Materials for today:
https://bit.ly/2Ti9gUM

Stage structure in variable environments

Review: Projection in stable environments

$$\begin{bmatrix} a_{11} & \cdots & a_{1j} \\ \vdots & \ddots & \vdots \\ a_{i1} & \cdots & a_{ij} \end{bmatrix} \begin{bmatrix} n_1 \\ \vdots \\ n_j \end{bmatrix}_{t=0} = \begin{bmatrix} n_1 \\ \vdots \\ n_j \end{bmatrix}_{t=1}$$

Review: Projection in stable environments

Time

$$\begin{bmatrix} a_{11} & \cdots & a_{1j} \\ \vdots & \ddots & \vdots \\ a_{i1} & \cdots & a_{ij} \end{bmatrix} \begin{bmatrix} n_1 \\ \vdots \\ n_j \end{bmatrix}_{t=0} = \begin{bmatrix} n_1 \\ \vdots \\ n_j \end{bmatrix}_{t=1}$$

$$\begin{bmatrix} a_{11} & \cdots & a_{1j} \\ \vdots & \ddots & \vdots \\ a_{i1} & \cdots & a_{ij} \end{bmatrix} \begin{bmatrix} n_1 \\ \vdots \\ n_j \end{bmatrix}_{t=1} = \begin{bmatrix} n_1 \\ \vdots \\ n_j \end{bmatrix}_{t=2}$$

$$\begin{bmatrix} a_{11} & \cdots & a_{1j} \\ \vdots & \ddots & \vdots \\ a_{i1} & \cdots & a_{ij} \end{bmatrix} \begin{bmatrix} n_1 \\ \vdots \\ n_j \end{bmatrix}_{t=2} = \begin{bmatrix} n_1 \\ \vdots \\ n_j \end{bmatrix}_{t=3}$$

Review: Projection in stable environments

$$\mathbf{A}\,\mathbf{n}_0 = \mathbf{n}_1$$

$$\mathbf{A}\,\mathbf{n}_1 = \mathbf{n}_2$$

$$\mathbf{A}\,\mathbf{n}_2 = \mathbf{n}_3$$

Time

Review: Projection in stable environments

$$\mathbf{n}_1 = \mathbf{A}\mathbf{n}_0$$

$$\mathbf{n}_2 = \mathbf{A}\mathbf{n}_1$$

$$= \mathbf{A}\left(\mathbf{A}\mathbf{n}_0\right)$$

$$\mathbf{n}_t = \mathbf{A}\left(\ldots\left(\mathbf{A}\mathbf{n}_0\right)\right)$$

If you begin with certainties,
you shall end in doubts,
but if you will be content to begin with doubts,
you shall end in certainties.

—Francis Bacon
(or Shakespeare?)

Components of a stochastic population model

- **Demographic model:** Demographic rates under different environmental conditions
- **Environmental model:** A way of generating the sequence of environmental conditions through time

A simple demographic model

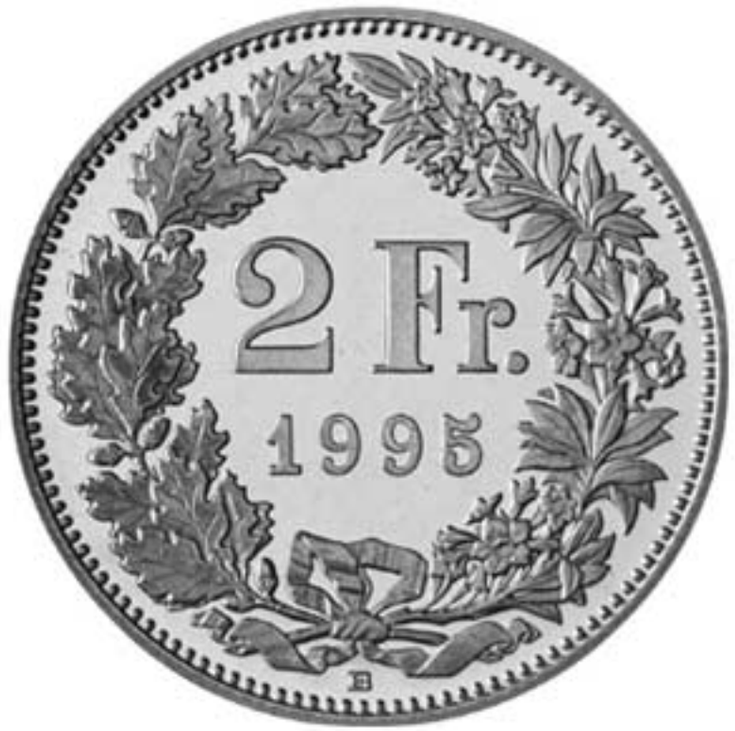
Environment *h*

$$\mathbf{A}_h = \begin{pmatrix} 0.2 & 0.2 \\ 1.0 & 0 \end{pmatrix} = \underset{\text{high}}{\mathbf{U}_h} \begin{pmatrix} 0 & 0 \\ 1.0 & 0 \end{pmatrix} + \underset{\text{low}}{\mathbf{F}_h} \begin{pmatrix} 0.2 & 0.2 \\ 0 & 0 \end{pmatrix}$$

