Materials for today:
https://bit.ly/2Ti9gUM

Structured populations

i. Endless varieties most beautiful & why we care

ii. Life cycle → Model

iii. Basic matrix model analysis

iv. Exercises

Conservation

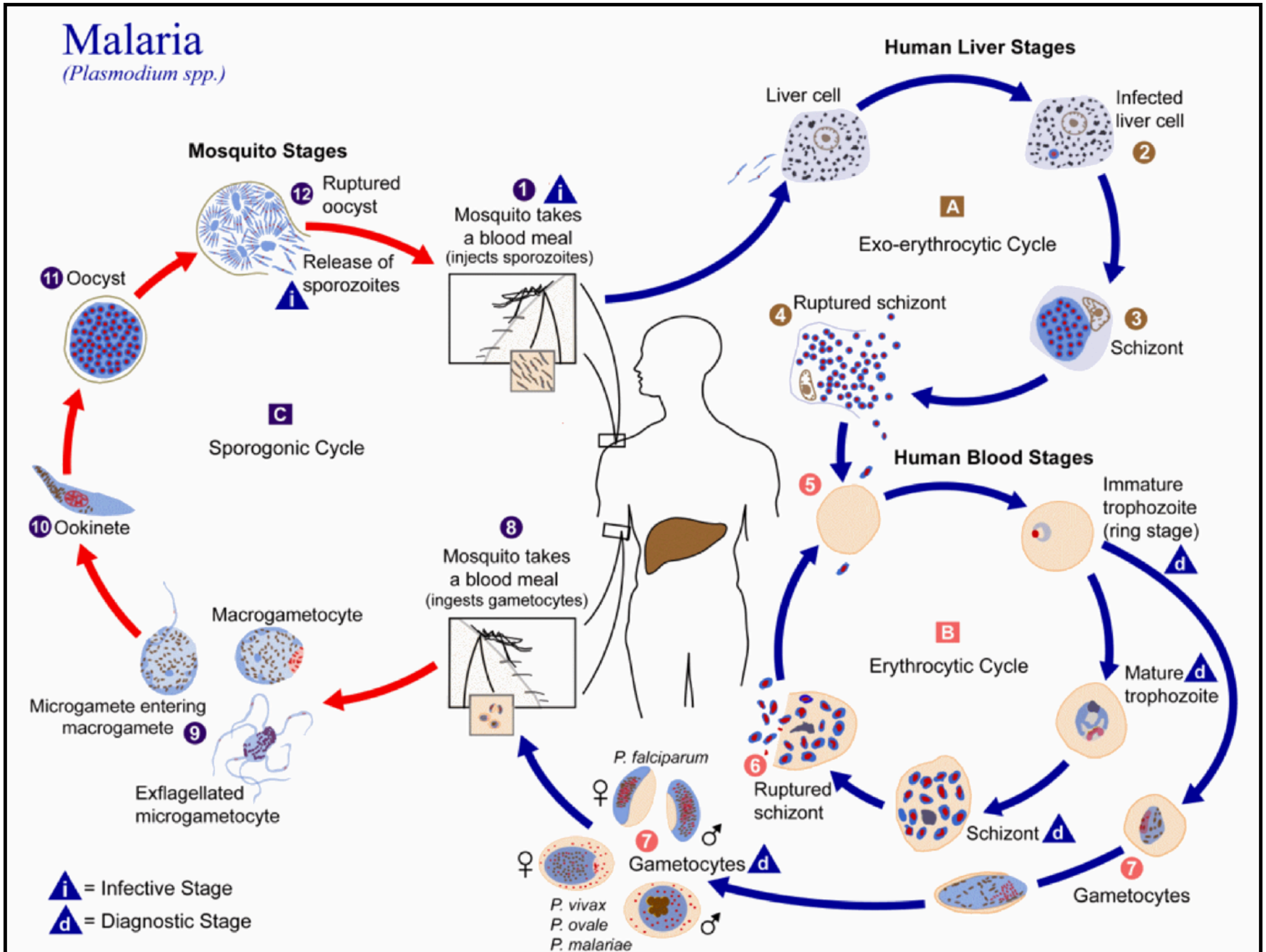
Which life stage would be best to **help**?

Conservation

Which life stage would be best to **harm**?

Disease

Where would an intervention be **most effective**?



Structured populations

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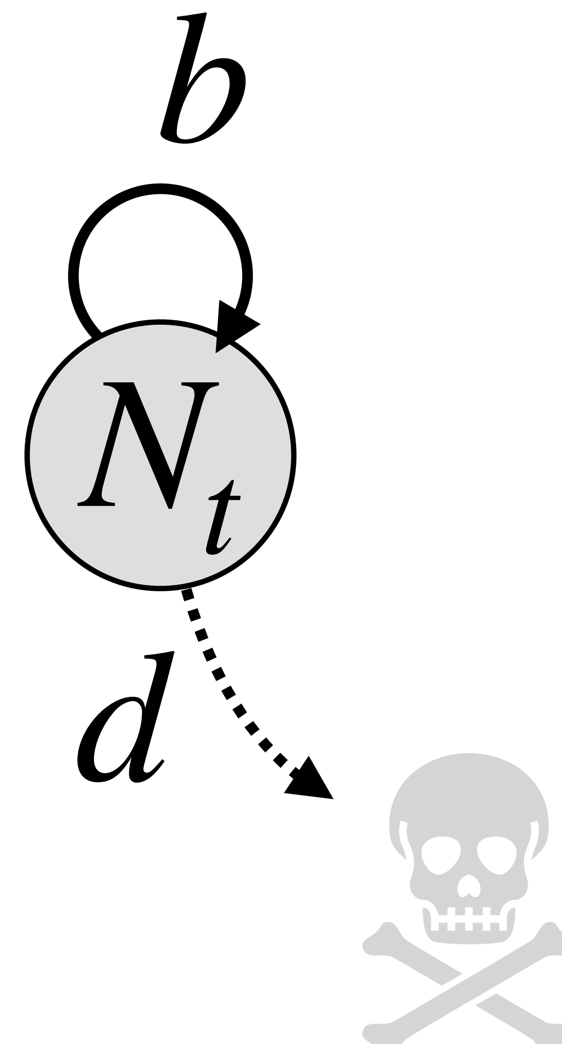
Unstructured population models

