
```
% Benjamin Stutzke
% ENAE 423
% Homework 8
```

Problem 3

Free vibration of slender body

```
syms s real

E=1;
Area=1;
L=1;
m=1;
alpha = 1.5;

nedof = 2;
net = 4;
ntdof = net+1;
iconm = [1 2;2 3;3 4;4 5];

elength = L/net;
factorm = m*elength/6;
factorik = E*Area/elength;
em = [2 1; 1 2]*factorm;
ek = [1 -1; -1 1]*factorik;

gm = zeros(ntdof, ntdof);
gk = zeros(ntdof, ntdof);
gf = zeros(ntdof, 1);

nrdoф = 5;
nreduced = [1 2 3 4 5];
gmr = zeros(nrdoф, nrdoф);
gkr = zeros(nrdoф, nrdoф);
gfr = zeros(nrdoф, 1);

for lnum = 1:net
    iconv(:) = iconm(lnum, :);
    gk(iconv(:), iconv(:)) = gk(iconv(:), iconv(:))+ek(:, :);
    gm(iconv(:), iconv(:)) = gm(iconv(:), iconv(:))+em(:, :);
end

gf(1) = 1;
gk(:, 1) = gk(:, 1) + (alpha*E*Area/L)*gf;

fprintf("Global Stiffness Matrix:\n");
disp(gk);
fprintf("Global Mass Matrix:\n");
disp(gm);
```

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gkr(:, :) = gk(nreduced(:, :), nreduced(:, :));
gmr(:, :) = gm(nreduced(:, :), nreduced(:, :));

[vec, p] = eig(gkr, gmr);
gv=zeros(ntdof,nrdof);
gv(nreduced(:, :))=vec(:, :, :);

nmode = 5;
msign = [1 -1 1 1 1];

beta = [0.9882, 3.5422, 6.5097, 9.5801, 12.684];

for i=1:nmode
    fprintf("\nMode %d\n", i);
    fprintf('Exact Natural Frequency = %.4f sqrt(EA/mL^2)\n', beta(i));
    fprintf('Natural Frequency = %.4f sqrt(EA/mL^2)\n', vpa(sqrt(p(i,i))));
end

figure;
hold on

N(1) = 1-s;
N(2) = s;
N = [N(1) N(2)];

np = 500;

for j=1:np
    s1 = (j-1)/np;
    N1 = subs(N, s, s1);

    u1(j) = (N1(1)*gv(1,i)+N1(2)*gv(2,i));
    u2(j) = (N1(1)*gv(2,i)+N1(2)*gv(3,i));
    u3(j) = (N1(1)*gv(3,i)+N1(2)*gv(4,i));
    u4(j) = (N1(1)*gv(4,i)+N1(2)*gv(5,i));
end

uall = [u1 u2 u3 u4];
uabs = abs(uall);
umax = max(uabs);
uscaled = uall/umax;
uscaled = msign(i) * uscaled;

nptotal = np*net;
delx = 1/nptotal;
endpoint = 1-delx;
xbar = 0:delx:endpoint;

uexact = cos(beta(i)*(1-xbar/L));

plot(xbar, uexact, 'r', 'LineWidth', 1);
plot(xbar, uscaled, '--k', 'LineWidth', 1);
legend('Exact', 'FEA');
xlabel("x/L");
title(sprintf("Mode %d", i));

```

