$$
\begin{bmatrix}
a_{1,1} & a_{1,2} & a_{1,3} & \cdots & a_{1,n} \\
 & \vdots & & \ddots & \vdots \\
a_{\frac{n}{2},1} & a_{\frac{n}{2},2} & a_{\frac{n}{2},3} & \cdots & a_{\frac{n}{2},n} \\
 & & \vdots & & \\
a_{n,1} & a_{n,2} & a_{n,3} & \dots\dots\dots \quad \dots\dots & a_{n,n}
\end{bmatrix}
\times
\begin{bmatrix}
b_{1,1} & b_{1,2} & b_{1,3} & \cdots & b_{1,n} \\
 & \vdots & & \ddots & \vdots \\
b_{\frac{n}{2},1} & b_{\frac{n}{2},2} & b_{\frac{n}{2},3} & \cdots & b_{\frac{n}{2},n} \\
 & & \vdots & & \\
b_{n,1} & b_{n,2} & b_{n,3} & \dots\dots\dots \quad \dots\dots & b_{n,n}
\end{bmatrix}
=
\begin{bmatrix}
c_{1,1} & c_{1,2} & c_{1,3} & \cdots & c_{1,n} \\
 & \vdots & & \ddots & \vdots \\
c_{\frac{n}{2},1} & c_{\frac{n}{2},2} & c_{\frac{n}{2},3} & \cdots & c_{\frac{n}{2},n} \\
 & & \vdots & & \\
c_{n,1} & c_{n,2} & c_{n,3} & \dots\dots\dots \quad \dots\dots & c_{n,n}
\end{bmatrix}
$$

Matrix has n×m terms, or $n^2$ terms

$\Theta(n^2)$

$C_{i,j}=\sum_{k=1}^{n} a_{i,k} \times bk_{,j}$ $\Theta(n)$

$$\underset{2 \times 2}{\begin{array}{cc} & B \\ A & \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \\ C & D \end{array}} \times \underset{2 \times 2}{\begin{array}{cc} E & F \\ \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} & \\ G & H \end{array}}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} E & F \\ G & H \end{bmatrix}$$

$$\begin{bmatrix} A.E + B.G & A.F + B.H \\ C.E + D.G & C.F + D.H \end{bmatrix}$$

$$\begin{bmatrix} 5 + 14 & 6 + 16 \\ 15 + 28 & 18 + 32 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

$$\begin{array}{cc} & 2 \times 2 \\ & \begin{array}{cc} \phantom{1} & B \end{array} \\ \begin{array}{c} A \\ C \end{array} & \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \\ & \begin{array}{cc} \phantom{1} & D \end{array} \end{array} \times \begin{array}{c} 2 \times 2 \\ \begin{array}{cc} E & F \end{array} \\ \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \\ \begin{array}{cc} G & H \end{array} \end{array}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} E & F \\ G & H \end{bmatrix}$$

$$\begin{bmatrix} A.E + B.G & A.F + B.H \\ C.E + D.G & C.F + D.H \end{bmatrix}$$

$$\begin{bmatrix} 5 + 14 & 6 + 16 \\ 15 + 28 & 18 + 32 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

$$\begin{array}{c} A \qquad\qquad B \\ \left(\begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ \hline 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{array}\right) \\ C \qquad\qquad D \end{array} \begin{array}{c} E \qquad\qquad F \\ \left(\begin{array}{cc|cc} 4 & 3 & 1 & 2 \\ 1 & 3 & 2 & 1 \\ \hline 1 & 2 & 1 & 2 \\ 3 & 2 & 4 & 1 \end{array}\right) \\ G \qquad\qquad H \end{array} = \begin{pmatrix} 21 & 23 & 24 & 14 \\ 30 & 33 & 32 & 20 \\ 39 & 43 & 40 & 26 \\ 48 & 53 & 48 & 32 \end{pmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} E & F \\ G & H \end{bmatrix} \qquad \begin{bmatrix} \boxed{A.E + B.G} & A.F + B.H \\ C.E + D.G & C.F + D.H \end{bmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 1 & 3 \end{pmatrix} + \begin{pmatrix} 3 & 4 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 21 & 23 \\ 30 & 33 \end{pmatrix}$$

