rather than those specified in Chapter 9.

8.1.3—Flexural members designed under Section 8.1.1 or 8.1.2 shall also meet the requirements for deflection control in Section 9.5, and the requirements of Sections 10.4 through 10.7.

8.2—Required loading*

8.2.1—The design provisions of this Code are based on the assumption that structures shall be designed to resist all applicable loads. The loads shall be in accordance with the general requirements of the building code of which this Code forms a part, with such live load reductions as are permitted therein.

8.2.2—In design for wind and earthquake forces, the integral structural parts shall be designed to resist the total lateral loads.†

8.2.3—Consideration shall be given to the effects of forces due to prestressing, crane loads, vibration, impact, shrinkage, temperature changes, creep, and unequal settlement of supports.

8.3—Modulus of elasticity

8.3.1—The modulus of elasticity, E_c, for concrete may be taken as $w^{1.5}33\sqrt{f_c'}$, in psi, for values of w between 90 and 155 lb per cu ft. For normal weight concrete, E_c may be considered as $57{,}000\sqrt{f_c'}$.

8.3.2—The modulus of elasticity of nonprestressed steel reinforcement may be taken as 29,000,000 psi. The modulus of elasticity of prestressing steel shall be determined by tests or supplied by the manufacturer.

8.4—Frame analysis and design—General

8.4.1—All members of frames or continuous construction shall be designed for the maximum effects of the design loads as determined by the theory of elastic frames. The simplifying assumptions of Section 8.5 may be used.

8.4.2—Except for prestressed concrete, approximate methods of frame analysis may be used for buildings of usual types of construction, spans, and story heights. For two or more approximately equal spans (the larger of two adjacent spans not exceeding the shorter by more than 20 percent) with loads uniformly distributed, where the unit live load does not exceed three times the unit dead load, the following moments and shears may be used in design in lieu of more accurate analyses.

Positive moment
End spans
If discontinuous end is unrestrained .... $\frac{1}{11} w l_n^2$

If discontinuous end is integral with the support .... $\frac{1}{14} w l_n^2$

*The provisions in this Code are suitable for live, wind, and earthquake loads, such as those recommended in "Building Code Requirements for Minimum Design Loads in Building and Other Structures," ANSI A-58.1, of the American National Standards Institute.
†Special provisions for seismic design are given in Appendix A.

Interior spans .... $\frac{1}{16} wl_n^2$

Negative moment at exterior face of first interior support

Two spans .... $\frac{1}{9} wl_n^2$

More than two spans .... $\frac{1}{10} wl_n^2$

Negative moment at other faces of interior supports .... $\frac{1}{11} wl_n^2$

Negative moment at face of all supports for, (a) slabs with spans not exceeding 10 ft, and (b) beams and girders where ratio of sum of column stiffnesses to beam stiffness exceeds eight at each end of the span .... $\frac{1}{12} wl_n^2$

Negative moment at interior faces of exterior supports for members built integrally with their supports

Where the support is a spandrel beam or girder .... $\frac{1}{24} wl_n^2$

Where the support is a column .... $\frac{1}{16} wl_n^2$

Shear in end members at face of first interior support .... $1.15 \frac{wl_n}{2}$

Shear at face of all other supports .... $\frac{wl_n}{2}$

8.5—Frame analysis and design—Details

8.5.1 *Arrangement of live load*

8.5.1.1 The live load may be considered to be applied only to the floor or roof under consideration, and the far ends of the columns may be assumed as fixed.

8.5.1.2 Consideration may be limited to combinations of:

(1) Design dead load on all spans with full design live load on two adjacent spans; and

(2) Design dead load on all spans with full design live load on alternate spans.

8.5.2 *Span length*

8.5.2.1 The span length of members that are not built integrally with their supports shall be considered the clear span plus the depth of the slab or beam but need not exceed the distance between centers of supports.

8.5.2.2 In analysis of continuous frames, center-to-center distances shall be used in the determination of moments. Moments at faces of support may be used for design of beams and girders.

8.5.2.3 Solid or ribbed slabs with clear spans of not more than 10 ft that are built integrally with their supports may be designed as continuous slabs on knife edge supports with spans equal to the clear spans of the slab and the width of beams otherwise neglected.

8.5.3 *Stiffness*

8.5.3.1 Any reasonable assumptions may be adopted for computing the relative flexural and torsional stiffnesses of columns, walls, floors, and roof systems. The assumptions made shall be consistent throughout the analysis.

8.5.3.2 The effect of haunches shall be considered both in determining bending moments and in design of members.

8.5.4 *Columns*

8.5.4.1 Columns shall be designed to resist the axial forces from design loads on all floors and the maximum bending due to design loads on a single adjacent span of the floor under consideration. Account shall also be taken of the loading condition giving the maximum ratio of bending moment to axial load. In building frames, particular attention shall be given to the effect of unbalanced floor loads on both exterior and interior columns and of eccentric loading due to other causes. In computing moments in columns due to gravity loading, the far ends of columns which are monolithic with the structure may be considered fixed.

8.5.4.2 Resistance to bending moments at any floor level shall be provided by distributing the moment between the columns immediately above and below the given floor in proportion to their relative stiffnesses and conditions of restraint.

8.6—Redistribution of negative moments in continuous nonprestressed flexural members*

Except where approximate values for bending moments are used, the negative moments calculated by elastic theory at the supports of continuous flexural members for any assumed loading arrangement may each be increased or decreased by not more than

$$20\left(1 - \frac{\rho - \rho'}{\rho_b}\right) \text{ percent}$$

These modified negative moments shall be used for calculations of the moments at sections within the spans. Such an adjustment shall be made only when the section, at which the moment is reduced is so designed that ρ or $\rho - \rho'$ is equal to or less than $0.50\rho_b$, where

$$\rho_b = \frac{0.85\beta_1 f_c'}{f_y} \frac{87{,}000}{87{,}000 + f_y} \qquad (8\text{-}1)$$

*For criteria on moment redistribution for prestressed concrete members, see Section 18.12.

8.7—Requirements for T-beams

8.7.1—In T-beam construction, the slab and beam shall be built integrally or otherwise effectively bonded together.

8.7.2—The effective flange width to be used in the design of symmetrical T-beams shall not exceed one-fourth of the span length of the beam, and its overhanging width on either side of the web shall not exceed eight times the thickness of the slab nor one-half the clear distance to the next beam.

8.7.3—Isolated beams in which the T-form is used only for the purpose of providing additional compression area, shall have a flange thickness not less than one-half the width of the web and a total flange width not more than four times the width of the web.

8.7.4—For beams having a flange on one side only, the effective overhanging flange width shall not exceed 1/12 of the span length of the beam, nor six times the thickness of the slab, nor one-half the clear distance to the next beam.

8.7.5—Where the principal reinforcement in a slab which is considered as the flange of a T-beam (not a joist in concrete joist floors) is parallel to the beam, transverse reinforcement shall be provided in the top of the slab. This reinforcement shall be designed to carry the design load on the portion of the slab required for the flange of the T-beam. The flange shall be assumed to act as a cantilever. The spacing of the bars shall not exceed five times the thickness of the flange nor in any case 18 in.

8.8—Concrete joist floor construction

8.8.1—Concrete joist construction consists of a monolithic combination of regularly spaced ribs and a top slab arranged to span in one direction or two orthogonal directions.

8.8.2—The joist ribs shall be at least 4 in. wide, spaced not more than 30 in. clear, and of a depth not more than three and one-half times their minimum width.

8.8.3—Ribbed slab construction not meeting the limitations of Sections 8.8.1 and 8.8.2 shall be designed as slabs and beams.

8.8.4—When permanent burned clay or concrete tile fillers of material having a unit compressive strength at least equal to that of the specified strength of the concrete in the joists are used, the vertical shells of the fillers in contact with the joists may be included in the calculations involving shear or negative bending moment. No other portion of the fillers may be included in the design calculations.

8.8.5—The thickness of the concrete slab over the permanent fillers shall be not less than 1½ in., nor less than 1/12 of the clear distance between joists. In a one-way system reinforcement shall be provided in the slab at right angles to the joists equal to that required in Section 7.13.

8.8.6—Where removable forms or fillers not complying with Section 8.8.4 are used, the thickness of the concrete slab shall not be less than 1/12 of the clear distance between joists and in no case less than 2 in. Such slab shall be reinforced at right angles to the joists with at least the amount of reinforcement required for flexure, considering load concentrations, if any, but in no case shall the reinforcement be less than that required by Section 7.13.

8.8.7—Where the slab contains conduits or pipes as allowed in Section 6.3, the thickness shall not be less than 1 in. plus the total overall depth of such conduits or pipes at any point. Such conduits or pipes shall be so located as not to impair significantly the strength of the construction.

8.8.8—The shear stress, v_c, for joists may be taken as 10 percent greater than values given in Chapter 11. Shear capacity may be increased by use of web reinforcement or by widening the ends of the joists.

8.9—Separate floor finish

A floor finish shall not be included as a part of a structural member unless it is placed monolithically with the floor slab or it meets the requirements of Chapter 17. All concrete floor finishes may be considered as part of the required cover or total thickness for nonstructural considerations.

8.10—Alternate design method*

Nonprestressed members may be designed in accordance with the following provisions. Where ϕ occurs it shall be taken as 1.

8.10.1 *Flexure in members without axial loads*—The straight-line theory of stress and strain in flexure shall be used and the following assumptions shall be made:

1. A section plane before bending remains plane after bending; strains vary as the distance from the neutral axis.

2. The stress-strain relation for concrete is a straight line under service loads within the allowable working stresses. Stresses vary as the distance from the neutral axis except for deep beams (Section 10.7).

3. The steel takes all the tension due to flexure.

4. The modular ratio, $n = E_s/E_c$, may be taken as the nearest whole number (but not less than 6). Except in calculations for deflections, the value of n for lightweight concrete shall be assumed to be the same as for normal weight concrete of the same strength. Members so de-

*For notation referred to in this section, see the section referred to or Appendix B.

signed shall be proportioned for an allowable extreme compression fiber stress in the concrete of $0.45f_c'$.

The allowable tensile stress in the reinforcement shall not be greater than 20,000 psi for Grade 40 or Grade 50 steel, and 24,000 psi for Grade 60 steel or for steels with yield strengths greater than 60,000 psi. For main reinforcement, ⅜ in. or less in diameter, in one-way slabs of not more than 12-ft span, the allowable stresses may be increased to 50 percent of the specified yield strength, but not to exceed 30,000 psi.

In doubly reinforced beams and slabs, an effective modular ratio of $2E_s/E_c$ shall be used to transform the compression reinforcement for stress computations. The allowable compressive stress in such reinforcement shall not be greater than the allowable tensile stress.

8.10.2 *Compression members with or without flexure* — The combined axial load and moment capacity of compression members shall be taken as 40 percent of that computed in accordance with the provisions of Chapter 10. The effect of the slenderness shall be considered as prescribed in Sections 10.10 and 10.11. The term P_u in Eq. (10-5) shall be replaced by 2.5 times the design axial load.

Walls shall be designed in accordance with Chapter 14 with combined axial load and moment capacity taken as 40 percent of that computed under the provisions of Chapter 14, or Section 10.16. with ϕ to be taken as 1.0 in Eq. (14-1).

8.10.3 *Shear, torsion, and bearing* — The allowable concrete stresses and the limiting maximum stresses for shear and torsion shall be 55 percent for beams, joists, walls, and one-way slabs and 50 percent for two-way slabs and footings, respectively, of the stresses given in this Code. The computed axial stress on the gross section shall be multiplied by 2 and substituted for N_u/A_g or N_u/l_wh in invoking the provisions of Section 11.4.3, 11.4.4, 11.7.6, or 11.16.2. The allowable stress in the reinforcement shall be that given in Section 8.10.1, except f_y shall be used in computing minimum areas of reinforcement by Eq. (11-1) and (11-21). In Eq. (11-26), the computed shear shall be multiplied by 2 and substituted for V_u. The allowable stresses for bearing shall be 35 percent of the stresses given in Section 10.14.

8.10.4 *Development of reinforcement* — Development of reinforcement shall be as required in Chapter 12, except that computed shears shall be multiplied by 2.0 and substituted for V_u. In computing M_t, the quantity $(d - a/2)$ may be taken as $0.85d$. Where the A_s provided is more than twice that required, the stress may be considered as always less than $0.5f_y$ for the purpose of satisfying provisions relating to splices.

8.10.5—Members may be proportioned for 75 percent of the capacities required by other parts of this section when considering wind or earthquake forces combined with other loads provided the resulting section is not less than that required for the combination of dead and live load.

8.10.6 — All other applicable provisions of the Code apply equally to this method of design, except those of Section 8.6.

CHAPTER 9—STRENGTH AND SERVICEABILITY REQUIREMENTS

9.0—Notation

A_g = gross area of section, sq in.

A_s = area of nonprestressed tension reinforcement, sq in.

A_s' = area of compression reinforcement, sq in.

d' = distance from extreme compression fiber to centroid of compression reinforcement, in.

d_s = distance from centroid of tension reinforcement to the tensile face of the member, in.

D = dead loads, or their related internal moments and forces

E = load effects of earthquake, or their related internal moments and forces

E_c = modulus of elasticity of concrete, psi. See Section 8.3.1

f_c' = specified compressive strength of concrete, psi

$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi

f_{ct} = average splitting tensile strength of lightweight aggregate concrete, psi

f_r = modulus of rupture of concrete, psi

f_y = specified yield strength of nonprestressed reinforcement, psi

F = lateral or vertical pressure of liquids, or their related internal moments and forces

h = overall thickness of member, in.

H = lateral earth pressure, or its related internal moments and forces

I_{cr} = moment of inertia of cracked section transformed to concrete

I_e = effective moment of inertia for computation of deflection

I_g = moment of inertia of gross concrete section about the centroidal axis, neglecting the reinforcement

l = span length of beam or one-way slab, as defined in Section 8.5.2; clear projection of cantilever, in.

l_n = length of clear span in long direction of two-way construction, measured face-to-face of columns in slabs without beams and face-to-face of beams or other supports in other cases

L = live loads, or their related internal moments and forces

M_a = maximum moment in member at stage for which deflection is being computed

M_{cr} = cracking moment. See Section 9.5.2.2

P_b = axial load capacity at simultaneous assumed ultimate strain of concrete and yielding of tension steel (balanced conditions)

P_u = axial design load in compression member

T = cumulative effects of temperature, creep, shrinkage, and differential settlement

U = required strength to resist design loads or their related internal moments and forces

W = wind load, or its related internal moment and forces

y_t = distance from centroidal axis of gross section, neglecting the reinforcement, to extreme fiber in tension

α = ratio of flexural stiffness of beam section to the flexural stiffness of a width of slab bounded laterally by the centerline of the adjacent panel, if any, on each side of the beam. See Chapter 13

α_m = average value of α for all beams on the edges of a panel

β = ratio of clear spans in long to short direction of two-way construction

β_s = ratio of length of continuous edges to total perimeter of a slab panel

ϕ = capacity reduction factor. See Section 9.2

9.1—General

9.1.1—Structures and structural members shall be designed to have strengths at all sections at least equal to the structural effects of the design loads and forces in such combinations as are stipulated in this Code.

9.1.2—Members shall also meet all other requirements of the Code to insure adequate performance at service load levels.

9.2—Strength

9.2.1—The strength of a member or cross section in terms of load, moment, shear, or stress shall be taken as the strength calculated in accordance with the requirements and assumptions of this Code, including a capacity reduction factor, ϕ. The following values for ϕ shall be used:

9.2.1.1 Bending in reinforced concrete, with or without axial tension, and for axial tension 0.90

9.2.1.2 Axial compression or axial compression combined with bending

(a) Reinforced members with spiral reinforcement conforming to Section 10.9.2 .... 0.75

(b) Other reinforced members 0.70

(c) With f_y not exceeding 60,000 psi the values given in (a) and (b) may be increased linearly to 0.90 as P_u decreases from $0.10f_c'A_g$ to zero for sections with symmetrical reinforcement and $(h - d' - d_s)/h$ not less than 0.70.

(d) The values given in (a) and (b) may be increased linearly to 0.90 as P_u decreases from $0.10f_c'A_g$ or P_b, whichever is smaller, to zero for sections with small axial compression not satisfying (c)

9.2.1.3 Shear and torsion 0.85

9.2.1.4 Bearing on concrete (See also Section 18.11.3) 0.70

9.2.1.5 Bending in plain concrete .. 0.65

9.2.2—Development lengths specified in Chapter 12 do not require a ϕ factor.

9.3—Required strength

9.3.1—The required strength U provided to resist dead load D and live load L shall be at least equal to

$$U = 1.4D + 1.7L \qquad (9\text{-}1)$$

9.3.2—In the design of a structure or member, if resistance to the structural effects of a specified wind load W must be included in the design, the following combinations of D, L, and W shall be investigated in determining the greatest required strength U

$$U = 0.75\,(1.4D + 1.7L + 1.7W) \qquad (9\text{-}2)$$

where the cases of L having its full value or being completely absent shall both be checked to determine the most severe condition, and

$$U = 0.9D + 1.3W \qquad (9\text{-}3)$$

but in any case the strength of the member or structure shall not be less than required by Eq. (9-1)

9.3.3—If resistance to specified earthquake loads or forces E must be included in the design, the requirements of Section 9.3.2 shall apply, except that $1.1E$ shall be substituted for W.

9.3.4—If lateral earth pressure H must be included in design, the strength U shall be at least equal to $1.4D + 1.7L + 1.7H$, but where D or L reduce the effect of H, the corresponding coefficients shall be taken as 0.90 for D and zero for L.

9.3.5—For lateral pressures from liquids F, the provisions for Section 9.3.4 shall apply, except that $1.4F$ shall be substituted for $1.7H$. The vertical pressure of liquids shall be considered as dead load, with due regard to variation in liquid depth.

9.3.6—Impact effects if any, shall be included with the live load L.

9.3.7—Where the structural effects of differential settlement, creep, shrinkage, or temperature change may be significant, they shall be included with the dead load D and the strength U shall be at least equal to $0.75\ (1.4D + 1.7L)$. Estimations of differential settlement, creep, shrinkage, or temperature change shall be based on a realistic assessment of such effects occurring in service.

9.4—Design strengths for reinforcement

Designs shall not be based on a yield strength f_y in excess of 80,000 psi, except for prestressing tendons.

9.5—Control of deflections

9.5.1 *General*—Reinforced concrete members subject to bending shall be designed to have adequate stiffness to limit deflections or any deformations which may adversely affect the strength or serviceability of the structure at service loads.

TABLE 9.5(a)—MINIMUM THICKNESS OF BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE COMPUTED*

Member	Minimum thickness, h			
	Simply supported	One end continuous	Both ends continuous	Cantilever
	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections.			
Solid one-way slabs	$l/20$	$l/24$	$l/28$	$l/10$
Beams or ribbed one-way slabs	$l/16$	$l/18.5$	$l/21$	$l/8$

*The span length l is in inches.

The values given in this table shall be used directly for nonprestressed reinforced concrete members made with normal weight concrete ($w = 145$ pcf) and Grade 60 reinforcement. For other conditions, the values shall be modified as follows:

(a) For structural lightweight concrete having unit weights in the range 90-120 lb per cu ft, the values in the table shall be multiplied by $1.65 - 0.005w$ but not less than 1.09 where w is the unit weight in lb per cu ft.

(b) For nonprestressed reinforcement having yield strengths other than 60,000 psi, the values in the table shall be multiplied by $0.4 + f_y/100{,}000$.

9.5.2 *Nonprestressed one-way construction*

9.5.2.1 Minimum thickness. The minimum thicknesses stipulated in Table 9.5(a) shall apply for one-way construction unless the computation of deflection indicates that lesser thickness may be used without adverse effects.

9.5.2.2 Computation of immediate deflection. Where deflections are to be computed, those which occur immediately on application of load shall be computed by the usual methods or formulas for elastic deflections. Unless values are obtained by a more comprehensive analysis, deflections shall be computed taking the modulus of elasticity for concrete as specified in Section 8.3.1 for normal weight or lightweight concrete and taking the effective moment of inertia as follows, but not greater than I_g.

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \quad \text{(9-4)}$$

where

$$M_{cr} = \frac{f_r I_g}{y_t} \quad \text{(9-5)}$$

and

$$f_r = 7.5\sqrt{f_c'}$$

When lightweight aggregate concretes are used, one of the following modifications shall apply:

(a) The equation for f_r shall be modified by substituting $f_{ct}/6.7$ for $\sqrt{f_c'}$, but the value of $f_{ct}/6.7$ used shall not exceed $\sqrt{f_c'}$. The value of f_{ct} shall be specified and the concrete proportioned in accordance with Section 4.2.

(b) When f_{ct} is not specified, the equation for f_r shall be multiplied by 0.75 for "all-lightweight" concrete, and 0.85 for "sand-lightweight" concrete. Linear interpolation may be used when partial sand replacement is used.

For continuous spans, the effective moment of inertia may be taken as the average of the values obtained from Eq. (9-4) for the critical positive and negative moment sections.

9.5.2.3 Computation of long-time deflection. Unless values are obtained by a more comprehensive analysis, the additional long-time deflection for both normal weight and lightweight concrete flexural members shall be obtained by multiplying the immediate deflection caused by the sustained load considered, computed in accordance with Section 9.5.2.2, by the factor

$$[2 - 1.2(A_s'/A_s)] \geq 0.6$$

9.5.2.4 Allowable deflection. The deflection computed in accordance with Sections 9.5.2.2 and 9.5.2.3 shall not exceed the limits stipulated in Table 9.5(b).

TABLE 9.5(b)—MAXIMUM ALLOWABLE COMPUTED DEFLECTIONS

Type of member	Deflection to be considered	Deflection limitation
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to the live load, L	$\frac{l^{*}}{180}$
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to the live load, L	$\frac{l}{360}$
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection which occurs after attachment of the nonstructural elements, the sum of the long-time deflection due to all sustained loads and the immediate deflection due to any additional live load†	$\frac{l^{‡}}{480}$
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		$\frac{l^{§}}{240}$

*This limit is not intended to safeguard against ponding. Ponding should be checked by suitable calculations of deflection, including the added deflections due to ponded water, and considering long-time effects of all sustained loads, camber, construction tolerances, and reliability of provisions for drainage.

†The long-time deflection shall be determined in accordance with Section 9.5.2.3 or 9.5.4.2 but may be reduced by the amount of deflection which occurs before attachment of the nonstructural elements. This amount shall be determined on the basis of accepted engineering data relating to the time-deflection characteristics of members similar to those being considered.

‡This limit may be exceeded if adequate measures are taken to prevent damage to supported or attached elements.

§But not greater than the tolerance provided for the nonstructural elements. This limit may be exceeded if camber is provided so that the total deflection minus the camber does not exceed the limitation.

9.5.3 *Nonprestressed two-way construction*

9.5.3.1 Minimum thickness. The minimum thicknesses of slabs or other two-way construction for floors designed in accordance with the provisions of Chapter 13, and having a ratio of long to short span not exceeding two, shall be governed by Eq. (9-6), (9-7), and (9-8), and the other provisions of this section

$$h = \frac{l_n\,(800 + 0.005f_y)}{36{,}000 + 5000\beta\left[\alpha_m - 0.5\,(1 - \beta_s)\left(1 + \frac{1}{\beta}\right)\right]} \tag{9-6}$$

but not less than

$$h = \frac{l_n\,(800 + 0.005f_y)}{36{,}000 + 5000\beta\,(1 + \beta_s)} \tag{9-7}$$

The thickness need not be more than

$$h = \frac{l_n\,(800 + 0.005f_y)}{36{,}000} \tag{9-8}$$

However, the thickness shall be not less than the following values:

For slabs without beams or drop panels . . 5 in.
For slabs without beams, but with drop panels satisfying Section 9.5.3.2 . . . . 4 in.
For slabs having beams on all four edges with a value of α_m at least equal to 2.0 . . . . 3½ in.

9.5.3.2 Drop panels. For slabs without beams but with drop panels extending in each direction, from the center line of support, a distance equal to at least one-sixth the span length measured from center to center of supports in that direction, and a projection below the slab of at least $h/4$, the thickness required by Eq. (9-6), (9-7), or (9-8) may be reduced by 10 percent.

9.5.3.3 Edge beams. At discontinuous edges, an edge beam shall be provided having a stiffness such that the value of α is at least 0.80, or the minimum thickness required by Eq. (9-6), (9-7) or (9-8), or Section 9.5.3.2 shall be increased by at least 10 percent in the panel having a discontinuous edge.

9.5.3.4 Computation of immediate deflection. Thicknesses less than those required in this section may be used only if it is shown by computation that the deflection will not exceed the limits in Table 9.5(b). Deflections shall be computed taking into account the size and shape of the panel, the conditions of support, and the nature of restraints at the panel edges. For such computations, the modulus of elasticity of the concrete shall be as specified in Section 8.3.1. The effective moment of inertia shall be that given by Eq. (9-4); other values may be used if they result in predictions of deflection in reasonable agreement with the results of comprehensive tests. Long-time deflections shall be computed in accordance with Section 9.5.2.3.

9.5.4 *Prestressed concrete*

9.5.4.1 Computation of immediate deflection. For prestressed concrete flexural members designed in accordance with the requirements of Chapter 18, deflections shall be calculated and the usual methods or formulas using the moment of inertia of the gross concrete section may be applied for uncracked sections.

9.5.4.2 Computation of long-time deflection. The additional long-time deflection of prestressed concrete members shall be computed taking into account the stresses in the concrete and steel under the sustained load and including the effects of creep and shrinkage of the concrete and relaxation of the steel.

9.5.4.3 Allowable deflection. The deflection computed in accordance with Sections 9.5.4.1

and 9.5.4.2 shall not exceed the limits stipulated in Table 9.5 (b).

