Induction

Class Attendance

https://forms.gle/thGUVx9emRLS4Xki7

Use the secret number:

32154

Recurrence

Procedure T (n: size of input) **defined as:**
If n<1 **then exit**

Do work of amount f(n) # this work is to accumulate information from subproblems

T(n/**b**)
T(n/**b**)
…….. Repeat for a total of **a** times
T(n/**b**)

a

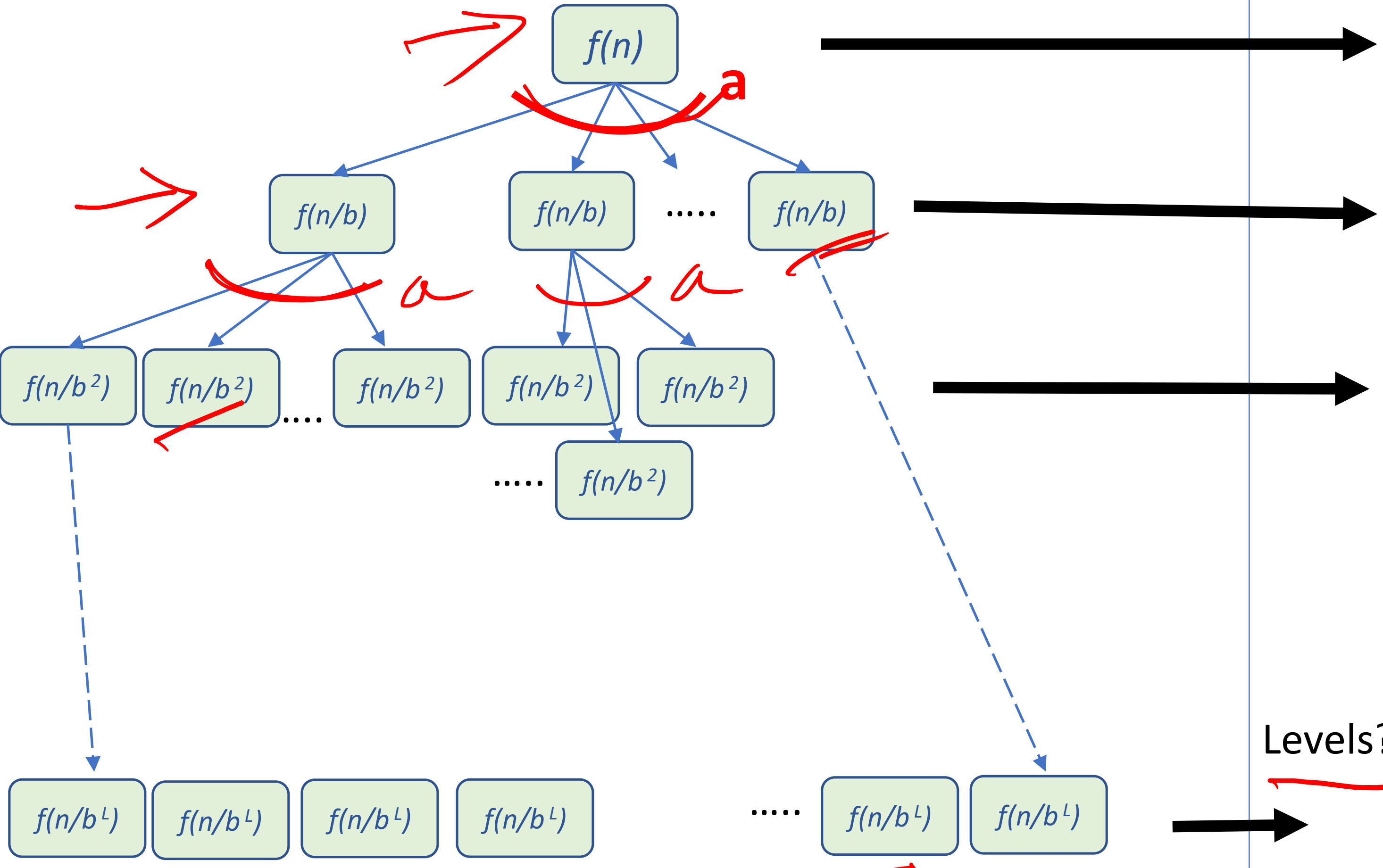
End procedure

$b < n$

$$T(n)=aT\left(\frac{n}{b}\right)+f(n)$$

Master Theorem

$$T(n)=aT(n/b)+f(n)$$



$f(n)$

$af(n/b)$

$a^2f(n/b^2)$

Levels? $L=\log_b n$

$a^Lf(n/b^L)$

Karatsuba (ab,cd)

1	7	8	9	×	1	4	3	2

a b c d

BASE CASE:

return b × d if inputs are 1 digit

ELSE:

$b=2$

$a=3$

Compute ac=karatsuba(a,c) → $T(\frac{n}{2})$

Compute bd=karatsuba(b,d) → $T(\frac{n}{2})$

Compute t=karatsuba((a+b),(c+d)) → $T(\frac{n}{2}) + 2n$

mid=t-ac-bd → **2n**

$f(n) = 8n$

Return ac × ((10)$^{\text{(number of digit of a)}}$)^2 +mid×(10)$^{\text{(number of digit of a)}}$ +bd } **4n steps**

$$T(n)=3T(\frac{n}{2}) + \underbrace{8n}_{f(n)}$$

Ignoring issue of carries

MergeSort(A,start,end)

1. **If start < end**
2. **q ← ⌊ start + end ⌋ /2**
3. Mergesort(A, start, q)
   Mergesort(A,q+1,end)
4. **Merge(A,start,q,end)**
5. **Else** *base case*

