

Arbitrage

Bitcoin Charts


Linear Scale Log Scale

Zoom 1d 7d 1m 3m 1y YTD ALL

From Apr 27, 2017 To Sep 18, 2019



Bitcoin Charts

Linear Scale Log Scale  

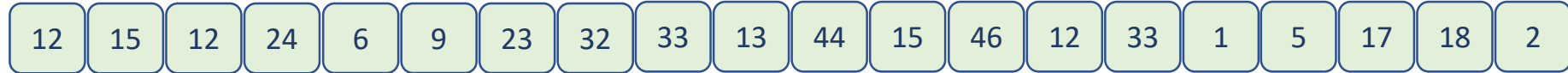
Zoom 1d 7d 1m 3m 1y YTD ALL

From Apr 27, 2017 To Sep 18, 2019



Arbitrage

Input: array of n numbers



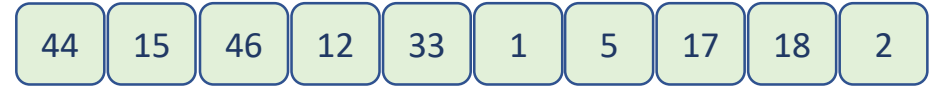
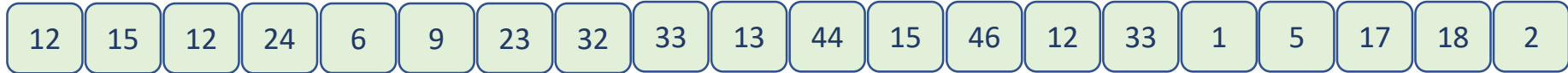
Goal: to find index i, j , s.t., $i < j$ which Maximize $A[j] - A[i]$

$\theta(n \log n)$

Arbitrage

Input: array of n numbers

Goal: to find index i, j , s.t., $i < j$ which Maximize $A[j] - A[i]$



Arbit(A[1.....n])

Base case if $|A| \leq 2$: return

LG=arbit(left(A))

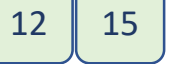
RG=arbit(right(A))

Minlg=min(left(A))

Maxrg=max(right(A))

Return max{Maxrg-Minlg, LG, RG}

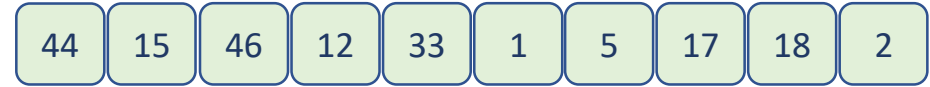
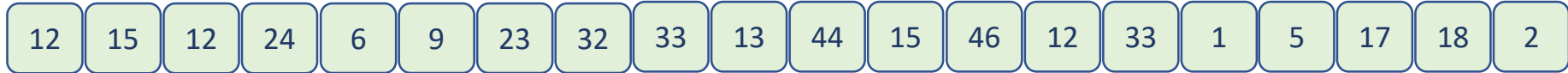
Example base case:



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Arbit($A[1.....n]$)

Base case if $|A| \leq 2$: return

LG=arbit(left(A))

RG=arbit(right(A))

Minlg=min(left(A))

Maxrg=max(right(A))

Return max{Maxrg-Minlg, LG, RG}

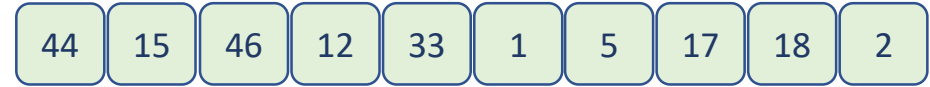
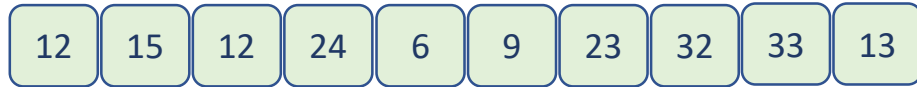
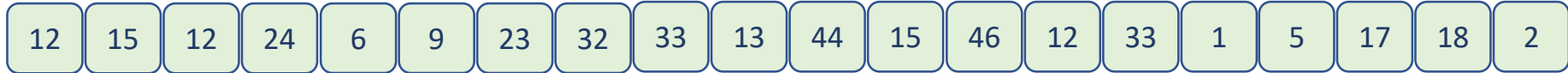
$T(n) = 2T(n/2) + \theta(n) = \theta(n \log n)$

Better approach?