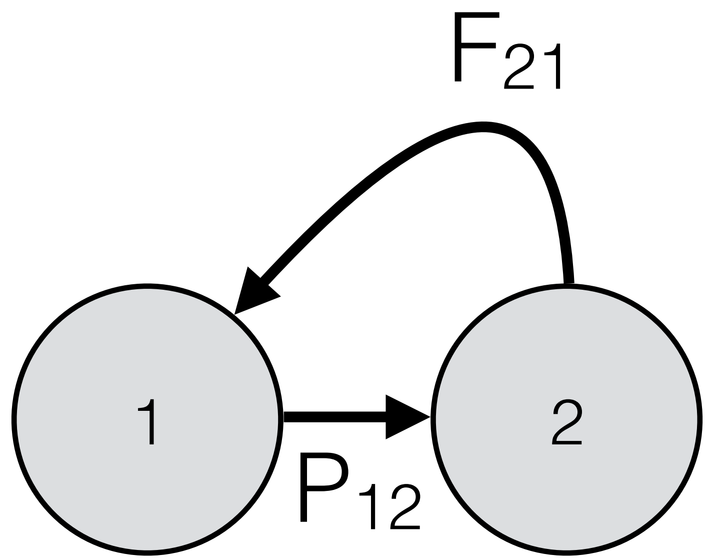
$$
\begin{aligned}
N_{t+1} &= bN_t - dN_t \\
&= (b - d)N_t \\
&= \lambda N_t
\end{aligned}
$$

Finite rate of increase



A simple structured life cycle

Verbascum thapsus



A simple structured life cycle

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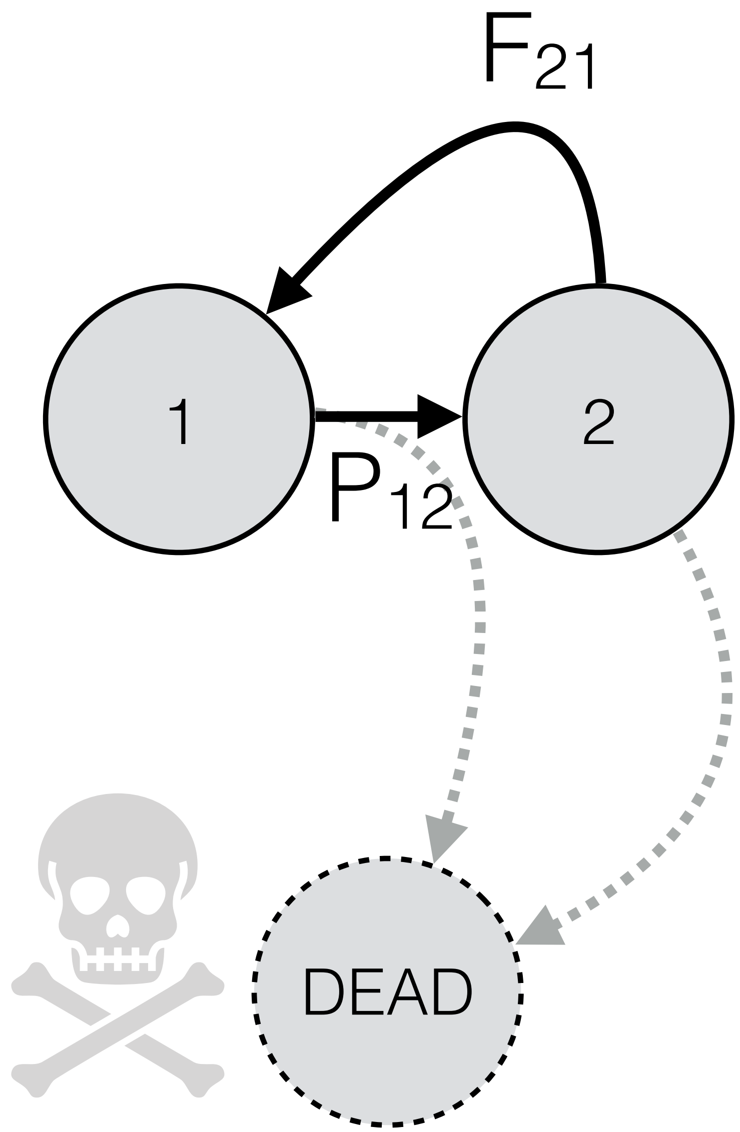
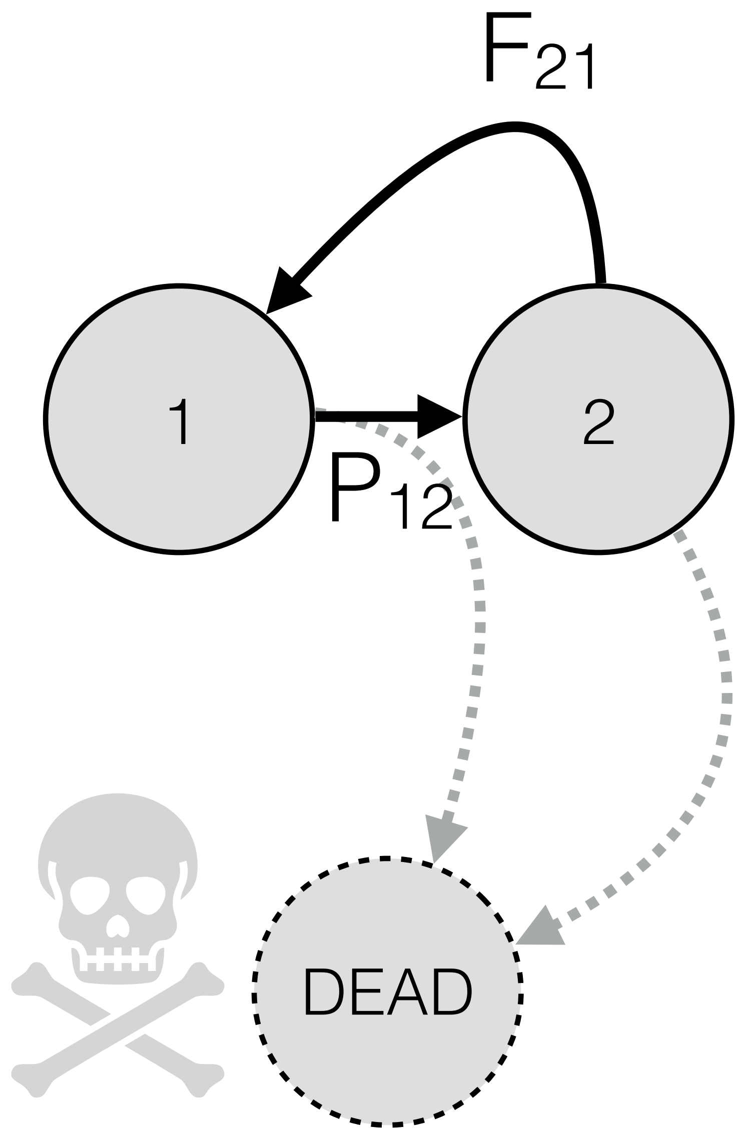