9.5.5 *Composite members*

9.5.5.1 Shored construction. If composite members are supported during construction in such a manner that, after removal of temporary supports, the dead load is resisted by the full composite section, the composite member may be considered equivalent to a cast-in-place member for the purposes of deflection calculation. For nonprestressed members, the portion of the member in compression shall determine whether the values given in Table 9.5 (a) for normal weight or lightweight concrete shall apply. If deflection is calculated, account should be taken of the curvatures resulting from differential shrinkage of the precast and cast-in-place components, and of the axial creep effects in a prestressed concrete member.

9.5.5.2 Unshored construction. If the thickness of a nonprestressed precast member meets the requirements of Table 9.5 (a), deflection need not be computed. If the thickness of a nonprestressed composite member meets the requirements of Table 9.5 (a), deflection occurring after the member becomes composite need not be calculated, but the long-time deflection of the precast member should be investigated for the magnitude and duration of load prior to the beginning of effective composite action.

9.5.5.3 Allowable deflection. The deflection computed in accordance with the requirements of Sections 9.5.5.1 and 9.5.5.2 shall not exceed the limits stipulated in Table 9.5 (b).

CHAPTER 10—FLEXURE AND AXIAL LOADS

10.0—Notation

a = depth of equivalent rectangular stress block, defined by Section 10.2.7

A = effective tension area of concrete surrounding the main tension reinforcing bars and having the same centroid as that reinforcement, divided by the number of bars, sq in. When the main reinforcement consists of several bar sizes the number of bars shall be computed as the total steel area divided by the area of the largest bar used

A_c = area of core of spirally reinforced column measured to the outside diameter of the spiral, sq in.

A_g = gross area of section, sq in.

A_s = area of nonprestressed tension reinforcement, sq in.

A_t = area of structural steel or tubing in a composite section

A_1 = loaded area

A_2 = maximum area of the portion of the supporting surface that is geometrically similar to and concentric with the loaded area

b = width of compression face of member

c = distance from extreme compression fiber to neutral axis

C_m = a factor relating the actual moment diagram to an equivalent uniform moment diagram

d = distance from extreme compression fiber to centroid of tension reinforcement, in.

d_c = thickness of concrete cover measured from the extreme tension fiber to the center of the bar located closest thereto

e = eccentricity of design load parallel to axis measured from the centroid of the section. It may be calculated by conventional methods of frame analysis

E_c = modulus of elasticity of concrete, psi. See Section 8.3.1

E_s = modulus of elasticity of steel, psi. See Section 8.3.2

EI = flexural stiffness of compression members. See Eq. (10-7) and Eq. (10-8)

f_c' = specified compressive strength of concrete, psi

f_s = calculated stress in reinforcement at service loads, ksi

f_y = specified yield strength of nonprestressed reinforcement, psi

h = overall thickness of member, in.

I_g = moment of inertia of gross concrete section about the centroidal axis, neglecting the reinforcement

I_{se} = moment of inertia of reinforcement about the centroidal axis of the member cross section

I_t = moment of inertia of structural steel or tubing in a cross section about the centroidal axis of the member cross section

k = effective length factor for compression members

l_u = unsupported length of compression member

M_c = moment to be used for design of compression member

M_1 = value of smaller design end moment on compression member calculated from a conventional elastic frame analysis, positive if member is bent in single curvature, negative if bent in double curvature

M_2 = value of larger design end moment on compression member calculated from a conventional elastic frame analysis, always positive

P_c = critical load. See Section 10.11.5

P_u = axial design load in compression member

r = radius of gyration of the cross section of a compression member

z = a quantity limiting distribution of flexural reinforcement. See Section 10.6

β_1 = a factor defined in Section 10.2.7

β_d = the ratio of maximum design dead load moment to maximum design total load moment, always positive

δ = moment magnification factor for columns. See Section 10.11.5

ρ = A_s/bd = ratio of nonprestressed tension reinforcement

ρ_b = reinforcement ratio producing balanced conditions. See Section 10.3.3

ρ_s = ratio of volume of spiral reinforcement to total volume of core (out-to-out of spirals) of a spirally reinforced concrete or composite column

ϕ = capacity reduction factor. See Section 9.2

10.1—Scope

This chapter covers the design of members subject to flexure or to axial loads or to both flexure and axial loads.

10.2—Assumptions

10.2.1—The strength design of members for flexure and axial loads shall be based on the assumptions given in this section, and on satisfaction of the applicable conditions of equilibrium and compatibility of strains.

10.2.2—Strain in the reinforcing steel and concrete shall be assumed directly proportional to the distance from the neutral axis.

10.2.3—The maximum usable strain at the extreme concrete compression fiber shall be assumed equal to 0.003.

10.2.4—Stress in reinforcement below the specified yield strength, f_y, for the grade of steel used shall be taken as E_s times the steel strain. For strains greater than that corresponding to f_y, the stress in the reinforcement shall be considered independent of strain and equal to f_y.

10.2.5—Tensile strength of the concrete shall be neglected in flexural calculations of reinforced concrete, except when meeting the requirements of Section 18.4.

10.2.6—The relationship between the concrete compressive stress distribution and the concrete strain may be assumed to be a rectangle, trapezoid, parabola, or any other shape which results in prediction of strength in substantial agreement with the results of comprehensive tests.

10.2.7—The requirements of Section 10.2.6 may be considered satisfied by an equivalent rectangular concrete stress distribution which is defined as follows: A concrete stress of $0.85f_c'$ shall be assumed uniformly distributed over an equivalent compression zone bounded by the edges of the cross section and a straight line located parallel to the neutral axis at a distance $a = \beta_1 c$ from the fiber of maximum compressive strain. The distance c from the fiber of maximum strain to the neutral axis is measured in a direction perpendicular to that axis. The fraction β_1 shall be taken as 0.85 for strengths, f_c', up to 4000 psi and shall be reduced continuously at a rate of 0.05 for each 1000 psi of strength in excess of 4000 psi.

10.3—General principles and requirements

10.3.1—The design of cross sections subject to flexure or combined flexure and axial load shall be based on stress and strain compatibility using the assumptions in Section 10.2.

10.3.2—For flexural members, and for members under combined flexure and axial load controlled by Section 9.2.1.2(d), the reinforcement ratio, ρ, shall not exceed 0.75 of that ratio which would produce balanced conditions for the section under flexure without axial load.

10.3.3—Balanced conditions exist at a cross section when the tension reinforcement reaches its specified yield strength, f_y, just as the concrete in compression reaches its assumed ultimate strain of 0.003.

10.3.4—All cross sections subject to a compression load shall be designed for the applied moments which can accompany this loading condition, including slenderness effects according to the requirements of Sections 10.10 and 10.11.

10.3.5—Compression reinforcement in conjunction with additional tension reinforcement may be used to increase the capacity of a flexural member.

10.3.6—All members subjected to a compression load shall be designed for the eccentricity e corresponding to the maximum moment which can accompany this loading condition, but not less than 1 in., or $0.05h$ for spirally reinforced or composite steel encased compression members, or $0.10h$ for tied compression members, about either principal axis. Slenderness effects shall be included according to the requirements of Sections 10.10 and 10.11. For precast members, the minimum design eccentricity may be reduced to not less than 0.6 in. provided that the manufacturing and erection tolerances are limited to one-third of the minimum design eccentricity.

10.4—Distance between lateral supports of flexural members

The spacing of lateral supports for a beam shall not exceed 50 times the least width b of compression flange or face. Effects of lateral eccentricity of load shall be taken into account in determining the spacing of lateral supports.

10.5—Minimum reinforcement of flexural sections

10.5.1—At any section of a flexural member (except slabs of uniform thickness) where positive reinforcement is required by analysis, the ratio ρ supplied shall not be less than that given by

$$\rho_{min} = \frac{200}{f_y} \qquad (10\text{-}1)$$

unless the area of reinforcement provided at every section, positive or negative, is at least one-third greater than that required by analysis. In T-beams and joists where the stem is in tension, the ratio ρ shall be computed for this purpose using the width of the stem.

10.5.2—In structural slabs of uniform thickness, the minimum amount of reinforcement in the direction of the span shall not be less than that required for shrinkage and temperature reinforcement (see Section 7.13).

10.6—Distribution of flexural reinforcement in beams and one-way slabs

10.6.1—This section prescribes rules for the distribution of flexural reinforcement in beams and in one-way slabs; that is, slabs reinforced to resist flexural stresses in only one direction. The distribution of reinforcement in two-way slabs shall be as required in Section 13.5.

10.6.2—Only deformed reinforcement shall be used. Tension reinforcement shall be well distributed in the zones of maximum concrete tension. Where flanges are in tension, a part of the main tension reinforcement shall be distributed over the effective flange width or a width equal to one-tenth of the span, whichever is smaller. If the effective flange width exceeds one-tenth of the span, some longitudinal reinforcement shall be provided in the outer portions of the flange.

10.6.3—When the design yield strength f_y for tension reinforcement exceeds 40,000 psi, the cross sections of maximum positive and negative moment shall be so proportioned that the quantity z given by

$$z = f_s \sqrt[3]{d_c A} \qquad (10\text{-}2)$$

does not exceed the values given by Section 10.6.4. The calculated flexural stress in the reinforcement at service loads f_s, in kips per sq in., shall be computed as the bending moment divided by the product of the steel area and the internal moment arm. In lieu of such computations, f_s may be taken as 60 percent of the specified yield strength f_y.

10.6.4—The quantity z shall not exceed 175 kips per in. for interior exposure and 145 kips per in. for exterior exposure. Eq. (10-2) does not apply to structures subjected to very aggressive exposure or designed to be watertight; special precautions are required and must be investigated for such cases.

10.6.5—If the depth of the web exceeds 3 ft, longitudinal reinforcement having a total area at least equal to 10 percent of the main tension steel area shall be placed near the faces of the web and distributed in the zone of flexural tension with a spacing not more than 12 in. or the width of the web, whichever is less. Such reinforcement may be taken into account in computation of the strength only if a strain compatibility analysis is made to determine stresses in the individual bars.

10.7—Deep flexural members

Flexural members with overall depth-span ratios greater than 2/5 for continuous spans, or 4/5 for simple spans, shall be designed as deep beams taking account of nonlinear distribution of stress and lateral buckling.

The design of such members for shear effects shall be in accordance with Section 11.9. The minimum horizontal and vertical reinforcement in the faces shall be the greater of the requirements of Section 11.9.6 or Section 14.2. The minimum principal tension reinforcement shall conform to Section 10.5.

10.8—Limiting dimensions for compression members

10.8.1 *Isolated compression member with multiple spirals*—If two or more interlocking spirals are used in a compression member, the outer boundary of the compression member shall be taken at a distance outside the extreme limits of the spiral equal to the requirements of Section 7.14.1.

10.8.2 *Compression members built monolithically with wall*—For a spiral compression member built monolithically with a concrete wall or pier, the outer boundary of the compression member's section shall be taken either as a circle at least 1½ in. outside the compression member spiral or as a square or rectangle, the sides of which are at least 1½ in. outside the spiral or spirals.

10.8.3 *Equivalent circular compression members*—As an exception to the general procedure of utilizing the full gross area of the compression member section, it shall be permissible to design a circular compression member and to build it with a square, octagonal, or other shaped section of the same least lateral dimension. In such case, the allowable load, the gross area considered, and the required percentages of reinforcement shall be taken as those of the circular compression member.

10.8.4 *Limits of section*—In a compression member which has a larger cross section than required by considerations of loading, a reduced effective area A_g not less than one-half of the total area may be used for determining minimum steel area and load capacity.

10.9—Limits for reinforcement of compression members

10.9.1—The longitudinal reinforcement for non-composite compression members shall be not less than 0.01 nor more than 0.08 times the gross area of the section. The minimum number of longitudinal reinforcing bars in compression members shall be six for bars in a circular arrangement and four for bars in a rectangular arrangement.

10.9.2—The ratio of spiral reinforcement ρ_s shall be not less than the value given by

$$\rho_s = 0.45 \left(\frac{A_g}{A_c} - 1\right) \frac{f_c'}{f_y} \qquad (10\text{-}3)$$

where f_y is the specified yield strength of spiral reinforcement but not more than 60,000 psi.

10.10—Slenderness effects in compression members

10.10.1—The design of compression members shall be based on forces and moments determined from an analysis of the structure. Such an analysis shall take into account the influence of axial loads and variable moment of inertia on member stiffness and fixed-end moments, the effect of deflections on the moments and forces, and the effects of the duration of the loads.

10.10.2—In lieu of the procedure described in Section 10.10.1, the design of compression members may be based on the approximate procedure presented in Section 10.11. The detailed requirements of Section 10.11 do not need to be applied if design is carried out according to Section 10.10.1.

10.11—Approximate evaluation of slenderness effects

10.11.1—The unsupported length l_u of a compression member shall be taken as the clear distance, between floor slabs, girders, or other members capable of providing lateral support for the compression member. Where capitals or haunches are present, the unsupported length shall be measured to the lower extremity of the capital or haunch in the plane considered.

10.11.2—The radius of gyration r may be taken equal to 0.30 times the overall dimension in the direction in which stability is being considered for rectangular compression members, and 0.25 times the diameter for circular compression members. For other shapes, r may be computed for the gross concrete section.

10.11.3—For compression members braced against sidesway, the effective length factor k shall be taken as 1.0, unless an analysis shows that a lower value may be used. For compression members not braced against sidesway, the effective length factor k shall be determined with due consideration of cracking and reinforcement on relative stiffness, and shall be greater than 1.0.

10.11.4—For compression members braced against sidesway, the effects of slenderness may be neglected when kl_u/r is less than $34 - 12M_1/M_2$. For compression members not braced against sidesway, the effects of slenderness may be neglected when kl_u/r is less than 22. For all compression members with kl_u/r greater than 100, an analysis as defined in Section 10.10.1 shall be made.

10.11.5—Compression members shall be designed using the design axial load from a conventional frame analysis and a magnified moment M_c defined by Eq. (10-4).

$$M_c = \delta M_2 \qquad (10\text{-}4)$$

where

$$\delta = \frac{C_m}{1 - \dfrac{P_u}{\phi P_c}} \geq 1.0 \qquad (10\text{-}5)$$

and

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} \qquad (10\text{-}6)$$

In lieu of a more precise calculation, EI in Eq. (10-6) may be taken either as

$$EI = \frac{\dfrac{E_c I_g}{5} + E_s I_{se}}{1 + \beta_d} \qquad (10\text{-}7)$$

or conservatively

$$EI = \frac{\dfrac{E_c I_g}{2.5}}{1 + \beta_d} \qquad (10\text{-}8)$$

In Eq. (10-5), for members braced against sidesway and without transverse loads between supports C_m may be taken as

$$C_m = 0.6 + 0.4\,\frac{M_1}{M_2} \qquad (10\text{-}9)$$

but not less than 0.4.

For all other cases C_m shall be taken as 1.0.

10.11.5.1 In frames not braced against sidesway, the value of δ shall be computed for the entire story assuming all columns to be loaded.

In Eq. (10-5), P_u and P_c shall be taken as the summation of ΣP_u and ΣP_c for all of the columns in the story. In designing each column within the story, δ shall be taken as the larger value computed for the entire story or computed for the individual column assuming its ends to be braced against sidesway.

10.11.5.2 When compression members are subject to bending about both principal axes, the moment about each axis shall be amplified by δ, computed from the corresponding conditions of restraint about that axis.

10.11.6—When design of compression members is governed by the minimum eccentricities specified in Section 10.3.6, M_2 in Eq. (10-4) shall be based on the specified minimum eccentricity, with conditions of curvature determined by either of the following:

(a) When the actual computed eccentricities are less than the specified minimum, the computed end moments may be used to evaluate the conditions of curvature.

(b) If computations show that there is no eccentricity at both ends of the member, conditions of curvature shall be based on a ratio of M_1/M_2 equal to one.

10.11.7—In structures which are not braced against sidesway, the flexural members shall be designed for the total magnified end moments of the compression members at the joint.

10.12—Axially loaded members supporting flat slabs

All axially loaded members supporting flat slabs shall be designed as provided in this chapter and in accordance with the additional requirements of Chapter 13.

10.13—Transmission of column loads through floor system

When the specified strength of concrete in columns exceeds that specified for the floor system by more than 40 percent, transmission of load shall be provided by one of the following:

(a) Concrete of the strength specified for the column shall be placed in the floor for an area four times the column areas about the column, well integrated into floor concrete, and placed in accordance with Section 6.4.2.

(b) The capacity of the column through the floor system shall be computed using the weaker concrete strength with vertical dowels and spirals as required.

(c) For columns laterally supported on four sides by beams of approximately equal depth or by slabs, the capacity may be computed by using an assumed concrete strength in the column formulas equal to 75 percent of the column concrete strength plus 35 percent of the floor concrete strength.

10.14—Bearing

10.14.1—Bearing stresses shall not exceed $0.85\phi f_c'$, except as provided below.

10.14.2—When the supporting surface is wider on all sides than the loaded area, the permissible bearing stress on the loaded area may be multiplied by $\sqrt{A_2/A_1}$, but not more than 2.

10.14.3—When the supporting surface is sloped or stepped, A_2 may be taken as the area of the lower base of the largest frustrum of a right pyramid or cone contained wholly within the support and having for its upper base the loaded area, and having side slopes of 1 vertical to 2 horizontal.

10.14.4—This section does not apply to post-tensioning anchorages.

10.15—Composite compression members

10.15.1—Composite compression members shall include all concrete compression members reinforced longitudinally with structural steel shape, pipe, or tubing with or without longitudinal bars.

10.15.2—The strength of composite compression members shall be computed for the same limiting conditions applicable to ordinary reinforced concrete members. Any direct compression load capacity assigned to the concrete in a member must be transferred to the concrete by members or brackets in direct bearing on the compression member concrete. All compression load capacity not assigned to the concrete shall be developed by direct connection to the structural steel shape, pipe, or tube.

10.15.3—For slenderness calculations, the radius of gyration of the composite section shall be not greater than the value given by

$$r = \sqrt{\frac{\frac{1}{5}E_cI_g + E_sI_t}{\frac{1}{5}E_cA_g + E_sA_t}} \qquad (10\text{-}10)$$

For computing P_c in Eq. (10-6), EI of the composite section shall be not greater than

$$EI = \frac{\frac{E_cI_g}{5} + E_sI_t}{1 + \beta_d} \qquad (10\text{-}11)$$

10.15.4—Where the composite compression member consists of a steel encased concrete core, the thickness of the steel encasement shall be greater than

$$b\sqrt{\frac{f_y}{3E_s}}, \text{ for each face of width } b \qquad (10\text{-}12)$$

or

$$h\sqrt{\frac{f_y}{8E_s}}, \text{ for circular sections of diameter } h \quad (10\text{-}13)$$

Longitudinal bars within the encasement may be considered in computing A_t and I_t.

10.15.5—Where the composite compression member consists of a spiral bound concrete encasement around a structural steel core, f_c' shall be not less than 2500 psi and spiral reinforcement shall conform to Eq. (10-3). The design yield strength of the structural steel core shall be the specified minimum yield strength for the grade of structural steel used but not to exceed 50,000 psi. Longitudinal reinforcing bars within the spiral shall be not less than 0.01 nor more than 0.08 times the net concrete section and may be considered in computing A_t and I_t.

10.15.6—Where the composite compression member consists of laterally tied concrete around a structural steel core, f_c' shall be not less than 2500 psi and the design yield strength of the structural steel core shall be the specified minimum yield strength for the grade of structural steel used but not to exceed 50,000 psi. Lateral ties shall extend completely around the steel core. Lateral ties shall be #5 bars, or smaller bars having a diameter not less than 1/50 the longest side or diameter of the cross section, but not smaller than #3. The vertical spacing of lateral ties shall not exceed one-half the least width of the cross section, or 48 tie bar diameters, or 16 longitudinal bar diameters. Welded wire fabric of equivalent area may be used.

Longitudinal reinforcing bars within the ties, not less than 0.01 nor more than 0.08 times the net concrete section, shall be provided. These shall be spaced not greater than one-half the least width of the cross section. A longitudinal bar shall be placed at each corner of a rectangular cross section. Bars placed within the lateral ties may be considered in computing A_t for strength calculations but not I_t for slenderness calculations.

10.16—Special provisions for walls

10.16.1—Walls may be designed under the provisions of this chapter with the limitations and exceptions of this section, or under Chapter 14.

10.16.2—The minimum ratio of vertical reinforcement to gross concrete area shall be:

(a) 0.0012 for deformed bars not larger than #5 and with a specified yield strength of 60,000 psi or greater, or

(b) 0.0015 for other deformed bars, or

(c) 0.0012 for welded wire fabric not larger than ⅝ in. in diameter.

10.16.3—Vertical reinforcement shall be spaced not farther apart than three times the wall thickness nor 18 in.

10.16.4—Vertical reinforcement need not be provided with lateral ties if such reinforcement is 0.01 times the gross concrete area or less, or where such reinforcement is not required as compression reinforcement.

10.16.5—The minimum ratio of horizontal reinforcement to gross concrete area shall be:

(a) 0.0020 for deformed bars not larger than #5 and with a specified yield strength of 60,000 psi or greater or

(b) 0.0025 for other deformed bars, or

(c) 0.0020 for welded wire fabric not larger than ⅝ in. in diameter.

10.16.6—Horizontal reinforcement shall be spaced not farther apart than one and one-half times the wall thickness nor 18 in.

CHAPTER 11—SHEAR AND TORSION

11.0 — Notation

a = shear span, distance between concentrated load and face of support

A_g = gross area of section, sq in.

A_h = area of shear reinforcement parallel to the main tension reinforcement, sq in.

A_l = total area of longitudinal reinforcement to resist torsion, sq in.

A_{ps} = area of prestressed reinforcement in tension zone

A_s = area of nonprestressed tension reinforcement, sq in.

A_t = area of one leg of a closed stirrup resisting torsion within a distance s, sq in.

A_v = area of shear reinforcement within a distance s, or area of shear reinforcement perpendicular to main reinforcement within a distance s for deep beams, sq in.

A_{vf} = area of shear-friction reinforcement, sq in.

A_{vh} = area of shear reinforcement parallel to the main tension reinforcement within a distance s_2, sq in.

b = width of compression face of member

b_o = periphery of critical section for slabs and footings

b_w = web width, or diameter of circular section, in.

c_1 = size of rectangular or equivalent rectangular column, capital, or bracket measured in the direction in which moments are being determined

c_2 = size of rectangular or equivalent rectangular column, capital, or bracket measured transverse to the direction in which moments are being determined

d = distance from extreme compression fiber to centroid of tension reinforcement, in.

f_c' = specified compressive strength of concrete, psi

$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi

f_{ct} = average splitting tensile strength of lightweight aggregate concrete, psi

f_d = stress due to dead load, at the extreme fiber of a section at which tensile stresses are caused by applied load, psi

f_{pc} = compressive stress in the concrete, after all prestress losses have occurred, at the centroid of the cross section resisting the applied loads or at the junction of the web and flange when the centroid lies in the flange, psi. (In a composite member, f_{pc} will be the resultant compressive stress at the centroid of the composite section, or at the junction of the web and flange when the centroid lies within the flange, due to both prestress and to the bending moments resisted by the precast member acting alone)

f_{pe} = compressive stress in concrete due to prestress only after all losses, at the extreme fiber of a section at which tensile stresses are caused by applied loads, psi

f_{pu} = ultimate strength of prestressing steel, psi

f_y = specified yield strength of nonprestressed reinforcement, psi

h = overall thickness of member, in.

h_v = total depth of shearhead cross section

h_w = total height of wall from its base to its top

I = moment of inertia of section resisting externally applied design loads

l_n = clear span measured face-to-face of supports

l_v = length of shearhead arm from centroid of concentrated load or reaction

l_w = horizontal length of wall

M_{cr} = cracking moment. See Section 9.5.2.2

M_m = modified bending moment

M_{max} = maximum bending moment due to externally applied design loads

M_p = required full plastic moment of shearhead cross section

M_u = applied design load moment at a section, in.-lb

M_v = moment resistance contributed by shearhead reinforcement

N_u = design axial load normal to the cross section occurring simultaneously with V_u to be taken as positive for compression, negative for tension, and to include the effects of tension due to shrinkage and creep

N_u = design tensile force on bracket or corbel acting simultaneously with V_u

s = shear or torsion reinforcement spacing in a direction parallel to the longitudinal reinforcement

s_1 = spacing of vertical reinforcement in a wall

s_2 = shear or torsion reinforcement spacing in a direction perpendicular to the longitudinal reinforcement—or spacing of horizontal reinforcement in a wall

T_u = design torsional moment

v_c = nominal permissible shear stress carried by concrete

v_{ci} = shear stress at diagonal cracking due to all design loads, when such cracking is the result of combined shear and moment

v_{cw} = shear stress at diagonal cracking due to all design loads, when such cracking is the result of excessive principal tensile stresses in the web

v_{tc} = nominal permissible torsion stress carried by concrete

v_{tu} = nominal total design torsion stress

v_u = nominal total design shear stress

V_d = shear force at section due to dead load

V_i = shear force at section occurring simultaneously with M_{max}

V_p = vertical component of the effective prestress force at the section considered

V_u = total applied design shear force at section

x = shorter overall dimension of a rectangular part of a cross section

x_1 = shorter center-to-center dimension of a closed rectangular stirrup

y = longer overall dimension of a rectangular part of a cross section

y_t = distance from the centroidal axis of gross section, neglecting the reinforcement, to the extreme fiber in tension

y_1 = longer center-to-center dimension of a closed rectangular stirrup

α = angle between inclined web bars and longitudinal axis of member

α_t = a coefficient as a function of y_1/x_1. See Section 11.8.2

α_v = ratio of stiffness of shearhead arm to surrounding composite slab section. See Section 11.11.2

μ = coefficient of friction. See Section 11.15

ρ = A_s/bd = ratio of nonprestressed tension reinforcement

ρ_h = the ratio of horizontal shear reinforcement area to the gross concrete area of a vertical section

ρ_n = the ratio of vertical shear reinforcement area to the gross concrete area of a horizontal section

ρ_v = $(A_s + A_h)/bd$

ρ_w = A_s/b_wd

ϕ = capacity reduction factor. See Section 9.2

11.1—General reinforcement requirements

11.1.1—A minimum area of shear reinforcement shall be provided in all reinforced, prestressed, and nonprestressed concrete flexural members except:

(a) Slabs and footings

(b) Concrete joist floor construction defined by Section 8.8

(c) Beams where the total depth does not exceed 10 in., two and one-half times the thickness of the flange, or one-half the width of the web, whichever is greater

(d) Where v_u is less than one-half of v_c

This requirement may be waived if it is shown by test that the required ultimate flexural and shear capacity can be developed when shear reinforcement is omitted.