$$\left(\begin{array}{cc|cc} \overset{A}{1} & 2 & \overset{B}{3} & 4 \\ 2 & 3 & 4 & 5 \\ \hline 3 & 4 & 5 & 6 \\ \underset{C}{4} & 5 & \underset{D}{6} & 7 \end{array}\right)\left(\begin{array}{cc|cc} \overset{E}{4} & 3 & \overset{F}{1} & 2 \\ 1 & 3 & 2 & 1 \\ \hline 1 & 2 & 1 & 2 \\ 3 & \underset{G}{2} & \underset{H}{4} & 1 \end{array}\right) = \begin{pmatrix} \boxed{\begin{matrix} 21 & 23 \\ 30 & 33 \end{matrix}} & \begin{matrix} 24 & 14 \\ 32 & 20 \end{matrix} \\ \begin{matrix} 39 & 43 \\ 48 & 53 \end{matrix} & \begin{matrix} 40 & 26 \\ 48 & 32 \end{matrix} \end{pmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} E & F \\ G & H \end{bmatrix} \qquad \begin{bmatrix} \boxed{A.E + B.G} & A.F + B.H \\ C.E + D.G & C.F + D.H \end{bmatrix}$$

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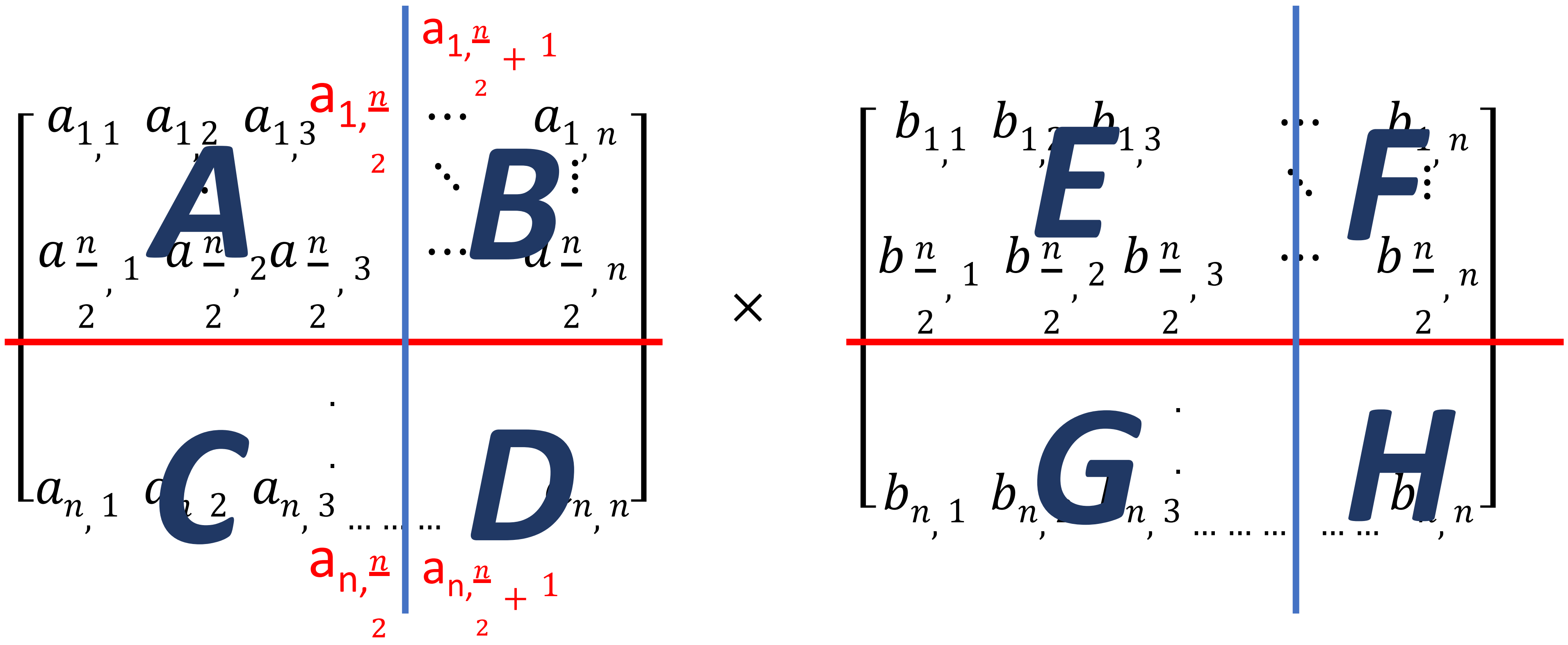
$$\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 1\cdot 4 + 2\cdot 1 & 1\cdot 3 + 2\cdot 3 \\ 2\cdot 4 + 3\cdot 1 & 2\cdot 3 + 3\cdot 3 \end{pmatrix} = \begin{bmatrix} 6 & 9 \\ 11 & 15 \end{bmatrix}$$