Stage-specific equations

Verbascum thapsus



Stage-specific equations

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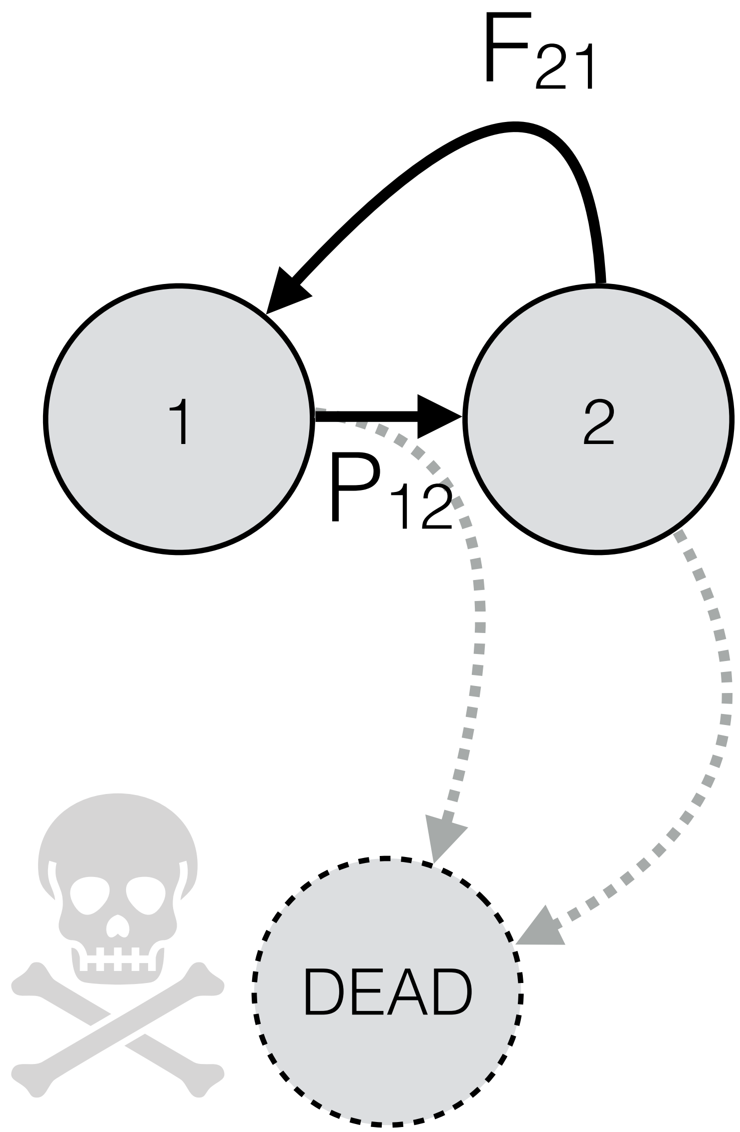