$T(n) = 2*T(n/2)+n$

1

$2*T(n/2)$

n

$b=2$

$a=2$

$f(n)=n$

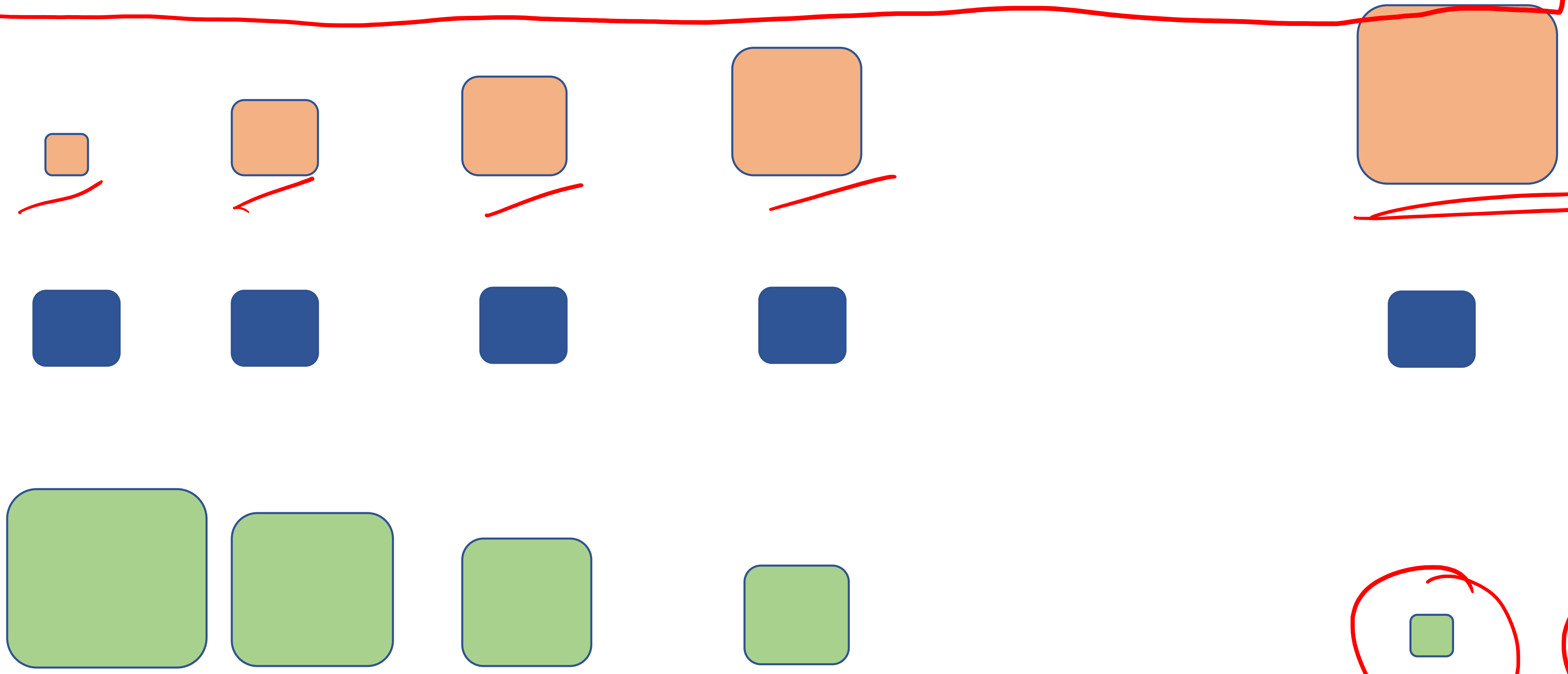
Master Theorem

$$T(n)=f(n)+af\left(\frac{n}{b}\right)+a^2f\left(\frac{n}{b^2}\right)+a^3f\left(\frac{n}{b^3}\right)+\ldots\ldots\ldots\ldots+a^Lf\left(\frac{n}{b^L}\right)$$

Case 1:

Case 2:

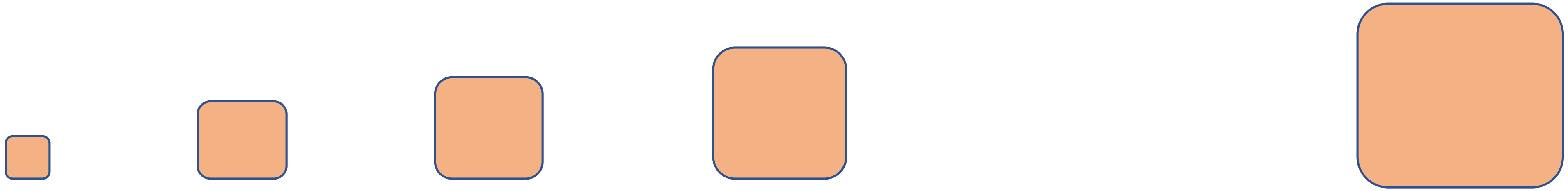
Case 3:



Master Theorem

$$T(n)=f(n)+af\left(\frac{n}{b}\right)+a^2f\left(\frac{n}{b^2}\right)+a^3f\left(\frac{n}{b^3}\right)+\ldots\ldots\ldots\ldots+a^Lf\left(\frac{n}{b^L}\right)$$

Case 1:



$f(n)=O(n^{\log_b a-\varepsilon}),\ \varepsilon>0$ then, $T(n)=\Theta(n^{\log_b a})$