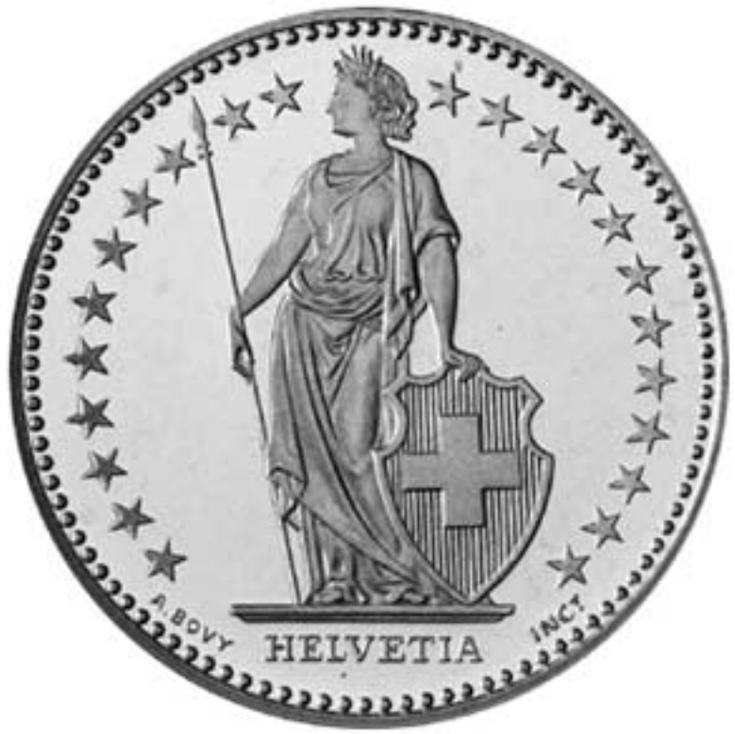
Environment *t*

$$\mathbf{A}_t = \begin{pmatrix} 0.1 & 3.0 \\ 0.2 & 0 \end{pmatrix} = \underset{\text{low}}{\mathbf{U}_t} \begin{pmatrix} 0 & 0 \\ 0.2 & 0 \end{pmatrix} + \underset{\text{high}}{\mathbf{F}_t} \begin{pmatrix} 0.1 & 3.0 \\ 0 & 0 \end{pmatrix}$$