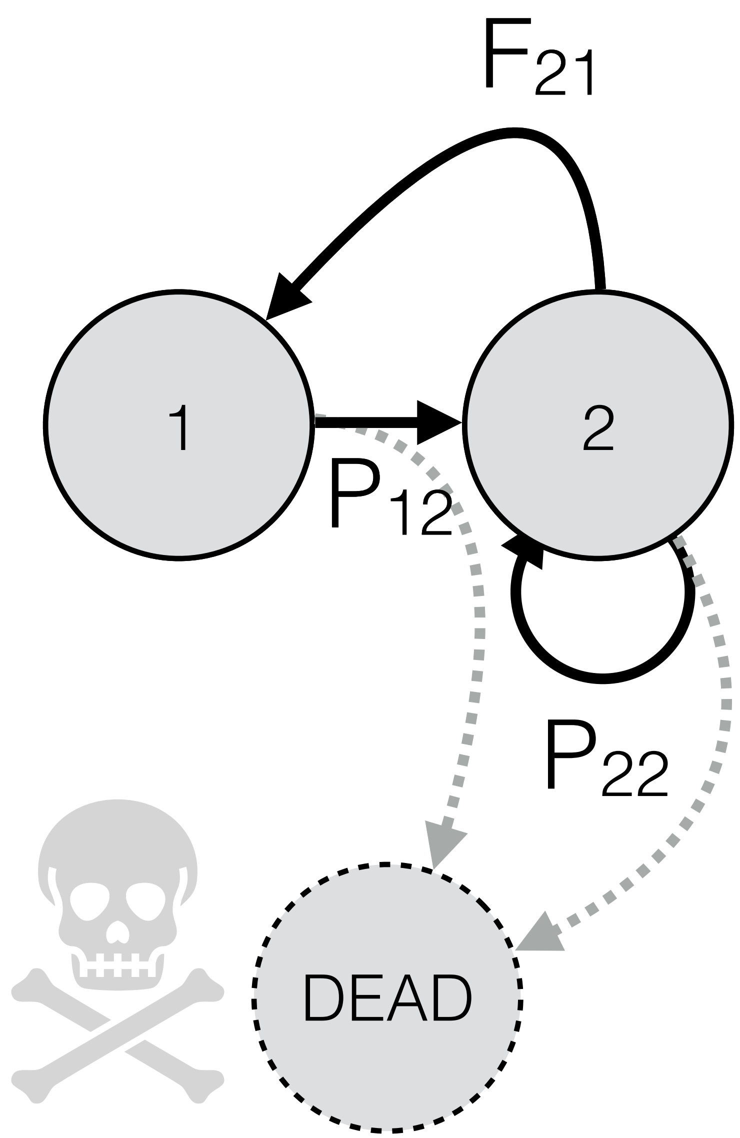
$$N_{1(t+1)} = F_{21}N_{2(t)}$$

$$N_{2(t+1)} = P_{12}N_{1(t)}$$

$$N_{\mathrm{DEAD(t+1)}} = (1 - P_{12})N_{1(t)} + N_{2(t)}$$

Stage-specific equations

Verbascum thapsus



$$N_{1(t+1)} = F_{21}N_{2(t)}$$

$$N_{2(t+1)} = P_{12}N_{1(t)} + P_{22}N_{2(t)}$$

$$N_{\mathrm{DEAD(t+1)}} = (1 - P_{12})N_{1(t)} + (1 - P_{22})N_{2(t)}$$

RESEARCH

PERSISTENT CHEMICALS

Predicting global killer whale population collapse from PCB pollution

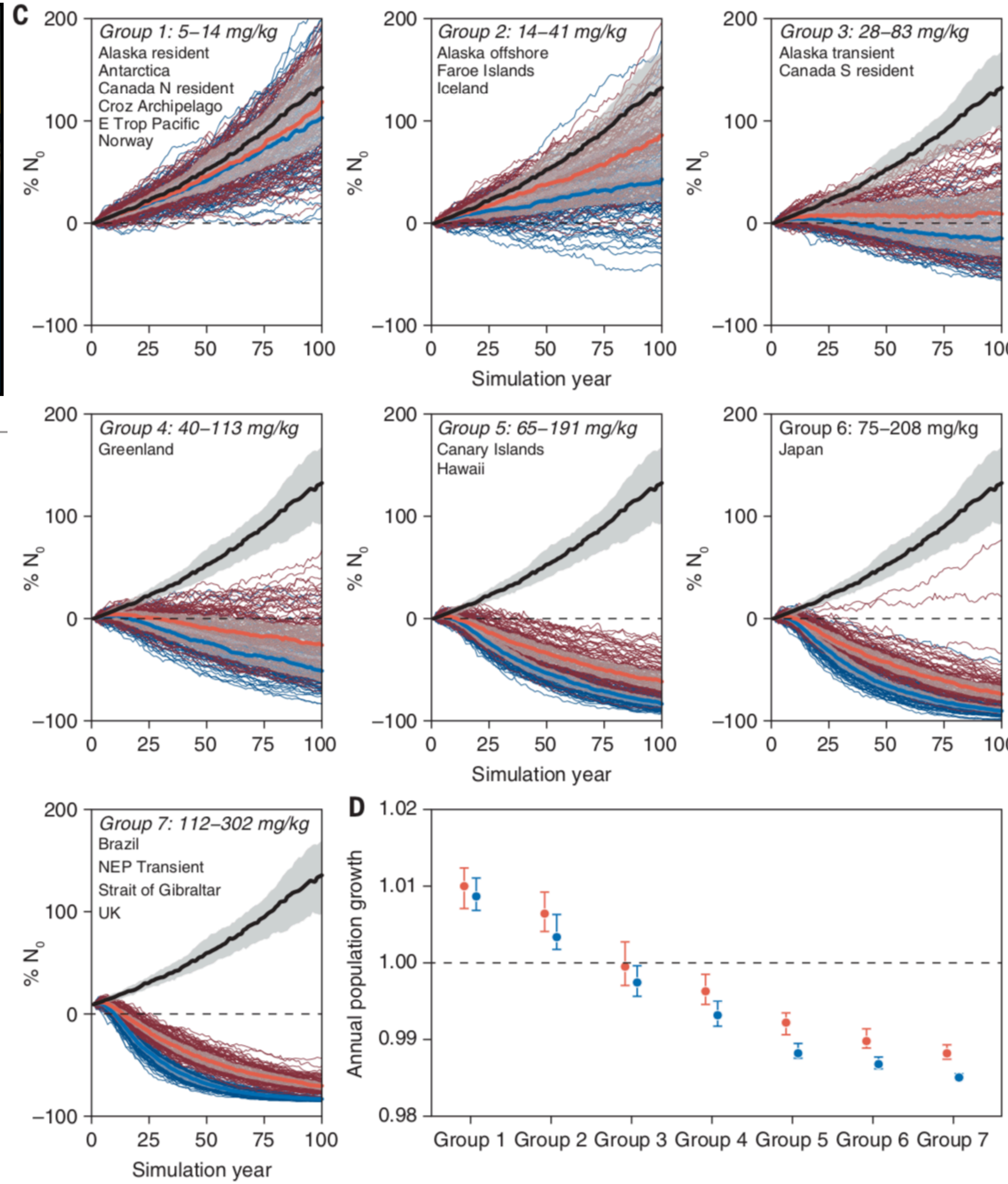
Jean-Pierre Desforges[1*], Ailsa Hall[2*], Bernie McConnell[2], Aqqalu Rosing-Asvid[3], Jonathan L. Barber[4], Andrew Brownlow[5], Sylvain De Guise[6,7], Igor Eulaers[1], Paul D. Jepson[8], Robert J. Letcher[9], Milton Levin[6], Peter S. Ross[10], Filipa Samarra[11], Gísli Víkingsson[11], Christian Sonne[1], Rune Dietz[1*]

Killer whales (*Orcinus orca*) are among the most highly polychlorinated biphenyl (PCB)–contaminated mammals in the world, raising concern about the health consequences of current PCB exposures. Using an individual-based model framework and globally available data on PCB concentrations in killer whale tissues, we show that PCB-mediated effects on reproduction and immune function threaten the long-term viability of >50% of the world's killer whale populations. PCB-mediated effects over the coming 100 years predicted that killer whale populations near industrialized regions, and those feeding at high trophic levels regardless of location, are at high risk of population collapse. Despite a near-global ban of PCBs more than 30 years ago, the world's killer whales illustrate the troubling persistence of this chemical class.