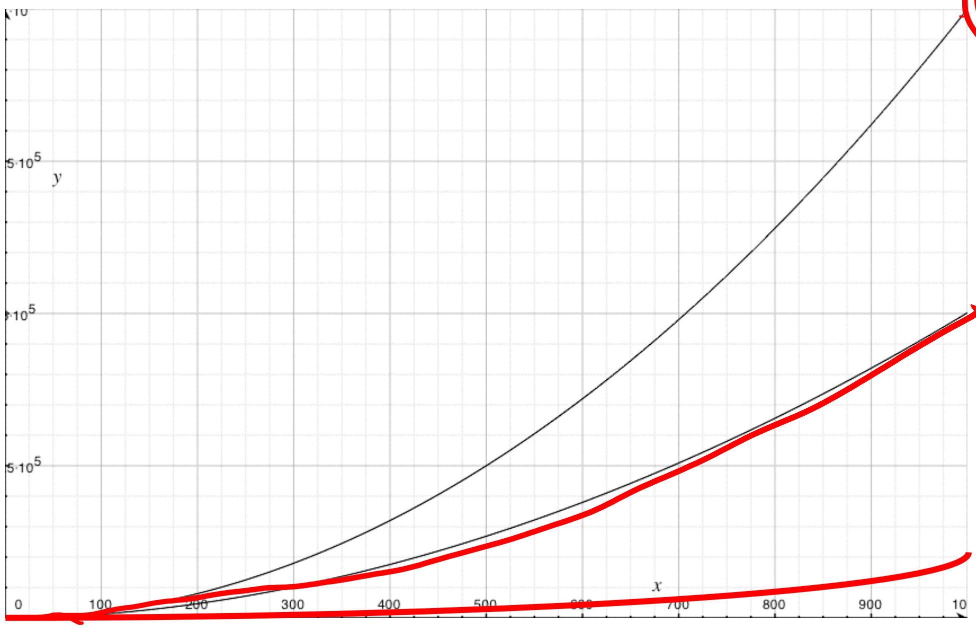
Master Theorem

$$T(n)=f(n)+af\left(\frac{n}{b}\right)+a^2f\left(\frac{n}{b^2}\right)+a^3f\left(\frac{n}{b^3}\right)+\ldots\ldots\ldots\ldots+a^Lf\left(\frac{n}{b^L}\right)$$

Case 1:

$f(n)=O(n^{\log_b a-\varepsilon})$, $\varepsilon>0$ then, $T(n)=\theta(n^{\log_b a})$

$f = n^{2-\epsilon}$



n^2

$N^{1.97}=n^{2-\varepsilon}$ where $\varepsilon=0.03$

$N^{1.6}=n^{2-\varepsilon}$ where $\varepsilon=0.4$

Karatsuba (ab,cd)

$$\underbrace{1\ 7}_{a}\ \underbrace{8\ 9}_{b} \times \underbrace{1\ 4}_{c}\ \underbrace{3\ 2}_{d}$$

BASE CASE:

return b × d if inputs are 1 digit

ELSE:

Compute ac=karatsuba(a,c) → $T(\frac{n}{2})$

Compute bd=karatsuba(b,d) → $T(\frac{n}{2})$

Compute t=karatsuba((a+b),(c+d)) → $T(\frac{n}{2}) + 2n$

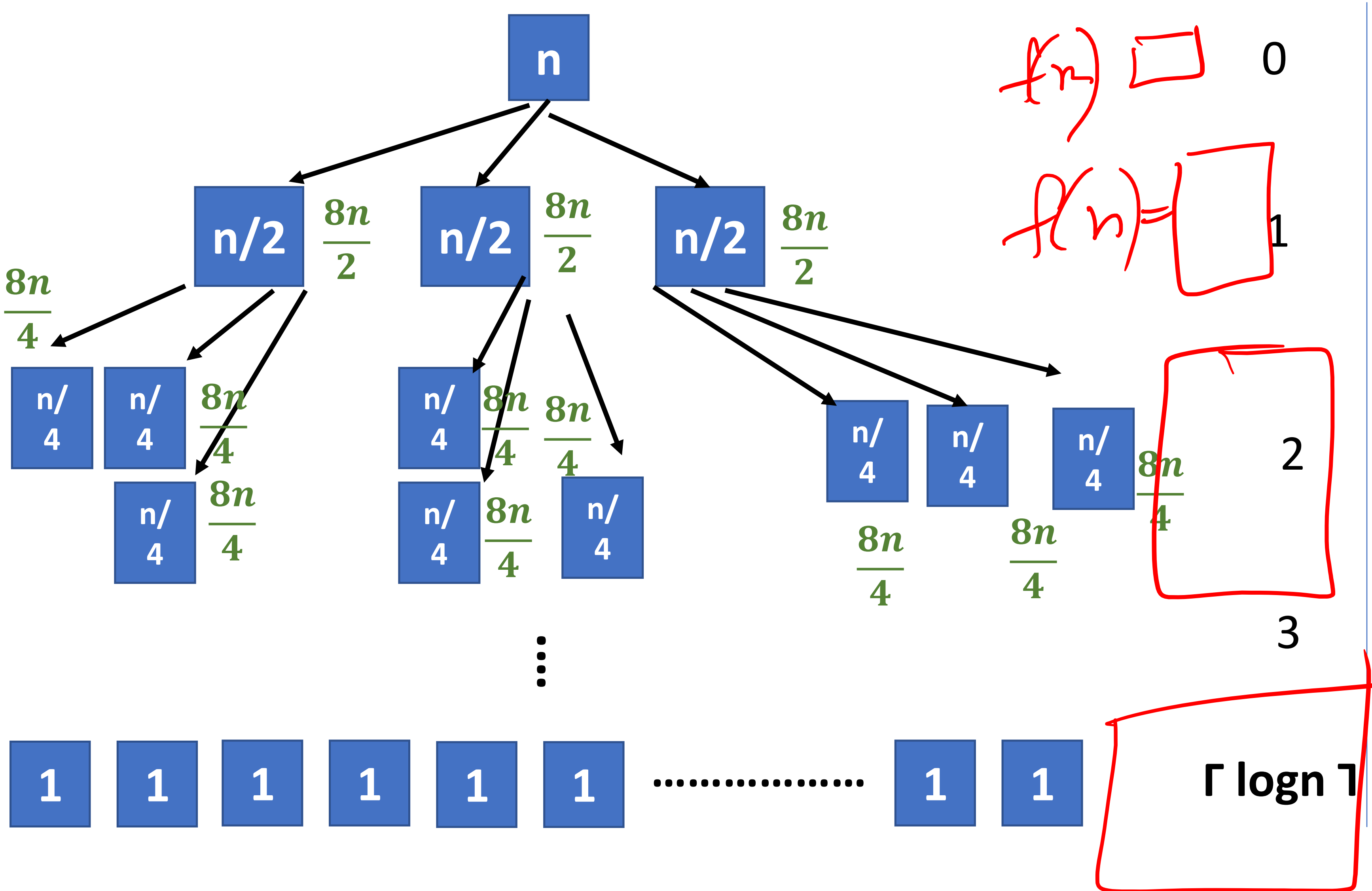
mid=t-ac-bd → 2n

$T(n)=3T(\frac{n}{2}) + 8n$

Ignoring issue of carries

Return ac × $((10)^{(\text{number of digit of a})})^2$ +mid×$(10)^{(\text{number of digit of a})}$ +bd } **4n steps**