```
grid on
hold off
end

Global Stiffness Matrix:
 5.5000   -4.0000      0      0      0
 -4.0000    8.0000   -4.0000      0      0
      0   -4.0000    8.0000   -4.0000      0
      0       0   -4.0000    8.0000   -4.0000
      0       0      0   -4.0000    4.0000
```

Global Mass Matrix:

| | | | | |
|--------|--------|--------|--------|--------|
| 0.0833 | 0.0417 | 0 | 0 | 0 |
| 0.0417 | 0.1667 | 0.0417 | 0 | 0 |
| 0 | 0.0417 | 0.1667 | 0.0417 | 0 |
| 0 | 0 | 0.0417 | 0.1667 | 0.0417 |
| 0 | 0 | 0 | 0.0417 | 0.0833 |

Mode 1

Exact Natural Frequency = 0.9882 sqrt(EA/mL^2)

Natural Frequency = 0.9908 sqrt(EA/mL^2)

Mode 2

Exact Natural Frequency = 3.5422 sqrt(EA/mL^2)

Natural Frequency = 3.6601 sqrt(EA/mL^2)

Mode 3

Exact Natural Frequency = 6.5097 sqrt(EA/mL^2)

Natural Frequency = 7.2401 sqrt(EA/mL^2)

Mode 4

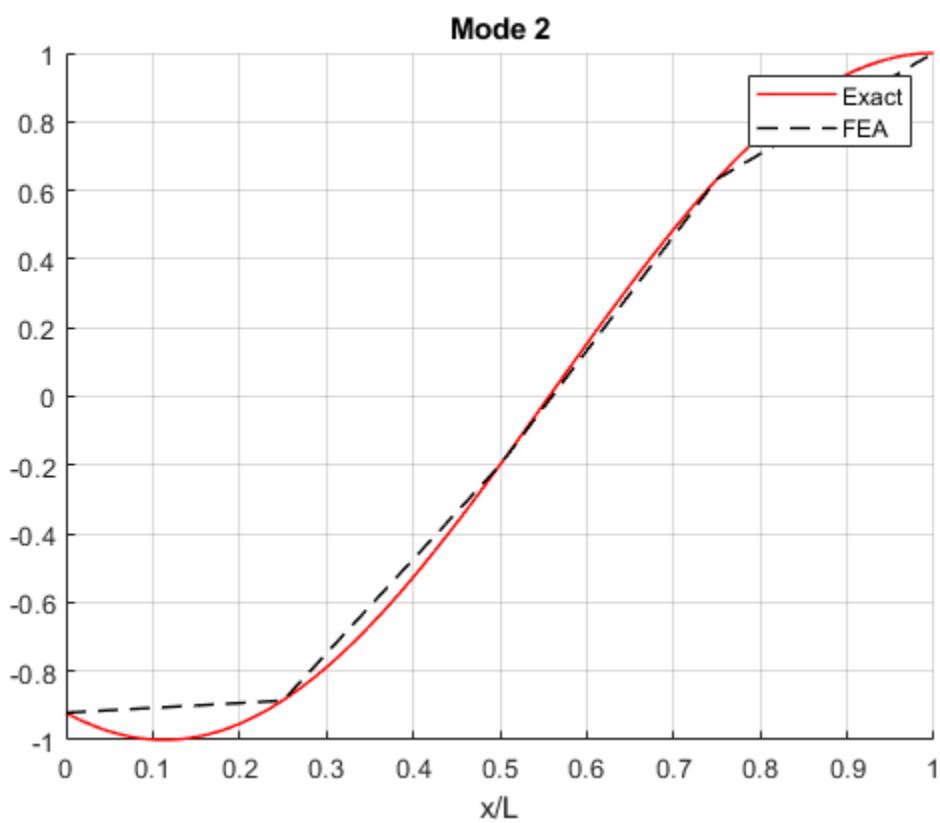