No PCB

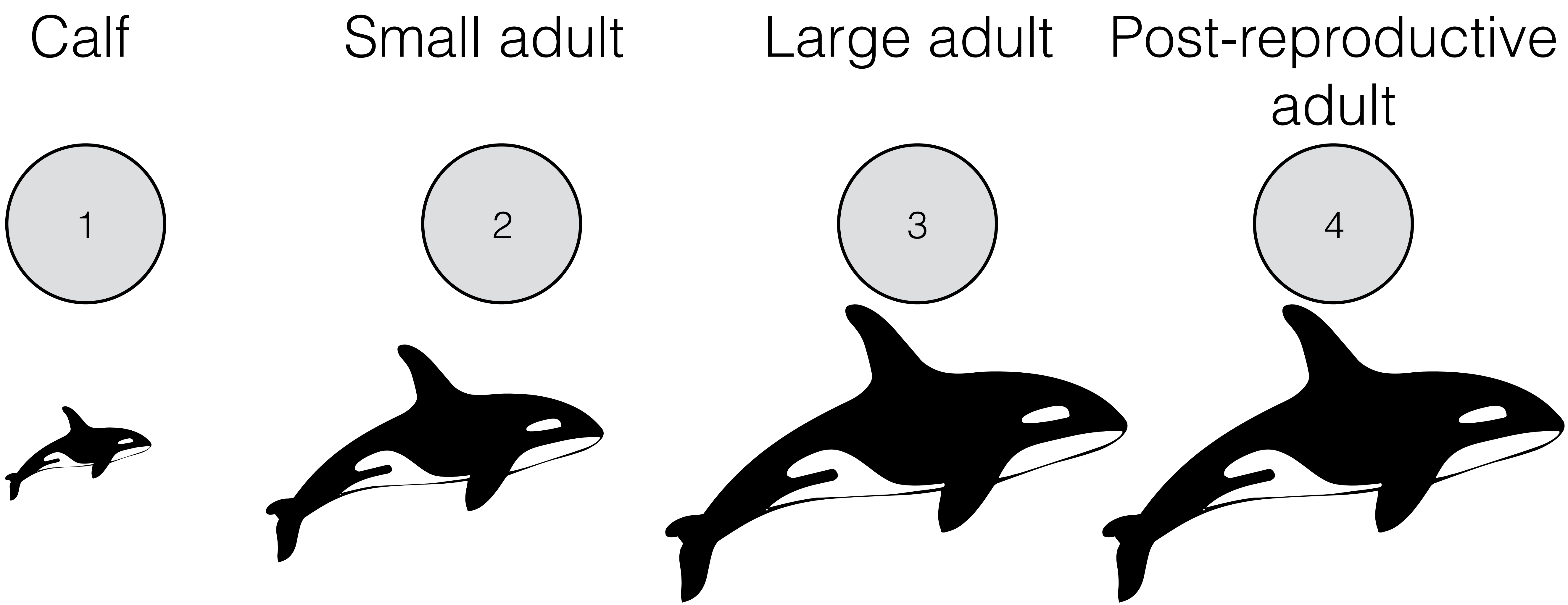
Reproductive effects

Reproduction + Immunity effects



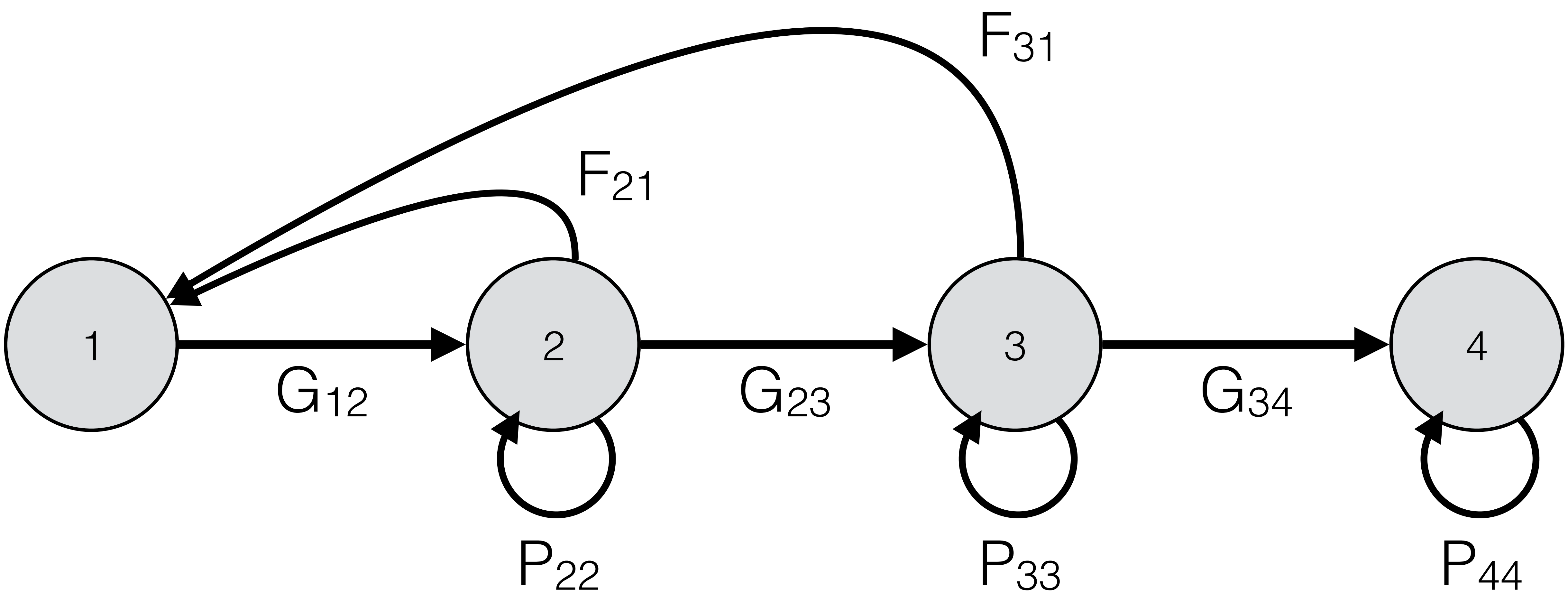
Science 28 Sept 2018

Group exercise: Orca life cycle

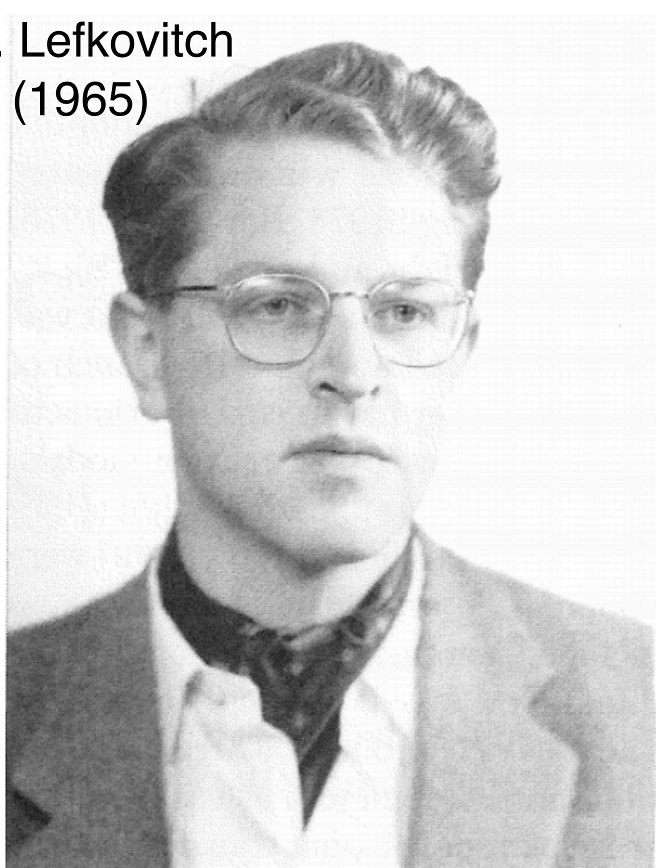


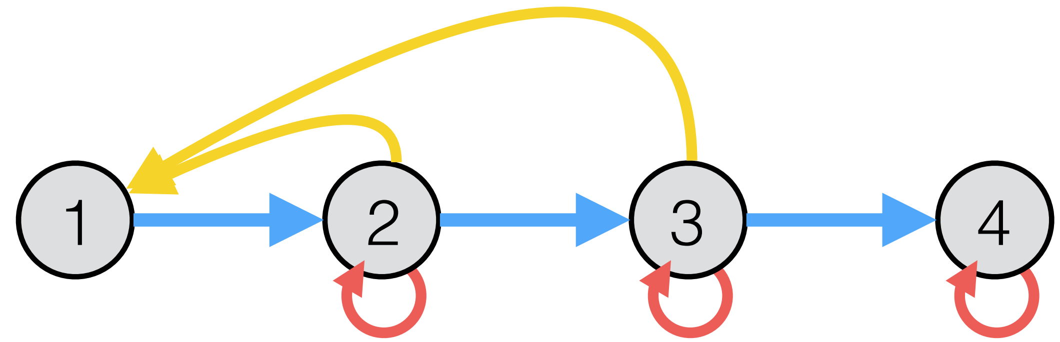
Brault & Caswell 1993 Ecology

Group exercise: Orca life cycle