A simple environmental model



tails



heads

Let’s generate a sequence…

Projection in stochastic environments

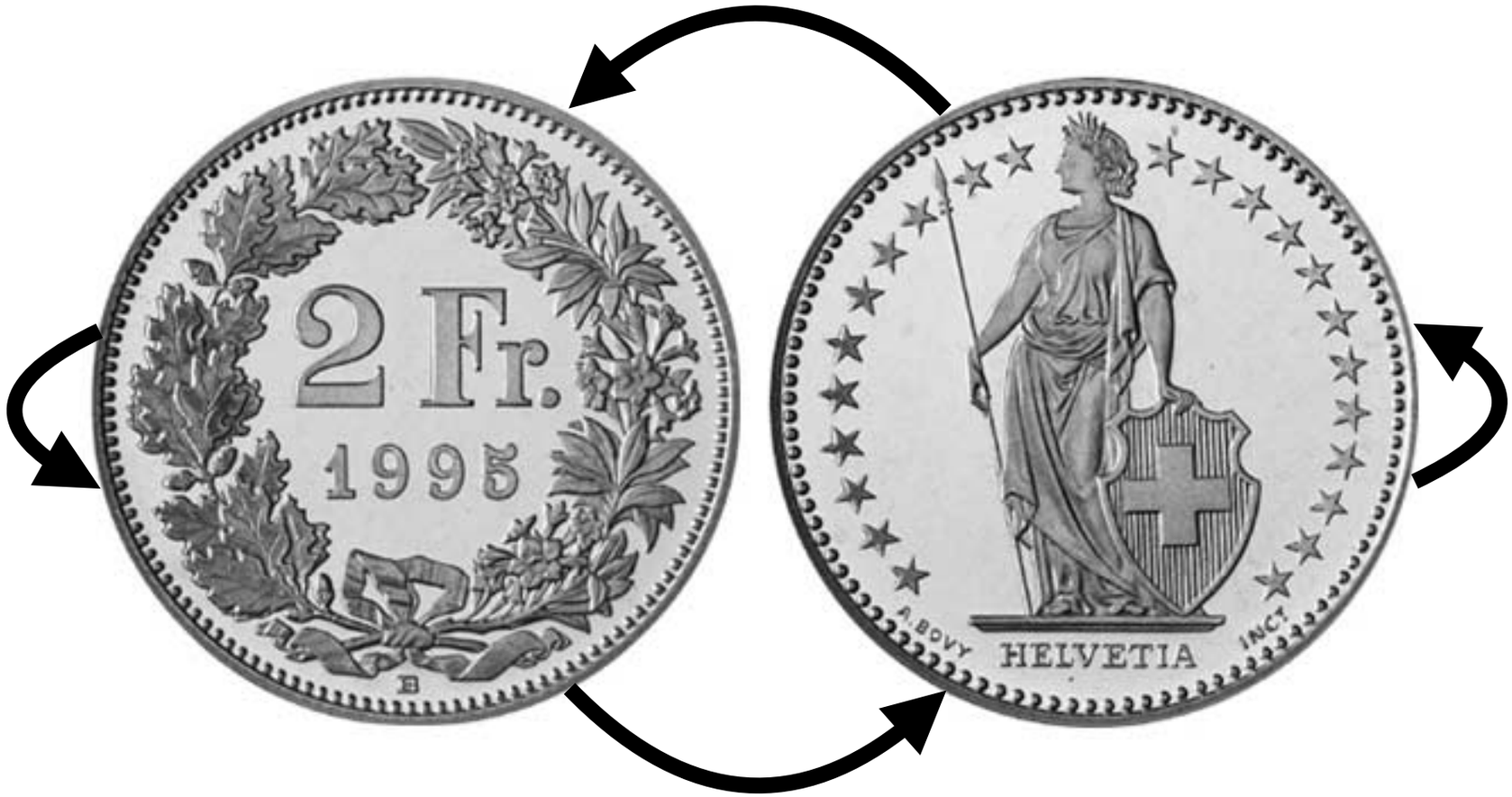
$$\mathbf{n}_1 = \mathbf{A}_0 \mathbf{n}_0$$

$$\mathbf{n}_2 = \mathbf{A}_1 \mathbf{n}_1$$

$$= \mathbf{A}_1 \left(\mathbf{A}_0 \mathbf{n}_0 \right)$$

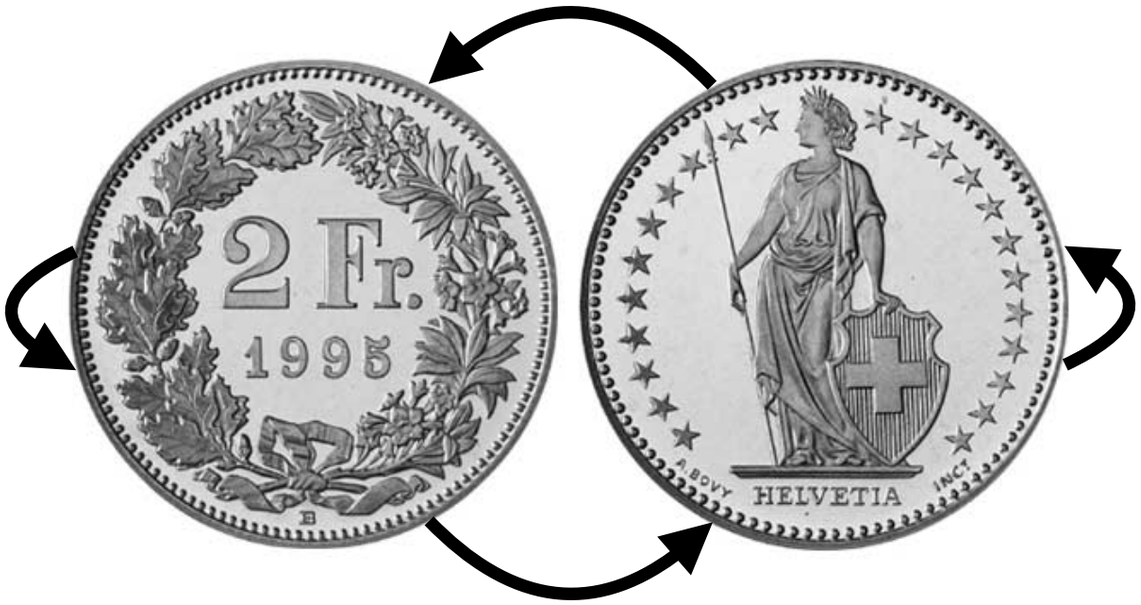
$$\mathbf{n}_t = \mathbf{A}_t \left(\ldots \left(\mathbf{A}_0 \mathbf{n}_0 \right) \right)$$