$$\begin{pmatrix} 3 & 4 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 3\cdot 1 + 4\cdot 3 & 3\cdot 2 + 4\cdot 2 \\ 4\cdot 1 + 5\cdot 3 & 4\cdot 2 + 5\cdot 2 \end{pmatrix} = \begin{bmatrix} 15 & 14 \\ 19 & 18 \end{bmatrix}$$

$$\begin{pmatrix} 6 & 9 \\ 11 & 15 \end{pmatrix} + \begin{pmatrix} 15 & 14 \\ 19 & 18 \end{pmatrix} = \begin{pmatrix} 21 & 23 \\ 30 & 33 \end{pmatrix}$$

$$\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \cdots & a_{1,n} \\ & \vdots & & \ddots & \vdots \\ a_{\frac{n}{2},1} & a_{\frac{n}{2},2} & a_{\frac{n}{2},3} & \cdots & a_{\frac{n}{2},n} \\ & & \vdots & & \\ a_{n,1} & a_{n,2} & a_{n,3} & \ldots\ldots\ldots\ \ldots\ldots & a_{n,n} \end{bmatrix} \times \begin{bmatrix} b_{1,1} & b_{1,2} & b_{1,3} & \cdots & b_{1,n} \\ & \vdots & & \ddots & \vdots \\ b_{\frac{n}{2},1} & b_{\frac{n}{2},2} & b_{\frac{n}{2},3} & \cdots & b_{\frac{n}{2},n} \\ & & \vdots & & \\ b_{n,1} & b_{n,2} & b_{n,3} & \ldots\ldots\ldots\ \ldots\ldots & b_{n,n} \end{bmatrix} = \begin{bmatrix} c_{1,1} & c_{1,2} & c_{1,3} & \cdots & c_{1,n} \\ & \vdots & & \ddots & \vdots \\ c_{\frac{n}{2},1} & c_{\frac{n}{2},2} & c_{\frac{n}{2},3} & \cdots & c_{\frac{n}{2},n} \\ & & \vdots & & \\ c_{n,1} & c_{n,2} & c_{n,3} & \ldots\ldots\ldots\ \ldots\ldots & c_{n,n} \end{bmatrix}$$

Divide each matrix into 4 matrices that are $\frac{n}{2} \times \frac{n}{2}\ in\ size$




Divide each matrix into 4 matrices that are $\frac{n}{2} \times \frac{n}{2}$ *in size*

**Where each of these us an $\frac{n}{2} \times \frac{n}{2}$ matrix**

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} E & F \\ G & H \end{bmatrix} \qquad \begin{bmatrix} A.E + B.G & A.F + B.H \\ C.E + D.G & C.F + D.H \end{bmatrix}$$

$$T(n) = 8T\left(\frac{n}{2}\right) + 4\left(\frac{n}{2}\right)^2$$

**Where each of these us an $\frac{n}{2} \times \frac{n}{2}$ matrix**

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} E & F \\ G & H \end{bmatrix} \qquad \begin{bmatrix} A.E + B.G & A.F + B.H \\ C.E + D.G & C.F + D.H \end{bmatrix}$$

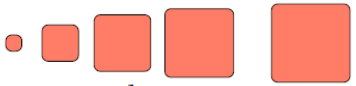
T(n)=8T($\frac{n}{2}$) +4($\frac{n}{2}$)$^2$

**Where each of these us an $\frac{n}{2} \times \frac{n}{2}$ matrix**

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} E & F \\ G & H \end{bmatrix}$$

$$\begin{bmatrix} A.E + B.G & A.F + B.H \\ C.E + D.G & C.F + D.H \end{bmatrix}$$

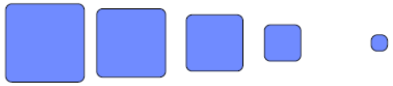
$$T(n) = aT(n/b) + f(n)$$

case 1: 

$f(n) = O(n^{\log_b a - \epsilon}), \epsilon > 0$

case 2: 