$$T(n)=3T\left(\frac{n}{2}\right)+8n$$



8n

$$3\times\frac{8n}{2}=\left(\frac{3}{2}\right)^{1}\times 8n$$

$$9\times\frac{8n}{4}=\left(\frac{3}{2}\right)^{2}\times 8n$$

$$27\times\frac{8n}{8}=\left(\frac{3}{2}\right)^{3}\times 8n$$

$$\left(\frac{3}{2}\right)^{\lceil \log n\rceil}\times 8n$$

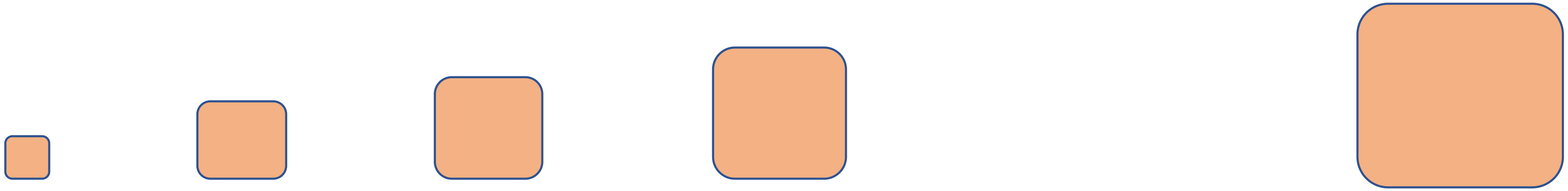
Karatsuba with Master Theorem

$a = 3$
$b = 2$

$$T(n)=f(n)+af(\frac{n}{b})+ a^2f(\frac{n}{b^2}) + a^3f(\frac{n}{b^3})+\dots\dots\dots\dots + a^Lf(\frac{n}{b^L})$$

$f(n) = 8n$

Case 1:



Case 1 $\Rightarrow$ $\boxed{f(n)=O(n^{\log_b a-\varepsilon}),\ \varepsilon>0}$ then, $T(n)=\theta(n^{\log_b a})$

$\varepsilon > 0$

$\log_2 3 = 1.6$

$8n^{1.6-0.01}$

T(n)=3T(n/2)+8n

a=3 b=2

$f(n)=8n < 8n^{\log_2 3-\varepsilon}$

1.6 ε=0.01

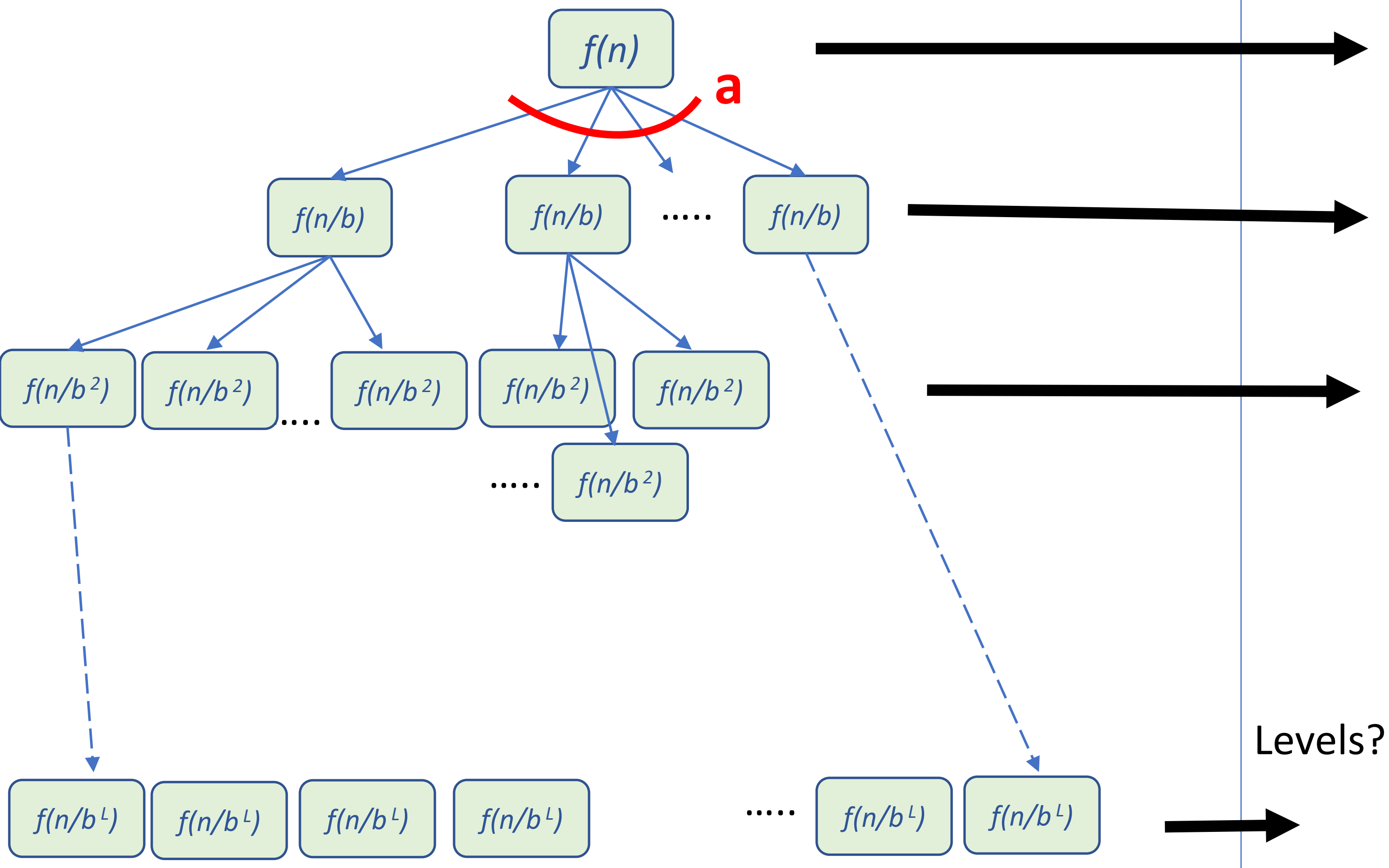
$8n^{1.59}$

$f(n)=O(n^{\log_3 2-\varepsilon})$ $T(n) = \theta(n^{\log_2 3})$ And according to master theorem: $T(n)= \theta (n^{\log_3 2-\varepsilon})$ $8n <$