A simple environmental model



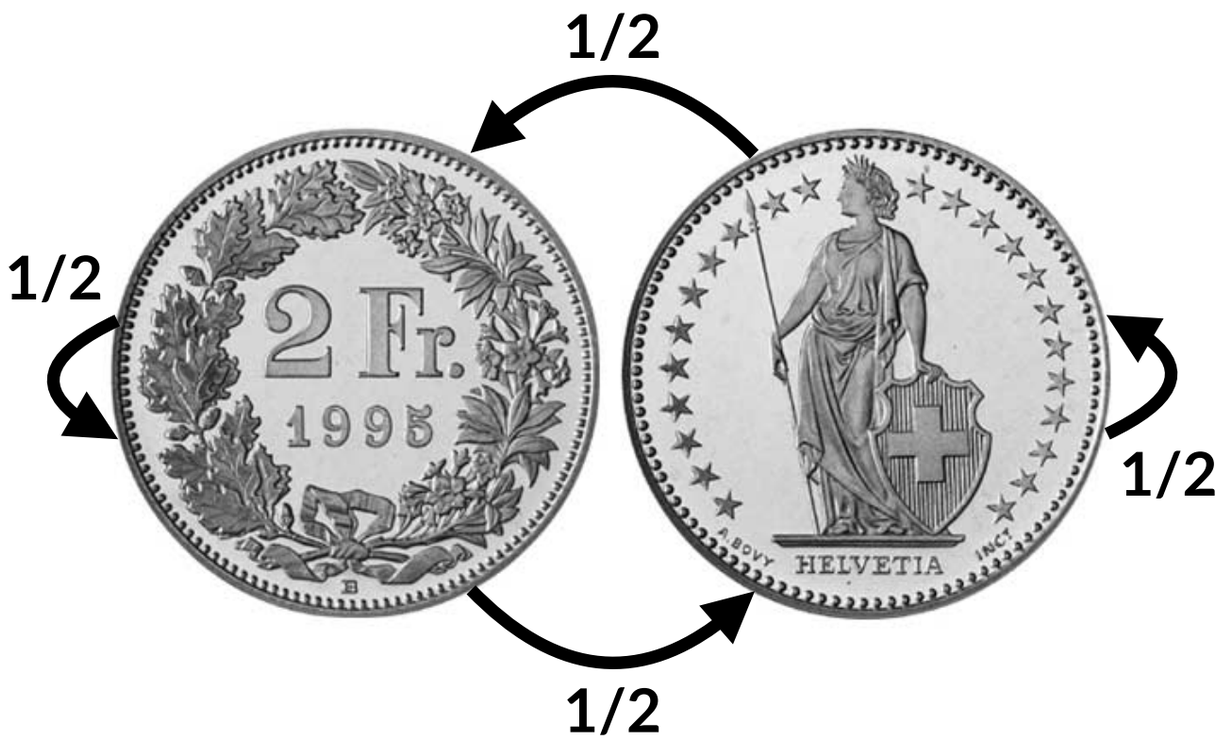
tails **h**eads

A simple environmental model



Environment *t+1* \ Environment *t*	Tails	Heads
Tails		
Heads		

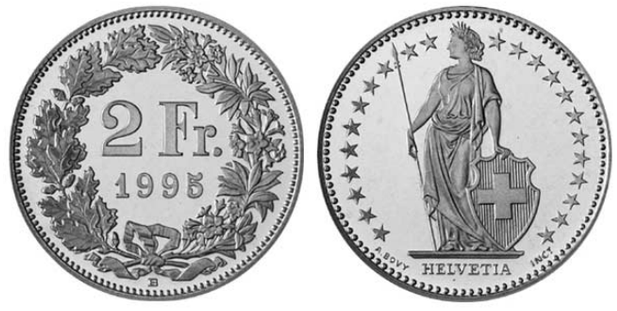
A simple environmental model



Environment $t+1$ \ Environment t	Tails	Heads
Tails	1/2	1/2
Heads	1/2	1/2

Stochastic population growth, λ_s

Environment:



$$\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$$

Demography:

$$\begin{pmatrix} 0.1 & 3.0 \\ 0.2 & 0 \end{pmatrix} \quad \begin{pmatrix} 0.2 & 0.2 \\ 1.0 & 0 \end{pmatrix}$$

$$\lambda = 0.83 \qquad \lambda = 0.56$$

stochastic.R

Extreme ice
Ice
Normal

Assignment:

- Uploaded to EDOZ & GitHub by 17:00 today
- Due 20 March 2019 by 23:59
- Email to william.petry@usys.ethz.ch