$f(n) = \Theta(n^{\log_b a})$

case 3: 

$f(n) = \Omega(n^{\log_b a + \epsilon}), \epsilon > 0$

and c<1 s.t $af(n/b) < cf(n)$

Then:

$T(n) = \Theta(n^{\log_b a})$

$T(n) = \Theta(n^{\log_b a} \log n)$

$T(n) = \Theta(f(n))$

$$T(n)=8T\left(\frac{n}{2}\right) +4\left(\frac{n}{2}\right)^2$$

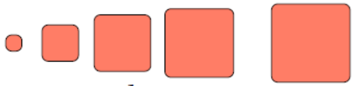
Where each of these us an $\frac{n}{2} \times \frac{n}{2}$ matrix

**$T(n)= \theta(n^3)$**

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} E & F \\ G & H \end{bmatrix}$$

$$\begin{bmatrix} A.E + B.G & A.F + B.H \\ C.E + D.G & C.F + D.H \end{bmatrix}$$

$$T(n) = aT(n/b) + f(n)$$

case 1: 

$f(n) = O(n^{\log_b a - \epsilon}), \epsilon > 0$

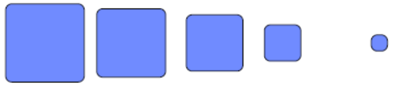
Then:

$T(n) = \Theta(n^{\log_b a})$

case 2: 

$f(n) = \Theta(n^{\log_b a})$

$T(n) = \Theta(n^{\log_b a} \log n)$

case 3: 

$f(n) = \Omega(n^{\log_b a + \epsilon}), \epsilon > 0$

and c<1 s.t $af(n/b) < cf(n)$

$T(n) = \Theta(f(n))$

$$T(n)=8T(\frac{n}{2}) +4(\frac{n}{2})^2$$

Case 1: $f(n)=n^2$

$n^{\log_2 8} = n^{\log_2(2^3)} = n^3 > f(n)$ , $\varepsilon=0.01$

**Where each of these us an $\frac{n}{2} \times \frac{n}{2}$ matrix**

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} E & F \\ G & H \end{bmatrix} \qquad \begin{bmatrix} A.E + B.G & A.F + B.H \\ C.E + D.G & C.F + D.H \end{bmatrix}$$

T(n)=8T($\frac{n}{2}$) +4($\frac{n}{2}$)$^2$

**T(n)= θ(n$^3$)**

Where each of these us an $\frac{n}{2} \times \frac{n}{2}$ matrix

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} E & F \\ G & H \end{bmatrix} \qquad \begin{bmatrix} A.E + B.G & A.F + B.H \\ C.E + D.G & C.F + D.H \end{bmatrix}$$

***Strassen***

P3+P4=CE+DE-DE+DG=CE+DG

P1=A(F-H)

P2=(A+B)H

P3=(C+D)E

P4=D(G-E)

P5=(A+D)(E+H)

P6=(B-D)(G+H)

P7=(A-C)(E+F)

**P1+P2=AF-AH+AH+BH=AF+BH**

Where each of these us an $\frac{n}{2} \times \frac{n}{2}$ matrix

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} E & F \\ G & H \end{bmatrix} \qquad \begin{bmatrix} A.E + B.G & A.F + B.H \\ C.E + D.G & C.F + D.H \end{bmatrix}$$

***Strassen***

P1=A(F-H)

P2=(A+B)H

P3=(C+D)E

P4=D(G-E)

P5=(A+D)(E+H)

P6=(B-D)(G+H)

P7=(A-C)(E+F)

P5+P4-P2+P6

=AE+AH+DE +DH

+DG-DE

-AH -BH

+BG –DG +BH-DH

**P5+P1-P3-P7**

Where each of these us an $\frac{n}{2} \times \frac{n}{2}$ matrix

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} E & F \\ G & H \end{bmatrix}$$

$$\begin{array}{cc} P5+P4-P2+P6 & P1+P2 \\ \left[\begin{matrix} A.E + B.G \\ C.E + D.G \end{matrix}\right. & \left.\begin{matrix} A.F + B.H \\ C.F + D.H \end{matrix}\right] \\ P3+P4 & P5+P1-P3-P7 \end{array}$$

***Strassen***

P1=A(F-H)

P2=(A+B)H

P3=(C+D)E

P4=D(G-E)

P5=(A+D)(E+H)

P6=(B-D)(G+H)