$f(n) = O(n^{1.59})$

Master Theorem

$$T(n)=aT(n/b)+f(n)$$



$f(n)$

$af(n/b)$

$a^2f(n/b^2)$

Levels? $L=\log_b n$

$a^Lf(n/b^L)$

$n=1$

Master Theorem

$T(n)=f(n)+af(\frac{n}{b})+a^2f(\frac{n}{b^2})+a^3f(\frac{n}{b^3})+\ldots\ldots\ldots\ldots+a^Lf(\frac{n}{b^L})$

$f(n)=O(n^{\log_b a-\varepsilon}) \rightarrow f(n) < c^* n^{\log_b a-\varepsilon}$

CASE 1: Upper bound

CASE 1:

$f(n)=O(n^{\log_b a-\varepsilon})$, $\varepsilon>0$ then, $T(n)=\theta(n^{\log_b a})$

$\frac{n}{b^L}=n$

$T(n) \leq c\, n^{\log_b a-\varepsilon} + a.c\,(\frac{n}{b})^{\log_b a-\varepsilon} + a^2.c\,(\frac{n}{b^2})^{\log_b a-\varepsilon} + \ldots\ldots + a^{L-1}.c\,(\frac{n}{b^{L-1}})^{\log_b a-\varepsilon} + a^L f(1)$

$$T(n) \leq cn^{\log_b a-\epsilon}\left[1+\frac{a}{b^{\log_b a-\epsilon}}+\frac{a^2}{(b^2)^{\log_b a-\epsilon}}+\frac{a^{L-1}}{(b^{L-1})^{\log_b a-\epsilon}}\right]+n^{\log_b a}c$$

(2) $a^L = a^{\log_b n} = (n^{\log_n a})^{\log_b n} = n^{\log_b a}$, using $\log_n a = \frac{\log_b a}{\log_b n} \Rightarrow \log_b a = \log_n a \log_b n$

(1)

Master Theorem

$$T(n)=f(n)+af(\frac{n}{b})+a^2f(\frac{n}{b^2})+a^3f(\frac{n}{b^3})+\ldots\ldots\ldots\ldots+a^Lf(\frac{n}{b^L})$$

CASE 1:

$f(n)=O(n^{\log_b a-\varepsilon})$, $\varepsilon>0$ then, $T(n)=\theta(n^{\log_b a})$

$f(n)=O(n^{\log_b a-\varepsilon}) \rightarrow f(n) < c^* n^{\log_b a-\varepsilon}$

CASE 1: Upper bound

$$T(n)\leq c\, n^{\log_b a-\varepsilon}+a.c\,(\frac{n}{b})^{\log_b a-\varepsilon}+a^2.c\,(\frac{n}{b^2})^{\log_b a-\varepsilon}+\ldots\ldots+a^{L-1}.c\,(\frac{n}{b^{L-1}})^{\log_b a-\varepsilon}+a^Lf(1)$$

$$T(n)\leq cn^{\log_b a-\epsilon}\left[1+\frac{a}{b^{\log_b a-\epsilon}}+\frac{a^2}{(b^2)^{\log_b a-\epsilon}}+\frac{a^{L-1}}{(b^{L-1})^{\log_b a-\epsilon}}\right]+n^{\log_b a}c$$

② $a^L = a^{\log_b n} = (n^{\log_n})^{a \log_b n} = n^{\log_b a}$, using $\log_n a = \dfrac{\log_b a}{\log_b n} \Rightarrow \log_b a = \log_n a \log_b n$

① $\dfrac{a^i}{(b^i)^{\log_b(a)-\epsilon}} = \dfrac{a^i}{(b^{\log_b(a)-\epsilon})^i} = \dfrac{a^i}{\dfrac{(b^{\log_b a})^i}{(b^\epsilon)^i}} = \dfrac{a^i}{\dfrac{a^i}{b^{\epsilon i}}} = b^{\epsilon i}$

Master Theorem

CASE 1:

$f(n)=O(n^{\log_b a-\varepsilon})$, $\varepsilon>0$ then, $T(n)=\theta(n^{\log_b a})$

$$T(n)=f(n)+af(\frac{n}{b})+a^2f(\frac{n}{b^2})+a^3f(\frac{n}{b^3})+\ldots\ldots\ldots\ldots+a^Lf(\frac{n}{b^L})$$

$f(n)=O(n^{\log_b a-\varepsilon}) \rightarrow f(n) < c^* \, n^{\log_b a-\varepsilon}$

$$T(n) \le c\, n^{\log_b a-\varepsilon} + a.c\,(\frac{n}{b})^{\log_b a-\varepsilon} + a^2.c\,(\frac{n}{b^2})^{\log_b a-\varepsilon} + \ldots\ldots + a^{L-1}.c\,(\frac{n}{b^{L-1}})^{\log_b a-\varepsilon} + a^L f(1)$$

$$T(n) \le cn^{\log_b a-\epsilon}\left[1+\frac{a}{b^{\log_b a-\epsilon}}+\frac{a^2}{(b^2)^{\log_b a-\epsilon}}+\frac{a^{L-1}}{(b^{L-1})^{\log_b a-\epsilon}}\right]+n^{\log_b a}\,c$$

$$T(n) \le cn^{\log_b a-\epsilon}\left[1+b^{\epsilon}+b^{2\epsilon}+\cdots+b^{\epsilon(L-1)}\right]+n^{\log_b a}\,c$$