The transition matrix





$$N_{1,t+1} = F_{21}N_{2,t} + F_{31}N_{3,t}$$

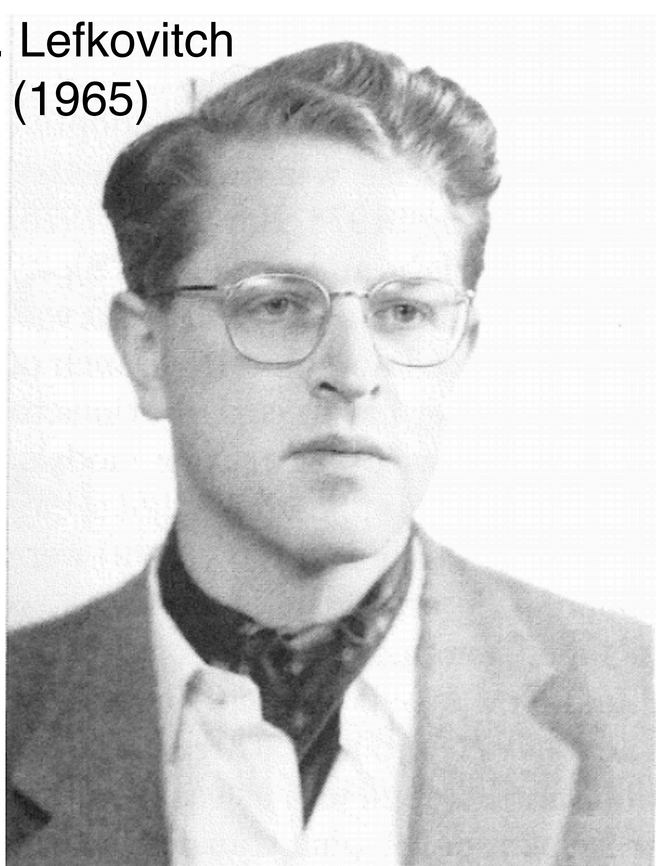
$$N_{2,t+1} = G_{12}N_{1,t} + P_{22}N_{2,t}$$

$$N_{3,t+1} = G_{23}N_{2,t} + P_{33}N_{3,t}$$

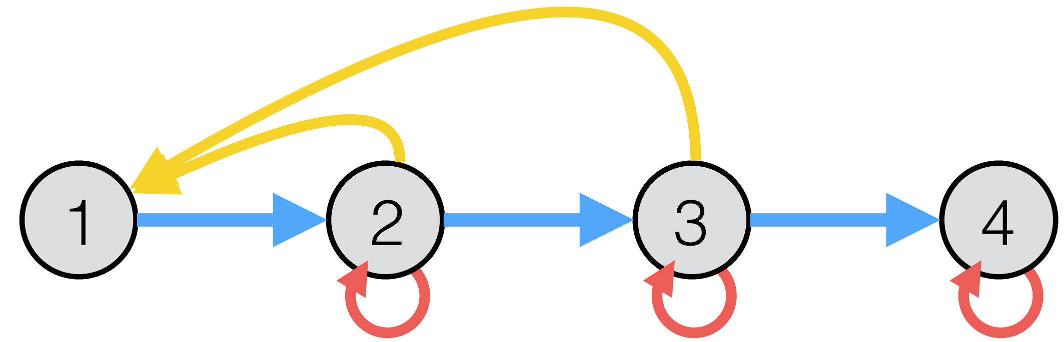
$$N_{4,t+1} = G_{34}N_{3,t} + P_{44}N_{4,t}$$

The transition matrix

P.H. Leslie (1945)