11.1.2—Where shear reinforcement is required by Section 11.1.1 or by calculations, and the nominal torsion stress v_{tu} does not exceed $1.5\sqrt{f_c'}$, the minimum area in square inches shall be

$$A_v = 50\frac{b_ws}{f_y} \tag{11-1}$$

for prestressed and nonprestressed members where b_w and s are in inches. Alternatively, a minimum area

$$A_v = \frac{A_{ps}}{80}\frac{f_{pu}}{f_y}\frac{s}{d}\sqrt{\frac{d}{b_w}} \tag{11-2}$$

may be used for prestressed members having an effective prestress force at least equal to 40 percent of tensile strength of the flexural reinforcement.

Where the nominal torsion stress v_{tu} is greater than $1.5\sqrt{f_c'}$, and where web reinforcement is required by Section 11.1.1 or by calculations, the minimum area of closed stirrups provided shall be

$$A_v + 2A_t = 50\frac{b_ws}{f_y}$$

11.1.3—The design yield strength of shear and torsion reinforcement shall not exceed 60,000 psi.

11.1.4—Shear reinforcement may consist of:

(a) Stirrups perpendicular to the axis of the member

(b) Welded wire fabric with wires located perpendicular to the axis of the member

Where shear reinforcement is required and is placed perpendicular to the axis of the member, it shall be spaced not further apart than $0.50d$ in nonprestressed concrete and $0.75h$ in prestressed concrete, but not more than 24 in.

11.1.5—For reinforced concrete members without prestressing, shear reinforcement may also consist of:

(a) Stirrups making an angle of 45 deg or more with the longitudinal tension bars

(b) Longitudinal bars with a bent portion making an angle of 30 deg or more with the longitudinal tensile bars

(c) Combinations of stirrups and bent bars

(d) Spirals

Inclined stirrups and bent bars shall be so spaced that every 45 deg line, extending toward the reaction from the middepth of the member, $0.50d$, to the longitudinal tension bars, shall be crossed by at least one line of web reinforcement.

11.1.6—Torsion reinforcement where required by Section 11.7 shall consist of closed stirrups, closed ties, or spirals combined with longitudinal bars.

11.1.7—Stirrups and other bars or wires used as shear or torsion reinforcement shall extend to a distance d from the extreme compression fiber and shall be anchored at both ends according to Sections 7.1 and 12.13 to develop the design yield strength of the reinforcement.

11.2—Shear strength

11.2.1—The nominal shear stress v_u shall be computed by:

$$v_u = \frac{V_u}{\phi b_wd} \tag{11-3}$$

The distance d shall be taken from the extreme compression fiber to the centroid of the longitudinal tension reinforcement, but need not be taken less than $0.80h$ for prestressed concrete members. For circular sections, d need not be taken less than the distance from the extreme compression fiber to the centroid of the longitudinal reinforcement in the opposite half of the member.

11.2.2—When the reaction, in the direction of the applied shear, introduces compression into the end region of the member, sections located less than a distance d from the face of the support may be designed for the same v_u as that computed at a distance d; for prestressed concrete, sections located at a distance less than $h/2$ may be designed for the shear computed at $h/2$.

11.2.3—The shear stress carried by the concrete, v_c, shall be calculated according to Section 11.4 or 11.5. Wherever applicable, the effects of inclined flexural compression in variable-depth members may be included, and effects of axial tension due to restrained shrinkage and creep shall be considered.

11.2.4—When v_u exceeds v_c, shear reinforcement shall be provided according to Section 11.6.

11.2.5—For deep beams, slabs, walls, brackets, and corbels the special provisions of Sections 11.9 through 11.16 shall apply.

11.3—Lightweight concrete shear and torsion stresses

The provisions of this chapter for nominal shear stress v_c and nominal torsion stress v_{tc} carried by the concrete apply to normal weight concrete. When lightweight aggregate concretes are used, one of the following modifications shall apply:

11.3.1—The provisions for v_c and v_{tc} shall be modified by substituting $f_{ct}/6.7$ for $\sqrt{f_c'}$, but the value of $f_{ct}/6.7$ used shall not exceed $\sqrt{f_c'}$. The value of f_{ct} shall be specified and the concrete proportioned in accordance with Section 4.2.

11.3.2—When f_{ct} is not specified, all values of $\sqrt{f_c'}$ affecting v_c, v_{tc}, and M_{cr} shall be multiplied by 0.75 for "all-lightweight" concrete, and 0.85 for "sand-lightweight" concrete. Linear interpolation may be used when partial sand replacement is used.

11.4—Nominal permissible shear stress for nonprestressed concrete members

11.4.1—The shear stress carried by the concrete, v_c, shall not exceed $2\sqrt{f_c'}$ unless a more detailed analysis is made in accordance with Section 11.4.2 or 11.4.3. For members subjected to axial load or torsion, v_c shall not exceed values given in Sections 11.4.3 through 11.4.5.

11.4.2—The nominal shear stress v_c shall not exceed:

$$v_c = 1.9\sqrt{f_c'} + 2500\rho_w \frac{V_u d}{M_u} \qquad (11\text{-}4)$$

but v_c shall not be greater than $3.5\sqrt{f_c'}$. M_u is the bending moment occurring simultaneously with V_u at the section considered, but $V_u d/M_u$ shall not be taken greater than 1.0 in computing v_c from Eq. (11-4).

11.4.3—For members subjected to axial compression, Eq. (11-4) may be used, except that M_m shall be substituted for M_u, and M_m shall be permitted to have values less than $V_u d$

$$M_m = M_u - N_u \frac{(4h - d)}{8} \qquad (11\text{-}5)$$

Alternatively, v_c may be computed by:

$$v_c = 2\left(1 + 0.0005 \frac{N_u}{A_g}\right)\sqrt{f_c'} \qquad (11\text{-}6)$$

However, v_c shall not exceed:

$$v_c = 3.5\sqrt{f_c'}\sqrt{1 + 0.002 \frac{N_u}{A_g}} \qquad (11\text{-}7)$$

The quantity N_u/A_g shall be expressed in psi.

11.4.4—For members subjected to significant axial tension, web reinforcement shall be designed to carry the total shear, unless a more detailed analysis is made using

$$v_c = 2\left(1 + 0.002 \frac{N_u}{A_g}\right)\sqrt{f_c'} \qquad (11\text{-}8)$$

where N_u is negative for tension. The quantity N_u/A_g shall be expressed in psi.

11.4.5—At cross sections subjected to a nominal torsion stress, v_{tu}, exceeding $1.5\sqrt{f_c'}$, computed by Eq. (11-16), v_c shall not exceed

$$v_c = \frac{2\sqrt{f_c'}}{\sqrt{1 + \left(\frac{v_{tu}}{1.2 v_u}\right)^2}} \qquad (11\text{-}9)$$

11.5—Nominal permissible shear stress for prestressed concrete members

11.5.1—For members having an effective prestress force at least equal to 40 percent of the tensile strength of the flexural reinforcement, unless a more detailed analysis is made in accordance with Section 11.5.2, the nominal shear stress carried by the concrete, v_c, shall not exceed

$$v_c = 0.6\sqrt{f_c'} + 700 \frac{V_u d}{M_u} \qquad (11\text{-}10)$$

but v_c need not be taken less than $2\sqrt{f_c'}$ nor shall v_c be greater than $5\sqrt{f_c'}$. M_u is the bending moment occurring simultaneously with V_u, but $V_u d/M_u$ shall not be taken greater than 1.0. When applying Eq. (11-10), d shall be the distance from the extreme compression fiber to the centroid of the prestressing tendons.

11.5.2—Except as allowed in Section 11.5.1 the shear stress v_c shall be computed as the lesser of v_{ci} or v_{cw}.

$$v_{ci} = 0.6\sqrt{f_c'} + \frac{V_d + \left(\frac{V_i M_{cr}}{M_{max}}\right)}{b_w d} \qquad (11\text{-}11)$$

but need not be taken less than $1.7\sqrt{f_c'}$, where

$$M_{cr} = (I/y_t)\ (6\sqrt{f_c'} + f_{pe} - f_d)$$

$$v_{cw} = 3.5\sqrt{f_c'} + 0.3 f_{pc} + \frac{V_p}{b_w d} \qquad (11\text{-}12)$$

Alternatively, v_{cw} may be taken as the shear stress corresponding to a multiple of dead load plus live load which results in a computed principal tensile stress of $4\sqrt{f_c'}$ at the centroidal axis of the member, or at the intersection of the flange and the web when the centroidal axis is in the flange. In a composite member, the principal tensile stress shall be computed using the cross section which resists live load.

11.5.2.1 In Eq. (11-11) and (11-12), d shall be the distance from the extreme compression fiber to the centroid of the prestressing tendons or $0.8h$, whichever is greater.

11.5.2.2 The values of M_{max} and V_i in Eq. (11-11) shall be computed from the load distribution causing maximum moment to occur at the section.

11.5.3—In a pretensioned member in which the section at a distance $h/2$ from the face of the support is closer to the end of the beam than the transfer length of the tendons, the reduced prestress shall be considered when calculating v_{cw}. This value of v_{cw} shall also be taken as the maximum limit for Eq. (11-10). The prestress force may be assumed to vary linearly from zero at the end of the tendon to a maximum at a distance from the end of the tendon equal to the transfer length, assumed to be 50 diameters for strand and 100 diameters for single wire.

11.6—Design of shear reinforcement

11.6.1—Shear reinforcement shall conform to the general requirements of Section 11.1. When shear reinforcement perpendicular to the longitudinal axis is used, the required area of shear reinforcement shall be not less than

$$A_v = \frac{(v_u - v_c)\, b_w s}{f_y} \qquad \text{(11-13)}$$

11.6.2—When inclined stirrups or bent bars are used as shear reinforcement in reinforced concrete members, the following provisions apply:

11.6.2.1 When inclined stirrups are used, the required area shall be not less than

$$A_v = \frac{(v_u - v_c)\, b_w s}{f_y\,(\sin\alpha + \cos\alpha)} \qquad \text{(11-14)}$$

11.6.2.2 When shear reinforcement consists of a single bar or a single group of parallel bars, all bent up at the same distance from the support, the required area shall be not less than

$$A_v = \frac{(v_u - v_c)\, b_w d}{f_y \sin\alpha} \qquad \text{(11-15)}$$

in which $(v_u - v_c)$ shall not exceed $3\sqrt{f_c'}$.

11.6.2.3 When shear reinforcement consists of a series of parallel bent-up bars or groups of parallel bent-up bars at different distances from the support, the required area shall be not less than that computed by Eq. (11-14).

11.6.2.4 Only the center three-fourths of the inclined portion of any longitudinal bar that is bent shall be considered effective for shear reinforcement.

11.6.2.5 Where more than one type of shear reinforcement is used to reinforce the same portion of the web, the required area shall be computed as the sum for the various types separately. In such computations, v_c shall be included only once.

11.6.3—When $(v_u - v_c)$ exceeds $4\sqrt{f_c'}$, the maximum spacings given in Sections 11.1.4 and 11.1.5 shall be reduced by one-half.

11.6.4—The value of $(v_u - v_c)$ shall not exceed $8\sqrt{f_c'}$.

11.7—Combined torsion and shear for nonprestressed members

11.7.1—Torsion effects shall be included for shear and bending whenever the nominal torsion stress v_{tu} exceeds $1.5\sqrt{f_c'}$. Otherwise, torsion effects may be neglected.

11.7.2—For members with rectangular or flanged sections, v_{tu} shall be computed by

$$v_{tu} = \frac{3T_u}{\phi \Sigma x^2 y} \qquad \text{(11-16)}$$

The sum $\Sigma x^2 y$ shall be taken for the component rectangles of the section, but the overhanging flange width used in design shall not exceed three times the thickness of the flange.

11.7.3—A rectangular box section may be taken as a solid section, provided that the wall thickness h is at least $x/4$. A box section with a wall thickness less than $x/4$, but greater than $x/10$, may also be taken as a solid section except that $\Sigma x^2 y$ shall be multiplied by $4h/x$. When h is less than $x/10$, the stiffness of the wall shall be considered. Fillets shall be provided at interior corners of all box sections.

11.7.4—Sections located less than a distance d from the face of the support may be designed for the same torsion, v_{tu}, as that computed at a distance d.

11.7.5—The nominal torsion stress carried by the concrete, v_{tc}, in reinforced concrete members shall not exceed

$$v_{tc} = \frac{2.4\sqrt{f_c'}}{\sqrt{1 + \left(1.2\,\dfrac{v_u}{v_{tu}}\right)^2}} \qquad \text{(11-17)}$$

11.7.6—For members subjected to significant axial tension, torsion reinforcement shall be designed to carry the total torque, unless a more detailed analysis is made in which v_{tc} given by Eq. (11-17) and v_c given by Eq. (11-9) shall be multiplied by $(1 + 0.002N_u/A_g)$, where N_u is negative for tension.

11.7.7—The torsion stress v_{tu} shall not exceed

$$\frac{12\sqrt{f_c'}}{\sqrt{1+\left(1.2\,\frac{v_u}{v_{tu}}\right)^2}} \qquad (11\text{-}18)$$

11.8—Design of torsion reinforcement

11.8.1—Torsion reinforcement, where required, shall be provided in addition to reinforcement required to resist shear, flexure, and axial forces. The reinforcement required for torsion may be combined with that required for other forces, provided the area furnished is the sum of the individually required areas and the most restrictive requirements for spacing and placement are met.

11.8.2—The required area of closed stirrups shall be computed by

$$A_t = \frac{(v_{tu} - v_{tc})\,s\Sigma x^2 y}{3\,\alpha_t\,x_1\,y_1\,(f_y)} \qquad (11\text{-}19)$$

where $\alpha_t = [0.66 + 0.33\,(y_1/x_1)]$, but not more than 1.50.

11.8.3—The spacing of closed stirrups shall not exceed $(x_1 + y_1)/4$, or 12 in., whichever is the smaller.

11.8.4—The required area of longitudinal bars shall be computed by

$$A_l = 2A_t\,\frac{x_1 + y_1}{s} \qquad (11\text{-}20)$$

or by

$$A_l = \left[\frac{400xs}{f_y}\left(\frac{v_{tu}}{v_{tu}+v_u}\right) - 2A_t\right]\left(\frac{x_1+y_1}{s}\right) \qquad (11\text{-}21)$$

whichever is the greater. The value of A_l computed by Eq. (11-21) need not exceed that obtained by substituting

$$\frac{50b_w s}{f_y} \text{ for } 2A_t$$

11.8.5—The spacing of longitudinal bars, not less than #3 in size, distributed around the perimeter of the stirrups, shall not exceed 12 in. At least one longitudinal bar shall be placed in each corner of the stirrups.

11.8.6—Torsion reinforcement shall be provided at least a distance $(d+b)$ beyond the point theoretically required.

11.9—Special provisions for deep beams

11.9.1—These provisions apply when l_n/d is less than 5 and the members are loaded at the top or compression face.

11.9.2—The nominal shear stress v_c carried by the concrete shall be determined by

$$v_c = \left(3.5 - 2.5\,\frac{M_u}{V_u d}\right) \times \left(1.9\sqrt{f_c'} + 2500\,\rho_w\,\frac{V_u d}{M_u}\right) \qquad (11\text{-}22)$$

except that the term

$$\left(3.5 - 2.5\frac{M_u}{V_u d}\right)$$

shall not exceed 2.5, and v_c shall not exceed $6\sqrt{f_c'}$. M_u and V_u are the bending moment and shear occurring simultaneously at the critical section defined by Section 11.9.3. In lieu of Eq. (11-22), v_c may be taken as $2\sqrt{f_c'}$.

11.9.3—The critical section for shear measured from the face of the support shall be taken at 0.15 l_n for uniformly loaded beams and $0.50a$ for beams with concentrated loads, but not greater than d. Shear reinforcement required at the critical section shall be used throughout the span.

11.9.4—The shear stress v_u shall not exceed $8\sqrt{f_c'}$ when l_n/d is less than 2. When l_n/d is between 2 and 5, v_u shall not exceed

$$v_u = \frac{2}{3}\left(10 + \frac{l_n}{d}\right)\sqrt{f_c'} \qquad (11\text{-}23)$$

11.9.5—The area of shear reinforcement shall be computed from

$$\frac{A_v}{s}\left(\frac{1+\frac{l_n}{d}}{12}\right) + \frac{A_{vh}}{s_2}\left(\frac{11-\frac{l_n}{d}}{12}\right) = \frac{(v_u - v_c)\,b_w}{f_y} \qquad (11\text{-}24)$$

11.9.6—The area of shear reinforcement A_v perpendicular to the main reinforcement shall not be less than $0.0015bs$, and s shall not exceed $d/5$ or 18 in. The area of shear reinforcement A_{vh} parallel to the main reinforcement shall not be less than $0.0025bs_2$, and s_2 shall not exceed $d/3$ or 18 in.

11.10—Special provisions for slabs and footings

11.10.1—The shear strength of slabs and footings in the vicinity of concentrated loads or reactions is governed by the more severe of two conditions:

(a) The slab or footing acting essentially as a wide beam, with a potential diagonal crack extending in a plane across the entire width. This case shall be considered in accordance with Sections 11.1 through 11.6.

(b) Two-way action for the slab or footing, with potential diagonal cracking along the surface of a truncated cone or pyramid around the concentrated load or reaction. In this case, the slab or footing shall be designed as specified in Sections 11.10.2 and 11.10.3.

11.10.2—The critical section for two-way action shall be perpendicular to the plane of the slab and located so that its periphery is a minimum and approaches no closer than $d/2$ to the periphery of the concentrated load or reaction area.

11.10.3—The nominal shear stress for two-way action shall be computed by

$$v_u = \frac{V_u}{\phi b_o d} \qquad (11\text{-}25)$$

in which V_u and b_o are taken at the critical section specified in Section 11.10.2. The shear stress v_u shall not exceed $v_c = 4\sqrt{f_c'}$ unless shear reinforcement is provided. A maximum increase of 50 percent in v_u is permitted if shear reinforcement is provided in accordance with Section 11.11.1, and a maximum increase of 75 percent is permitted if shearhead reinforcement is provided in accordance with Section 11.11.2.

11.11—Shear reinforcement in slabs and footings

11.11.1—Shear reinforcement consisting of bars or wires anchored in accordance with Section 12.13 may be provided in slabs. For design of such shear reinforcement, shear stresses shall be investigated at the critical section defined in Section 11.10.2 and at successive sections more distant from the support; and the shear stress, v_c, carried by the concrete at any section shall not exceed $2\sqrt{f_c'}$. Where v_u exceeds v_c, the shear reinforcement shall be provided according to Section 11.6.

11.11.2—Shear reinforcement within the slab consisting of steel I or channel shapes shall be designed in accordance with the following provisions, which do not apply where shear is transferred to a column at an edge or a corner of a slab. At exterior columns, special designs are required.

11.11.2.1 Each shearhead shall consist of steel shapes fabricated by welding into four identical arms at right angles and continuous through the column section. The ends of shearheads may be cut at angles not less than 30 deg with the horizontal, provided that the plastic moment capacity of the remaining tapered section is adequate to resist the shear force attributed to that arm of the shearhead. The ratio α_v between the stiffness for each shearhead arm and that for the surrounding composite cracked slab section of width $(c_2 + d)$ shall not be less than 0.15. All compression flanges of the steel shapes shall be located within $0.3d$ of the compression surface of the concrete slab. The steel shapes shall not be deeper than 70 times their web thickness.

11.11.2.2 The full plastic moment of resistance M_p required for each arm of the shearhead shall be computed by

$$M_p = \frac{V_u}{\phi 8}\left[h_v + \alpha_v\left(l_v - \frac{c_1}{2}\right)\right] \qquad (11\text{-}26)$$

where ϕ is the capacity reduction factor for flexure and l_v is the minimum length of each shearhead arm required to comply with the requirements of Sections 11.11.2.3 and 11.11.2.4.

11.11.2.3 The critical slab section shall be perpendicular to the plane of the slab. The section shall cross each shearhead arm three-quarters of the distance, $l_v - (c_1/2)$, from the column face to the end of the shearhead, and it shall be so located that its periphery is a minimum. However, the critical section need not approach closer than $d/2$ to the periphery of the column.

11.11.2.4 The shear stress v_u shall not exceed $4\sqrt{f_c'}$ on the critical section specified in Section 11.11.2.3.

11.11.2.5 The shearhead may be assumed to contribute a resisting moment M_v to each column strip of the slab computed by

$$M_v = \frac{\phi \alpha_v V_u}{8}\left(l_v - \frac{c_1}{2}\right) \qquad (11\text{-}27)$$

where ϕ is the capacity reduction factor for flexure, and l_v is the length of each shearhead arm actually provided. However, M_v shall exceed neither 30 percent of the total moment resistance required for each column strip of the slab, nor the change in column strip moment over the length l_v, nor the value of M_p given by Eq. (11-26).

11.12—Openings in slabs

When openings in slabs and footings are located at a distance less than ten times the thickness of the slab from a concentrated load or reaction, or when openings in flat slabs are located within the column strips as defined in Chapter 13, the critical sections specified in Sections 11.10.2 and 11.11.2.3 shall be modified as follows:

(a) For slabs without shearheads, that part of the periphery of the critical section which is enclosed by radial projections of the openings to the centroid of the loaded area shall be considered ineffective.

(b) For slabs with shearheads, one-half of that part of the periphery specified in (a) shall be considered ineffective.

11.13—Transfer of moments to columns

11.13.1—Shear forces exerted by unbalanced loads at connection to columns shall be considered in the design of lateral reinforcement in the column. Lateral reinforcement not less than that required by Eq. (11-1) shall be provided within the connections, except those not part of a primary seismic load-resisting system which are restrained on four sides by beams or slabs of approximately equal depth.

11.13.2—When unbalanced gravity load, wind, earthquake, or other lateral forces cause transfer of bending moment between slab and column, a fraction of the moment given by

$$\left(1 - \frac{1}{1 + \frac{2}{3}\sqrt{\frac{c_1 + d}{c_2 + d}}}\right)$$

shall be considered transferred by eccentricity of the shear about the centroid of the critical section defined in Section 11.10.2. Shear stresses shall be taken as varying linearly about the centroid of the critical section and the shear stress v_u shall not exceed $4\sqrt{f_c'}$.

11.14—Special provisions for brackets and corbels

11.14.1—These provisions apply to brackets and corbels having a shear-span-to-depth ratio, a/d, of unity or less. When the shear-span-to-depth ratio a/d is one-half or less, the design provisions of Section 11.15 may be used in lieu of Eq. (11-28) and (11-29), except that all limitations on quantity and spacing of reinforcement in Section 11.14 shall apply. The distance d shall be measured at a section adjacent to the face of the support, but shall not be taken greater than twice the depth of the corbel or bracket at the outside edge of the bearing area.

11.14.2—The shear stress shall not exceed

$$v_u = \left[6.5 - 5.1\sqrt{\frac{N_u}{V_u}}\right]\left[1 - 0.5\frac{a}{d}\right] \times \left\{1 + \left[64 + 160\sqrt{\left(\frac{N_u}{V_u}\right)^3}\right]\rho\right\}\sqrt{f_c'} \tag{11-28}$$

where ρ shall not exceed $0.13f_c'/f_y$ and N_u/V_u shall not be taken less than 0.20. The tensile force N_u shall be regarded as a live load even when it results from creep, shrinkage, or temperature change.

11.14.3—When provisions are made to avoid tension due to restrained shrinkage and creep, so that the member is subject to shear and moment only, v_u shall not exceed

$$v_u = 6.5\left(1 - 0.5\frac{a}{d}\right)\left(1 + 64\rho_v\right)\sqrt{f_c'} \tag{11-29}$$

where

$$\rho_v = \frac{A_s + A_h}{bd}$$

but not greater than

$$0.20\frac{f_c'}{f_y}$$

and A_h shall not exceed A_s.

11.14.4—Closed stirrups or ties parallel to the main tension reinforcement having a total cross-sectional area A_h not less than $0.50A_s$ shall be uniformly distributed within two-thirds of the effective depth adjacent to the main tension reinforcement.

11.14.5—The ratio $\rho = A_s/bd$ shall not be less than $0.04\ (f_c'/f_y)$.

11.15—Shear-friction

11.15.1—These provisions apply where it is inappropriate to consider shear as a measure of diagonal tension, and particularly in design of reinforcing details for precast concrete structures.

11.15.2—A crack shall be assumed to occur along the shear path. Relative displacement shall be considered resisted by friction maintained by shear-friction reinforcement across the crack. This reinforcement shall be approximately perpendicular to the assumed crack.

11.15.3—The shear stress v_u shall not exceed $0.2f_c'$, nor 800 psi.

11.15.4—The required area of reinforcement A_{vf} shall be computed by

$$A_{vf} = \frac{V_u}{\phi f_y \mu} \tag{11-30}$$

The design yield strength f_y shall not exceed 60,000 psi. The coefficient of friction, μ, shall be 1.4 for concrete cast monolithically, 1.0 for concrete placed against hardened concrete, and 0.7 for concrete placed against as-rolled structural steel.