Master Theorem

$$\boxed{1+b^{\varepsilon}+b^{2\varepsilon}+b^{3\varepsilon}+\ldots\ldots+b^{(L-1)\varepsilon}=\frac{b^{\varepsilon L}-1}{b^{\varepsilon}-1}} = \frac{\boxed{n^{\epsilon}-1}}{b^{\epsilon}-1}$$

$$① \; \frac{a^i}{(b^i)^{\log_b(a)-\epsilon}} = \frac{a^i}{(b^{\log_b(a)-\epsilon})^i} = \frac{a^i}{\frac{(b^{\log_b a})^i}{(b^{\epsilon})^i}} = \frac{a^i}{\frac{a^i}{b^{\epsilon i}}} = b^{\epsilon i}$$

$$② \; a^L = a^{\log_b n} = (n^{\log_n})^{a \log_b n} = n^{\log_b a}, \quad \text{using} \quad \log_n a = \frac{\log_b a}{\log_b n} \Rightarrow \log_b a = \log_n a \log_b n$$

$$③ \; b^{\epsilon L} = (b^{\log_b n})^{\epsilon} = \boxed{n^{\epsilon}}$$

Master Theorem

$\frac{c}{c_1} = c'$

CASE 1:

$f(n)=O(n^{\log_b a-\varepsilon})$, $\varepsilon>0$ then, $T(n)=\theta(n^{\log_b a})$

$$T(n)=f(n)+af\left(\frac{n}{b}\right)+a^2f\left(\frac{n}{b^2}\right)+a^3f\left(\frac{n}{b^3}\right)+\ldots\ldots\ldots\ldots+a^Lf\left(\frac{n}{b^L}\right)$$

$f(n)=O(n^{\log_b a-\varepsilon}) \rightarrow f(n) < c^* \, n^{\log_b a-\varepsilon}$

$$T(n) \leq c\, n^{\log_b a-\varepsilon} + a.c\left(\frac{n}{b}\right)^{\log_b a-\varepsilon} + a^2.c\left(\frac{n}{b^2}\right)^{\log_b a-\varepsilon} + \ldots\ldots + a^{L-1}.c\left(\frac{n}{b^{L-1}}\right)^{\log_b a-\varepsilon} + a^L f(1)$$

$$T(n) \leq cn^{\log_b a-\epsilon}\left[1+\frac{a}{b^{\log_b a-\epsilon}}+\frac{a^2}{(b^2)^{\log_b a-\epsilon}}+\frac{a^{L-1}}{(b^{L-1})^{\log_b a-\epsilon}}\right]+n^{\log_b a}\,c$$

$$T(n) \leq cn^{\log_b a-\epsilon}\left[1+b^{\epsilon}+b^{2\epsilon}+\cdots+b^{\epsilon(L-1)}\right]+n^{\log_b a}\,c$$

$$T(n) \leq cn^{\log_b a-\epsilon}\boxed{\left[\frac{b^{\epsilon L}-1}{b^{\epsilon}-1}\right]}+n^{\log_b a}\,c$$

c_1

$$T(n) \leq c'n^{\log_b a-\epsilon}[n^{\epsilon}-1]+n^{\log_b a}c = O(n^{\log_b a})$$

$c' n^{\log_b a-\epsilon+\epsilon}$

$c' n^{\log_b a-\epsilon} + n^{\log_b a} c$

$\boxed{c'' n^{\log_b a}} - \cancel{c' n^{\log_b a-\epsilon}}$

$\boxed{c'' n^{\log_b a}}$

Master Theorem

CASE 1:

$f(n)=O(n^{\log_b a-\varepsilon})$, $\varepsilon>0$ then, $T(n)=\theta(n^{\log_b a})$

$$T(n)=f(n)+af(\frac{n}{b})+a^2f(\frac{n}{b^2})+a^3f(\frac{n}{b^3})+\ldots\ldots\ldots\ldots+a^Lf(\frac{n}{b^L})$$

$f(n)=O(n^{\log_b a-\varepsilon}) \rightarrow f(n) < c^* n^{\log_b a-\varepsilon}$

$$T(n)\le c\, n^{\log_b a-\varepsilon}+a.c\,(\frac{n}{b})^{\log_b a-\varepsilon}+a^2.c\,(\frac{n}{b^2})^{\log_b a-\varepsilon}+\ldots\ldots+a^{L-1}.c\,(\frac{n}{b^{L-1}})^{\log_b a-\varepsilon}+a^Lf(1)$$

$$T(n)\le cn^{\log_b a-\epsilon}\left[1+\frac{a}{b^{\log_b a-\epsilon}}+\frac{a^2}{(b^2)^{\log_b a-\epsilon}}+\frac{a^{L-1}}{(b^{L-1})^{\log_b a-\epsilon}}\right]+n^{\log_b a}c$$

$$T(n)\le cn^{\log_b a-\epsilon}\left[1+b^{\epsilon}+b^{2\epsilon}+\cdots+b^{\epsilon(L-1)}\right]+n^{\log_b a}c$$

$$T(n)\le cn^{\log_b a-\epsilon}\left[\frac{b^{\epsilon L}-1}{b^{\epsilon}-1}\right]+n^{\log_b a}c$$

$$T(n)\le c'n^{\log_b a-\epsilon}\left[n^{\epsilon}-1\right]+n^{\log_b a}c=O(n^{\log_b a})$$

Master Theorem

$1+b^{\varepsilon}+b^{2\varepsilon}+b^{3\varepsilon}+\ldots\ldots+b^{(L-1)\varepsilon}=\frac{b^{\varepsilon L}-1}{b^{\varepsilon}-1} = \frac{n^{\epsilon}-1}{b^{\epsilon}-1}$