L.P. Lefkovitch (1965)



$$
\begin{aligned}
N_{1,t+1} &= F_{21}N_{2,t} + F_{31}N_{3,t} \\
N_{2,t+1} &= G_{12}N_{1,t} + P_{22}N_{2,t} \\
N_{3,t+1} &= G_{23}N_{2,t} + P_{33}N_{3,t} \\
N_{4,t+1} &= G_{34}N_{3,t} + P_{44}N_{4,t}
\end{aligned}
$$

$$
\begin{bmatrix} N_{1,t+1} \\ N_{2,t+1} \\ N_{3,t+1} \\ N_{4,t+1} \end{bmatrix} = \begin{bmatrix} 0 & F_{21} & F_{31} & 0 \\ G_{12} & P_{22} & 0 & 0 \\ 0 & G_{23} & P_{33} & 0 \\ 0 & 0 & G_{34} & P_{44} \end{bmatrix} \begin{bmatrix} N_{1,t} \\ N_{2,t} \\ N_{3,t} \\ N_{4,t} \end{bmatrix}
$$

Death is an absorbing state

$$\begin{array}{cc|c|c}
 & & \multicolumn{2}{c}{\textbf{Time } t} \\
 & & \textbf{Alive} & \textbf{Dead} \\
\hline
\textbf{Time } t\textbf{+1} & \textbf{Alive} & \begin{bmatrix} 0 & F_{21} & F_{31} & 0 \\ G_{12} & P_{22} & 0 & 0 \\ 0 & G_{23} & P_{33} & 0 \\ 0 & 0 & G_{34} & P_{44} \end{bmatrix} & 0 \\
\hline
 & \textbf{Dead} & \left[1-\sum_{j=1}^{n} a_{1j} \quad \dots \quad 1-\sum_{j=1}^{n} a_{4j}\right] & 1
\end{array}$$

U = survival & growth F = fecundity C = clonality

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.9775 & 0.9111 & 0 & 0 \\ 0 & 0.0736 & 0.9534 & 0 \\ 0 & 0 & 0.0452 & 0.9804 \end{bmatrix} + \begin{bmatrix} 0 & 0.0043 & 0.1132 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sum_{i=1}^{n} a_{ij} = \begin{bmatrix} 0.9775 & 0.9847 & 0.9986 & 0.9804 \end{bmatrix}$$

Stage-specific
survival rates

Brault & Caswell 1993 Ecology

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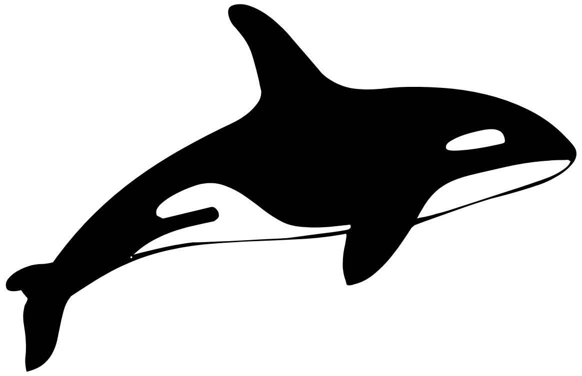
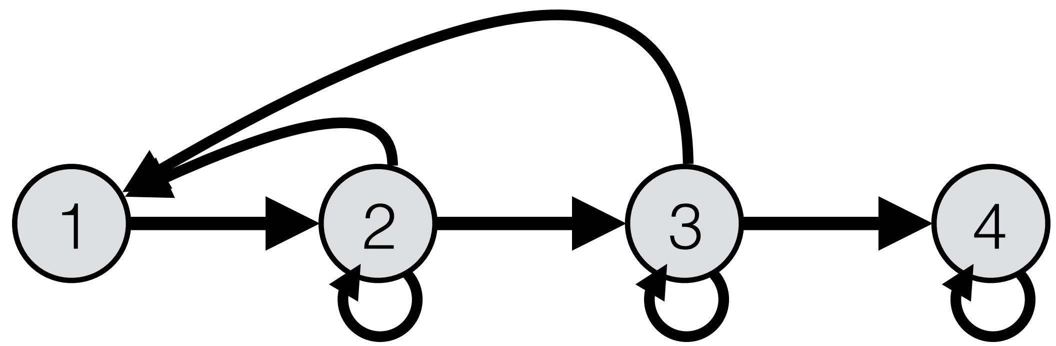


Projection



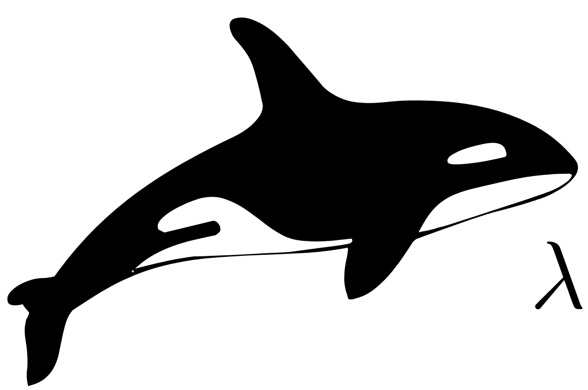
$$\begin{bmatrix} N_{1,t+1} \\ N_{2,t+1} \\ N_{3,t+1} \\ N_{4,t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0.0043 & 0.1132 & 0 \\ 0.9775 & 0.9111 & 0 & 0 \\ 0 & 0.0736 & 0.9534 & 0 \\ 0 & 0 & 0.0452 & 0.9804 \end{bmatrix} \begin{bmatrix} N_{1,t} \\ N_{2,t} \\ N_{3,t} \\ N_{4,t} \end{bmatrix}$$

Matrix multiply,
& iterate

Brault & Caswell 1993 Ecology

Equilibrium: Population growth (λ)

$$\begin{bmatrix} \mathrm{a}_{11} & \cdots & \mathrm{a}_{1n} \\ \vdots & \ddots & \vdots \\ \mathrm{a}_{n1} & \cdots & \mathrm{a}_{nn} \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix} = \lambda \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$$



$$\lambda = 1.025$$

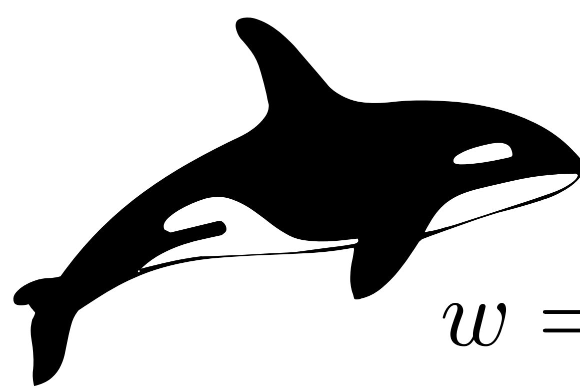
Brault & Caswell 1993 Ecology

	Population size	Population decline	Population growth rate (λ)
Critically endangered	–	≥ 80% 10 years	$\lambda \leq (1-0.8)^{\frac{1}{10}}$ ≤ 0.851
	N < 250	≥ 25% 3 years	≤ 0.909
Endangered	–	≥ 50% 10 years	≤ 0.933
	N < 2500	≥ 20% 5 years	≤ 0.956
Vulnerable	–	≥ 20% 10 years	≤ 0.978
	N < 10000	≥ 10% 10 years	≤ 0.990