11.15.5—Direct tension across the assumed crack shall be provided for by additional reinforcement.

11.15.6—The shear-friction reinforcement shall be well distributed across the assumed crack and shall be adequately anchored on both sides by embedment, hooks, or welding to special devices.

11.15.7—When shear is transferred between concrete placed against hardened concrete, the interface shall be rough with a full amplitude of approximately ¼ in. When shear is transferred between as-rolled steel and concrete, the steel shall be clean and without paint.

11.16—Special provisions for walls

11.16.1—Design for horizontal shear forces in the plane of the wall shall be in accordance with Section 11.16. The nominal shear stress, v_u, shall be computed by

$$v_u = \frac{V_u}{\phi hd} \tag{11-31}$$

where d shall be taken equal to $0.8l_w$. A larger value of d, equal to the distance from the extreme compression fiber to the center of force of all reinforcement in tension, may be used when determined by a strain compatibility analysis.

11.16.2—The shear stress carried by the concrete, v_c, shall not be taken greater than the lesser value computed from

$$v_c = 3.3\sqrt{f_c'} + \frac{N_u}{4l_w h} \tag{11-32}$$

and

$$v_c = 0.6\sqrt{f_c'} + \frac{l_w\left(1.25\sqrt{f_c'} + 0.2\frac{N_u}{l_w h}\right)}{\frac{M_u}{V_u} - \frac{l_w}{2}} \tag{11-33}$$

where N_u is negative for tension.

However, v_c may be taken as $2\sqrt{f_c'}$ if N_u is compression or Section 11.4.4 may be applied if N_u is tension.

11.6.3—Sections located closer to the base than a distance $l_w/2$ or one-half of the wall height, whichever is less, may be designed for the same v_c as that computed at a distance $l_w/2$ or one-half the height.

11.16.4—When v_u is less than $v_c/2$, reinforcement shall be provided in accordance with the provisions below or in accordance with Chapter 14. When v_u exceeds $v_c/2$, wall reinforcement for resisting shear shall conform to Sections 11.16.4.1 and 11.16.4.2.

11.16.4.1 The area of horizontal shear reinforcement shall be not less than that computed by Eq. (11-13). The ratio, ρ_h, of horizontal shear reinforcement area to the gross concrete area of vertical sections shall be at least 0.0025. The spacing of horizontal shear reinforcement shall not exceed $l_w/5$, $3h$, nor 18 in.

11.16.4.2 The ratio of vertical shear reinforcement area to gross concrete area of horizontal section shall be not less than

$$\rho_n = 0.0025 + 0.5\left(2.5 - \frac{h_w}{l_w}\right)(\rho_h - 0.0025) \qquad (11\text{-}34)$$

nor 0.0025, but need not be greater than the value of ρ_h required by Section 11.16.4.1. The spacing of vertical shear reinforcement shall not exceed $l_w/3$, $3h$, nor 18 in.

11.16.5—The total design shear stress, v_u, at any section shall not exceed $10\sqrt{f_c'}$.

11.16.6—Design for shear forces perpendicular to the face of the wall shall be in accordance with provisions for slabs in Section 11.10.

CHAPTER 12—DEVELOPMENT OF REINFORCEMENT

12.0—Notation

a = depth of equivalent rectangular stress block, defined by Section 10.2.7
A_b = area of an individual bar, sq in.
A_s = area of nonprestressed tension reinforcement, sq in.
A_v = area of shear reinforcement within a distance s
A_w = area of a deformed wire, sq in.
b_w = web width, or diameter of circular section, in.
d = distance from extreme compression fiber to centroid of tension reinforcement, in.
d_b = nominal diameter of bar, wire, or prestressing strand, in.
f_c' = specified compressive strength of concrete, psi
$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi
f_{ct} = average splitting tensile strength of lightweight aggregate concrete, psi
f_h = tensile stress developed by a standard hook, psi
f_{ps} = calculated stress in prestressing steel at design load, psi
f_{se} = effective stress in prestressing steel, after losses, psi
f_y = specified yield strength of nonprestressed reinforcement, psi
l_a = additional embedment length at support or at point of inflection, in.
l_d = development length, in.
l_e = equivalent embedment length, in.
M_t = theoretical moment strength, in.-lb, of a section
= $A_s f_y (d - a/2)$
n = number of cross wires in anchorage zone of welded deformed wire fabric
s = spacing of stirrups, in.
s_w = spacing of deformed wires, in.
V_u = total applied design shear force at section
β_b = ratio of area of bars cut off to total area of bars at the section
ξ = constant for standard hook

12.1—Development requirements—General

12.1.1—The calculated tension or compression in the reinforcement at each section shall be developed on each side of that section by embedment length or end anchorage or a combination thereof. For bars in tension, hooks may be used in developing the bars.

12.1.2—Tension reinforcement may be anchored by bending it across the web and making it continuous with the reinforcement on the opposite face of the member, or anchoring it there.

12.1.3—The critical sections for development of reinforcement in flexural members are at points of maximum stress and at points within the span where adjacent reinforcement terminates, or is bent. The provisions of Section 12.2.3 must also be satisfied. Splices must meet the stricter requirements of Section 7.6.

12.1.4—Reinforcement shall extend beyond the point at which it is no longer required to resist flexure for a distance equal to the effective depth of the member or 12 bar diameters, whichever is greater, except at supports of simple spans and at the free end of cantilevers.

12.1.5—Continuing reinforcement shall have an embedment length not less than the development length l_d beyond the point where bent or terminated tension reinforcement is no longer required to resist flexure.

12.1.6—Flexural reinforcement shall not be terminated in a tension zone unless one of the following conditions is satisfied:

12.1.6.1 The shear at the cutoff point does not exceed two-thirds that permitted, including the shear strength of furnished web reinforcement.

12.1.6.2 Stirrup area in excess of that required for shear and torsion is provided along each terminated bar over a distance from the termination point equal to three-fourths the effective depth of the member. The excess stirrups shall be proportioned such that their $(A_v/b_w s) f_y$ is not less than 60 psi. The resulting spacing s shall not exceed $d/8\beta_b$ where β_b is the ratio of the area of bars cut off to the total area of bars at the section.

12.1.6.3 For #11 and smaller bars, the continuing bars provide double the area required for flexure at the cutoff point and the shear does not exceed three-fourths that permitted.

12.2—Positive moment reinforcement

12.2.1—At least one-third the positive moment reinforcement in simple members and one-fourth the positive moment reinforcement in continuous members shall extend along the same face of the member into the support, and in beams at least 6 in.

12.2.2—When a flexural member is part of the primary lateral load resisting system, the positive reinforcement required to be extended into the support by Section 12.2.1 shall be anchored to develop its yield stress in tension at the face of the support.

12.2.3—At simple supports and at points of inflection, positive moment tension reinforcement shall be limited to a diameter such that l_d computed for f_y by Section 12.5 does not exceed

$$\frac{M_t}{V_u} + l_a$$

M_t is the computed flexural strength assuming all reinforcement at the section to be stressed to f_y. V_u is the maximum applied shear at the section. l_a at a support shall be the sum of the embedment length beyond the center of the support and the equivalent embedment length of any furnished hook or mechanical anchorage. l_a at a point of inflection shall be limited to the effective depth of the member or $12d_b$, whichever is greater. The value M_t/V_u in the development length limitation may be increased 30 percent when the ends of the reinforcement are confined by a compressive reaction.

12.3—Negative moment reinforcement

12.3.1—Tension reinforcement in a continuous, restrained, or cantilever member, or in any member of a rigid frame, shall be anchored in or through the supporting member by embedment length, hooks, or mechanical anchorage.

12.3.2—Negative moment reinforcement shall have an embedment length into the span as required by Sections 12.1.1 and 12.1.4.

12.3.3—At least one-third the total reinforcement provided for negative moment at the support shall have an embedment length beyond the point of inflection not less than the effective depth of the member, $12d_b$, or one-sixteenth of the clear span whichever is greater.

12.4—Special members

Adequate end anchorage shall be provided for tension reinforcement in flexural members where reinforcement stress is not directly proportional to moment, such as: sloped, stepped, or tapered footings; brackets; deep beams; or members in which the tension reinforcement is not parallel to the compression face.

12.5—Development length of deformed bars and deformed wire in tension

The development length l_d, in inches, of deformed bars and deformed wire in tension shall be computed as the product of the basic development length of (a) and the applicable modification factor or factors of (b), (c), and (d), but l_d shall be not less than 12 in.

(a) The basic development length shall be:

For #11 or smaller bars ..... $0.04A_b f_y/\sqrt{f_c'}$*
but not less than .... $0.0004 d_b f_y$†
For #14 bars .... $0.085 f_y/\sqrt{f_c'}$‡
For #18 bars .... $0.11 f_y/\sqrt{f_c'}$‡
For deformed wire .... $0.03 d_b f_y/\sqrt{f_c'}$

(b) The basic development length shall be multiplied by the applicable factor or factors for:

Top reinforcement§ 1.4
Bars with f_y greater than 60,000 psi .... $2 - \dfrac{60{,}000}{f_y}$

*The constant carries the unit of 1/in.
†The constant carries the unit of in.²/lb
‡The constant carries the unit of in.
§Top reinforcement is horizontal reinforcement so placed that more than 12 in. of concrete is cast in the member below the bar.

(c) When lightweight aggregate concrete is used, the basic development lengths in (a) shall be multiplied by 1.33 for "all lightweight" concrete and 1.18 for "sand-lightweight" concrete with linear interpolation when partial sand replacement is used, or the basic development length may be multiplied by $6.7\sqrt{f_c'}/f_{ct}$, but not less than 1.0, when f_{ct} is specified and the concrete is proportioned in accordance with Section 4.2. The factors of (b) and (d) shall also be applied.

(d) The basic development length, modified by the appropriate requirements of (b) and (c), may be multiplied by the applicable factor or factors for:

Reinforcement being developed in the length under consideration and spaced laterally at least 6 in. on center and at least 3 in. from the side face of the member 0.8

Reinforcement in a flexural member in excess of that required .... (A_s required) / (A_s provided)

Bars enclosed within a spiral which is not less than ¼ in. diameter and not more than 4 in. pitch 0.75

12.6—Development length of deformed bars in compression

The development length l_d for bars in compression shall be computed as $0.02 f_y d_b / \sqrt{f_c'}$ but shall not be less than $0.0003 f_y d_b$ or 8 in. Where excess bar area is provided, the l_d length may be reduced by the ratio of required area to area provided. The development length may be reduced 25 percent when the reinforcement is enclosed by spirals not less than ¼ in. in diameter and not more than 4 in. pitch.

12.7—Development length of bundled bars

The development length of each bar of bundled bars shall be that for the individual bar, increased by 20 percent for a three-bar bundle, and 33 percent for a four-bar bundle.

12.8—Standard hooks

12.8.1—Standard hooks shall be considered to develop a tensile stress in bar reinforcement $f_h = \xi\sqrt{f_c'}$ where ξ is not greater than the values in Table 12.8.1. The value of ξ may be increased 30 percent where enclosure is provided perpendicular to the plane of the hook.

TABLE 12.8.1 — ξ VALUES

Bar size	f_y = 60 ksi		f_y = 40 ksi
	Top bars	Other bars	All bars
#3 to #5	540	540	360
#6	450	540	360
#7 to #9	360	540	360
#10	360	480	360
#11	360	420	360
#14	330	330	330
#18	220	220	220

12.8.2—An equivalent embedment length l_e shall be computed using the provisions of Section 12.5(a) by substituting f_h for f_y and l_e for l_d.

12.8.3—Hooks shall not be considered effective in adding to the compressive resistance of reinforcement.

12.9—Combination development length

Development length l_d may consist of a combination of the equivalent embedment length of a hook or mechanical anchorage plus additional embedment length of the reinforcement.

12.10—Development of welded wire fabric

12.10.1—The yield strength of smooth longitudinal wires of welded wire fabric shall be considered developed by embedding at least two cross wires with the closer one at least 2 in. from the point of critical section. An embedment of one cross wire at least 2 in. from the point of critical section may be considered to develop half the yield strength.

12.10.2—The development length of welded deformed wire fabric may be computed as a deformed wire in Section 12.5 by substituting $(f_y - 20{,}000n)$* for f_y, where n is the number of cross wires within the development length which are at least 2 in. from the critical section. The minimum development length shall be $250A_w/s_w$ where A_w is the area of one tension wire and s_w is the spacing of wires.

12.11—Development length of prestressing strand

12.11.1—Three- or seven-wire pretensioning strand shall be bonded beyond the critical section for a development length, in inches, not less than

$$\left(f_{ps} - \frac{2}{3} f_{se}\right) d_b$$

where d_b is the nominal diameter in inches, f_{ps} and f_{se} are expressed in kips per square inch, and the expression in the parenthesis is used as a constant without units.

Investigation may be limited to those cross sections nearest each end of the member which

*The 20,000 has units of psi.

are required to develop their full strength under the specified design load.

12.11.2—Where bonding of the strand does not extend to the end of the member, the bonded development length specified in Section 12.11.1 shall be doubled.

12.12—Mechanical anchorage

Any mechanical device capable of developing the strength of the reinforcement without damage to the concrete may be used as anchorage. Test results showing the adequacy of such devices shall be presented to the Building Official.

12.13—Anchorage of web reinforcement

12.13.1—Web reinforcement shall be carried as close to the compression and tension surfaces of the member as cover requirements and the proximity of other steel will permit, and in any case the ends of single leg, simple U-, or multiple U-stirrup shall be anchored by one of the following means:

12.13.1.1 A standard hook plus an effective embedment of $0.5l_d$. The effective embedment of a stirrup leg shall be taken as the distance between the middepth of the member $d/2$ and the start of the hook (point of tangency).

12.13.1.2 Embedment above or below the middepth, $d/2$, of the beam on the compression side for a full development length l_d but not less than 24 bar diameters.

12.13.1.3 Bending around the longitudinal reinforcement through at least 180 deg. Hooking or bending stirrups around the longitudinal reinforcement shall be considered effective anchorage only when the stirrups make an angle of at least 45 deg with deformed longitudinal bars.

12.13.1.4 For each leg of welded plain wire fabric forming simple U-stirrups, either:

(a) Two longitudinal wires running at a 2 in. spacing along the beam at the top of the U.

(b) One longitudinal wire not more than $d/4$ from the compression face and a second wire closer to the compression face and spaced at least 2 in. from the first. The second wire may be beyond a bend or on a bend which has an inside diameter of at least 8 wire diameters.

12.13.2—Between the anchored ends, each bend in the continuous portion of a transverse simple U- or multiple U-stirrup shall enclose a longitudinal bar.

12.13.3—Longitudinal bars bent to act as web reinforcement shall, in a region of tension, be continuous with the longitudinal reinforcement and in a compression zone shall be anchored, above or below the middepth $d/2$ as specified for development length in Section 12.5 for that part of f_y which is needed to satisfy Eq. (11-14).

12.13.4—Pairs of U-stirrups or ties so placed as to form a closed unit shall be considered properly spliced when the laps are $1.7l_d$. In members at least 18 in. deep, such splices having A_bf_y not more than 9000 lb per leg may be considered adequate if the legs extend the full available depth of the member.

PART 5—STRUCTURAL SYSTEMS OR ELEMENTS

CHAPTER 13—SLAB SYSTEMS WITH MULTIPLE SQUARE OR RECTANGULAR PANELS

13.0—Notation

c_2 = size of rectangular or equivalent rectangular column, capital, or bracket measured transverse to the direction in which moments are being determined

C = cross-sectional constant to define the torsional properties. See Eq. (13-7)

E_{cb} = modulus of elasticity for beam concrete

E_{cc} = modulus of elasticity for column concrete

E_{cs} = modulus of elasticity for slab concrete

h = overall thickness of member, in.

I_b = moment of inertia about centroidal axis of gross section of a beam as defined in Section 13.1.5

I_c = moment of inertia of gross cross section of columns

I_s = moment of inertia about centroidal axis of gross section of slab
= $h^3/12$ times width of slab specified in definitions of α and β_t

K_b = flexural stiffness of beam; moment per unit rotation

K_c = flexural stiffness of column; moment per unit rotation

K_{ec} = flexural stiffness of an equivalent column; moment per unit rotation. See Eq. (13-5)

K_s = flexural stiffness of slab; moment per unit rotation
K_t = torsional stiffness of torsional member; moment per unit rotation
l_c = height of column, center-to-center of floors or roof
l_n = length of clear span, in the direction moments are being determined, measured face-to-face of supports
l_1 = length of span in the direction moments are being determined, measured center-to-center of supports
l_2 = length of span transverse to l_1, measured center-to-center of supports
M_o = total static design moment
w = design load per unit area
w_d = design dead load per unit area
w_l = design live load per unit area
x = shorter overall dimension of a rectangular part of a cross section
y = longer overall dimension of a rectangular part of a cross section
α = ratio of flexural stiffness of beam section to the flexural stiffness of a width of slab bounded laterally by the center line of the adjacent panel, if any, on each side of the beam
$= \dfrac{E_{cb}I_b}{E_{cs}I_s}$
α_c = ratio of flexural stiffness of the columns above and below the slab to the combined flexural stiffness of the slabs and beams at a joint taken in the direction moments are being determined
$= \dfrac{\Sigma K_c}{\Sigma(K_s + K_b)}$
α_{ec} = ratio of flexural stiffness of the equivalent column to the combined flexural stiffness of the slabs and beams at a joint taken in the direction moments are being determined
$= \dfrac{K_{ec}}{\Sigma(K_s + K_b)}$
α_{min} = minimum α_c to satisfy Section 13.3.6.1 (a)
α_1 = α in the direction of l_1
α_2 = α in the direction of l_2
β_a = ratio of dead load per unit area to live load per unit area (in each case without load factors)
β_t = ratio of torsional stiffness of edge beam section to the flexural stiffness of a width of slab equal to the span length of the beam, center-to-center of supports
$= \dfrac{E_{cb}C}{2E_{cs}I_s}$
δ_s = factor defined by Eq. (13-4). See Section 13.3.6.1

13.1—Scope and definitions

13.1.1—The provisions of this chapter govern the design of slab systems reinforced for flexure in more than one direction with or without beams between supports. Solid slabs and slabs with recesses or pockets made by permanent or removable fillers between ribs or joists in two directions are included under this definition. Slabs with paneled ceilings are also included under this definition provided the panel of reduced thickness lies entirely within the middle strips, and is at least two-thirds the thickness of the remainder of the slab, exclusive of the drop panel, and is not less than 4 in. thick. The thicknesses shall satisfy requirements of Section 9.5.3.

13.1.2—A column strip is a design strip with a width of $0.25l_2$ but not greater than $0.25l_1$ on each side of the column center line. The strip includes beams, if any.

13.1.3—A middle strip is a design strip bounded by two column strips.

13.1.4—A panel is bounded by column or wall center lines on all sides.

13.1.5—For monolithic or fully composite construction, the beam includes that portion of the slab on each side of the beam extending a distance equal to the projection of the beam above or below the slab, whichever is greater, but not greater than four times the slab thickness.

13.1.6—The slab may be supported on walls, columns, or beams. No portion of a column capital shall be considered for structural purposes which lies outside the largest right circular cone or pyramid with a 90 deg vertex which can be included within the outlines of the supporting element.

13.2—Design procedures

13.2.1—A slab system may be designed by any procedure satisfying the conditions of equilibrium and geometrical compatibility provided it is shown that the strength furnished is at least that required considering Sections 9.2 and 9.3, and that all serviceability conditions, including the specified limits on deflections, are met.

13.2.2—A slab system, including the slab and any supporting beams, columns, and walls, may be designed directly by either of the procedures described in this chapter: The Direct-Design Method (Section 13.3) or The Equivalent Frame Method (Section 13.4).

13.2.3—The slabs and beams shall be proportioned for the design bending moments prevailing at every section.

13.2.4—When unbalanced gravity load, wind, earthquake, or other lateral loads cause transfer of bending moment between slab and column, the flexural stresses on the critical section shall be investigated by analysis, and the cross section proportioned according to the requirements of Section 11.13.2. Concentration of reinforcement over the column head by closer spacing or additional reinforcement may be used to resist the moment on this section. A slab width between lines that are one-half slab or drop panel thickness, $h/2$, on each side of the column or capital may be considered effective.

13.2.5—Design for the transmission of load from the slab to the supporting walls and columns through shear and torsion shall be in accordance with Chapter 11.

13.3—Direct design method

13.3.1 *Limitations*

13.3.1.1 There shall be a minimum of three continuous spans in each direction.

13.3.1.2 The panels shall be rectangular with the ratio of the longer to shorter spans within a panel not greater than 2.0.

13.3.1.3 The successive span lengths in each direction shall not differ by more than one-third of the longer span.

13.3.1.4 Columns may be offset a maximum of 10 percent of the span, in direction of the offset, from either axis between center lines of successive columns.

13.3.1.5 The live load shall not exceed three times the dead load.

13.3.1.6 If a panel is supported by beams on all sides, the relative stiffness of the beams in the two perpendicular directions

$$\frac{\alpha_1 l_2^2}{\alpha_2 l_1^2} \qquad (13\text{-}1)$$

shall not be less than 0.2 nor greater than 5.0.

13.3.1.7 Variations from the limitations of this section may be considered acceptable if demonstrated by analysis that the requirements of Section 13.2.1 are satisfied.

13.3.2 *Total static design moment for a span*

13.3.2.1 The total static design moment for a span shall be determined in a strip bounded laterally by the center line of the panel on each side of the center line of the supports. The absolute sum of the positive and average negative bending moments in each direction shall be not less than

$$M_o = \frac{w l_2 l_n^2}{8} \qquad (13\text{-}2)$$

Where the transverse span of the panels on either side of the center line of supports varies, l_2 shall be taken as the average of the transverse spans. When the span adjacent and parallel to an edge is being considered, the distance from the edge to the panel center line shall be substituted for l_2 in Eq. (13-2).

13.3.2.2 The clear span l_n shall extend from face to face of columns, capitals, brackets, or walls. The value of l_n used in Eq. (13-2) shall be not less than $0.65 l_1$. Circular supports shall be treated as square supports having the same area.

13.3.3 *Negative and positive design moments*

13.3.3.1 The negative design moment shall be located at the face of rectangular supports. Circular supports shall be treated as square supports having the same area.

13.3.3.2 In an interior span, the total static design moment M_o shall be distributed as follows:

Negative design moment 0.65
Positive design moment 0.35

13.3.3.3 In an end span, the total static design moment M_o shall be distributed as follows:

Interior negative design moment

$$0.75 - \frac{0.10}{1 + \frac{1}{\alpha_{ec}}}$$

Positive design moment

$$0.63 - \frac{0.28}{1 + \frac{1}{\alpha_{ec}}}$$

Exterior negative design moment

$$\frac{0.65}{1 + \frac{1}{\alpha_{ec}}}$$

where α_{ec} is computed for the exterior column.

13.3.3.4 The negative moment section shall be designed to resist the larger of the two interior negative design moments determined for the spans framing into a common support unless an analysis is made to distribute the unbalanced moment in accordance with the stiffnesses of the adjoining elements.

13.3.4 *Design moments and shears in column and middle strips and beams*

13.3.4.1 The column strips shall be proportioned to resist the following portions in percent of the interior negative design moment:

l_2/l_1	0.5	1.0	2.0
$(\alpha_1 l_2/l_1) = 0$	75	75	75
$(\alpha_1 l_2/l_1) \geqq 1.0$	90	75	45

Linear interpolations shall be made between the values shown.

13.3.4.2 The column strip shall be proportioned to resist the following portions in percent of the exterior negative design moment:

l_2/l_1		0.5	1.0	2.0
$(\alpha_1 l_2/l_1) = 0$	$\beta_t = 0$	100	100	100
	$\beta_t \geqq 2.5$	75	75	75
$(\alpha_1 l_2/l_1) \geqq 1.0$	$\beta_t = 0$	100	100	100
	$\beta_t \geqq 2.5$	90	75	45

Linear interpolations shall be made between the values shown.

Where the exterior support consists of column or wall extending for a distance equal to or greater than three-quarters of the l_2 used to compute M_o, the exterior negative moment shall be considered to be uniformly distributed across l_2.

13.3.4.3 The column strip shall be proportioned to resist the following portions in percent of the positive design moment:

l_2/l_1	0.5	1.0	2.0
$(\alpha_1 l_2/l_1) = 0$	60	60	60
$(\alpha_1 l_2/l_1) \geqq 1.0$	90	75	45

Linear interpolations shall be made between the values shown.

13.3.4.4 The beam shall be proportioned to resist 85 percent of the column strip moment if $(\alpha_1 l_2/l_1)$ is equal to or greater than 1.0. For values of $(\alpha_1 l_2/l_1)$ between 1.0 and zero, the proportion of moment to be resisted by the beam shall be obtained by linear interpolation between 85 and zero percent. Moments caused by loads applied on the beam and not considered in the slab design shall be determined directly. The slab in the column strip shall be proportioned to resist that portion of the design moment not resisted by the beam.

13.3.4.5 That portion of the design moment not resisted by the column strip shall be proportionately assigned to the corresponding half middle strips. Each middle strip shall be proportioned to resist the sum of the moments assigned to its two half middle strips. The middle strip adjacent to and parallel with an edge supported by a wall shall be proportioned to resist twice the moment assigned to the half middle strip corresponding to the first row of interior supports.

13.3.4.6 A design moment may be modified by 10 percent provided the total static design moment for the panel in the direction considered is not less than that required by Eq. (13-2).

13.3.4.7 Beams with $(\alpha_1 l_2/l_1)$ equal to or greater than 1.0 shall be proportioned to resist the shear caused by loads in tributary areas bounded by 45 deg lines drawn from the corners of the panels and the center line of the panels parallel to the long sides. For values of $(\alpha_1 l_2/l_1)$ less than 1.0, the shear on the beam may be obtained by linear interpolation, assuming that for $\alpha = 0$ the beams carry no load. In addition, all beams shall be proportioned to resist the shear caused by directly applied loads.

