```
% Benjamin Stutzke
% ENAE 423
% Homework 8
```

Problem 2

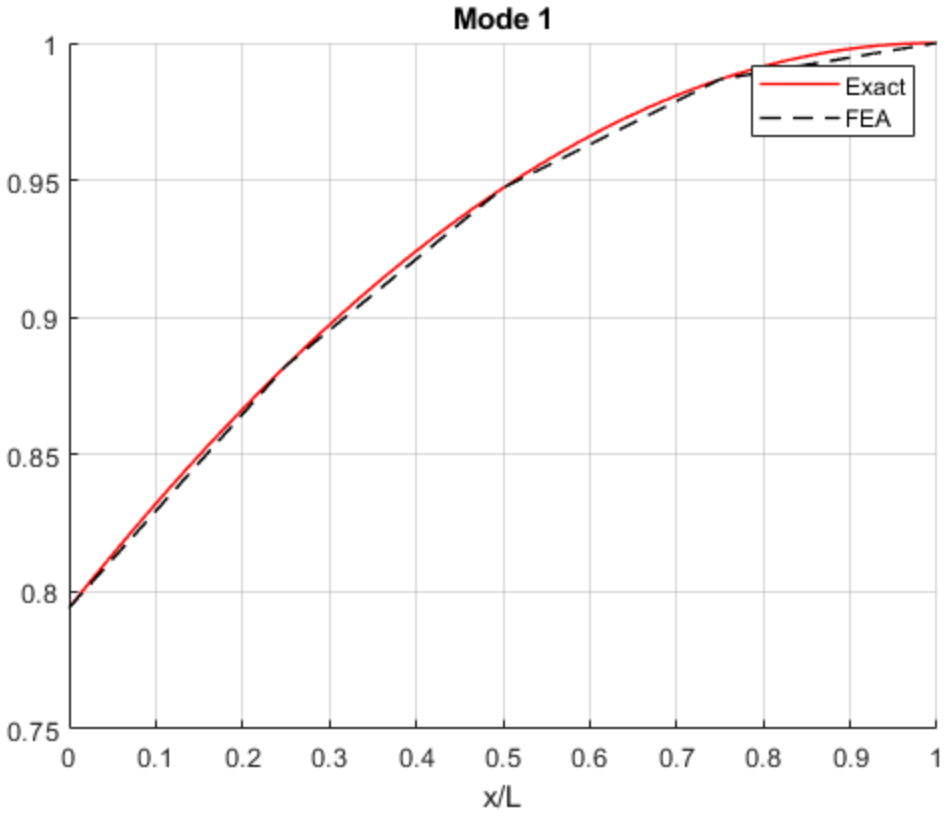
Free vibration of slender body

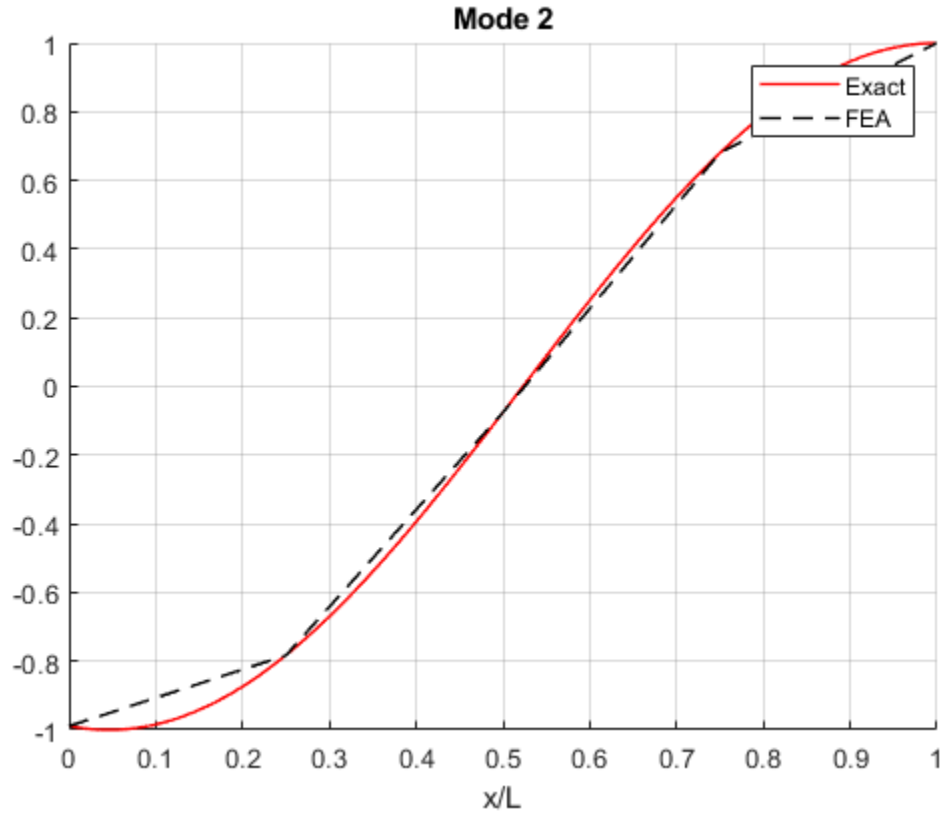
```
syms s real

E=1;
Area=1;
L=1;
m=1;
alpha = 0.5;

nedof = 2;
net = 4;
ntdof = net+1;
iconm = [1 2;2 3;3 4;4 5];

elength = L/net;
factorm = m*elength/6;
factork = E*Area/elength;
em = [2 1; 1 2]*factorm;
ek = [1 -1; -1 1]*factork;

gm = zeros(ntdof, ntdof);
gk = zeros(ntdof, ntdof);
gf = zeros(ntdof, 1);

nrdof = 5;
nreduced = [1 2 3 4 5];
gmr = zeros(nrdof, nrdof);
gkr = zeros(nrdof, nrdof);
gfr = zeros(nrdof, 1);

for lnum = 1:net
    iconv(:) = iconm(lnum, :);
    gk(iconv(:), iconv(:)) = gk(iconv(:), iconv(:))+ek(:,:);
    gm(iconv(:), iconv(:)) = gm(iconv(:), iconv(:))+em(:,:);
end

gf(1) = 1;
gk(:, 1) = gk(:, 1) + (alpha*E*Area/L)*gf;

fprintf("Global Stiffness Matrix:\n");
disp(gk);
fprintf("Global Mass Matrix:\n");
disp(gm);
```

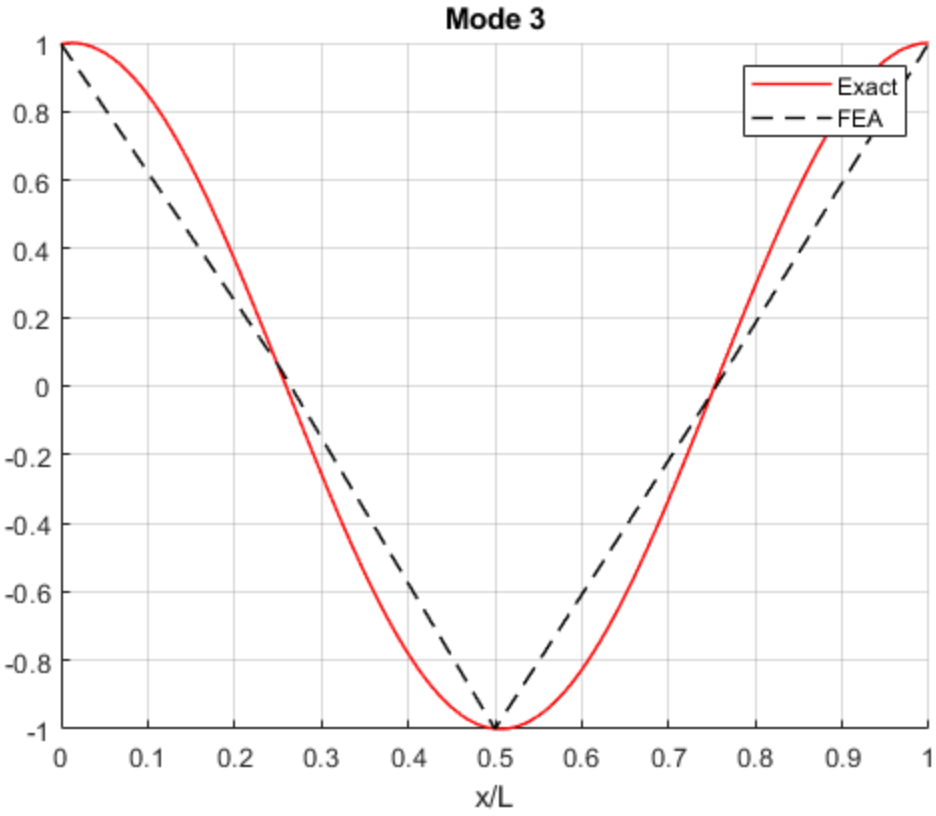
```
gkr(:,:) = gk(nreduced(:), nreduced(:));
gmr(:,:) = gm(nreduced(:), nreduced(:));

[vec, p] = eig(gkr, gmr);
gv=zeros(ntdof,nrdof);