IUCN
Caswell 2001

Equilibrium: Stage structure (*w*)

$$\begin{bmatrix} \mathrm{a}_{11} & \cdots & \mathrm{a}_{n1} \\ \vdots & \ddots & \vdots \\ \mathrm{a}_{1n} & \cdots & \mathrm{a}_{nn} \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix} = \lambda \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$$



$$w = \begin{bmatrix} 0.0370 \\ 0.3161 \\ 0.3229 \\ 0.3240 \end{bmatrix} = \begin{bmatrix} 3.70\% \\ 31.61\% \\ 32.29\% \\ 32.40\% \end{bmatrix}$$

Brault & Caswell 1993 Ecology

Structured vs. unstructured population dynamics

orca.R

Shiny app
[matrixProjector.R]

https://rstudio.cloud/spaces/12144/project/239821

Which stages have the biggest impact on λ?

seaturtle.R

https://rstudio.cloud/spaces/12144/project/239821

$$\begin{bmatrix} 0 & 0 & 0 & 4.665 & 61.896 \\ 0.675 & 0.703 & 0 & 0 & 0 \\ 0 & 0.047 & 0.657 & 0 & 0 \\ 0 & 0 & 0.019 & 0.682 & 0 \\ 0 & 0 & 0 & 0.061 & 0.8091 \end{bmatrix}$$

No intervention:

$\lambda = 0.952$ Endangered

Crowder *et al.* 1994 Ecol. Appl.

Protect hatchlings

No intervention:

$$\begin{bmatrix} 0 & 0 & 0 & 4.665 & 61.896 \\ 0.675 & 0.703 & 0 & 0 & 0 \\ 0 & 0.047 & 0.657 & 0 & 0 \\ 0 & 0 & 0.019 & 0.682 & 0 \\ 0 & 0 & 0 & 0.061 & 0.8091 \end{bmatrix}$$

$$\lambda_{\text{control}} = 0.952$$

Endangered

Hatchling protection:

$$\begin{bmatrix} 0 & 0 & 0 & 4.665 & 61.896 \\ 1.000 & 0.703 & 0 & 0 & 0 \\ 0 & 0.047 & 0.657 & 0 & 0 \\ 0 & 0 & 0.019 & 0.682 & 0 \\ 0 & 0 & 0 & 0.061 & 0.8091 \end{bmatrix}$$

$$\lambda_{\text{hatchling protection}} = 0.974$$

Vulnerable

Crowder *et al.* 1994 Ecol. Appl.

Turtle Exclusion Device

No intervention:

$$\begin{bmatrix} 0 & 0 & 0 & 4.665 & 61.896 \\ 0.675 & 0.703 & 0 & 0 & 0 \\ 0 & 0.047 & 0.657 & 0 & 0 \\ 0 & 0 & 0.019 & 0.682 & 0 \\ 0 & 0 & 0 & 0.061 & 0.8091 \end{bmatrix}$$

$$\begin{bmatrix} 0.675 & 0.750 & 0.676 & 0.743 & 0.8091 \end{bmatrix}$$

$$\lambda_{\text{control}} = 0.952$$

Endangered

Turtle Exclusion Device (TED):

$$\begin{bmatrix} 0 & 0 & 0 & 4.665 & 61.896 \\ 0.675 & 0.703 & 0 & 0 & 0 \\ 0 & 0.047 & 0.757 & 0 & 0 \\ 0 & 0 & 0.022 & 0.769 & 0 \\ 0 & 0 & 0 & 0.069 & 0.877 \end{bmatrix}$$

$$\begin{bmatrix} 0.675 & 0.750 & 0.77918 & 0.83814 & 0.876703 \end{bmatrix}$$

$$\lambda_{\text{TED}} = 1.027$$

Recovering

Crowder *et al.* 1994 Ecol. Appl.

Sensitivity

"Fixed change in λ for a fixed change in a_{ij}"

$$\frac{\partial \lambda}{\partial \mathrm{a}_{ij}} = \frac{v_i w_j}{w \cdot v}$$

$$\begin{bmatrix} \mathrm{a}_{11} & \dots & \mathrm{a}_{1n} \\ \vdots & \ddots & \vdots \\ \mathrm{a}_{n1} & \dots & \mathrm{a}_{nn} \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix} = \lambda \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$$

$$\begin{bmatrix} v_1 & \dots & v_n \end{bmatrix} \begin{bmatrix} \mathrm{a}_{11} & \dots & \mathrm{a}_{1n} \\ \vdots & \ddots & \vdots \\ \mathrm{a}_{n1} & \dots & \mathrm{a}_{nn} \end{bmatrix} = \lambda_L \begin{bmatrix} v_1 & \dots & v_n \end{bmatrix}$$

Elasticity

"Proportional change in λ for a proportional change in a_{ij}"

$$e_{ij} = \frac{\mathrm{a}_{ij}}{\lambda}\frac{\partial\lambda}{\partial\mathrm{a}_{ij}}$$

$$
\begin{bmatrix}
0 & 0 & 0 & 4.665 & 61.896 \\
0.675 & 0.703 & 0 & 0 & 0 \\
0 & 0.047 & 0.657 & 0 & 0 \\
0 & 0 & 0.019 & 0.682 & 0 \\
0 & 0 & 0 & 0.061 & 0.8091
\end{bmatrix}
$$

Sensitivity

0	0	0	0.002	0.001
0.082	0.222	0	0	0
0	1.173	0.187	0	0
0	0	2.901	0.204	0
0	0	0	0.768	0.329

Elasticity

0	0	0	0.009	0.049
0.058	0.164	0	0	0
0	0.058	0.129	0	0
0	0	0.058	0.147	0
0	0	0	0.049	0.28

Crowder *et al.* 1994 Ecol. Appl.

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Next week:
Noisy environments