13.3.4.8 The shear stresses in the slab may be computed on the assumption that the load is distributed to the supporting beams in accordance with Section 13.3.4.7. The total shear occurring on the panel shall be accounted for.

13.3.4.9 The shear stresses shall satisfy the requirements of Chapter 11.

13.3.4.10 Edge beams or the edges of the slab shall be proportioned to resist in torsion their share of the exterior negative design moments.

13.3.5 *Moments in columns and walls*

13.3.5.1 Columns and walls built integrally with the slab system shall resist moments arising from loads on the slab system.

13.3.5.2 At an interior support, the supporting elements above and below the slab shall resist the moment specified by Eq. (13-3) in direct proportion to their stiffnesses unless a general analysis is made.

$$M = \frac{0.08\,[\,(w_d + 0.5\,w_l)\,l_2\,l_n^2 - w_d'\,l_2'\,(l_n')^2]}{1 + \dfrac{1}{\alpha_{ec}}} \qquad (13\text{-}3)$$

where w_d', l_2' and l_n' refer to the shorter span.

13.3.6 *Provisions for effects of pattern loadings*

13.3.6.1 Where the ratio of dead load to live load, β_a, is less than 2.0, one of the following conditions shall be satisfied:

TABLE 13.3.6.1—MINIMUM α_{min}

β_a	Aspect ratio l_2/l_1	Relative beam stiffness, α: 0	0.5	1.0	2.0	4.0
2.0	0.5-2.0	0	0	0	0	0
1.0	0.5	0.6	0	0	0	0
	0.8	0.7	0	0	0	0
	1.0	0.7	0.1	0	0	0
	1.25	0.8	0.4	0	0	0
	2.0	1.2	0.5	0.2	0	0
0.5	0.5	1.3	0.3	0	0	0
	0.8	1.5	0.5	0.2	0	0
	1.0	1.6	0.6	0.2	0	0
	1.25	1.9	1.0	0.5	0	0
	2.0	4.9	1.6	0.8	0.3	0
0.33	0.5	1.8	0.5	0.1	0	0
	0.8	2.0	0.9	0.3	0	0
	1.0	2.3	0.9	0.4	0	0
	1.25	2.8	1.5	0.8	0.2	0
	2.0	13.0	2.6	1.2	0.5	0.3

(a) The sum of flexural stiffnesses of the columns above and below the slab shall be such that α_c is not less than the minimum α_{min} specified in Table 13.3.6.1.

(b) If the columns do not satisfy (a), the design positive bending moments in the panels supported by those columns shall be multiplied by the coefficient δ_s determined from Eq. (13-4).

$$\delta_s = 1 + \frac{2 - \beta_a}{4 + \beta_a}\left(1 - \frac{\alpha_c}{\alpha_{min}}\right) \qquad (13\text{-}4)$$

13.4—Equivalent frame method

13.4.1 *Assumptions*—In design by the equivalent frame method the following assumptions shall be used and all sections of slabs and supporting members shall be proportioned for the moments and shears thus obtained.

13.4.1.1 The structure shall be considered to be made up of equivalent frames on column lines taken longitudinally and transversely through the building. Each frame consists of a row of equivalent columns or supports and slab-beam strips, bounded laterally by the center line of the panel on each side of the center line of the columns or supports. Frames adjacent and parallel to an edge shall be bounded by the edge and the center line of the adjacent panel.

13.4.1.2 Each such frame may be analyzed in its entirety, or, for vertical loading, each floor thereof and the roof may be analyzed separately with its columns as they occur above and below, the columns being assumed fixed at their remote ends. Where slab-beams are thus analyzed separately, it may be assumed in determining the bending moment at a given support that the slab-beam is fixed at any support two panels distant therefrom provided the slab continues beyond that point.

13.4.1.3 The moment of inertia of the slab-beam or column at any cross section outside of the joint or column capital may be based on the cross-sectional area of the concrete. Variation in the moments of inertia of the slab-beams and columns along their axes shall be taken into account.

13.4.1.4 The moment of inertia of the slab-beam from the center of the column to the face of the column, bracket, or capital shall be assumed equal to the moment of inertia of the slab-beam at the face of the column, bracket, or capital divided by the quantity $(1 - c_2/l_2)^2$ where c_2 and l_2 are measured transverse to the direction moments are being determined.

13.4.1.5 The equivalent column shall be assumed to consist of the actual columns above and below the slab-beam plus an attached torsional member transverse to the direction in which moments are being determined and extending to the bounding lateral panel center lines on each side of the column. The flexibility (inverse of the stiffness) of the equivalent column shall be taken as the sum of the flexibilities of the columns above and below the slab-beam and the flexibility of the torsional member.

$$\frac{1}{K_{ec}} = \frac{1}{\Sigma K_c} + \frac{1}{K_t} \qquad (13\text{-}5)$$

In computing the stiffness of the column K_c, the moment of inertia shall be assumed infinite from the top to the bottom of the slab-beam at the joint.

The attached torsional members shall be assumed to have a constant cross section throughout their length consisting of the larger of:

(a) A portion of the slab having a width equal to that of the column, bracket, or capital in the direction in which moments are being considered.

(b) For monolithic or fully composite construction, the portion of the slab specified in (a) plus that part of the transverse beam above and below the slab.

(c) The transverse beam as defined in Section 13.1.5.

The stiffness K_t of the torsional member shall be calculated by the following expression:

$$K_t = \Sigma \frac{9E_{cs}C}{l_2\left(1 - \frac{c_2}{l_2}\right)^3} \qquad (13\text{-}6)$$

where c_2 and l_2 relate to the transverse spans on each side of the column. The constant C in Eq. (13-6) may be evaluated for the cross section by dividing it into separate rectangular parts and carrying out the following summation:

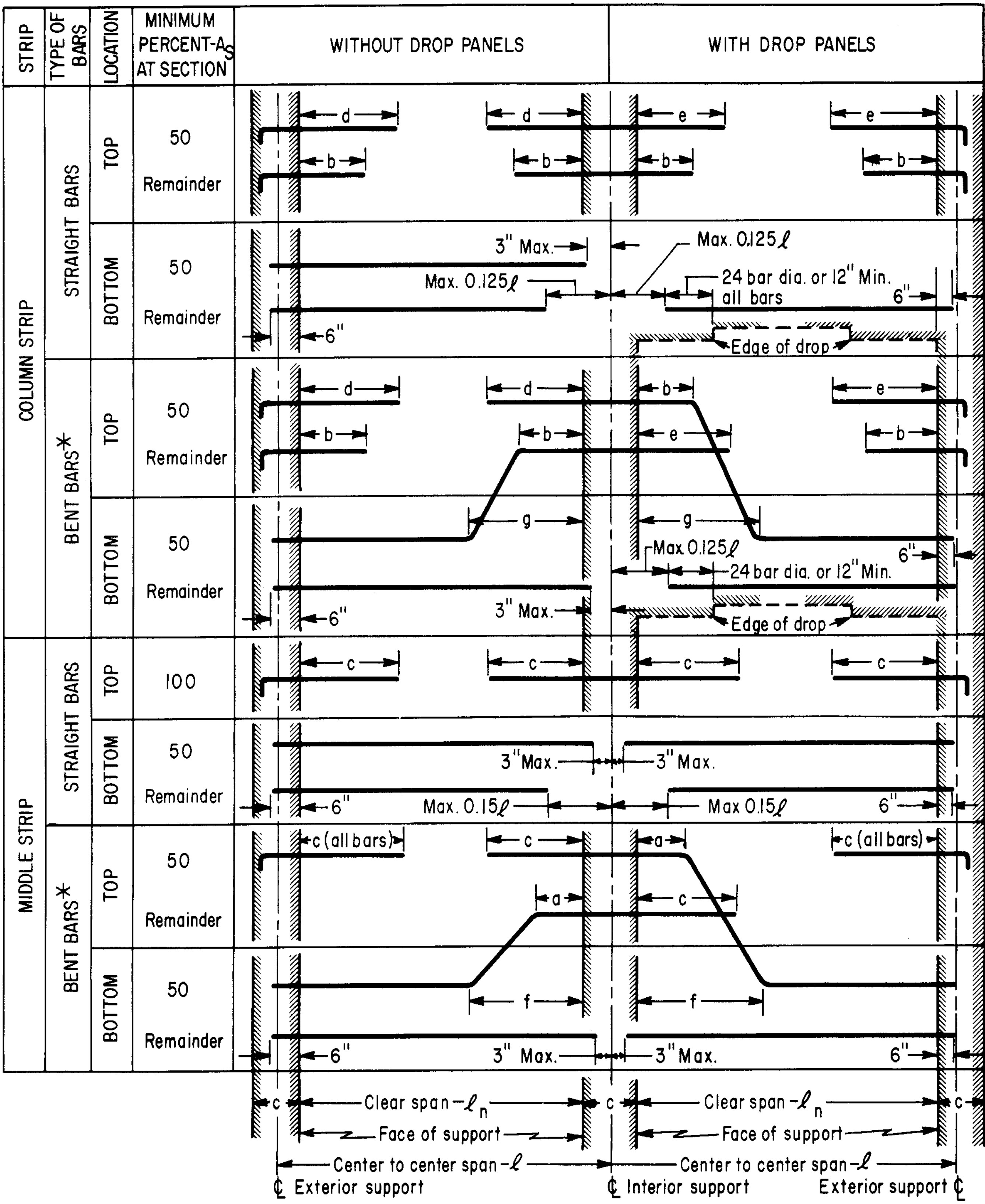
$$C = \Sigma\left(1 - 0.63\,\frac{x}{y}\right)\frac{x^3y}{3} \qquad (13\text{-}7)$$

Where beams frame into the column in the direction moments are being determined, the value of K_t as computed by Eq. (13-6) shall be multiplied by the ratio of the moment of inertia of the slab with such beam to the moment of inertia of the slab without such beam.

13.4.1.6 Where metal column capitals are used, account may be taken of their contributions to stiffness and resistance to bending and to shear.

13.4.1.7 The change in length of columns and slabs due to direct stress, and deflections due to shear, may be neglected.

13.4.1.8 When the loading pattern is known, the structure shall be analyzed for that load. When the live load is variable but does not exceed three-quarters of the dead load, or the nature of the live load is such that all panels will be loaded simultaneously, the maximum moments may be assumed to occur at all sections when full design live load is on the entire slab system. For other conditions, maximum positive moment near midspan of a panel may be assumed to occur when three-quarters of the full design



* Bent bars at exterior supports may be used if a general analysis is made

	BAR LENGTH FROM FACE OF SUPPORT						
	MINIMUM LENGTH					MAXIMUM LENGTH	
MARK	a	b	c	d	e	f	g
LENGTH	$0.14\ell_n$	$0.20\ell_n$	$0.22\ell_n$	$0.30\ell_n$	$0.33\ell_n$	$0.20\ell_n$	$0.24\ell_n$

Fig. 13.5.6—Minimum length of slab reinforcement, slabs without beams (See Section 12.2.1 regarding extending reinforcing into supports)

live load is on the panel and on alternate panels; and maximum negative moment in the slab at a support may be assumed to occur when three-quarters of the full design live load is on the adjacent panels only. In no case shall the design moments be taken as less than those occurring with full design live load on all panels.

13.4.2 *Negative design moment*—At interior supports, the critical section for negative moment, in both the column and middle strips, shall be taken at the face of rectilinear supports, but in no case at a distance greater than $0.175 l_1$ from the center of the column. At exterior supports provided with brackets or capitals, the critical section for negative moment in the direction perpendicular to the edge shall be taken at a distance from the face of the supporting element not greater than one-half the projection of the bracket or capital beyond the face of the supporting element. Circular or regular polygon shaped supports shall be treated as square supports having the same area.

13.4.3 *Distribution of panel moments*—Bending at critical sections across the slab-beam strip of each frame may be distributed to the column strips, middle strips, and beams as specified in Section 13.3.4 if the requirement of Section 13.3.1.6 is satisfied.

13.4.4 *Column moments*—Moments determined for the equivalent columns in the frame analysis shall be used in the design of the columns.

13.4.5 *Sum of positive and average negative moments*—Slabs within the limitations of Section 13.3, when designed by the equivalent frame method, may have the resulting analytical moments reduced in such proportion that the numerical sum of the positive and average negative bending moments used in design need not exceed the value obtained from Eq. (13-2).

13.5—Slab reinforcement

13.5.1—The spacing of the bars at critical sections shall not exceed two times the slab thickness, except for those portions of the slab area which may be of cellular or ribbed construction. In the slab over the cellular spaces, reinforcement shall be provided as required by Section 7.13.

13.5.2—In exterior spans, all positive reinforcement perpendicular to the discontinuous edge shall extend to the edge of the slab and have embedment, straight or hooked, of at least 6 in. in spandrel beams, walls, or columns. All negative reinforcement perpendicular to the discontinuous edge shall be bent, hooked, or otherwise anchored, in spandrel beams, walls, or columns, to be developed at the face of the support according to the provisions of Chapter 12. Where the slab is not supported by a spandrel beam or wall, or where the slab cantilevers beyond the support, anchorage of reinforcement may be within the slab.

13.5.3—The area of reinforcement shall be determined from the bending moments at the critical sections but shall not be less than required by Section 7.13.

13.5.4—In slabs supported on beams having a value of α greater than 1.0, special reinforcement shall be provided at exterior corners in both the bottom and top of the slab. This reinforcement shall be provided for a distance in each direction from the corner equal to one-fifth the longer span.

The reinforcement in both the top and bottom of the slab shall be sufficient to resist a moment equal to the maximum positive moment per foot of width in the slab. The direction of the moment is parallel to the diagonal from the corner in the top of the slab and perpendicular to the diagonal in the bottom of the slab. In either the top or bottom of the slab, the reinforcement may be placed in a single band in the direction of the moment or in two bands parallel to the sides of the slab.

13.5.5—Where a drop panel is used to reduce the amount of negative reinforcement over the column of a flat slab, such drop shall extend in each direction, from the center line of support, a distance equal to at least one-sixth the span length measured from center to center of supports in that direction, and the projection below the slab should be at least one-quarter the thickness of the slab beyond the drop. For determining reinforcement, the thickness of the drop panel below the slab shall not be assumed to be more than one-fourth the distance from the edge of the drop panel to the edge of the column capital.

13.5.6—In addition to the other requirements of this section, reinforcement shall have the minimum lengths given in Fig. 13.5.6. Where adjacent spans are unequal, the extension of negative reinforcement beyond the face of the support as prescribed in Fig. 13.5.6 shall be based on the requirements of the longer span.

13.6—Openings in the slab system

13.6.1—Openings of any size may be provided in the slab system if it is shown by analysis that the strength furnished is at least that required with consideration of Sections 9.2 and 9.3, and that all serviceability conditions, including the specified limits on deflections, are met.

13.6.2—Openings conforming to the following requirements may be provided in slab systems not having beams without special analysis as required in Section 13.6.1.

(a) Openings of any size may be placed in the area within the middle half of the span in each direction, provided the total amount of reinforcement required for the panel without the opening is maintained.

(b) In the area common to two column strips, not more than one-eighth of the width of strip in either span shall be interrupted by the openings. The equivalent of reinforcement interrupted shall be added on all sides of the openings.

(c) In the area common to one column strip and one middle strip, not more than one-quarter of the reinforcement in either strip shall be interrupted by the opening. The equivalent of reinforcement interrupted shall be added on all sides of the openings.

(d) The shear requirements of Chapter 11 shall be satisfied.

CHAPTER 14—WALLS

14.0—Notation

A_g = gross area of section, sq in.
f_c' = specified compressive strength of concrete, psi
h = overall thickness of member, in.
l_c = vertical distance between supports
P_u = axial design load in compression member
ϕ = capacity reduction factor. See Section 9.2

14.1—Structural design of walls

14.1.1—Walls shall be designed for any lateral or other loads to which they are subjected. Proper provisions shall be made for eccentric loads and wind forces. Unless designed in accordance with Section 14.1.2, walls subject to combined flexure and axial load shall be designed under the provisions of Section 10.16.

14.1.2—Walls in which the resultant of the loads falls within the middle third of its thickness may be considered reasonably concentrically loaded and may be designed in accordance with the provisions of Section 14.2. The limits of thickness and quantity of reinforcement required by Section 14.2 may be waived where structural analysis shows adequate strength and stability.

14.2—Empirical design of walls

(a) Reinforced concrete bearing walls carrying reasonably concentric loads may be designed by the empirical provisions of this section when they conform to all the limitations given herein.

(b) The capacity of the wall shall be:

$$P_u = 0.55\,\phi\, f_c'\, A_g\left[1-\left(\frac{l_c}{40h}\right)^2\right] \qquad (14\text{-}1)$$

where $\phi = 0.70$.

(c) The length of the wall to be considered as effective for each concentrated load shall not exceed the center-to-center distance between loads, nor shall it exceed the width of the bearing plus four times the wall thickness.

(d) Reinforced concrete bearing walls shall have a thickness of at least 1/25 of the unsupported height or width, whichever is the shorter.

(e) Reinforced concrete bearing walls of buildings shall be not less than 6 in. thick for the uppermost 15 ft of their height; and for each successive 25 ft downward, or fraction thereof, the minimum thickness shall be increased 1 in. Reinforced concrete bearing walls of two-story dwellings may be 6 in. thick throughout their height, provided that the permissible load in Eq. (14-1) is not exceeded.

(f) The area of the horizontal reinforcement of reinforced concrete walls shall be not less than 0.0025 and that of the vertical reinforcement not less than 0.0015 times the area of the wall. These values may be reduced to 0.0020 and 0.0012, respectively, if the reinforcement is not larger than ⅝ in. in diameter and consists of either welded wire fabric or deformed bars with a specified yield strength of 60,000 psi or greater.

(g) Walls more than 10 in. thick, except for basement walls, shall have the reinforcement for each direction placed in two layers parallel with the faces of the wall. One layer consisting of not less than one-half and not more than two-thirds the total required shall be placed not less than 2 in. nor more than one-third the thickness of the wall from the exterior surface. The other layer, comprising the balance of the required reinforcement, shall be placed not less than ¾ in. and not more than one-third the thickness of the wall from the interior surface. Bars, if used, shall not be less than #3 bars, nor shall they be spaced more than 18 in. on centers. Welded wire reinforcement for walls shall be in flat sheet form.

(h) In addition to the minimum as prescribed in (f) there shall be not less than two #5 bars around all window or door openings. Such bars shall extend at least 24 in. beyond the corner of the openings.

(i) Reinforced concrete walls shall be anchored to the floors, or to the columns, pilasters, buttresses, and intersecting walls.

(j) Panel and enclosure walls of reinforced concrete shall have a thickness of not less than 4 in. and not less than 1/30 the distance between the supporting or enclosing members.

(k) Exterior basement walls, foundation walls, fire walls, and party walls shall not be less than 8 in. thick.

(l) Where reinforced concrete bearing walls consist of studs or ribs tied together by reinforced concrete members at each floor level, the studs may be considered as columns.

14.3—Walls as grade beams

Walls designed as grade beams shall have top and bottom reinforcement as required by the provisions of Chapter 10. Portions exposed above grade, in addition shall meet the requirements of Section 10.16 or Section 14.2.

CHAPTER 15—FOOTINGS

15.0—Notation

d_p = diameter of the pile at footing base

$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi

β = ratio of long side to short side of a footing

ϕ = capacity reduction factor. See Section 9.2

15.1—Scope

15.1.1—The requirements prescribed in Sections 15.2 through 15.9 apply to isolated footings and, where applicable, to combined footings.

15.1.2—Additional general procedures for the design of combined footings are given in Section 15.10.

15.2—Loads and reactions

15.2.1—Footings shall be proportioned to sustain the applied loads and induced reactions without exceeding the limits prescribed elsewhere in this Code and as further provided in this chapter.

15.2.2—Axial forces, shears, and bending moments applied to the footing shall fully and safely be transferred to the supporting soil.

15.2.3—For footings on piles, computations for moments and shears may be based on the assumption that the reaction from any pile is concentrated at the center of the pile.

15.2.4—The base area of the footing or the number and arrangement of the piles shall be determined by using the external forces and moments* transmitted by the footing, and the allowable soil pressure or allowable pile capacity selected through principles of soil mechanics.

15.3—Sloped or stepped footings

15.3.1—In sloped or stepped footings, the angle of slope or the depth and location of steps shall be such that the design requirements are satisfied at every section.

15.3.2—Sloped or stepped footings that are designed as a unit shall be constructed to assure action as a unit.

15.4—Bending moment

15.4.1—The external moment on any section shall be determined by passing a vertical plane completely through the footing, and computing the moment of the forces acting over the entire area of the footing on one side of said plane.

15.4.2—The greatest bending moment to be used in the design of an isolated footing shall be the moment computed in the manner prescribed in Section 15.4.1 at sections located as follows:

(a) At the face of the column, pedestal, or wall, for footings supporting a concrete column, pedestal, or wall

(b) Halfway between the middle and the edge of the wall, for footings under masonry walls

(c) Halfway between the face of the column or pedestal and the edge of the steel base, for footings under steel bases.

15.4.3—In one-way reinforced footings and in two-way square reinforced footings, the reinforcement shall be distributed uniformly across the full width of the footing.

15.4.4—In two-way rectangular footings, the reinforcement in the long direction shall be distributed uniformly across the full width of the

*External forces and moments are those resulting from the unfactored loads (*D*, *L*, *W*, and *E*) specified in the General Code of which these requirements form a part.

footing. In the short direction the reinforcement determined by Eq. (15-1) shall be uniformly distributed across a band width b centered with respect to the center line of the column or pedestal and having a width equal to the length of the short side of the footing. The remainder of the reinforcement shall be uniformly distributed in the outer portions of the footing.

$$\frac{\text{Reinforcement in band width } b}{\text{Total reinforcement in short direction}} = \frac{2}{(\beta + 1)} \qquad (15\text{-}1)$$

where β is the ratio of the long side to the short side of the footing.

15.5—Shear and development of reinforcement

15.5.1—For computation of shear in footings see Chapter 11. The location of the critical section for shear shall be measured from the face of a column, wall, or pedestal or, in the case of a member on a steel base plate, from the section described in Section 15.4.2 (c).

15.5.2—For computation of development of reinforcement see Chapter 12.

15.5.3—Critical sections for development of reinforcement shall be assumed at the same locations as prescribed in Section 15.4.2; also at all other vertical planes where changes of section or of reinforcement occur.

15.5.4—The reinforcement at any section shall develop the calculated tension or compression on each side of that section by proper embedment length, end anchorage, hooks (for tension only), or combinations thereof.

15.5.5—In computing the external shear on any section through a footing supported on piles, the entire reaction from any pile whose center is located $d_p/2$ or more outside the section shall be assumed as producing shear on the section. The reaction from any pile whose center is located $d_p/2$ or more inside the section shall be assumed as producing no shear on the section. For intermediate positions of the pile center, the portion of the pile reaction to be assumed as producing shear on the section shall be based on straight-line interpolation between full value at $d_p/2$ outside the section and zero value at $d_p/2$ inside the section.

15.6—Transfer of stress at base of column or pedestal

15.6.1—All axial forces, shears, and bending moments applied at the base of a column or pedestal shall be transferred to the top of the supporting pedestal or footing by compression in the concrete and by reinforcement. In the case of uplift, the entire tensile force shall be resisted by the reinforcement.

15.6.2—The bearing stress on the concrete contact area of the supporting and supported member shall not exceed the permissible bearing stress for either surface as given in Section 10.14.

15.6.3—Where the permissible bearing stress on the concrete in the supporting or supported member would be exceeded, developed reinforcement shall be provided for the excess force, either by extending the longitudinal bars into the supporting member, or by dowels. See also Section 15.6.5.

15.6.4—Where transfer of force is accomplished by reinforcement, the development length of the reinforcement shall be sufficient to transfer the compression or tension to the supporting member in accordance with Chapter 12.

15.6.5—Extended longitudinal reinforcement or dowels of at least 0.5 percent of the cross-sectional area of the supported column or pedestal and a minimum of four bars shall be supplied. Where dowels are used their diameter shall not exceed the diameter of the column bars by more than 0.15 in.

15.6.6—In sloped or stepped footings, the supporting area for bearing shall be taken in accordance with Section 10.14.

15.6.7—Shear keys or other devices shall be used where necessary to transmit transverse forces between column and footing.

15.6.8—#14 and #18 column bars in compression only can be dowelled at the footings with bars of smaller size of the necessary area. The dowel shall extend into the column a distance equal to the development length of the #14 or #18 bar and into the footing a distance equal to the development length of the dowel.

15.7—Pedestals and footings of unreinforced concrete

15.7.1—The maximum compressive stress on an unreinforced concrete pedestal shall not exceed the permissible bearing stress. Where this stress is exceeded, reinforcement shall be provided and the member designed as a reinforced concrete column.

15.7.2—The depth and width of a pedestal or footing of unreinforced concrete on soil shall be such that the flexural tensile stress in the concrete shall not exceed $5.0\phi\sqrt{f_c'}$ if the load factors and ϕ factor of Chapter 9 are used or $1.6\sqrt{f_c'}$ if the alternate method of Section 8.1.2 is used. The average shear stress for unreinforced concrete shall not exceed $2.0\sqrt{f_c'}$ for beam action and $4.0\sqrt{f_c'}$ for two-way action if load and ϕ factors are used.

15.7.3—Footings on piles shall not be made of unreinforced concrete.

15.8—Footings supporting round or regular polygon shaped columns

In computing the stresses in footings which support a round or regular polygon shaped concrete column or pedestal, the "face" of the column or pedestal may be taken as the side of a square having an area equal to the area enclosed within the perimeter of the column or pedestal.

15.9—Minimum edge thickness

15.9.1—In unreinforced concrete footings on soil, the thickness at the edge shall be not less than 8 in.