gv(nreduced(:),:)=vec(:,:);

nmode = 5;
msign = [-1 -1 1 1 1];

beta = [0.65327, 3.2923, 6.3616, 9.4775, 12.606];

for i=1:nmode
    fprintf("\nMode %d\n", i);
    fprintf('\nExact Natural Frequency = %.4f sqrt(EA/mL^2)\n', beta(i));
    fprintf('\nNatural Frequency = %.4f sqrt(EA/mL^2)\n', vpa(sqrt(p(i,i))));

    figure;
    hold on

    N(1) = 1-s;
    N(2) = s;
    N = [N(1) N(2)];

    np = 500;

    for j=1:np
        s1 = (j-1)/np;
        N1 = subs(N, s, s1);

        u1(j) = (N1(1)*gv(1,i)+N1(2)*gv(2,i));
        u2(j) = (N1(1)*gv(2,i)+N1(2)*gv(3,i));
        u3(j) = (N1(1)*gv(3,i)+N1(2)*gv(4,i));
        u4(j) = (N1(1)*gv(4,i)+N1(2)*gv(5,i));
    end

    uall = [u1 u2 u3 u4];
    uabs = abs(uall);
    umax = max(uabs);
    uscaled = uall/umax;
    uscaled = msign(i) * uscaled;

    nptotal = np*net;
    delx = 1/nptotal;
    endpoint = 1-delx;
    xbar = 0:delx:endpoint;

    uexact = cos(beta(i)*(1-xbar/L));

    plot(xbar, uexact, 'r', 'LineWidth', 1);
    plot(xbar, uscaled, '--k', 'LineWidth', 1);
    legend('Exact', 'FEA');
    xlabel("x/L");
    title(sprintf("Mode %d", i));
```