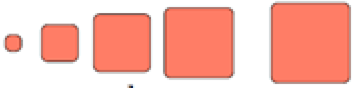
P7=(A-C)(E+F)

$$T(n)=7T(\frac{n}{2})+18(\frac{n}{2})^2$$

$$=\theta(n^{\log_2 7})$$

$$\sim n^{2.805}$$

$$T(n) = aT(n/b) + f(n)$$

case 1: 

$f(n) = O(n^{\log_b a - \epsilon}), \epsilon > 0$

Then:

$T(n) = \Theta(n^{\log_b a})$

# Taking this idea further?

***Strassen:*** used 7 matrix multiplication of size $\frac{n}{2} \times \frac{n}{2}$

**Laderman '75**

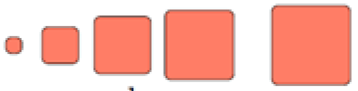
- Used 23 matrix multiplication of size $\frac{n}{3} \times \frac{n}{3}$

$$T(n) = 23T(\tfrac{n}{3}) + c(\tfrac{n}{2})^2$$

$$= \theta(n^{\log_3 23})$$

$$\sim n^{2.854}$$

$$T(n) = aT(n/b) + f(n)$$

case 1: 

$f(n) = O(n^{\log_b a - \epsilon}), \epsilon > 0$

Then:

$T(n) = \Theta(n^{\log_b a})$

# Taking this idea further?

***Strassen:*** used 7 matrix multiplication of size $\frac{n}{2} \times \frac{n}{2}$

**Laderman '75**

- Used 23 matrix multiplication of size $\frac{n}{3} \times \frac{n}{3}$

$T(n)=23T(\frac{n}{3})+c(\frac{n}{2})^2$

$\theta(n^{\log_3 21}) \sim 2.771$

$=\theta(n^{\log_3 23})$

$\sim n^{2.854}$

# Taking this idea further?

***Victor pan 1978:***

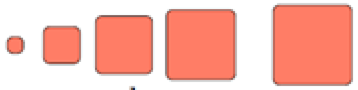
used 143640 matrix multiplication of size $\frac{n}{70} \times \frac{n}{70}$