Arbitrage

Input: array of n numbers

Goal: to find index i, j , s.t., $i < j$ which Maximize $A[j] - A[i]$



Arbit(A[1.....n])

Base case if $|A| \leq 2$: return

LG=arbit(left(A)) $T(n/2)$

RG=arbit(right(A)) $T(n/2)$

Minlg=min(left(A)) $\Theta(n)$

Maxrg=max(right(A)) $\Theta(n)$

Return max{Maxrg-Minlg, LG, RG}

$$T(n) = 2T(n/2) + \theta(n) = \theta(n \log n)$$

Big idea?

We can pass information about min/max through the return values!

Arbitrage

Input: array of n numbers

Goal: to find index i, j , s.t., $i < j$ which Maximize $A[j] - A[i]$

Arbit(A[1.....n])

Base case if $|A| \leq 2$: return

LG, MinL, MaxL=arbit(left(A))

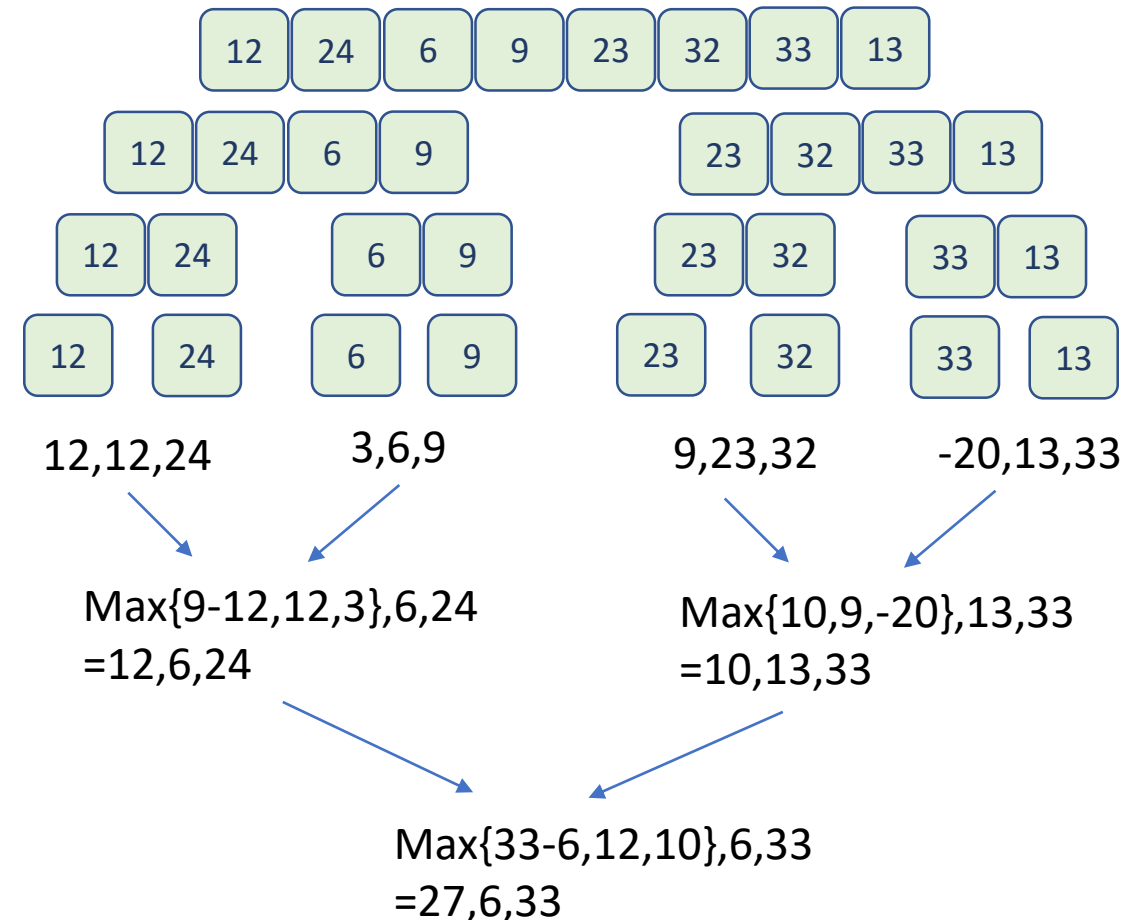
RG, MinR, MaxR=arbit(right(A))