15.9.2—In reinforced concrete footings, the thickness at the edge above the bottom reinforcement shall be not less than 6 in. for footings on soil, nor less than 12 in. for footings on piles.

15.10—Combined footings and mats

The following requirements apply to combined footings and mats supporting more than one column or wall:

(a) All assumptions with respect to the distribution of the soil pressure shall be consistent with the properties of the soil and the structure and with established principles of soil mechanics.

(b) Design of combined footings and mats shall conform with the appropriate chapters of this Code.

CHAPTER 16—PRECAST CONCRETE

16.1—Scope

All provisions of this Code shall apply to precast members except for specific variations given in this chapter. The specific variations apply only to precast concrete members manufactured under plant controlled conditions.

16.2—Design

16.2.1—Design shall consider all loading and restraint conditions from initial fabrication to completion of the structure, including form removal, storage, transportation, and erection. In cases where the structure does not behave monolithically, the effects at all interconnected and adjoining details shall be considered to assure proper performance of the system. The effects of initial and long-time deflections shall be considered, including the effects on interconnected elements.

16.2.2—Design of joints and bearings shall include the effects of all forces to be transmitted, including shrinkage, creep, temperature, elastic deformation, wind, and earthquake. All details shall be designed to provide for manufacturing and erection tolerances and temporary erection stresses.

16.3—Bearing and nonbearing wall panels

16.3.1—Bearing and nonbearing precast walls shall be designed in accordance with Chapter 10 or 14.

16.3.2—Where panels are designed to span horizontally to columns or isolated footings, the ratio of height to thickness shall not be limited, provided the effect of deep beam action, buckling, and deflections are provided for in the design in accordance with Section 10.7.

16.4—Details

16.4.1—All details of reinforcement, connections, bearing seats, inserts, anchors, concrete cover, openings, lifting devices, fabrication, and erection tolerances shall be shown on the shop drawings.

16.4.2—Lifting devices shall have a capacity sufficient to support four times the appropriate portion of the members dead weight. The inclination of the lifting force shall be considered.

16.5—Identification and marking

Each precast member shall be marked to indicate its location in the structure, its top surface, and date of fabrication. Identification marks shall correspond to the placing plans.

16.6—Transportation, storage, and erection

16.6.1—During curing, form removal, storage, transportation, and erection, precast members shall not be overstressed, warped, or otherwise damaged or have the camber adversely affected.

16.6.2—Precast members shall be adequately braced and supported during erection to insure proper alignment and safety until permanent connections are completed.

CHAPTER 17—COMPOSITE CONCRETE FLEXURAL MEMBERS

17.0—Notation

b_v = the width of the cross section being investigated for horizontal shear
d = distance from extreme compression fiber to centroid of tension reinforcement, in.
v_{dh} = design horizontal shear stress at any cross section, psi
v_h = permissible horizontal shear stress, psi
V_u = total applied design shear force at section
ϕ = capacity reduction factor. See Section 9.2

17.1—Scope

17.1.1—Composite concrete flexural members consist of concrete elements constructed in separate placements but so interconnected that the elements respond to loads as a unit.

17.1.2—The provisions of all other chapters apply to composite concrete flexural members, except as specifically modified herein.

17.2—General considerations

17.2.1—The entire composite member or portions thereof may be used in resisting the shear and the bending moment. The individual elements shall be investigated for all critical stages of loading.

17.2.2—If the specified strength, unit weight, or other properties of the various components are different, the properties of the individual components, or the most critical values, shall be used in design.

17.2.3—In calculating the strength of a composite member, no distinction shall be made between shored and unshored members.

17.2.4—All elements shall be designed to support all loads introduced prior to the full development of the design strength of the composite member.

17.2.5—Reinforcement shall be provided as necessary to control cracking and to prevent separation of the components.

17.2.6—Composite members shall meet the requirements for control of deflections given in Section 9.5.5.

17.3—Shoring

When used, shoring shall not be removed until the supported elements have developed the design properties required to support all loads and limit deflections and cracking at the time of shoring removal.

17.4—Vertical shear

When the entire composite member is assumed to resist the vertical shear, the design shall be in accordance with the requirements of Chapter 11 as for a monolithically cast member of the same cross-sectional shape.

Web reinforcement shall be fully anchored into the components in accordance with Section 12.13. Extended and anchored web reinforcement may be included as ties for horizontal shear.

17.5—Horizontal shear

17.5.1—In the composite member, full transfer of the shear forces shall be assured at the interfaces of the separate components.

17.5.2—Full transfer of horizontal shear forces may be assumed when all of the following are satisfied: (a) the contact surfaces are clean and intentionally roughened, (b) minimum ties are provided in accordance with Section 17.6.1, (c) web members are designed to resist the entire vertical shear, and (d) all stirrups are fully anchored into all intersecting components.

Otherwise, horizontal shear shall be fully investigated.

17.5.3—The horizontal shear stress v_{dh} may be calculated at any cross section as

$$v_{dh} = \frac{V_u}{\phi b_v d} \qquad (17\text{-}1)$$

in which d is for the entire composite section. Alternatively, the actual compressive or tensile force in any segment may be computed, and provisions made to transfer that force as horizontal shear to the supporting element. The ϕ factor specified for shear shall be used with the compressive or tensile force.

17.5.4—The design shear force may be transferred at contact surfaces using the permissible horizontal shear stresses v_h stated below.

(a) When ties are not provided, but the contact surfaces are clean and intentionally roughened, permissible $v_h = 80$ psi

(b) When the minimum tie requirements of Section 17.6.1 are provided and the contact surfaces are clean but not intentionally roughened, permissible $v_h = 80$ psi

(c) When the minimum tie requirements of Section 17.6.1 are provided and the contact surfaces are clean and intentionally roughened, permissible $v_h = 350$ psi

(d) When v_{dh} exceeds 350 psi, design for horizontal shear shall be made in accordance with Section 11.15.

17.5.5—When tension exists perpendicular to any surface, shear transfer by contact may be assumed only when the minimum tie requirement of Section 17.6.1 are satisfied.

17.6—Ties for horizontal shear

17.6.1—When vertical bars or extended stirrups are used to transfer horizontal shear, the tie area shall not be less than that required by Section 11.1.2, and the spacing shall not exceed four times the least dimension of the supported element nor 24 in.

17.6.2—Ties for horizontal shear may consist of single bars, multiple leg stirrups, or the vertical legs of welded wire fabric. All ties shall be fully anchored into the components in accordance with Section 12.13.

17.7—Measure of roughness

Intentional roughness may be assumed only when the interface is roughened with a full amplitude of approximately ¼ in.

CHAPTER 18—PRESTRESSED CONCRETE

18.0—Notation

A = area of that part of the cross section between the flexural tension face and the center of gravity of the gross section

A_c = area of concrete at the cross section considered

A_{ps} = area of prestressed reinforcement in tension zone

A_s = area of nonprestressed tension reinforcement, sq in.

A_s' = area of compression reinforcement, sq in.

b = width of compression face of member

d = distance from extreme compression fiber to centroid of prestressing steel, or to combined centroid when nonprestressing tension reinforcement is included, in.

e = base of Napierian logarithms

f_c' = specified compressive strength of concrete, psi

f'_{ci} = compressive strength of concrete at time of initial prestress

f_{ps} = calculated stress in prestressing steel at design load, psi

f_{pu} = ultimate strength of prestressing steel, psi

f_{py} = specified yield strength of prestressing steel, psi

f_{se} = effective stress in prestressing steel, after losses, psi

f_y = specified yield strength of nonprestressed reinforcement, psi

K = wobble friction coefficient per foot of prestressing steel

l = length of prestressing steel element from jacking end to any point x

N_c = tensile force in the concrete under load of $D + 1.2L$

P_s = steel force at jacking end

P_x = steel force at any point x

α = total angular change of prestressing steel profile in radians from jacking end to any point x

μ = curvature friction coefficient

ρ = A_s/bd
= ratio of nonprestressed tension reinforcement

ρ' = A_s'/bd

ρ_p = A_{ps}/bd
= ratio of prestressed reinforcement

ϕ = capacity reduction factor. See Section 9.2

ω = $\rho f_y/f_c'$

ω' = $\rho' f_y/f_c'$

ω_p = $\rho_p f_{ps}/f_c'$

ω_w, ω_{pw}, ω_w' = reinforcement indices for flanged sections computed as for ω, ω_p, and ω' except that b shall be the web width, and the steel area shall be that required to develop the compressive strength of the web only

18.1—Scope

18.1.1—Provisions in this chapter apply to structural members prestressed with high strength steel meeting the requirements for prestressing steels in Sections 3.5.9 and 3.5.10.

18.1.2—All provisions of this Code not specifically excluded, and not in conflict with the provisions of this chapter, are to be considered applicable to prestressed concrete.

18.1.3—The following provisions shall not apply to prestressed concrete unless specifically noted: Sections 8.6, 8.7.2, 8.7.3, 8.7.4, 8.8, 10.3.2, 10.3.3, 10.3.6, 10.5, 10.9.1, and Chapters 13 and 14.

18.2—General considerations

18.2.1—Members shall meet the strength requirements specified in this Code. Design shall be based on strength and on behavior at service conditions at all load stages that may be critical during the life of the structure from the time the prestress is first applied.

18.2.2—Stress concentrations due to the prestressing shall be considered in the design.

18.2.3—The effects on the adjoining structure of elastic and plastic deformations, deflections, changes in length, and rotations caused by the prestressing shall be provided for. The effects of temperature and shrinkage shall be considered.

18.2.4—The possibility of buckling in a member between points where the concrete and the prestressing steel are in contact and of buckling in thin webs and flanges shall be considered.

18.3—Basic assumptions

18.3.1—In designing for strength, the assumptions provided in Section 10.2 shall apply, except Section 10.2.4 applies only to reinforcing steel conforming to Section 3.5.1, 3.5.6, or 3.5.8.

18.3.2—In investigating sections at service loads, after transfer of prestress and at cracking load, straight-line theory may be used with the following assumptions:

(a) Strains vary linearly with depth through the entire load range.

(b) At cracked sections, the ability of the concrete to resist tension is neglected.

(c) In calculations of section properties, prior to bonding of tendons, areas of the open ducts shall be deducted. The transformed area of bonded tendons and reinforcing steel may be included in pretensioned members and in post-tensioned members after grouting.

18.4—Permissible stresses in concrete—Flexural members

18.4.1—Flexural stresses immediately after transfer, before losses, shall not exceed the following:

(a) Compression .... $0.60f'_{ci}$

(b) Tension in members without bonded auxiliary reinforcement (unprestressed or prestressed) in the tension zone .... $3\sqrt{f'_{ci}}$

Where the calculated tensile stress exceeds this value, reinforcement shall be provided to resist the total tensile force in the concrete computed on the assumption of an uncracked section.

18.4.2—Stresses at service loads, after allowance for all prestress losses, shall not exceed the following:

(a) Compression .... $0.45f_c'$

(b) Tension in precompressed tension zone .... $6\sqrt{f_c'}$

(c) Tension in precompressed tension zone in members where computations based on the transformed cracked section and on bilinear moment-deflection relationships show that immediate and long-term deflections comply with requirements of Section 9.5 .... $12\sqrt{f_c'}$

18.4.3—The permissible stresses in Sections 18.4.1 and 18.4.2 may be exceeded when it is shown experimentally or analytically that performance will not be impaired.

18.5—Permissible stresses in steel

18.5.1—Due to jacking force .... $0.80f_{pu}$ but not greater than the maximum value recommended by the manufacturer of the steel or of the anchorages.

18.5.2—Pretensioning tendons immediately after transfer, or post-tensioning tendons immediately after anchoring .... $0.70f_{pu}$

18.6—Loss of prestress

18.6.1—To determine the effective prestress, allowance for the following sources of loss of prestress shall be considered:

(a) Slip at anchorage
(b) Elastic shortening of concrete
(c) Creep of concrete
(d) Shrinkage of concrete
(e) Relaxation of steel stress
(f) Frictional loss due to intended or unintended curvature in the tendons.

18.6.2—Friction losses in post-tensioned steel shall be based on experimentally determined wobble and curvature coefficients, and shall be verified during stressing operations. The values of coefficients assumed for design, and the acceptable ranges of jacking forces and steel elongations, shall be shown on the plans. These friction losses shall be calculated as follows:

$$P_s = P_x e^{(Kl + \mu\alpha)} \qquad (18\text{-}1)$$

When $(Kl + \mu\alpha)$ is not greater than 0.3, Eq. (18-2) may be used.

$$P_s = P_x\ (1 + Kl + \mu\alpha) \qquad (18\text{-}2)$$

18.6.3—When prestress in a member may be reduced through its connection with adjoining elements, such reduction shall be allowed for in the design.

18.7—Flexural strength

18.7.1—Flexural strength of members shall be computed by the strength design methods given in this Code. For prestressing steel, f_{ps} shall be substituted for f_y. In lieu of a more precise determination of f_{ps} based on strain compatibility, and provided that f_{se} is not less than $0.5f_{pu}$, the following approximate values shall be used:

(a) Bonded members

$$f_{ps} = f_{pu}\left(1 - 0.5\rho_p \frac{f_{pu}}{f_c'}\right) \qquad (18\text{-}3)$$

(b) Unbonded members

$$f_{ps} = f_{se} + 10{,}000 + \frac{f_c'}{100\rho_p} \qquad (18\text{-}4)$$

but not more than f_{py} or $f_{se} + 60{,}000$.

18.7.2—Nonprestressed reinforcement conforming to Section 3.5.1, 3.5.6, or 3.5.8, when used in combination with prestressed steel, may be considered to contribute to the tensile force in a member at design load moment an amount equal to its area times its yield strength. For other types of nonprestressed reinforcement, a strain compatibility analysis shall be made to determine its contribution to the tensile force.

18.8—Steel percentage

18.8.1—Except as provided in Section 18.8.2, the ratio of prestressed and nonprestressed steel used for calculation of flexural strength shall be such that ω_p, $(\omega + \omega_p - \omega')$, or $(\omega_w + \omega_{pw} - \omega_w')$ is not greater than 0.30.

18.8.2—When a steel ratio in excess of that specified in Section 18.8.1 is used, the design moment shall not exceed the moment strength calculated from equations based on the compression portion of the internal resisting moment couple.

18.8.3—The total amount of prestressed and nonprestressed reinforcement shall be adequate to develop a design load in flexure at least 1.2 times the cracking load calculated on the basis of the modulus of rupture specified in Section 9.5.2.2.

18.9—Minimum bonded reinforcement requirements

18.9.1—Some bonded reinforcement shall be provided in the precompressed tension zone of flexural members where the prestressing steel is unbonded. Such bonded reinforcement shall be distributed uniformly over the tension zone near the extreme tension fiber.

18.9.2—The minimum amount of bonded reinforcement A_s in beams and one-way slabs shall be:

$$A_s = \frac{N_c}{0.5f_y} \qquad (18\text{-}5)$$

or

$$A_s = 0.004A \qquad (18\text{-}6)$$

whichever is larger, where A = area of that part of the cross section between the flexural tension face and the center of gravity of the gross section, and N_c = tensile force in the concrete under load of $D + 1.2L$ and f_y shall not exceed 60,000 psi.

18.9.3—The minimum amount of bonded reinforcement A_s in two-way slabs shall be that required by Section 18.9.2. This requirement for bonded reinforcement in two-way slabs may be decreased where the tension in the precompressed tension zone at service loads does not exceed zero.

18.10—Repetitive loads

18.10.1—In unbonded construction subject to repetitive loads, special attention shall be given to the possibility of fatigue in the anchorages or couplers. See Section 18.20.

18.10.2—The possibility of inclined diagonal tension cracks forming under repetitive loading at appreciably smaller stresses than under static loading shall be taken into account in the design.

18.11—End regions

18.11.1—Reinforcement shall be provided when required in the anchorage zone to resist bursting, horizontal splitting, and spalling forces induced by the tendon anchorages. Regions of abrupt change in section shall be adequately reinforced.

18.11.2—End blocks shall be provided when required for end bearing or for distribution of concentrated prestressing forces.

18.11.3—Post-tensioning anchorages and the supporting concrete shall be designed to support the maximum jacking load at the concrete strength at time of prestressing and the end anchorage region shall be designed to develop the guaranteed ultimate tensile strength of the tendons at a ϕ of 0.90 for the concrete.

18.12—Continuity

Continuous beams and other statically indeterminate structures shall be designed for adequate strength and satisfactory behavior. Behavior shall be determined by elastic analysis, taking into account the reactions, moments, shears, and axial forces produced by prestressing, the effects of temperature, creep, shrinkage, axial deformation, restraint of attached structural elements, and foundation settlement.

The negative moments due to design dead and live loads calculated by elastic theory for any assumed loading arrangement, at the supports of continuous prestressed concrete beams with sufficient bonded steel to assure control of cracking, may be increased or decreased by not more than $20[1 - (\omega + \omega_p - \omega')/0.30]$ percent, provided that these modified negative moments are also used for final calculations of the moments at other sections in the span corresponding to the same loading condition. Such an adjustment shall only be made when the section at which the moment is reduced is so designed that ω_p, $(\omega + \omega_p - \omega')$, or $(\omega_w + \omega_{pw} - \omega_w')$, whichever is applicable, is equal to or less than 0.20. The effect of moments due to prestressing shall be neglected when calculating the design moments.

18.13—Slab systems

Slabs reinforced in more than one direction shall be analyzed and designed by a method which will account for column stiffnesses, rigidity of slab-column connection, and for the effect of prestressing in accordance with Section 18.12. Moment coefficients used for design of reinforced concrete slabs are not applicable.

18.14—Compression members—Combined axial load and bending

18.14.1—Members with average prestress of 225 psi or higher shall be subject to the other provisions of this section. Members with average prestress less than 225 psi shall have minimum reinforcement in accordance with Section 10.9.1 for columns or Section 10.16 for walls. Average prestress is defined as the total effective prestress force divided by the gross area of the concrete section.

18.14.2 *Combined axial load and bending*—Prestressed concrete members under combined axial load and bending, with or without nonprestressed reinforcement, shall be proportioned by the strength design methods given in this Code for members without prestressing. The effects of prestress, shrinkage, and creep shall also be included. The minimum amounts of reinforcement specified in Section 10.16 may be waived where average prestress is over 225 psi and a structural analysis shows adequate strength and stability.

18.14.3 *Lateral reinforcement*—Except for walls, all prestressing steel shall be enclosed by spirals conforming to Section 7.12.2 or closed lateral ties at least #3 in size. The spacing of the ties shall not exceed 48 tie diameters, or the least dimension of the column. Ties shall be located vertically not more than one-half a tie spacing above the floor or footing, and shall be spaced as provided herein to not more than one-half a tie spacing below the lowest horizontal reinforcement in the slab or drop panel above. Where beams or brackets provide enclosure on all sides of the column, the ties may be terminated not more than 3 in. below the lowest reinforcement in such beams or brackets.

18.15—Corrosion protection for unbonded tendons

Unbonded tendons shall be completely coated with suitable material to insure corrosion protection. Wrapping must be continuous over the entire zone to be unbonded, and shall prevent intrusion of cement paste or the loss of coating materials during casting operations.

18.16—Post-tensioning ducts

18.16.1—Ducts for grouted or unbonded tendons shall be mortar-tight and nonreactive with concrete, tendons, or the filler material.

18.16.2—To facilitate grout injection, the inside diameter of the ducts shall be at least ¼ in. larger than the diameter of the post-tensioning tendon or large enough to produce an internal area at least twice the gross area of the prestressing steel.

18.17—Grout for bonded tendons

18.17.1—Grout shall consist of portland cement and potable water, or portland cement, sand, and potable water. Suitable admixtures known to have no injurious effects on the steel or the concrete may be used to increase workability and to reduce bleeding and shrinkage. Calcium chloride shall not be used.

18.17.2—Sand, if used, shall conform to "Specifications for Aggregate for Masonry Mortar" (ASTM C 144) except that gradation may be modified as necessary to obtain increased workability.

18.17.3—Proportions of grouting materials shall be based on results of tests on fresh and hardened grout prior to beginning work. The water content shall be minimum necessary for proper placement but in no case more than 0.50 the content of cement by weight.

18.17.4—Grout shall be mixed in a high speed mechanical mixer and then passed through a strainer into pumping equipment which provides for recirculation.

18.17.5—Temperature of members at the time of grouting must be above 50 F and shall be maintained at this temperature for at least 48 hr.

18.18—Steel tendons

Burning or welding operations in the vicinity of prestressing steel shall be carefully performed, so that the prestressing steel shall not be subjected to excessive temperatures, welding sparks, or ground currents.

18.19—Application and measurement of prestressing force

18.19.1—Prestressing force shall be determined (1) by measuring tendon elongation and (2) either by checking jack pressure on a calibrated gage or load cell or by the use of a calibrated dynamometer. The cause of any difference in force determination which exceeds 5 percent shall be ascertained and corrected. Elongation requirements shall be taken from average load-elongation curves for the steel used.

18.19.2—Where transfer of force from the bulkheads of the pretensioning bed to the concrete is accomplished by flame cutting the prestressing steel, the cutting points and cutting sequence shall be predetermined to avoid undesired temporary stresses. Long lengths of exposed strands shall be cut near the member to minimize shock to the concrete.

18.19.3—The total loss of prestress due to unreplaced broken tendons shall not exceed 2 percent of the total prestress.

18.20—Post-tensioning anchorages and couplers

18.20.1—Anchorages for unbonded tendons and couplers shall develop the specified ultimate capacity of the tendons without exceeding anticipated set. Anchorages for bonded tendons shall develop at least 90 percent of the specified ultimate capacity of the tendons, when tested in an unbonded condition, without exceeding anticipated set. However, 100 percent of the specified ultimate capacity of the tendons shall be developed after the tendons are bonded in the member. Couplers shall be placed in areas approved by the Engineer and enclosed in housings long enough to permit the necessary movements.

18.20.2—Anchorage and end fittings shall be permanently protected against corrosion.

18.20.3—Anchor fittings for unbonded tendons shall be capable of transferring to the concrete a load equal to the capacity of the tendon under both static and cyclic loading conditions.

CHAPTER 19—SHELLS AND FOLDED PLATE MEMBERS

19.0—Notations

f_c' = specified compressive strength of concrete, psi

$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi

f_y = specified yield strength of nonprestressed reinforcement, psi

h = overall thickness of member, in.

ϕ = capacity reduction factor. See Section 9.2

19.1—Scope and definitions

19.1.1—The provisions of this chapter apply to the design of thin shell concrete structures and only to the thin shell portions of such structures.

19.1.2—All other provisions of this Code not specifically excluded and not in conflict with the provisions of this chapter are to be considered applicable.

19.1.3—Thin shells are curved or folded slabs whose thicknesses are small compared to their other dimensions. They are characterized by their three-dimensional load-carrying behavior which is determined by their geometrical shape, their boundary conditions, and the nature of the applied load.

19.1.4—Thin shells are usually bounded by supporting members and edge members with a capacity to stiffen the shell and distribute or carry load in composite action with the shell.

19.1.5—Elastic analysis is any structural analysis involving assumptions which are suitable approximations of three-dimensional elastic behavior.

19.2—Assumptions

19.2.1—In the analysis of thin shells, it may be assumed that the material is ideally elastic, homogeneous, and isotropic.

19.2.2—Poisson's ratio may be assumed equal to zero.

19.3—General considerations

19.3.1—Elastic behavior shall be the accepted basis for determining internal forces, displacements, and stability of thin shells. Equilibrium checks of internal forces and external loads shall be made to insure consistency of results.

19.3.2—Approximate methods of analysis which do not satisfy compatibility of strains or stresses in the shell may be used in cases where experience has shown them to provide safe designs.

19.3.3—Analysis based on the results of elastic model tests approved by the Building Official shall be considered as valid elastic analyses. When such model analysis is used, only those portions which significantly affect the items under study need be simulated. Every attempt shall be made to insure that these tests reveal the quantitative behavior of the prototype structure.

19.3.4—The thin shell elements shall be proportioned for the required strength in accordance with the provisions of this Code.

19.3.5—Supporting members shall be designed according to the applicable provisions of this Code. A portion of the shell equal to the flange width specified in Section 8.7 may be assumed to act with the supporting member. In such portions of the shell the reinforcement perpendicular to the supporting member shall be at least equal to that required for the flange of a T-beam by Section 8.7.5.

19.3.6—When investigating thin shells for stability, consideration shall be given to the possible reduction in the buckling capacity caused by large deflections, creep effects, and the deviation between the actual and theoretical shell surface.

19.4—Design strengths

Minimum specified compressive strength of concrete at 28 days f_c' shall be 3000 psi. Maximum specified yield strength of reinforcement f_y shall be 60,000 psi.

19.5—Reinforcement requirements

19.5.1—The area of reinforcement in square inches per foot of width of shell shall not exceed $7.2hf_c'/f_y$ nor $29{,}000\ h/f_y$. If the deviation of the reinforcement from the lines of principal stress is greater than 10 deg, the maximum area of reinforcement shall be one-half the above values.

19.5.2—Reinforcement shall be spaced not farther apart than five times the shell thickness, nor more than 18 in. Where the computed principal tensile stress in the concrete due to design loads exceeds $4\phi\sqrt{f_c'}$, the reinforcement shall be spaced not farther apart than three times the shell thickness.

19.5.3—Reinforcement shall be provided to resist the total principal tensile stress, but shall not be less than required by Section 7.13. Such reinforcement, assumed to act at the middle surface of the shell, may be placed either parallel to the lines of principal tensile stress, or in two or three directions in straight lines. In the regions of high tension, the reinforcement shall be placed in the general direction of the principal stress.

19.5.4—Reinforcement may be considered parallel to the line of principal stress when its direction does not deviate from this line by more than 15 deg. Where excess reinforcement is provided, the 15 deg deviation may be increased 1 deg for each 5 percent decrease in steel stress below the specified yield strength, f_y. Variations in the direction of the principal stress over the cross section of the shell due to moments need not be considered for the determination of the maximum deviation.