(1) $\frac{a^i}{(b^i)^{\log_b(a)-\epsilon}} = \frac{a^i}{(b^{\log_b(a)-\epsilon})^i} = \frac{a^i}{\frac{(b^{\log_b a})^i}{(b^{\epsilon})^i}} = \frac{a^i}{\frac{a^i}{b^{\epsilon i}}} = b^{\epsilon i}$

(2) $a^L = a^{\log_b n} = (n^{\log_n})^{a \log_b n} = n^{\log_b a}$, using $\log_n a = \frac{\log_b a}{\log_b n} \Rightarrow \log_b a = \log_n a \log_b n$

(3) $b^{\epsilon L} = (b^{\log_b n})^{\epsilon} = n^{\epsilon}$

Master Theorem

$$T(n)=f(n)+af(\frac{n}{b})+ a^2f(\frac{n}{b^2}) + a^3f(\frac{n}{b^3})+\ldots\ldots\ldots\ldots + a^Lf(\frac{n}{b^L})$$

> 1

CASE 1:
$f(n)=O(n^{\log_b a-\varepsilon})$, $\varepsilon>0$ then, $T(n)=\theta(n^{\log_b a})$

CASE 1: lower bound

$T(n) \geq a^L = n^{\log_b a}$
$T(n) = \Omega(n^{\log_b a})$

Master Theorem

$$T(n)=f(n)+af(\frac{n}{b})+ a^2f(\frac{n}{b^2}) + a^3f(\frac{n}{b^3})+\ldots\ldots\ldots\ldots + a^Lf(\frac{n}{b^L})$$

Case 2:

$f(n)=\theta(n^{\log_b a})$ then, $T(n)=\theta(n^{\log_b a} \log n)$

MergeSort(A,start,end)

$T(n) = 2*T(n/2)+n$

1	**If start < end**	
2	**q ← ⌊ start + end ⌋ /2**	1
3	Mergesort(A, start, q) Mergesort(A,q+1,end)	2* T(n/2)
4	**Merge(A,start,q,end)**	n
5	**Else** *base case*	

MergeSort with Master Theorem

$a = 2$

$b = 2$

$\boxed{f(n) = n}$

$T(n)= 2*T(n/2)+n$

Case 2:

$f(n)=\theta(n^{\log_b a})$ then, $T(n)=\theta(n^{\log_b a} \log n)$

$a=2$ $b=2$

$n^{\log_b a} = n^{\log_{\cancel{1}\,2} \cancel{1}\,2} = n = f(n)$

$f(n) = \theta(n^{\log_b a})$

So, $T(n)=\theta(n^{\log_1 1} \log n) =\theta(n\log n)$

$T(n) = \theta(n^{\log_{\cancel{2}} \cancel{2}} \log n)$

$= \theta(n \log n)$

Master Theorem

$(b^i)^{\log_b a} = (b^{\log_b a})^i = a^i$

CASE 2:

$\boxed{f(n)=\theta(n^{\log_b a})}$ then, $T(n)=\theta(n^{\log_b a}\log n)$

CASE 2: Upper bound

$$T(n)=f(n)+af(\frac{n}{b})+ a^2f(\frac{n}{b^2}) + a^3f(\frac{n}{b^3})+\ldots\ldots\ldots\ldots + a^Lf(\frac{n}{b^L})$$

$f(n)=\theta(n^{\log_b a}) \rightarrow f(n) < c^* n^{\log_b a}$

$$T(n) \leq c\, n^{\log_b a} + a.c\, (\frac{n}{b})^{\log_b a} + a^2.c\, (\frac{n}{b^2})^{\log_b a} + \ldots\ldots + a^{L-1}.c\, (\frac{n}{b^{L-1}})^{\log_b a} + a^L.c\, (\frac{n}{b^L})^{\log_b a}$$

$$\leq cn^{\log_b(a)}\left[1+\frac{a}{b^{\log_b(a)}}+\frac{a^2}{(b^2)^{\log_b(a)}}+\cdots+\frac{a^{L-1}}{(b^{L-1})^{\log_b(a)}}\right] + a^L.c\, (\frac{n}{b^L})^{\log_b a}$$

$$\leq c\, n^{\log_b a}\, [\underbrace{1+\frac{a}{a}+\frac{a^2}{a^2}+\frac{a^3}{a^3}+\ldots\ldots+\frac{a^{L-1}}{a^{L-1}}}_{1\ 2\ 3\ \ldots\ L-1}] + a^L.c\, (\frac{n}{b^L})^{\log_b a}$$

$$\leq c\, n^{\log_b a}\, [1+1+\ldots\ldots+1] + + a^L.c\, (\frac{n}{b^L})^{\log_b a}$$

$\log n$

$\boxed{L-1 \sim \log_b n}$

$$\leq c\, n^{\log_b a} \log_b n + \ldots ? \ldots \quad = O(n^{\log_b a} \log_b n)$$

Master Theorem

$$T(n)=f(n)+af(\frac{n}{b})+a^2f(\frac{n}{b^2})+a^3f(\frac{n}{b^3})+\ldots\ldots\ldots\ldots+a^Lf(\frac{n}{b^L})$$

$f(n)=\theta(n^{\log_b a}) \rightarrow f(n) \geq c^* n^{\log_b a}$

CASE 2:

$f(n)=\theta(n^{\log_b a})$ then, $T(n)=\theta(n^{\log_b a}\log n)$

CASE 2: Lower bound

$$T(n) \geq c\, n^{\log_b a} + a.c\,(\frac{n}{b})^{\log_b a} + a^2.c\,(\frac{n}{b^2})^{\log_b a} + \ldots\ldots + a^{L-1}.c\,(\frac{n}{b^{L-1}})^{\log_b a} + \cancel{a^L.c\,(\frac{n}{b^L})^{\log_b a}}$$

$$T(n) \geq cn^{\log_b(a)}\left[1+\frac{a}{b^{\log_b(a)}}+\frac{a^2}{(b^2)^{\log_b(a)}}+\cdots+\frac{a^{L-1}}{(b^{L-1})^{\log_b(a)}}\right]$$