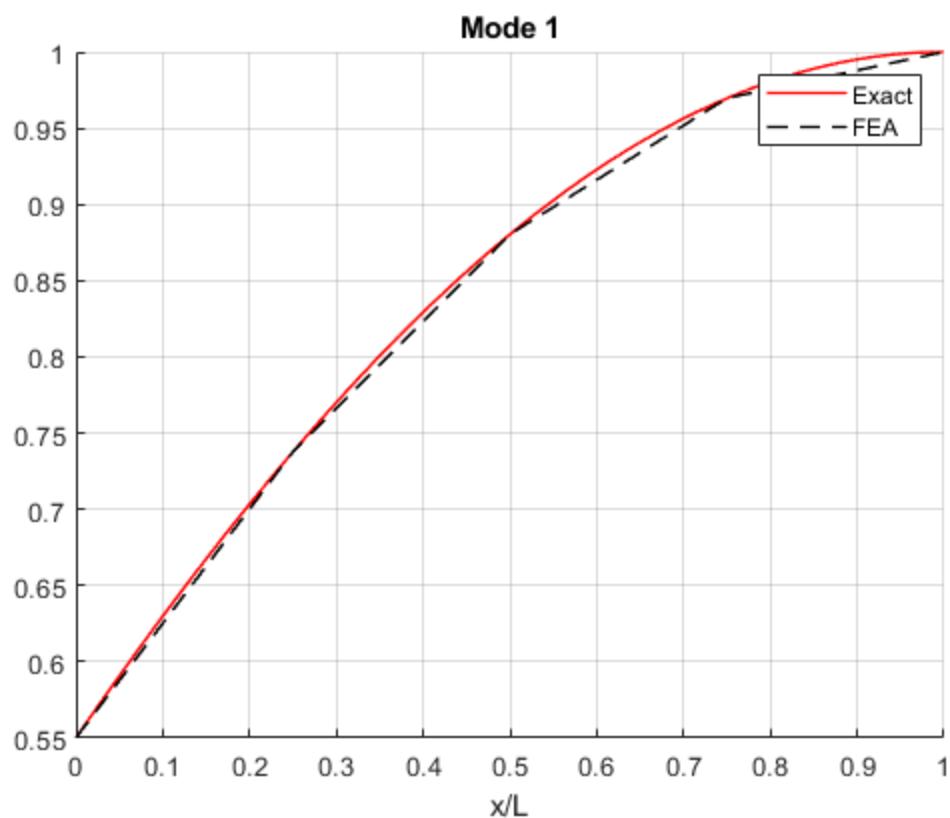
Exact Natural Frequency = 9.5801 sqrt(EA/mL^2)

Natural Frequency = 11.5677 sqrt(EA/mL^2)

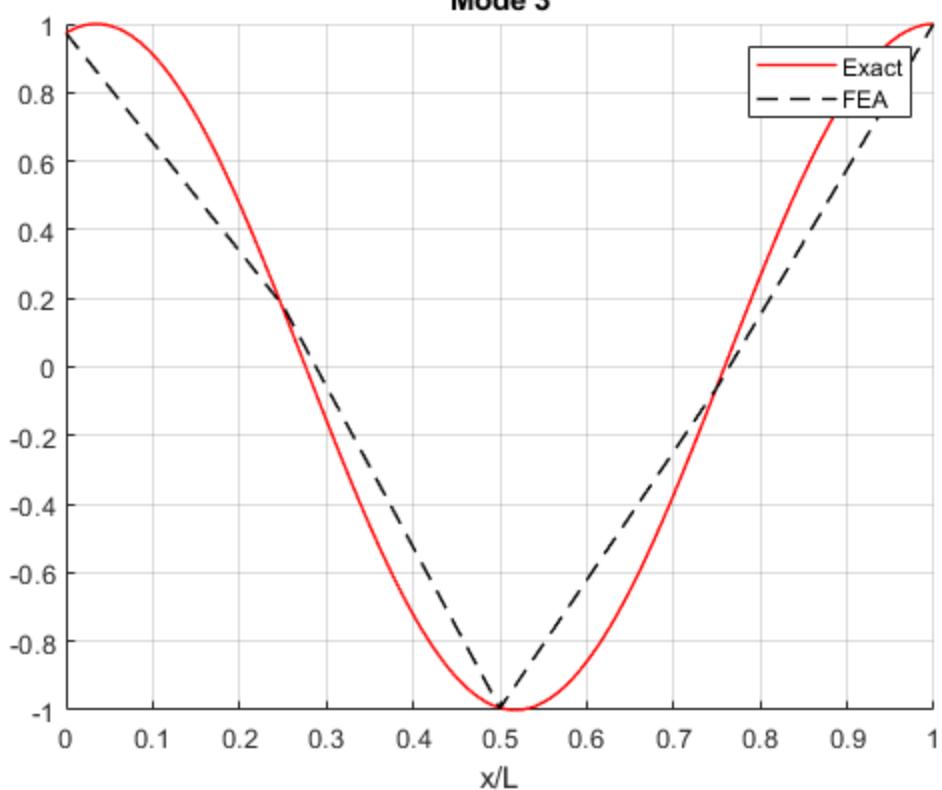
Mode 5

Exact Natural Frequency = 12.6840 sqrt(EA/mL^2)

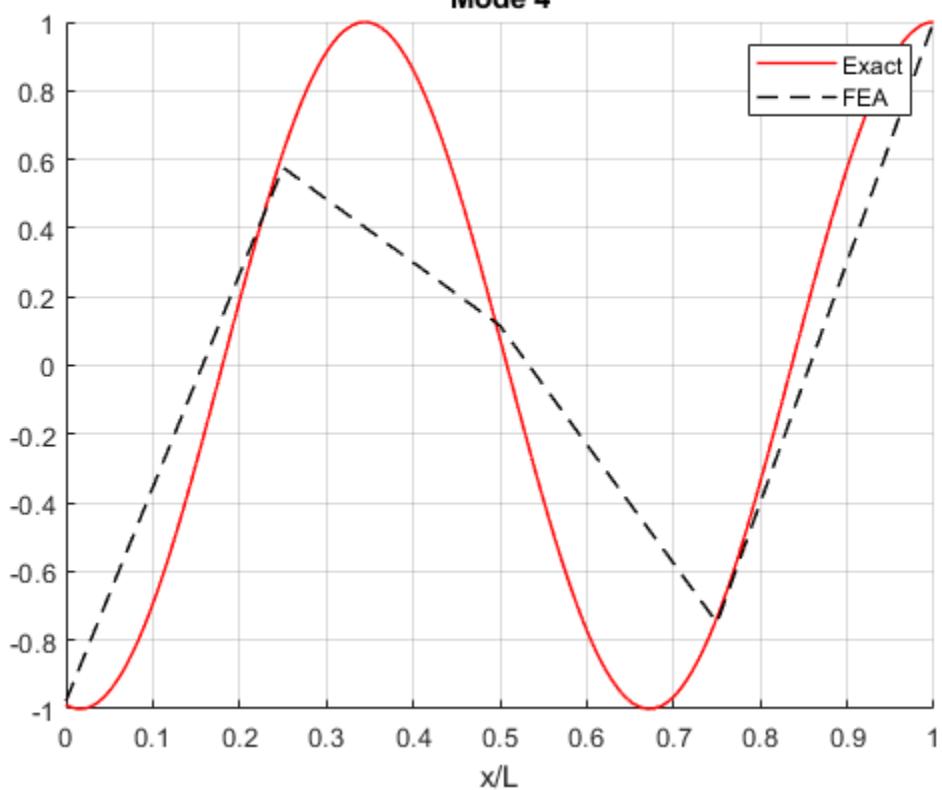
Natural Frequency = 14.0471 sqrt(EA/mL^2)

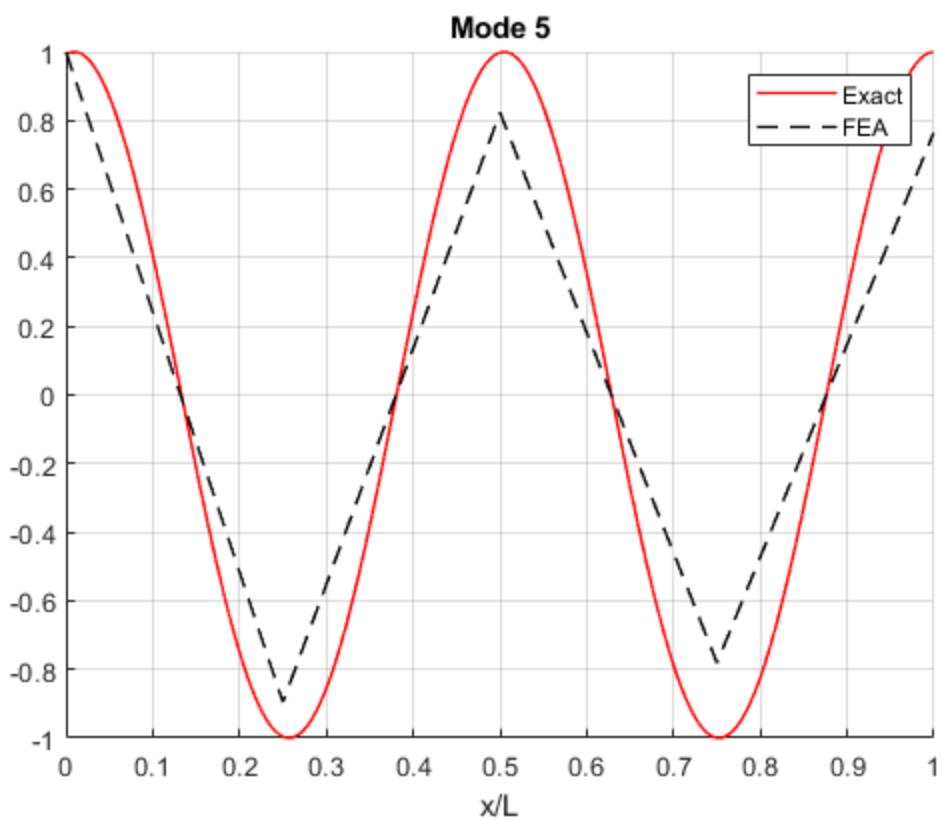


Mode 3



Mode 4





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