19.5.5—Nonprestressed reinforcement placed in more than one direction shall be proportioned to resist the components of the principal tensile stresses in each direction.

19.5.6—Where the tensile stresses vary greatly in magnitude over the shell, as in the case of cylindrical shells, the reinforcement resisting the total tension may be concentrated in the region of maximum tensile stress. However, the ratio of steel to concrete in any portion of the tensile zone shall not be less than 0.0035.

19.5.7—Reinforcement required to resist bending moments shall be proportioned with due regard to axial forces.

19.5.8—Splices in principal tensile reinforcement shall conform to the requirements of Sections 7.5 through 7.9.

19.5.9—Shell reinforcement at the junction of the shell and supporting members or edge members shall be anchored in or through the supporting member by embedment length, hooks, or mechanical anchorage in accordance with Chapter 12.

19.6—Prestressing

Where prestressing tendons are draped within a shell the design shall account for the force components on the shell resulting from the tendon profile not lying in one plane.

19.7—Construction

When necessary to base removal of formwork on modulus of elasticity because of stability or deflection considerations, the modulus of elasticity that must be developed by the concrete before formwork removal shall be determined by tests of field cured beams. The dimensions of the beam and test procedures shall be specified by the Engineer. The proportions and loading of these specimens shall insure action which is primarily flexural.

PART 6—SPECIAL CONSIDERATIONS

CHAPTER 20—STRENGTH EVALUATION OF EXISTING STRUCTURES

20.0—Notation

a = maximum deflection under test load of a member relative to a line joining the ends of the span, or of the free end of a cantilever relative to its support, in.

D = dead loads, or their related internal moments and forces

h = overall thickness of member, in.

l_t = span of member under load test (the shorter span of flat slabs and of slabs supported on four sides). The span, except as provided in Section 20.4.6(c), is the distance between the centers of the supports or the clear distance between supports plus the depth of the member, whichever is smaller, in.

L = live loads, or their related internal moments and forces

20.1—Strength evaluation—General*

If doubt develops concerning the safety of a structure or member, the Building Official may order a structural strength investigation by analysis or by means of load tests, or by a combination of these methods.

20.2—General requirements for analytical investigation

If the strength evaluation is by analytical means, a thorough field investigation shall be made of the dimensions and details of the members, properties of the materials, and other pertinent conditions of the structure as actually built. The analysis based on this investigation shall satisfy the Building Official that the load factors meet the requirements and intent of the remainder of this Code. See Section 20.6.

20.3—General requirements for load tests

20.3.1—If the strength evaluation is by load tests, a qualified engineer acceptable to the Building Official shall control the tests.

20.3.2—A load test shall not be made until the portion of the structure subjected to load is at least 56 days old. When the owner of the structure, the contractor, and all involved parties mutually agree, the test may be made at an earlier age.

20.3.3—When only a portion of a structure is to be load tested, the questionable portion shall be load tested in such a manner as to adequately test the suspected source of weakness.

20.3.4—Forty-eight hours prior to the application of the test load, a load to simulate the effect of the portion of the dead loads not already present shall be applied and shall remain in place until all testing has been completed.

20.4—Load tests on flexural members

20.4.1—When flexural members, including beams and slabs, are load tested, the additional provisions in this section shall apply.

20.4.2—Immediately prior to the application of the test load, the necessary initial readings shall be made as datum for the measurements of deflections caused by the application of the test load.

20.4.3—The portion of the structure selected for loading shall be subjected to a total load, including the dead loads already in place, equivalent to 0.85 $(1.4D + 1.7L)$. The test load shall be applied in not less than four approximately equal increments without shock to the structure and in a manner to avoid arching of the loading materials.

20.4.4—After the test load has been in position for 24 hr deflection readings shall be taken. The test load shall then be removed and readings of deflections shall be taken 24 hr after the removal of the test load.

20.4.5—If the portion of the structure tested shows visible evidence of failure it shall be considered to have failed the test and no retesting of the previously tested portion shall be permitted.

20.4.6—If the structure shows no visible evidence of failure it shall satisfy the following criteria:

(a) If the measured maximum deflection a of a beam, floor, or roof exceeds $l_t^2/20{,}000h$, the deflection recovery within 24 hr after the removal of the test load shall be at least 75 percent of the maximum deflection for nonprestressed concrete, or 80 percent for prestressed concrete.

(b) If the maximum deflection a is less than $l_t^2/20{,}000h$ the requirement on recovery of deflection in Section 20.4.6(a) shall be waived.

(c) In Sections 20.4.6(a) and (b), l_t for cantilevers shall be taken as twice the distance from the support to the end, and the deflection shall be adjusted for any movement of the support.

*For approval of special systems of design or construction see Section 1.4.

(d) Construction failing to show 75 percent recovery of the deflection may be retested. The second test loading shall not be made until at least 72 hr after removal of the first test load. The structure shall show no visible evidence of failure in the retest, and the recovery of deflection caused by the second test load shall be at least 80 percent. Prestressed concrete construction shall not be retested.

20.5—Members other than flexural members

Members other than flexural members shall preferably be investigated by analytical procedures.

20.6—Provision for lower load rating

If the structure under investigation does not satisfy the conditions or criteria of Sections 20.2 or 20.4.6, the Building Official may approve a lower load rating for the structure based on the results of the load test or analysis.

20.7—Safety

Load tests shall be conducted in such a manner as to provide for safety of life and structure during the test but any safety measures shall not interfere with the load test procedures or affect results.

APPENDICES

APPENDIX A—SPECIAL PROVISIONS FOR SEISMIC DESIGN

A.0—Notation

A_{ch} = area of rectangular core of column measured out-to-out of hoop

A_g = gross area of section, sq in.

A_s = area of nonprestressed tension reinforcement, sq in.

A_s' = area of compression reinforcement, sq in.

A_{sh} = area of transverse hoop bar (one leg)

A_v = area of shear reinforcement within a distance s

d = distance from extreme compression fiber to centroid of tension reinforcement, in.

f_c' = specified compressive strength of concrete, psi

f_r = modulus of rupture of concrete, psi

f_y = specifield yield strength of nonprestressed reinforcement, psi

h = overall thickness of member, in.

l_h = maximum unsupported length of rectangular hoop measured between perpendicular legs of the hoop or supplementary crossties

l_n = clear span, measured face-to-face of supports

P_b = axial load capacity at simultaneous assumed ultimate strain of concrete and yielding of tension steel (balanced conditions)

P_e = maximum design axial load acting on a column or wall during an earthquake

s = shear reinforcement spacing in the direction of the longitudinal reinforcement

s_h = center-to-center spacing of hoops

ρ = ratio of nonprestressed tension reinforcement
= A_s/bd

ρ_s = ratio of volume of spiral reinforcement to total volume of core (out-to-out of spirals)

A.1—Scope

A.1.1—Reinforced concrete structures shall conform to the minimum provisions of the body of this Code and to the special provisions of this appendix for the types of members and systems covered when a ductile moment-resisting space frame or equivalent must be provided by design for structures located in an area where an earthquake of such magnitude as to cause major damage to construction has a high probability of occurrence during the lifetime of the structure.

A.1.2—The provisions of this appendix apply to monolithic or composite special ductile frames with cast-in-place beam-column connections and to special shear walls used with special ductile frames.

A.1.3—The requirements for seismic load resisting members under this appendix are based on the strength method of design.

A.1.4—Other structural systems and methods of design may be used when shown by analysis and design, based on accepted engineering principles, to provide adequate strength and ductility to resist the anticipated seismic movements.

A.2—Definitions

Confined region—Region with transverse reinforcement conforming to the required area and spacing of Sections A.5.9 and A.5.10 or Section A.6.4 and regions within beam-column connections conforming to Section A.7.

Hoop—A one-piece closed tie or continuously wound tie not less than #3 in size, the ends of which have a standard 135 deg bend with a 10-bar-diameter extension, that encloses the longitudinal reinforcement.

Plastic hinge—Region where ultimate moment capacity in a member may be developed and maintained with corresponding significant inelastic rotation as main tensile steel elongates beyond yield strain.

Special ductile frame—A structural frame composed of reinforced concrete flexural members and columns with cast-in-place connections designed and detailed to accommodate reversible lateral displacements after the formation of plastic hinges.

Special shear wall—A reinforced concrete shear wall designed and detailed in accordance with this appendix.

Stirrup-tie—A closed stirrup conforming to the definition of a hoop.

Supplementary crosstie—Tie with a standard semicircular hook at each end conforming to requirements of Section A.6.4.3.

A.3—General requirements

A.3.1—The interaction of all structural and nonstructural elements which affect the response of the building to seismic accelerations shall be considered in the analysis of the structure. The consequences of failure of elements which are not a part of the primary system for resisting seismic forces shall also be considered.

A.3.2—Frames, shear walls, and combinations of frames and shear walls shall be designed to resist forces from seismic accelerations. Floor and roof systems shall be designed to act as horizontal structural elements to transfer forces to frames or shear walls.

A.3.3—The minimum specified concrete strength f_c' shall be 3000 psi. The maximum specified yield strength of reinforcement f_y shall be 60,000 psi. Grade of reinforcement used shall be only that specified; substitution of higher grades shall not be permitted.

A.4—Assumptions

The requirements of this appendix assume that special ductile frames composed of flexural members and columns, with or without special shear walls, will be forced into lateral deformations sufficient to create reversible plastic hinges by the action of the most severe earthquake. The moment capacity of plastic hinges may be taken as equal to the moment strength computed by the provisions of Chapter 10.

A.5—Flexural members of special ductile frames

A.5.1—The maximum ρ shall not exceed 0.50 of the ratio producing balanced conditions as defined by Section 10.3.2 and 10.3.3. Both top and bottom reinforcement shall consist of not less than two bars and shall have a minimum ρ of $200/f_y$ throughout the entire length of the member.

A.5.2—At least one-third of the tension reinforcement provided for negative moment at the support shall extend anchorage distance beyond the extreme position of the point of inflection but not less than $0.25l_n$ from the face of the support. One-quarter of the larger amount of the tension reinforcement required at either end of the beam shall be continuous throughout the top of the beam.

A.5.3—The positive moment capacity of flexural members at column connections shall not be less than 50 percent of the negative moment capacity.

A.5.4—Top and bottom reinforcement reaching a column in flexural members framing into opposite sides of the column shall be continuous through the column where possible. When a top or bottom bar cannot be continued through the column due to variations in beam cross section, it shall be anchored in conformance with Section A.5.5.

A.5.5—A flexural member framing into a column where there is no flexural member on the opposite side shall have top and bottom reinforcement extended to the far face of the confined region and anchored to develop the specified yield strength. Development length shall be computed beginning at the near face of the column. Every bar shall terminate with a standard 90 deg hook or with such a hook and additional bar extension where needed to provide the required development length.

A.5.6—The length of anchorage in confined regions shall be not less than two-thirds the development length computed by Sections 12.5(a) and 12.5(c). The length of anchorage in other regions shall be not less than that required by Chapter 12. The anchorage length of flexural reinforcement shall be not less than 16 in. in either case.

A.5.7—Web reinforcement shall be provided to develop the shears resulting from design gravity loads on the member and from the moment capacities of plastic hinges at the ends of the member produced by lateral displacement.

A.5.8—Web reinforcement perpendicular to the longitudinal reinforcement shall be provided throughout the length of the member. The minimum size stirrup shall be #3, and the maximum spacing shall be $d/2$.

A.5.9—Within a distance equal to four times the effective depth d from the end of the member, the amount of web reinforcement shall not be less than

$$A_v \frac{d}{s} = 0.15A_s' \text{ or } 0.15A_s \qquad \text{(A-1)}$$

whichever is larger, and the spacing shall not exceed $d/4$. The two stirrups at the end of a member framing into a column shall be stirrup-ties. The first stirrup-tie shall be located within 3 in. of the column face.

A.5.10—Stirrup-ties spaced not farther apart than 16 bar diameters or 12 in. shall be provided when bars are required to act as compression reinforcement. Such stirrup-ties at beam ends shall be provided for a distance of at least twice the effective beam depth d from the column face.

A.5.11—Where inelastic deformation of the frame may develop the moment strength of the member at sections away from the end of the member, the amount and spacing of web reinforcement at these sections shall conform to Sections A.5.9 and A.5.10.

A.5.12—Tension steel shall not be spliced by lapping in a region of tension or reversing stress unless stirrup-ties spaced according to the requirements of Section A.5.10 are provided at the lap. At least two stirrup-ties shall be provided at all splices. Splice lengths shall be at least 24 bar diameters or 12 in. minimum. No welded splice shall be placed within a distance d of a plastic hinge.

A.6—Special ductile frame columns subjected to axial loads and bending

A.6.1—The vertical reinforcement ratio in columns shall be limited to a minimum of 1.0 percent and a maximum of 6.0 percent. Section 10.8.4 does not apply.

A.6.2—At any beam-column connection the sum of the moment strengths of the columns at the design axial load shall be greater than the sum of the moment strengths of the beams along each principal plane at that connection unless the sum of the moment strengths of the confined cores of the columns is sufficient to resist the applied design loads. Beam-column connections at any level may be exempt from this limitation provided the remaining columns and connected flexural members comply and are capable of resisting the entire shear at that level accounting for changes in forces and torsion resulting from the action of the nonconforming connection.

A.6.3—Columns shall be designed and detailed as flexural members in accordance with the requirements of Section A.5 when

$$P_e \leq 0.4P_b \qquad \text{(A-2)}$$

A.6.4—The concrete core of a column shall be confined by special transverse-reinforcement as specified below when

$$P_e > 0.4P_b \qquad \text{(A-3)}$$

A.6.4.1 Confinement reinforcement consisting of spiral or hoop reinforcement shall be supplied above and below connections over a minimum length from the face of the connection at least equal to the overall depth h (h being the longer dimension in the case of rectangular columns or the diameter of a round column), 18 in., and one-sixth the clear height of the column.

A.6.4.2 Where a spiral is used the volumetric ratio ρ_s shall be not less than indicated by Eq. (10-3), but not less than $0.12f_c'/f_y$.

A.6.4.3 Where rectangular hoop reinforcement is used, the required area of the bar shall be computed by:

$$A_{sh} = \frac{l_h \, \rho_s \, s_h}{2} \qquad \text{(A-4)}$$

where ρ_s is the volumetric ratio required by Section A.6.4.2 with A_{ch} substituted for A_c and with f_y the yield strength of the hoop reinforcement. The center-to-center spacing between hoops or the pitch of continuous hoops shall not exceed 4 in. Minimum bar size shall be that required for ties by Section 7.12.3. Supplementary crossties of the same bar size as the hoop may be used to reduce the unsupported length, l_h. Each end of the supplementary crossties shall engage the periphery hoop with a standard semicircular hook, and shall be secured to a longitudinal bar to prevent displacement of the crosstie during construction. Minimum cover of supplementary crosstie reinforcement shall be ½ in.

A.6.4.4 Supplementary ties in addition to the hoops shall be provided if needed to satisfy Section A.6.6.

A.6.5—Where walls or stiff partitions do not continue from story to story, the columns supporting the wall or partition load shall have confinement reinforcement conforming to the requirements of Section A.6.4 throughout the column length.

A.6.6—Transverse reinforcement in the column shall be provided to ensure that the shear capacity of the member is at least equal to the applied shears at the formation of plastic hinges in the frame due to the combination of lateral displacement and design gravity loads. The shear permitted on the concrete shall be in accordance with Sections 11.4.3 or 11.4.4. Maximum spacing of shear reinforcement in columns shall be $d/2$.

A.6.7—Splices in vertical reinforcement shall conform with Chapter 7 but in no case shall a lap splice length be less than 30 bar diameters or 16 in. When continuity is established by welding or by mechanical devices not more than one-fourth of the bars shall be spliced at any level and the distance between levels of splicing of adjacent bars shall be not less than 12 in.

A.7—Beam-column connections in special ductile frames

A.7.1—Transverse reinforcement through the connection shall be proportioned according to the requirements of Sections A.6.4 and A.6.6. The design shear shall be equal to the maximum shear in the connection computed by an analysis taking into account the column shear and the shears developed from the yield forces in the beam reinforcement.

A.7.2—In connections having beams framing into four sides of the column, the transverse reinforcement in the connection may be one-half that required by Section A.7.1 if every beam has a width of not less than one-half the column width and a depth of not less than three-fourths that of the deepest beam framing into the support.

A.7.3—Where the axis of a flexural member does not intersect the axis of the column, the connection shall be designed and detailed to accommodate the shears, moments, and torsions caused by the offset.

A.8—Special shear walls

A.8.1—Special shear walls shall be proportioned to resist the combination of overturning moments, vertical loads, and shears. Adequate provisions shall be made to transfer wall moments, vertical loads, and shears to foundations or supports.

A.8.2—The minimum areas of distributed horizontal and vertical reinforcement of earthquake resisting shear walls shall each be not less than 0.0025 times the area of the gross section of the wall.

A.8.3—Reduced horizontal force factors, if permitted for the design of the building because of the inclusion of a ductile moment-resisting space frame or equivalent, shall not be used to calculate the shear reinforcement for a shear wall.

A.8.4—In a shear wall when $P_e \leqq 0.4P_b$ and the extreme fiber stress in tension computed in accordance with Eq. (9-2) and (9-3) on the gross cross section of the wall acting as an elastic, homogeneous material exceeds $0.15f_r$, the minimum area of vertical reinforcement concentrated near the ends of the wall shall be

$$A_s = \left(\frac{200}{f_y}\right) hd \qquad \text{(A-5)}$$

where d is the horizontal distance, in inches, from the extreme compression fiber to the centroid of such reinforcement. In no case shall the wall reinforcement be less than that required for axial load, moment, and shear.

A.8.5—The following design procedures are acceptable for a shear wall when $P_e > 0.4P_b$. Either a design method based on accepted engineering principles or the provisions of Sections A.8.5.1 and A.8.5.2 shall be used.

A.8.5.1 A shear wall shall have vertical boundary elements designed to carry all vertical stresses resulting from the design wall dead load, design tributary dead and live loads, and design horizontal forces.

A.8.5.2 Reinforcing steel in the boundary elements shall be confined with transverse reinforcement for their full length in conformity with Section A.6.4.

A.8.6—Construction joints in earthquake resisting shear walls shall be constructed in conformance with Section 6.4. The top of the hardened concrete shall be clean and rough before placing the next lift.

A.8.7—Splices in vertical reinforcement shall conform to the provisions of Section A.6.7.

APPENDIX B—NOTATION

a = depth of equivalent rectangular stress block, defined by Section 10.2.7—Chapters 8, 10, and 12

a = shear span, distance between concentrated load and face of support—Chapter 11

a = maximum deflection under test load of a member relative to a line joining the ends of the span, or of the free end of a cantilever relative to its support, in.—Chapter 20

A = effective tension area of concrete surrounding the main tension reinforcing bars and having the same centroid as that reinforcement, divided by the number of bars, sq in. When the main reinforcement consists of several bar sizes the number of bars shall be computed as the total steel area divided by the area of the largest bar used—Chapter 10

A = area of that part of the cross section between the flexural tension face and the center of gravity of the gross section—Chapter 18

A_b = area of an individual bar, sq in.—Chapter 12

A_c = area of core of spirally reinforced column measured to the outside diameter of the spiral, sq in.—Chapter 10 and Appendix A

A_c = area of concrete at the cross section considered—Chapter 18

A_{ch} = area of rectangular core of column measured out-to-out of hoop—Appendix A

A_g = gross area of section, sq in.—Chapters 8, 9, 10, 11, 14, and Appendix A

A_h = area of shear reinforcement parallel to the main tension reinforcement, sq in.—Chapter 11

A_l = total area of longitudinal reinforcement to resist torsion, sq in.—Chapter 11

A_{ps} = area of prestressed reinforcement in tension zone—Chapters 11 and 18

A_s = area of nonprestressed tension reinforcement, sq in.—Chapters 8, 9, 10, 11, 12, 18, and Appendix A

A_s' = area of compression reinforcement, sq in.—Chapters 8, 9, 18, and Appendix A

A_{sh} = area of transverse hoop bar (one leg)—Appendix A

A_t = area of structural steel or tubing in a composite section—Chapter 10

A_t = area of one leg of a closed stirrup resisting torsion within a distance *s*, sq in.—Chapter 11

A_v = area of shear reinforcement within a distance *s*, or area of shear reinforcement perpendicular to main reinforcement within a distance *s* for deep beams, sq in.—Chapter 11

A_v = area of shear reinforcement within a distance *s*—Chapter 12 and Appendix A

A_{vf} = area of shear-friction reinforcement, sq in.—Chapter 11

A_{vh} = area of shear reinforcement parallel to the main tension reinforcement within a distance s_2, sq in.—Chapter 11

A_w = area of a deformed wire, sq in.—Chapters 7 and 12

A_1 = loaded area—Chapter 10

A_2 = maximum area of the portion of the supporting surface that is geometrically similar to and concentric with the loaded area—Chapter 10

b = width of compression face of member—Chapters 8, 10, 11, and 18

b_o = periphery of critical section for slabs and footings—Chapter 11

b_v = the width of the cross section being investigated for horizontal shear—Chapter 17

b_w = web width, or diameter of circular section, in.—Chapters 11 and 12

c = distance from extreme compression fiber to neutral axis—Chapter 10

c_1 = size of rectangular or equivalent rectangular column, capital, or bracket measured in the direction in which moments are being determined—Chapter 11

c_2 = size of rectangular or equivalent rectangular column, capital, or bracket measured transverse to the direction in which moments are being determined—Chapters 11 and 13

C = cross-sectional constant to define the torsional properties. See Eq. (13-7)—Chapter 13

C_m = a factor relating the actual moment diagram to an equivalent uniform moment diagram—Chapter 10

d = distance from extreme compression fiber to centroid of tension reinforcement, in.—Chapters 7, 8, 10, 11, 12, 17, and Appendix A

d = distance from extreme compression fiber to centroid of prestressing steel, or to combined centroid when nonprestressing tension reinforcement is included, in.—Chapter 18

d' = distance from extreme compression fiber to centroid of compression reinforcement, in.—Chapter 9

d_b = nominal diameter of bar, wire, or prestressing strand, in.—Chapters 7 and 12

d_c = thickness of concrete cover measured from the extreme tension fiber to the center of the bar located closest thereto—Chapter 10

d_p = diameter of the pile at footing base—Chapter 15

d_s = distance from centroid of tension reinforcement to the tensile face of the member, in.—Chapter 9

D = dead loads, or their related internal moments and forces—Chapters 9 and 20

e = eccentricity of design load parallel to axis measured from the centroid of the section. It may be calculated by conventional methods of frame analysis—Chapter 10

e = base of Napierian logarithms—Chapter 18

E = load effects of earthquake, or their related internal moments and forces—Chapter 9

E_c = modulus of elasticity of concrete, psi. See Section 8.3.1—Chapters 8, 9, and 10

E_{cb} = modulus of elasticity for beam concrete—Chapter 13

E_{cc} = modulus of elasticity for column concrete—Chapter 13

E_{cs} = modulus of elasticity for slab concrete—Chapter 13

EI = flexural stiffness of compression members. See Eq. (10-7) and Eq. (10-8)—Chapter 10

E_s = modulus of elasticity of steel, psi. See Section 8.3.2—Chapters 8 and 10

f_c' = specified compressive strength of concrete, psi—Chapters 4, 7, 8, 9, 10, 11, 12, 14, 18, 19, and Appendix A

$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi—Chapters 7, 9, 11, 12, 15, and 19

f'_{ci} = compressive strength of concrete at time of initial prestress—Chapter 18

f_{ct} = average splitting tensile strength of lightweight aggregate concrete, psi—Chapters 4, 9, 11, and 12

f_d = stress due to dead load, at the extreme fiber of a section at which tensile stresses are caused by applied load, psi—Chapter 11

f_h = tensile stress developed by a standard hook, psi—Chapter 12

f_{pc} = compressive stress in the concrete, after all prestress losses have occurred, at the centroid of the cross section resisting the applied loads or at the junction of the web and flange when the centroid lies in the flange, psi. (In a composite member, f_{pc} will be the resultant compressive stress at the centroid of the composite section, or at the junction of the web and flange when the centroid lies within the flange, due to both prestress and to bending moments resisted by the precast member acting alone)—Chapter 11

f_{pe} = compressive stress in concrete due to prestress only after all losses, at the extreme fiber of a section at which tensile stresses are caused by applied loads, psi—Chapter 11

f_{ps} = calculated stress in prestressing steel at design load, psi—Chapters 12 and 18

f_{pu} = ultimate strength of prestressing steel, psi—Chapters 3, 11, and 18

f_{py} = specified yield strength of prestressing steel, psi—Chapter 18

f_r = modulus of rupture of concrete, psi—Chapter 9 and Appendix A

f_s = calculated stress in reinforcement at service loads, ksi—Chapter 10

f_{se} = effective stress in prestressing steel, after losses, psi—Chapters 12 and 18

f_y = specified yield strength of nonprestressed reinforcement, psi—Chapters 3, 7, 8, 9, 10, 11, 12, 18, 19, and Appendix A

F = lateral or vertical pressure of liquids, or their related internal moments and forces—Chapter 9

h = overall thickness of member, in.—Chapters 7, 8, 9, 10, 11, 13, 14, 19, 20, and Appendix A

h_v = total depth of shearhead cross section—Chapter 11

h_w = total height of wall from its base to its top—Chapter 11

H = lateral earth pressure, or its related internal moments and forces—Chapter 9

I = moment of inertia of section resisting externally applied design loads—Chapter 11

I_b = moment of inertia about centroidal axis of gross section of a beam as defined in Section 13.1.5—Chapter 13