$$\geq c\, n^{\log_b(a)}\left[1+\frac{a}{a}+\frac{a^2}{a^2}+\frac{a^3}{a^3}+\ldots\ldots+\frac{a^{L-1}}{a^{L-1}}\right]$$

$$\geq c\, n^{\log_b(a)}\,\underbrace{[1+1+\ldots\ldots+1]}_{L=\log_b n}$$

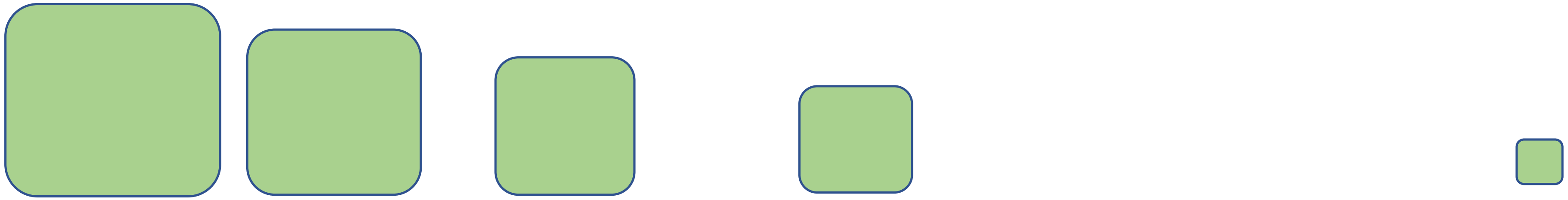
$$\geq c\, n^{\log_b(a)}\log_b n = \Omega(n^{\log_b a}\log_b n)$$

Master Theorem

$$T(n)=f(n)+af(\frac{n}{b})+ a^2f(\frac{n}{b^2}) + a^3f(\frac{n}{b^3})+\ldots\ldots\ldots\ldots + a^Lf(\frac{n}{b^L})$$

Case 2: 

$f(n)=\theta(n^{\log_b a})$ then, $T(n)=\theta(n^{\log_b a} \log n)$

Case 3: 

$f(n)=\Omega(n^{\log_b a+\varepsilon})$, $\varepsilon>0$, and $c<1$ s.t **$a\, f(n/b) < c\, f(n)$**

then, $T(n)=\theta(f(n))$

Master Theorem

$T(n)=f(n)+af(\frac{n}{b})+a^2f(\frac{n}{b^2})+a^3f(\frac{n}{b^3})+\ldots\ldots\ldots\ldots+a^Lf(\frac{n}{b^L})$

CASE 3 $f(n)=\Omega(n^{\log_b a+\varepsilon})$, $\varepsilon>0 \rightarrow f(n) > n^{\log_b a+\varepsilon}$ and **c<1 s.t** $\boxed{\mathbf{a\,f(n/b) < c\,f(n)}}$

$af(\frac{n}{b}) < c.f(n)$

$a^2f(\frac{n}{b^2}) \leq a.a.f(\frac{n/b}{b}) < a.c.f(\frac{n}{b}) < c.a.f(\frac{n}{b}) < c^2f(n)$

$a^3f(\frac{n}{b^3}) < c^3f(n)$

$a^if(\frac{n}{b^i}) < c^if(n)$

Master Theorem

$T(n)=f(n)+af(\frac{n}{b})+ a^2f(\frac{n}{b^2}) + a^3f(\frac{n}{b^3})+\ldots\ldots\ldots\ldots + a^Lf(\frac{n}{b^L})$

CASE 3 $f(n)=\Omega(n^{\log_b a+\varepsilon})$, $\varepsilon>0 \rightarrow f(n) > n^{\log_b a+\varepsilon}$ and **$c<1$ s.t $a\, f(n/b) < c\, f(n)$**

$$T(n) \le f(n)+ cf(n)+ c^2f(n) + c^3f(n)+ \ldots.. + c^Lf(n)$$
$$\le f(n)[1+c+c^2+\ldots\ldots.+ c^L]$$

$T(n) = O(f(n))$ because $1+c+c^2+\ldots.$ Is a constant

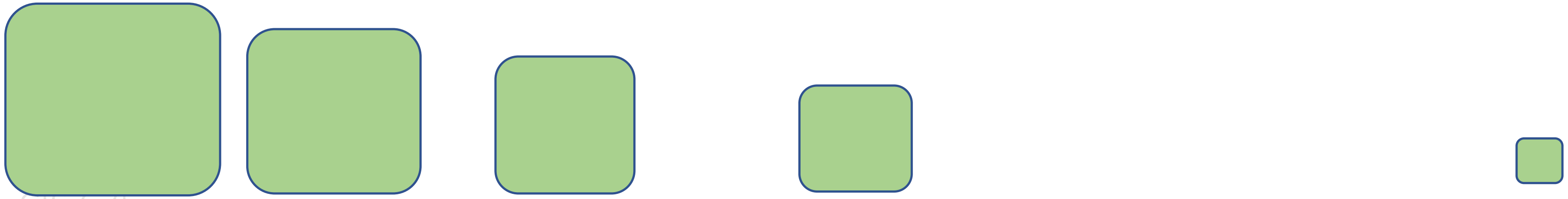
$af(\frac{n}{b}) < c.f(n)$

$a^2f(\frac{n}{b^2}) < a.a.f(\frac{n/b}{b}) < a.c.f(\frac{n}{b}) < c.a.f(\frac{n}{b}) < c^2f(n)$

$a^2f(\frac{n}{b^2}) < c^3f(n)$

So, why $c<1$?

Case 3:



Master Theorem

$T(n)=2T(\frac{n}{2})+n^2$

$f(n)=\Omega(n^{\log_b a+\varepsilon})$, $\varepsilon>0$, and $c<1$ s.t **$a\ f(n/b) < c\ f(n)$**
then, $T(n)=\theta(f(n))$

In this case, **$n^{\log_b a}=n$**

And $f(n)=n^2$, $f(n)=n^2> n^{\log_b a}$

$c<1$ s.t $a\ f(n/b) < c\ f(n)$ and all n sufficiently large

So, $T(n)= \theta(f(n))= \theta(n^2)$

Question