$$T(n)=143640\,T\left(\frac{n}{70}\right)+c\left(\frac{n}{70}\right)^2$$

$$=\theta\left(n^{\log_{70}143640}\right)$$

$$\sim n^{2.795}$$

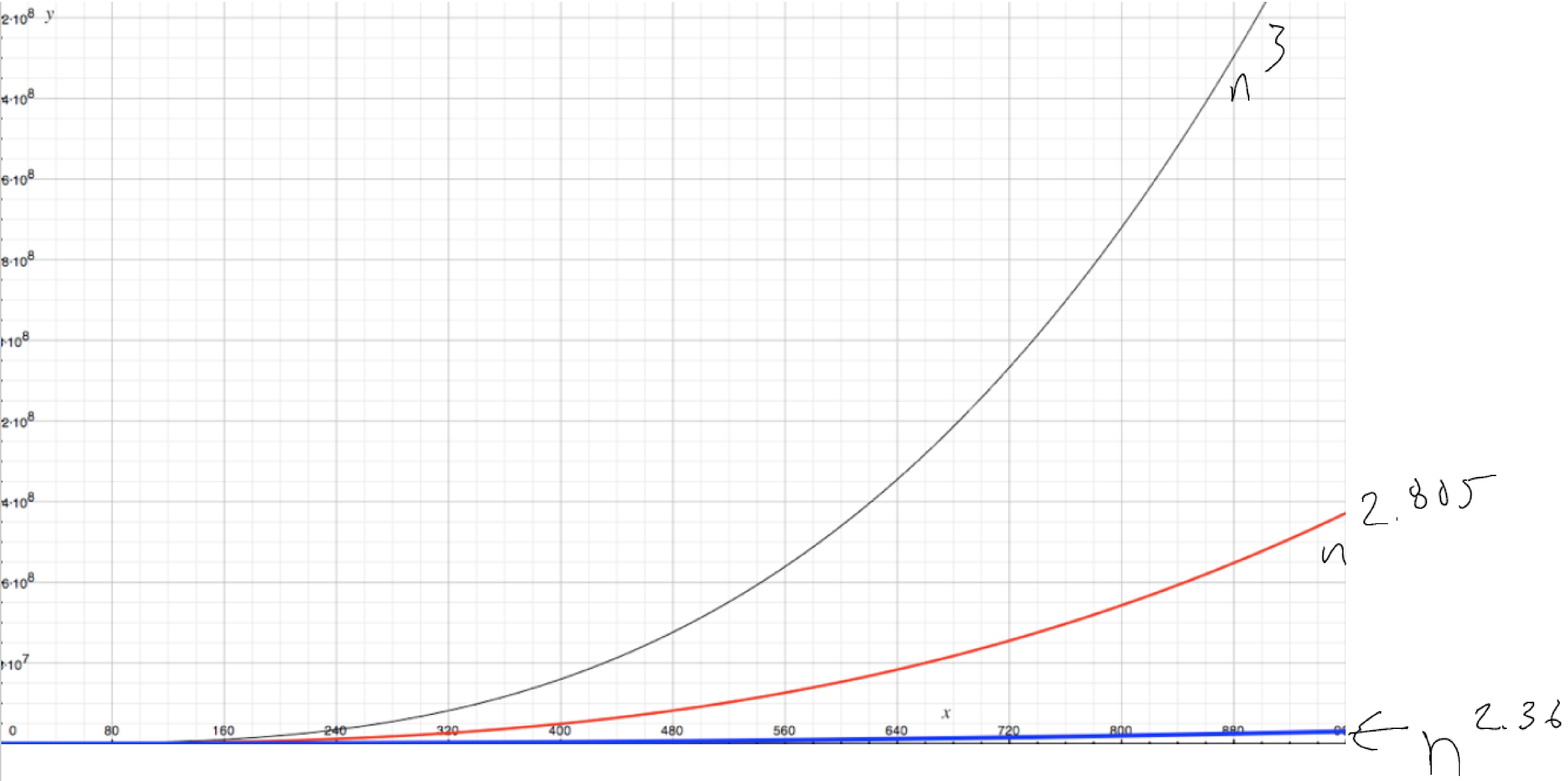
$$T(n) = aT(n/b) + f(n)$$

case 1: 

$f(n) = O(n^{\log_b a-\epsilon}), \epsilon > 0$

Then:

$T(n) = \Theta(n^{\log_b a})$


y
x
0
80
160
240
320
400
480
560
640
720
800
880
n^3
n^2.805
n^2.36

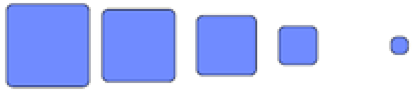
## PROBLEM 3 *Flip-flop (10 point)*

Consider the recurrence $T(n) = 2 \times T(\frac{n}{2}) + f(n)$ in which:

$$f(n) = \begin{cases} n^3 & \text{if } \lceil log(n) \rceil \text{ is even} \\ n^2 & \text{otherwise} \end{cases}$$

Show that $f(n) = \Omega(n^{log_b(a)+\epsilon})$. Explain why the third case of the Master's theorem stated above does not apply. Prove a $\theta$ bound for the recurrence.

**PROBLEM 3** *Flip-flop (10 point)*

case 3: 

$f(n) = \Omega(n^{\log_b a+\epsilon}), \epsilon > 0$

and c<1 s.t $af(n/b) < cf(n)$

$T(n) = \Theta(f(n))$

Consider the recurrence $T(n) = 2 \times T(\frac{n}{2}) + f(n)$ in which:

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Show that $f(n) = \Omega(n^{log_b(a)+\epsilon})$. Explain why the third case of the Master's theorem stated above does not apply. Prove a $\theta$ bound for the recurrence.

**Consider all n such that log($\lceil n \rceil$) is odd, and log($\lceil n/2 \rceil$) is even.**

**Now:**

**$2(f(n/2))=n^3/4$ but, $f(n)=n^2$ So, no C can make the inequality hold for n value's that goes toward infinity.**

## PROBLEM 3 *Flip-flop (10 point)*

Consider the recurrence $T(n) = 2 \times T(\frac{n}{2}) + f(n)$ in which:

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$T(n)=\theta(n^3)$

Induction: $T(n)<3n^3$     Suppose it holds for $n<n_0$

**Now consider for $n_0$:**

$$\begin{aligned} T(n_0) &= 2T(n_0/2) + f(n_0) \\ &\leq 2 \cdot 3(n_0/2)^3 + n_0^3 \\ &\leq n_0^3\left(\frac{3}{4}+1\right) < 3n_0^3 \end{aligned}$$

Upper bound proof!