I_c = moment of inertia of gross cross section of columns—Chapter 13

I_{cr} = moment of inertia of cracked section transformed to concrete—Chapter 9

I_e = effective moment of inertia for computation of deflection—Chapter 9

I_g = moment of inertia of gross concrete section about the centroidal axis, neglecting the reinforcement—Chapters 9 and 10

I_s = moment of inertia about centroidal axis of gross section of slab
= $h^3/12$ times width of slab specified in definitions of α and β_t—Chapter 13

I_{se} = moment of inertia of reinforcement about the centroidal axis of the member cross section—Chapter 10

I_t = moment of inertia of structural steel or tubing in a cross section about the centroidal axis of the member cross section—Chapter 10

k = effective length factor for compression members—Chapter 10

K = wobble friction coefficient per foot of prestressing steel—Chapter 18

K_b = flexural stiffness of beam; moment per unit rotation—Chapter 13

K_c = flexural stiffness of column; moment per unit rotation—Chapter 13

K_{ec} = flexural stiffness of an equivalent column; moment per unit rotation. See Eq. (13-5)—Chapter 13

K_s = flexural stiffness of slab; moment per unit rotation—Chapter 13

K_t = torsional stiffness of torsional member; moment per unit rotation—Chapter 13

l = span length of beam or one-way slab, as defined in Section 8.5.2; clear projection of cantilever, in.—Chapter 9

l = length of prestressing steel element from jacking end to any point x—Chapter 18

l_a = additional embedment length at support or at point of inflection, in.—Chapter 12

l_c = height of column, center-to-center of floors or roof—Chapter 13

l_c = vertical distance between supports—Chapter 14

l_d = development length, in. See Chapter 12—Chapters 7 and 12

l_e = equivalent embedment length, in.—Chapter 12

l_h = maximum unsupported length of rectangular hoop measured between perpendicular legs of the hoop or supplementary crossties—Appendix A

l_n = clear span for positive moment or shear and the average adjacent clear spans for negative moment—Chapter 8

l_n = length of clear span in long direction of two-way construction, measured face-to-face of columns in slabs without beams and face-to-face of beams or other supports in other cases—Chapter 9

l_n = length of clear span, in the direction moments are being determined, measured face-to-face of supports—Chapter 13

l_n = clear span measured face-to-face of supports—Chapter 11 and Appendix A

l_t = span of member under load test (the shorter span of flat slabs and of slabs supported on four sides). The span, except as provided in Section 20.4.6(c) is the distance between the centers of the supports or the clear distance between supports plus the depth of the member, whichever is smaller, in.—Chapter 20

l_u = unsupported length of compression member—Chapter 10

l_v = length of shearhead arm from centroid of concentrated load or reaction—Chapter 11

l_w = total lengths of wire extending beyond outermost cross wires, for each pair of spliced wires, in.—Chapter 7

l_w = horizontal length of wall—Chapters 8 and 11

l_1 = length of span in the direction moments are being determined measured center-to-center of supports—Chapter 13

l_2 = length of span transverse to l_1, measured center-to-center of supports—Chapter 13

L = live loads, or their related internal moments and forces—Chapters 9 and 20

M_a = maximum moment in member at stage for which deflection is being computed—Chapter 9

M_c = moment to be used for design of compression member—Chapter 10

M_{cr} = cracking moment. See Section 9.5.2.2—Chapters 9 and 11

M_m = modified bending moment—Chapter 11

M_{max} = maximum bending moment due to externally applied design loads—Chapter 11

M_o = total static design moment—Chapter 13

M_p = required full plastic moment of shearhead cross section—Chapter 11

M_t = theoretical moment strength, in.-lb, of a section

$= A_s f_y\left(d - \frac{a}{2}\right)$—Chapters 8 and 12

M_u = applied design load moment at a section, in.-lb—Chapter 11

M_v = moment resistance contributed by shearhead reinforcement—Chapter 11

M_1 = value of smaller design end moment on compression member calculated from a conventional elastic frame analysis, positive if member is bent in single curvature, negative if bent in double curvature—Chapter 10

M_2 = value of larger design end moment on compression member calculated from a conventional elastic frame analysis, always positive—Chapter 10

n = number of pairs of cross wires in splice —Chapter 7

n = modular ratio $= E_s/E_c$—Chapter 8

n = number of cross wires in anchorage zone of welded deformed wire fabric—Chapter 12

N_c = tensile force in the concrete under load of $D + 1.2L$—Chapter 18

N_u = design tensile force on bracket or corbel acting simultaneously with V_u—Chapter 11

N_u = design axial load normal to the cross section occurring simultaneously with V_u to be taken as positive for compression, negative for tension, and to include the effects of tension due to shrinkage and creep—Chapters 8 and 11

P_b = axial load capacity at simultaneous assumed ultimate strain of concrete and yielding of tension steel (balanced conditions)—Chapter 9 and Appendix A

P_c = critical load. See Section 10.11.5—Chapter 10

P_e = maximum design axial load acting on a column or wall during an earthquake—Appendix A

P_s = steel force at jacking end—Chapter 18

P_u = axial design load in compression member —Chapters 8, 9, 10, and 14

P_x = steel force at any point x—Chapter 18

r = radius of gyration of the cross section of a compression member—Chapter 10

s = tie spacing, in.—Chapter 7

s = shear or torsion reinforcement spacing in a direction parallel to the longitudinal reinforcement—Chapter 11

s = spacing of stirrups, in.—Chapter 12

s = shear reinforcement spacing in the direction of the longitudinal reinforcement —Appendix A

s_h = center-to-center spacing of hoops—Appendix A

s_w = spacing of deformed wires, in.—Chapters 7 and 12

s_1 = spacing of vertical reinforcement in a wall—Chapter 11

s_2 = shear or torsion reinforcement spacing in a direction perpendicular to the longitudinal reinforcement—or spacing of horizontal reinforcement in a wall—Chapter 11

T = cumulative effect of temperature, creep, shrinkage, and differential settlement—Chapter 9

T_u = design torsional moment—Chapter 11

U = required strength to resist design loads or their related internal moments and forces —Chapter 9

v_c = nominal permissible shear stress carried by concrete—Chapters 8 and 11

v_{ci} = shear stress at diagonal cracking due to all design loads, when such cracking is the result of combined shear and moment —Chapter 11

v_{cw} = shear stress at diagonal cracking due to all design loads, when such cracking is the result of excessive principal tensile stresses in the web—Chapter 11

v_{dh} = design horizontal shear stress at any cross section, psi—Chapter 17

v_h = permissible horizontal shear stress, psi—Chapter 17

v_{tc} = nominal permissible torsion stress carried by concrete—Chapter 11

v_{tu} = nominal total design torsion stress—Chapter 11

v_u = nominal total design shear stress—Chapter 11

v_d = shear force at section due to dead load—Chapter 11

v_l = shear force at section occurring simultaneously with M_{max}—Chapter 11

V_p = vertical component of the effective prestress force at the section considered—Chapter 11

V_u = total applied design shear force at section —Chapters 8, 11, 12, and 17

w = design load per unit length of beam or per unit area of slab—Chapter 8

w = weight of concrete, lb per cu ft—Chapters 8 and 9

w = design load per unit area—Chapter 13

w_d = design dead load per unit area—Chapter 13

w_l = design live load per unit area—Chapter 13

W = wind load, or its related internal moment and forces—Chapter 9

x = shorter overall dimension of a rectangular part of a cross section—Chapters 11 and 13

x_1 = shorter center-to-center dimension of a closed rectangular stirrup—Chapter 11

y = longer overall dimension of a rectangular part of a cross section—Chapters 11 and 13

y_t = distance from the centroidal axis of gross section, neglecting the reinforcement, to the extreme fiber in tension—Chapters 9 and 11

y_1 = longer center-to-center dimension of a closed rectangular stirrup—Chapter 11

z = a quantity limiting distribution of flexural reinforcement. See Section 10.6—Chapter 10

α (alpha) = ratio of flexural stiffness of beam section to the flexural stiffness of a width of slab bounded laterally by the center line of the adjacent panel, if any, on each side of the beam

$= \dfrac{E_{cb}I_b}{E_{cs}I_s}$ —Chapters 9 and 13

α = angle between inclined web bars and longitudinal axis of member—Chapter 11

α = total angular change of prestressing steel profile in radians from jacking end to any point x—Chapter 18

α_c = ratio of flexural stiffness of the columns above and below the slab to the combined flexural stiffness of the slabs and beams at a joint taken in the direction moments are being determined

$= \dfrac{\Sigma K_c}{\Sigma(K_s + K_b)}$ —Chapter 13

α_{ec} = ratio of flexural stiffness of the equivalent column to the combined flexural stiffness of the slabs and beams at a joint taken in the direction moments are being determined

$= \dfrac{K_{ec}}{\Sigma(K_s + K_b)}$ —Chapter 13

α_m = average value of α for all beams on the edges of a panel—Chapter 9

α_{min} = minimum α_c to satisfy Section 13.3.6.1(a) —Chapter 13

α_t = a coefficient as a function of y_1/x_1. See Section 11.8.2—Chapter 11

α_v = ratio of stiffness of shearhead arm to surrounding composite slab section. See Section 11.11.2—Chapter 11

α_1 = α in the direction of l_1—Chapter 13

α_2 = α in the direction of l_2—Chapter 13

β (beta) = ratio of clear spans in long to short direction of two-way construction—Chapter 9

β = ratio of long side to short side of a footing—Chapter 15

β_a = ratio of dead load per unit area to live load per unit area (in each case without load factors)—Chapter 13

β_b = ratio of area of bars cut off to total area of bars at the section—Chapter 12

β_d = the ratio of maximum design dead load moment to maximum design total load moment, always positive—Chapter 10

β_s = ratio of length of continuous edges to total perimeter of a slab panel—Chapter 9

β_t = ratio of torsional stiffness of edge beam section to the flexural stiffness of a width of slab equal to the span length of the beam, center-to-center of supports

$= \dfrac{E_{cb}C}{2E_{cs}I_s}$ —Chapter 13

β_1 = a factor defined in Section 10.2.7—Chapters 8 and 10

δ (delta) = moment magnification factor for columns. See Section 10.11.5—Chapter 10

δ_s = factor defined by Eq. (13-4). See Section 13.3.6.1—Chapter 13

μ (mu) = coefficient of friction. See Section 11.15—Chapter 11

μ = curvature friction coefficient—Chapter 18

ξ (xi) = constant for standard hook—Chapter 12

ρ (rho) = ratio of nonprestressed tension reinforcement

$= A_s/bd$—Chapters 8, 10, 11, 18, and Appendix A

ρ' = A_s'/bd—Chapters 8 and 18

ρ_b = reinforcement ratio producing balanced conditions. See Section 10.3.3—Chapters 8 and 10

ρ_h = the ratio of horizontal shear reinforcement area to the gross concrete area of a vertical section—Chapter 11

ρ_{min} = minimum reinforcement ratio—Chapter 10

ρ_n = the ratio of vertical shear reinforcement area to the gross concrete area of a horizontal section—Chapter 11

ρ_p = ratio of prestressed reinforcement

$= A_{ps}/bd$—Chapter 18

ρ_s = ratio of volume of spiral reinforcement to total volume of core (out-to-out of spirals) of a spirally reinforced concrete or composite column—Chapter 10

ρ_s = ratio of volume of spiral reinforcement to total volume of core (out-to-out of spirals)—Appendix A

ρ_v = $(A_s + A_h)/bd$—Chapter 11

ρ_w = A_s/b_wd—Chapter 11

ϕ (phi) = capacity reduction factor. See Section 9.2—Chapters 8, 9, 10, 11, 14, 15, 17, 18, and 19

ω (omega) = $\rho f_y/f_c'$—Chapter 18

ω' = $\rho' f_y/f_c'$—Chapter 18

ω_p = $\rho_p f_{ps}/f_c'$—Chapter 18

$\omega_w, \omega_{pw}, \omega_w'$ = reinforcement indices for flanged sections computed as for $\omega, \omega_p, \omega'$ except that b shall be the web width, and the steel area shall be that required to develop the compressive strength of the web only—Chapter 18

APPENDIX C—METRIC EQUIVALENTS

The following is not part of this standard, but metric equivalents of all the dimensional values in this Code and metric conversions of nonhomogeneous equations are given below for the convenience of users.

Note that concrete strengths are based on standard 6 x 12-in. (15 x 30-cm) cylinders and steel strength on the minimum specified yield strength.

The units are in the technical (MKS) system. To convert to the Systéme International, use the following equations:

1 kilogram (kg) = 9.806 Newtons (N)

1 kg per sq cm = 0.9806 bars

METRIC EQUIVALENTS OF DIMENSIONAL UNITS

Length

English	*Metric*
1 in.	2.54 cm
0.15 in.	3.81mm
¼ in.	6.35 mm
⅜ in.	9.53 mm
½ in.	1.27 cm
⅝ in.	1.59 cm
¾ in.	1.91 cm
1¼ in.	3.18 cm
1½ in.	3.81 cm
2 in.	5.08 cm
2½ in.	6.35 cm
3 in.	7.62 cm
3½ in.	8.89 cm
4 in.	10.16 cm
5 in.	12.70 cm
6 in.	15.24 cm
8 in.	20.32 cm
10 in.	25.40 cm
12 in.	30.5 cm
16 in.	40.6 cm
18 in.	45.7 cm
20 in.	50.8 cm
24 in.	61.0 cm
30 in.	76.2 cm
1 ft	0.3048 m
3 ft	0.915 m
10 ft	3.05 m
12 ft	3.66 m
15 ft	4.58 m
25 ft	7.63 m

Stress (pressure)

English psi	*Metric* kg per sq cm
1	0.07
50	3.52
60	4.22
80	5.62
120	8.44
150	10.5
200	14.1
250	17.6
300	21.1
350	24.6
400	28.1
500	35.2
550	38.7
600	42.2
700	49.2
800	56.2
900	63.3
1000	70.3
1200	84.4
2500	176.0
3000	211.0
3500	246.0
4000	281.0
4500	316.0
5000	352.0
20,000	1,406
24,000	1,687
30,000	2,109
40,000	2,812
50,000	3,516
60,000	4,219
80,000	5,625
29,000,000	2,039,000

Moment of inertia

$1 \text{ in.}^4 = 41.62 \text{ cm}^4$

Weight

English lb per cu ft	*Metric* kg per cu m
1	16
70	1121
90	1442
115	1842
120	1922
145	2323
155	2482

Temperature

English deg F	*Metric* deg C
40	4
50	10
60	16
80	27
90	32
95	35
150	65

Area

English	*Metric*
5000 sq ft	464.5 sq m

Wire size

English	*Metric*
D4	25.8 sq mm

No. 10 AS&W gage 3.43 mm dia.

Cylinder

English	Metric
6 x 12 in.	15 x 30 cm

Volume

English	Metric
50 cu yd	38.3 cu m
150 cu yd	114.8 cu m

Bar size and area

Bar size	Diameter, mm	Area, sq cm
#3	9.52	0.71
#4	12.70	1.29
#5	15.88	2.00
#6	19.05	2.84
#7	22.22	3.87
#8	25.40	5.10
#9	28.65	6.45
#10	32.26	8.19
#11	35.81	10.06
#14	43.00	14.52
#18	57.33	25.81

METRIC EQUIVALENTS OF LIMITING VALUES

Units

	English	Metric
Stress	psi	kg per sq cm
Size	in.	cm
Weight	lb per cu ft	kg per cu m
Area	sq in.	sq cm
Load	lb	kg

Stress

English	Metric
$\sqrt{f_c'}$	$0.265\sqrt{f_c'}$
$1.5\sqrt{f_c'}$	$0.398\sqrt{f_c'}$
$1.6\sqrt{f_c'}$	$0.422\sqrt{f_c'}$
$1.7\sqrt{f_c'}$	$0.451\sqrt{f_c'}$
$2.0\sqrt{f_c'}$	$0.530\sqrt{f_c'}$
$3.0\sqrt{f_c'}$	$0.795\sqrt{f_c'}$
$3.5\sqrt{f_c'}$	$0.928\sqrt{f_c'}$
$4.0\sqrt{f_c'}$	$1.06\ \sqrt{f_c'}$
$5.0\sqrt{f_c'}$	$1.33\ \sqrt{f_c'}$
$5.5\sqrt{f_c'}$	$1.46\ \sqrt{f_c'}$
$6.0\sqrt{f_c'}$	$1.59\ \sqrt{f_c'}$
$6.3\sqrt{f_c'}$	$1.67\ \sqrt{f_c'}$
$7.5\sqrt{f_c'}$	$1.99\ \sqrt{f_c'}$
$8.0\sqrt{f_c'}$	$2.12\ \sqrt{f_c'}$
$10.0\sqrt{f_c'}$	$2.65\ \sqrt{f_c'}$
$12.0\sqrt{f_c'}$	$3.18\ \sqrt{f_c'}$

Sec. 7.7.1.1

English	Metric
$0.005 f_y d_b$	$0.00711 f_y d_b$
$(0.0009 f_y - 24) d_b$	$(0.0128 f_y - 24) d_b$

Sec. 7.9

English	Metric
$\dfrac{0.045 d_b f_y}{\sqrt{f_c'}}$	$\dfrac{0.17 d_b f_y}{\sqrt{f_c'}}$
$0.045 d_b \left(\dfrac{f_y - 20{,}000 n}{\sqrt{f_c'}}\right)$	$0.17 d_b \left(\dfrac{f_y - 1406 n}{\sqrt{f_c'}}\right)$

Sec. 7.13

English	Metric
$\dfrac{0.0018 \times 60{,}000}{f_y}$	$\dfrac{0.0018 \times 4219}{f_y}$

Sec. 8.3.1

English	Metric
$w^{1.5} 33\sqrt{f_c'}$	$w^{1.5} 0.1368\sqrt{f_c'}$
$57{,}000\sqrt{f_c'}$	$15{,}100\sqrt{f_c'}$

Table 9.5(a) Footnote (a) and (b)

English	Metric
$1.65 - 0.005 w \geqq 1.09$	$1.65 - 0.000312 w \geqq 1.09$
$0.4 + f_y/100{,}000$	$0.4 + 0.0142 f_y$
(f_y in psi)	(f_y in kg per sq mm)

Sec. 10.5.1

English	Metric
$\dfrac{200}{f_y}$	$\dfrac{14.06}{f_y}$
(f_y in psi)	(f_y in kg per sq cm)

Sec. 10.6.4

English	Metric
175 kips per in.	31,250 kg per cm
145 kips per in.	25,850 kg per cm

Sec. 11.3.1

English	Metric
$\dfrac{f_{ct}}{6.7} \leqq \sqrt{f_c'}$	$\dfrac{f_{ct}}{6.7} \leqq 0.265\sqrt{f_c'}$

Sec. 11.7.6

English	Metric
$\left(1 + 0.002 \dfrac{N_u}{A_g}\right)$	$\left(1 + 0.0285 \dfrac{N_u}{A_g}\right)$

Sec. 12.5(a)

English	Metric
$\dfrac{0.04 A_b f_y}{\sqrt{f_c'}}$	$\dfrac{0.0594 A_b f_y}{\sqrt{f_c'}}$
$0.0004 d_b f_y$	$0.00569 d_b f_y$
$\dfrac{0.085 f_y}{\sqrt{f_c'}}$	$\dfrac{0.815 f_y}{\sqrt{f_c'}}$
$\dfrac{0.11 f_y}{\sqrt{f_c'}}$	$\dfrac{1.054 f_y}{\sqrt{f_c'}}$
$\dfrac{0.03 d_b f_y}{\sqrt{f_c'}}$	$\dfrac{0.113 d_b f_y}{\sqrt{f_c'}}$

Sec. 12.5(b)

English	Metric
$2 - \dfrac{60{,}000}{f_y}$	$2 - \dfrac{4219}{f_y}$

Sec. 12.5(c)

English	Metric
$\dfrac{6.7\sqrt{f_c'}}{f_{ct}}$	$\dfrac{1.775\sqrt{f_c'}}{f_{ct}}$

Sec. 12.6

English	Metric
$\dfrac{0.02 f_y d_b}{\sqrt{f_c'}}$	$\dfrac{0.0755 f_y d_b}{\sqrt{f_c'}}$
$0.0003 f_y d_b$	$0.00427 f_y d_b$

Sec. 12.8.1

$f_h = \xi\sqrt{f_c'}$	$f_h = \xi\sqrt{f_c'}$
220	58
330	87
360	95
420	111
450	119
480	127
540	143

Sec. 12.10.2

$(f_y - 20{,}000n)$ $\qquad$ $(f_y - 1406n)$

Sec. 12.11.1

$(f_{ps} - \frac{2}{3}f_{se})\,d_b$ (f_{ps} and f_{se} in ksi) $\qquad$ $0.01422\,(f_{ps} - \frac{2}{3}f_{se})\,d_b$ (f_{ps} and f_{se} in kg per sq cm)

Sec. 19.5.1

$$\frac{7.2hf_c'}{f_y} \qquad \frac{60hf_c'}{f_y}$$

nor $29{,}000\,\dfrac{h}{f_y}$ $\qquad$ nor $17{,}000\,\dfrac{h}{f_y}$ = sq cm per m

Sec. A.5.1

$$\frac{200}{f_y} \qquad \frac{14.06}{f_y}$$

METRIC CONVERSIONS OF NONHOMOGENEOUS EQUATIONS

Eq. (8-1)

$$\rho_b = 0.85\beta_1 \frac{f_c'}{f_y}\,\frac{6117}{6117 + f_y}$$

Eq. (9-6)

$$h = \frac{l_n\ (800 + 0.0712f_y)}{36{,}000 + 5000\beta\left[\alpha_m - 0.5\ (1 - \beta_s)\ \left(1 + \frac{1}{\beta}\right)\right]}$$

Eq. (9-7)

$$h = \frac{l_n\ (800 + 0.0712f_y)}{36{,}000 + 5000\beta\ (1 + \beta_s)}$$

Eq. (9-8)

$$h = \frac{l_n\ (800 + 0.0712f_y)}{36{,}000}$$

Eq. (11-1)

$$A_v = 3.52\ \frac{b_w s}{f_y}$$

Eq. (11-4)

$$v_c = 0.504\sqrt{f_c'} + 176\rho_w\ \frac{V_u d}{M_u}$$

Eq. (11-6)

$$v_c = 0.53\ \left(1 + 0.00712\ \frac{N_u}{A_g}\right)\sqrt{f_c'}$$

Eq. (11-7)

$$v_c = 0.928\sqrt{f_c'}\sqrt{1 + 0.0285\ \frac{N_u}{A_g}}$$

Eq. (11-8)

$$v_c = 0.53\ \left(1 + 0.0285\ \frac{N_u}{A_g}\right)\sqrt{f_c'}$$

Eq. (11-9)

$$v_c = \frac{0.53\sqrt{f_c'}}{\sqrt{1 + \left(\dfrac{v_{tu}}{1.2v_u}\right)^2}}$$

Eq. (11-10)

$$v_c = 0.159\sqrt{f_c'} + 49.2\ \frac{V_u d}{M_u}$$

Eq. (11-11)

$$v_{ci} = 0.159\sqrt{f_c'}\ + \frac{V_d + \left(\dfrac{V_i M_{cr}}{M_{max}}\right)}{b_w d}$$

where

$$M_{cr} = \frac{I}{y_t}\ (1.59\sqrt{f_c'} + f_{pe} - f_d)$$

Eq. (11-12)

$$v_{cw} = 0.928\sqrt{f_c'} + 0.3f_{pc} + \frac{V_p}{b_w d}$$

Eq. (11-17)

$$v_{tc} = \frac{0.636\sqrt{f_c'}}{\sqrt{1 + \left(\dfrac{1.2\ v_u}{v_{tu}}\right)^2}}$$

Eq. (11-18)

$$v_{tu} = \frac{3.18\sqrt{f_c'}}{\sqrt{1 + \left(1.2\ \dfrac{v_u}{v_{tu}}\right)^2}}$$

Eq. (11-21)

$$A_l = \left[\frac{28.1_{xs}}{f_y}\left(\frac{v_{tu}}{v_{tu} + v_u}\right) - 2A_t\right]\left(\frac{x_1 + y_1}{s}\right)$$

$$\frac{3.52b_w s}{f_y}\ \text{for } 2A_t$$

Eq. (11-22)

$$v_c = \left(3.5 - 2.5\ \frac{M_u}{V_u d}\right)\left(0.504\sqrt{f_c'} + 176\rho_w\ \frac{V_u d}{M_u}\right)$$

Eq. (11-23)

$$v_u = 0.177\ \left(10 + \frac{l_n}{d}\right)\sqrt{f_c'}$$

Eq. (11-28)

$$v_u = \left[6.5 - 5.1\ \sqrt{\frac{N_u}{V_u}}\right]\left[1 - 0.5\ \frac{a}{d}\right] \times \left\{1 + \left[64 + 160\ \sqrt{\left(\frac{N_u}{V_u}\right)^3}\ \right]\rho\right\} 0.265\sqrt{f_c'}$$

Eq. (11-29)

$$v_u = 1.72\ \left(1 - 0.5\ \frac{a}{d}\right)\left(1 + 64\rho_v\right)\sqrt{f_c'}$$

Eq. (11-32)

$$v_c = 0.87\sqrt{f_c'} + \frac{N_u}{4l_w h}$$

Eq. (11-33)

$$v_c = 0.159\sqrt{f_c'} + \frac{l_w\left(0.331\sqrt{f_c'} + 0.2\ \dfrac{N_u}{l_w h}\right)}{\dfrac{M_u}{V_u} - \dfrac{l_w}{2}}$$

Eq. (18-4)

$$f_{ps} = f_{se} + 703 + \frac{f_c'}{100\rho_p}$$

Eq. (A-5)

$$A_s = \left(\frac{14.06}{f_y}\right)hd$$

INDEX

Numbers indicate the major subsection in which the subject appears. The number to the left of the decimal in each is the chapter number.