Return $\max\{\text{MaxR} - \text{MinL}, \text{LG}, \text{RG}\}$,

$\min\{\text{MinL}, \text{MinR}\}$,

$\max\{\text{MaxL}, \text{MaxR}\}$

$T(n) = 2T(n/2) + \theta(1) = \theta(n)$



$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 5 + 14 & 6 + 16 \\ 15 + 28 & 18 + 32 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

$$\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \cdots & a_{1,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{2,1} & a_{2,2} & a_{2,3} & \cdots & a_{2,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & a_{n,3} & \cdots & a_{n,n} \end{bmatrix} \times \begin{bmatrix} b_{1,1} & b_{1,2} & b_{1,3} & \cdots & b_{1,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{2,1} & b_{2,2} & b_{2,3} & \cdots & b_{2,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{n,1} & b_{n,2} & b_{n,3} & \cdots & b_{n,n} \end{bmatrix} = \begin{bmatrix} c_{1,1} & c_{1,2} & c_{1,3} & \cdots & c_{1,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{2,1} & c_{2,2} & c_{2,3} & \cdots & c_{2,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{n,1} & c_{n,2} & c_{n,3} & \cdots & c_{n,n} \end{bmatrix}$$

Matrix has $n \times m$ terms, or n^2 terms

$$c_{i,j} = \sum_{k=1}^n a_{i,k} \times b_{k,j} \quad \Theta(n) \quad \Theta(n^2)$$

$$\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \cdots & a_{1,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{\frac{n}{2},1} & a_{\frac{n}{2},2} & a_{\frac{n}{2},3} & \cdots & a_{\frac{n}{2},n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & a_{n,3} & \cdots & a_{n,n} \end{bmatrix} \times \begin{bmatrix} b_{1,1} & b_{1,2} & b_{1,3} & \cdots & b_{1,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{\frac{n}{2},1} & b_{\frac{n}{2},2} & b_{\frac{n}{2},3} & \cdots & b_{\frac{n}{2},n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{n,1} & b_{n,2} & b_{n,3} & \cdots & b_{n,n} \end{bmatrix} = \begin{bmatrix} c_{1,1} & c_{1,2} & c_{1,3} & \cdots & c_{1,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{\frac{n}{2},1} & c_{\frac{n}{2},2} & c_{\frac{n}{2},3} & \cdots & c_{\frac{n}{2},n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{n,1} & c_{n,2} & c_{n,3} & \cdots & c_{n,n} \end{bmatrix}$$

Divide each matrix into 4 matrices that are $\frac{n}{2} \times \frac{n}{2}$ *in size*

$$\begin{bmatrix}
 a_{1,1} & a_{1,2} & a_{1,3} & \cdots & a_{1,n} \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
 a_{\frac{n}{2},1} & a_{\frac{n}{2},2} & a_{\frac{n}{2},3} & \cdots & a_{\frac{n}{2},n} \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
 a_{n,1} & a_{n,2} & a_{n,3} & \cdots & a_{n,n}
 \end{bmatrix}
 \times
 \begin{bmatrix}
 b_{1,1} & b_{1,2} & b_{1,3} & \cdots & b_{1,n} \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
 b_{\frac{n}{2},1} & b_{\frac{n}{2},2} & b_{\frac{n}{2},3} & \cdots & b_{\frac{n}{2},n} \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
 b_{n,1} & b_{n,2} & b_{n,3} & \cdots & b_{n,n}
 \end{bmatrix}$$

The first matrix is partitioned into four submatrices: A (top-left), B (top-right), C (bottom-left), and D (bottom-right). The second matrix is partitioned into four submatrices: E (top-left), F (top-right), G (bottom-left), and H (bottom-right).

Divide each matrix into 4 matrices that are $\frac{n}{2} \times \frac{n}{2}$ in size

Where each of these is an $\frac{n}{2} \times \frac{n}{2}$ matrix

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} E & F \\ G & H \end{bmatrix}$$

$$\begin{bmatrix} A.E + B.G & A.F + B.H \\ C.E + D.G & C.F + D.H \end{bmatrix}$$

$$T(n) = 8T\left(\frac{n}{2}\right) + 4\left(\frac{n}{2}\right)^2$$