PROBLEM 3 *Flip-flop (10 point)*

Consider the recurrence $T(n) = 2 \times T(\frac{n}{2}) + f(n)$ in which:

$$f(n) = \begin{cases} n^3 & \text{if } \lceil log(n) \rceil \text{ is even} \\ n^2 & \text{otherwise} \end{cases}$$

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Upper bound proof!

PROBLEM 3 *Flip-flop (10 point)*

Consider the recurrence $T(n) = 2 \times T(\frac{n}{2}) + f(n)$ in which:

$$f(n) = \begin{cases} n^3 & \text{if } \lceil log(n) \rceil \text{ is even} \\ n^2 & \text{otherwise} \end{cases}$$

$T(n)=\theta(n^3)$

Show that $f(n) = \Omega(n^{log_b(a)+\epsilon})$. Explain why the third case of the Master's theorem stated above does not apply. Prove a $\theta$ bound for the recurrence.

Induction: $T(n) > \frac{1}{8}n^3$

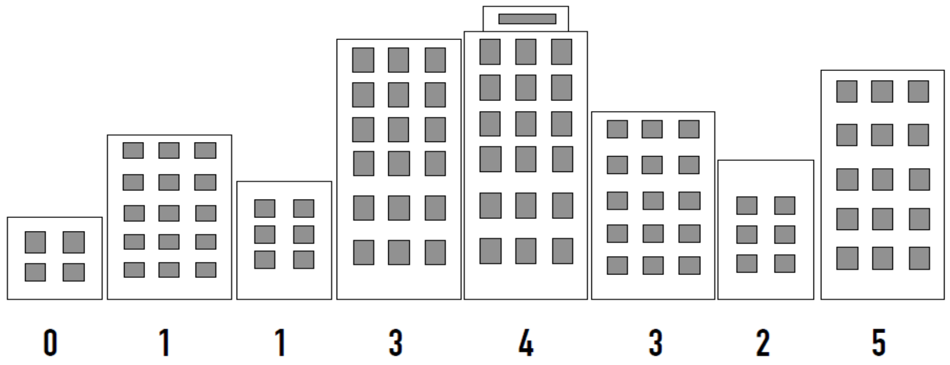
Suppose it holds for $n<n_0$

$log(\lceil n_0 \rceil)$ is even, or $log(\lceil n_0/2 \rceil)$ is even

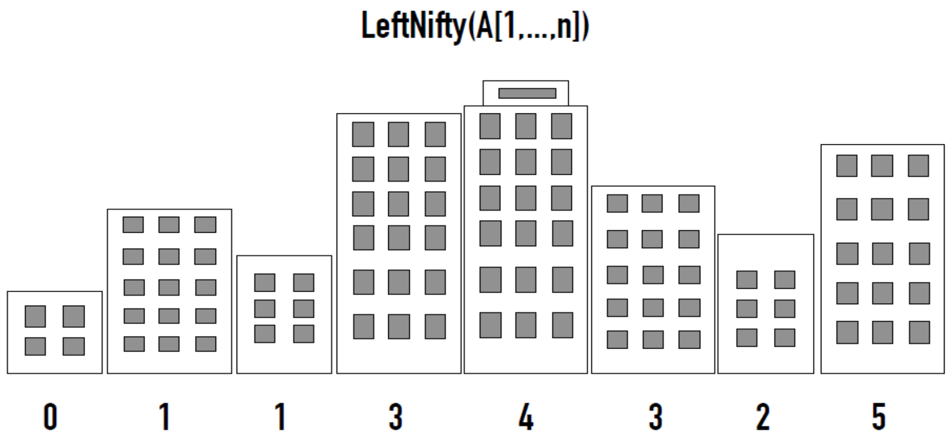
$$\begin{aligned} T(n_0) &= 2T(n_0/2) + f(n_0) \\ &= 2[2T(n_0/4) + f(n_0/2)] + f(n_0) \\ &= 4T(n_0/4) + 2f(n_0/2) + f(n_0) \\ &\geq n_0^3/4 > n_0^3/8 \end{aligned}$$

# Problem 4: Skyline problem

LeftNifty(A[1,....,n])