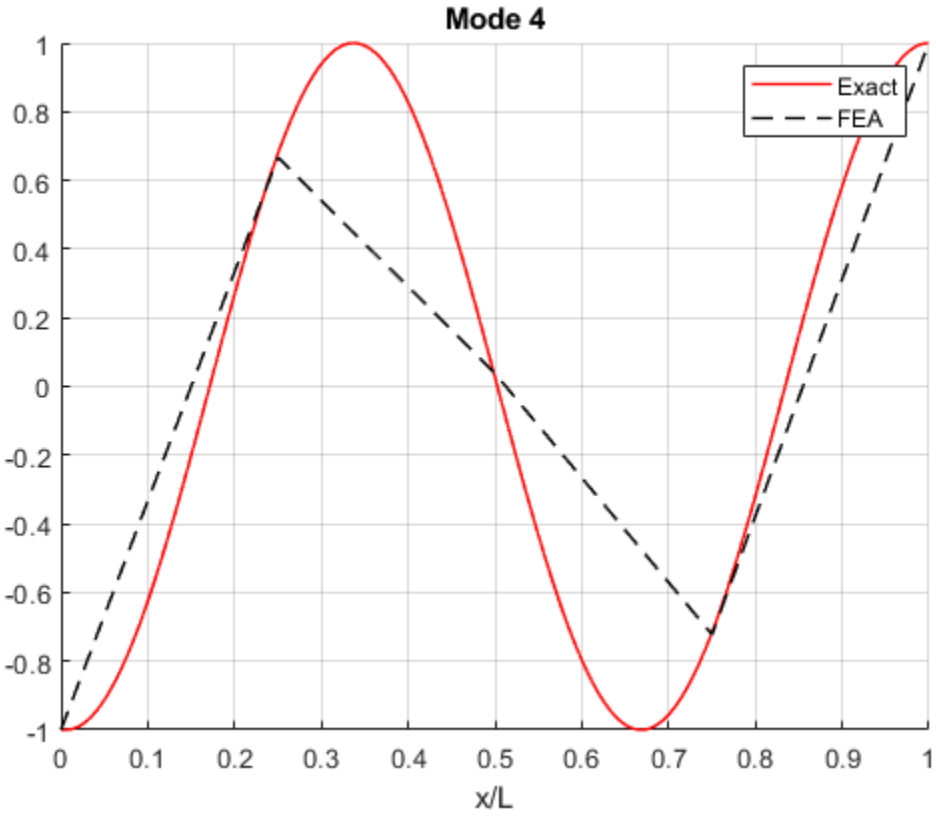
```
    grid on
    hold off
end
```

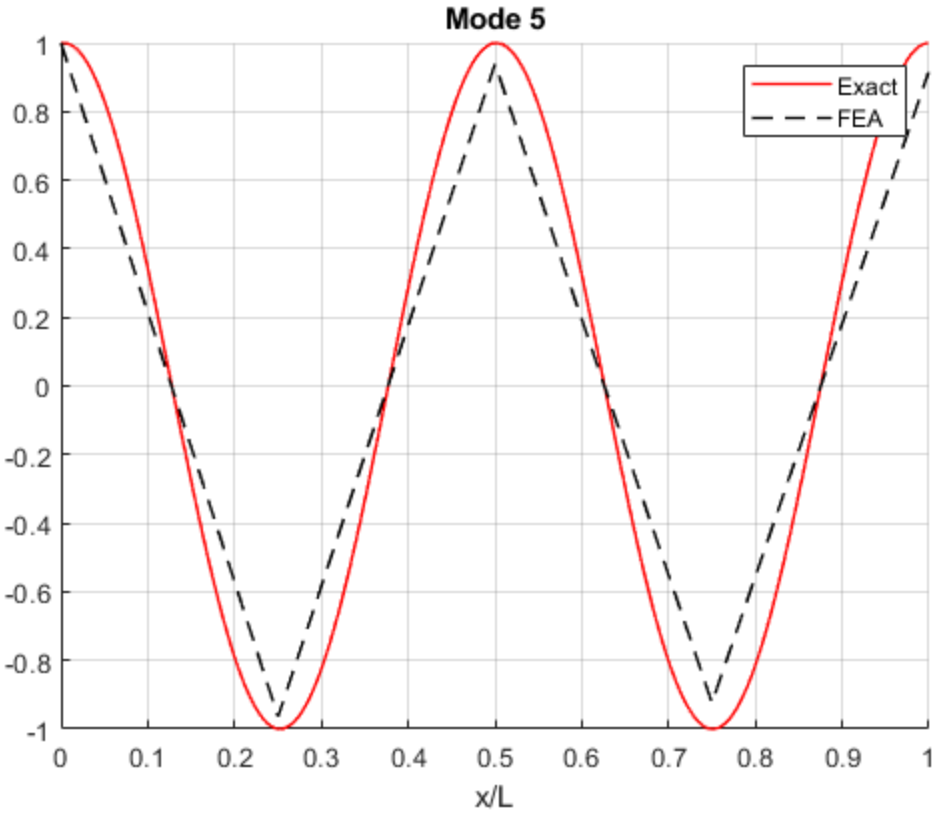
```
Global Stiffness Matrix:
    4.5000   -4.0000         0         0         0
   -4.0000    8.0000   -4.0000         0         0
         0   -4.0000    8.0000   -4.0000         0
         0         0   -4.0000    8.0000   -4.0000
         0         0         0   -4.0000    4.0000

Global Mass Matrix:
    0.0833    0.0417         0         0         0
    0.0417    0.1667    0.0417         0         0
         0    0.0417    0.1667    0.0417         0
         0         0    0.0417    0.1667    0.0417
         0         0         0    0.0417    0.0833


Mode 1

Exact Natural Frequency = 0.6533 sqrt(EA/mL^2)

Natural Frequency = 0.6540 sqrt(EA/mL^2)

Mode 2

Exact Natural Frequency = 3.2923 sqrt(EA/mL^2)

Natural Frequency = 3.3863 sqrt(EA/mL^2)

Mode 3

Exact Natural Frequency = 6.3616 sqrt(EA/mL^2)

Natural Frequency = 7.0354 sqrt(EA/mL^2)

Mode 4

Exact Natural Frequency = 9.4775 sqrt(EA/mL^2)

Natural Frequency = 11.3620 sqrt(EA/mL^2)

Mode 5

Exact Natural Frequency = 12.6060 sqrt(EA/mL^2)

Natural Frequency = 13.9135 sqrt(EA/mL^2)
```


Mode 1
Exact
FEA
1
0.95
0.9
0.85
0.8
0.75
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x/L


Mode 2
Exact
FEA
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x/L


Mode 3
Exact
FEA
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x/L


Mode 4
Exact
FEA
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x/L



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