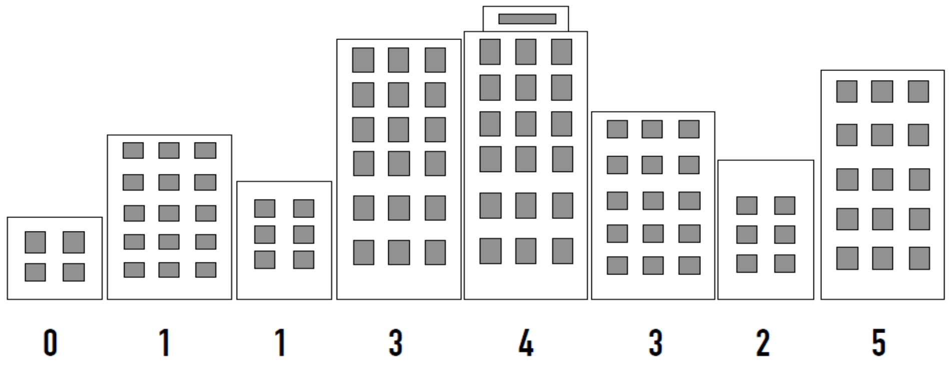


# *Problem 4: Skyline problem*



# Problem 4: Skyline problem

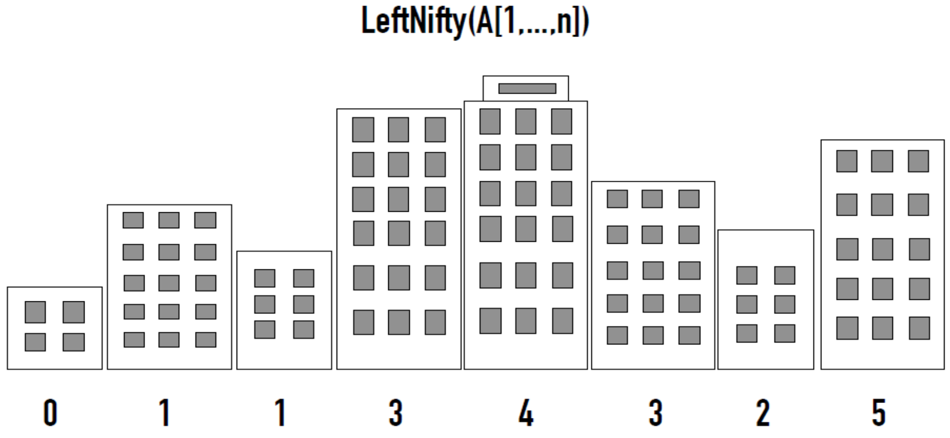
LeftNifty(A[1,....,n])



# *Problem 4: Skyline problem*

**Algorithm 1** $(n_\ell, n_r, C[1,\ldots,n]) \leftarrow \text{niftyness}(A[1,\ldots,n])$

If list has size 1 return $(0, 0, A)$.
$(n_\ell, n_r, L) \leftarrow \text{niftyness}(A[1,\ldots,n/2])$
$(m_\ell, m_r, R) \leftarrow \text{niftyness}(A[n/2+1,\ldots,n])$
$o_\ell \leftarrow \text{Combine-Left}(L[1,\ldots,n/2],\ R[1,\ldots,n/2])$
$o_r \leftarrow \text{Combine-Right}(L[1,\ldots,n/2],\ R[1,\ldots,n/2])$
Merge lists $L[1,\ldots,n/2], R[1,\ldots,n/2]$ into list $C$
**return** $(n_\ell + m_\ell + o_\ell, n_r + m_r + o_r, C)$



## PROBLEM 2 *Proof by Induction (6 point)*

Use the substitution method (thats the induction one) to prove the runtime of the following recurrence relation:

$T(n) = 2 \times T(\frac{n}{2}) + n^2$

How large does the constant c have to be?

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Suppose $T(i) <= c * i^2$ for all $i<k$

**Now We have to prove: $T(k) <= c * k^2$**

$T(k)=2*T(k/2)+K^2$

$T(k)<=2*[c*(k/2)^2]+k^2$

$T(k)<=2*c*(k^2/4)+k^2$

$T(k)<=c*(k^2/2)+k^2$

$T(k)<=(k^2)*(c/2+1)$ $<= k^2*C$

How, (c/2+1) has to be less than or equal to c?

$$\frac{c}{2} + 1 \leq c$$
$$c + 2 \leq 2 * c$$
$$2 \leq c$$
$$c \geq 2$$

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