

# Measurements of Lambda Cold Dark Matter ( $\Lambda$ CDM) parameters with Markov chain Monte Carlo method using Cosmic Microwave Background Radiation data

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(Dated: April 16, 2022)

The cosmic microwave background (CMB) is an important source of data on the early universe. They can fit to get many important cosmological parameters. In this project, we fit it to the standard model of the Lambda Cold Dark Matter ( $\Lambda$ CDM) to get the basic parameters, especially the density parameters, and the Hubble's constant  $H_0$ . We also apply a constrain on  $H_0$  from the redshift data of type Ia supernovae(SNe Ia). The density parameters from CMB data are  $\Omega_m = 0.3039 \pm 0.0090$ ,  $\Omega_\Lambda = 0.6961 \pm 0.0090$ , and the Hubble's constant  $H_0 = 67.7 \pm 1.1$ . From SNe Ia, we get  $H_0 = 69.97 \pm 0.45$ ,  $\Omega_m = 0.275 \pm 0.071$ ,  $\Omega_\Lambda = 0.71 \pm 0.12$ . The density parameters from the two models agree with each other. While combining  $H_0$  of these two, we get an  $H_0 = 69.4$ .

## I. INTRODUCTION

Lambda Cold Dark Matter ( $\Lambda$ CDM) [1] is currently widely used as a standard model of Big Bang, dark matter, and dark energy cosmology. It attempts to explain the cosmic microwave background radiation(CMB), the large-scale structure of the universe, and supernova observations of the accelerating expansion of the universe.

The simple  $\Lambda$ CDM model is based on six parameters: physical baryon density parameter  $\Omega_b h^2$ ; physical dark matter density parameter  $\Omega_c h^2$ ; the age of the universe  $t_0$ ; scalar spectral index  $n_s$ ; curvature fluctuation amplitude  $A_s$ ; and reionization optical depth  $\tau$ . These parameters are generally not computed by existing theories, but are usually obtained through data fitting, like the cosmic microwave background (CMB) map. Since now, data from the Planck spacecraft[2] is the most accurate source for some of the critical parameters.

With known  $t_0 = 13.799 \times 10^9 \text{ years}$  in model, we can fit the data using model from PICO [3] with Markov chain Monte Carlo (MCMC).[4] With these parameters, the Hubble constant  $H_0$  and the dark energy density  $\Omega_m$  can be readily calculated. These two parameters can be checked for consistency with the redshift relation for supernovae. Here we fit the distance modulus data of Type Ia Supernovae(SNe Ia) [5] with model  $\Lambda$ CDM from `astropy` still with MCMC.

### A. Theoretical Background

Hubble constant  $H_0$  is the constant of proportionality in the relation between the velocities of remote galaxies and their distances.[6] It expresses the rate at which the universe is expanding. Measurements of  $H_0$  derived from observations of supernovae (SNe) calibrated by the cosmic distance ladder, are currently in tension with the value of  $H_0$  inferred from Planck satellite observations of anisotropies in the CMB assuming a flat  $\Lambda$ CDM cosmological model.

In the case of the CMB, for instance, the calibration of  $H_0$  is derived directly from the measurement of the angular scale subtended by rs. CMB power spectrum measurements constrain the physical energy densities of baryons  $\Omega_b h^2$  and matter  $\Omega_c h^2$ , where  $h = \frac{H_0}{100 \text{ Km/s/Mpc}}$ . Then we can infer the dark energy density:

$$\Omega_m = \Omega_b + \Omega_c = \frac{\Omega_b h^2 + \Omega_c h^2}{\left(\frac{H_0}{100 \text{ Km/s/Mpc}}\right)^2}. \quad (1)$$

From the definitions of the cosmological parameters:

$$1 = \Omega_m + \Omega_\Lambda + \Omega_k, \quad (2)$$

Where  $\Omega_\Lambda$  is the Dark energy density parameter,  $\Omega_k$  is the curvature density. These density parameters are the ratios of the average density of matter and energy in the universe to the critical density. In the model of CMB, we are mainly assuming a flat universe, which means that this universe has no spatial curvature. thus we set  $\Omega_k = 0$ . In this case, we can obtain  $\Omega_\Lambda$  from

$$\Omega_\Lambda = 1 - \Omega_m. \quad (3)$$

To give further constraints on these parameters, we can fit the luminosity distance  $D_L$  of supernovae with redshift  $z$ . We are using specifically the type Ia supernovae, as they are standardizable candles.

## II. TECHNICAL APPROACH

To generate the posterior function for the MCMC, We set a uniform prior to limit the scope of our parameters. We modify the scope according to the result of the corner plot we generate with the MCMC simulation data. All data outside the scope is set to 0, or say  $-\infty$  by the log form.

The final parameter range we use for CMB is:

$$\begin{aligned} 1e - 9 &< A_s < 5e - 9, \\ 0.5 &< n_s < 1.5, \\ -0.1 &< \Omega_b h^2 < .2, \\ 0 &< \Omega_c h^2 < 0.5, \\ -0.5 &< \tau < 0.5, \\ 0 &< H_0 < 150. \end{aligned}$$

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The final parameter range we use for CMB is:

$$\begin{aligned} 50 < H_0 < 100, \\ 0.0 < \Omega_m < 0.6, \\ 0.1 < \Omega_\Lambda < 1.2. \end{aligned}$$

We use the likelihood function:

$$p(data|theta) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2} \frac{(D_{L,data} - D_{L,model})^2}{\sigma^2}\right).$$

Then add up the log function above to get the posterior. With the posterior function, we can run MCMC on some randomly generated Gaussian distributed data using the `emcee` package to get the sample.[7] The mean and standard deviation of the sample were calculated as our parameter result.

### III. RESULT

#### A. Parameters from CMB

The raw data of the CMB map from Planck gives data from multipole moment  $L$  from 0 to 2508. We ran it with MCMC for 4500 iterations with 30 runners. The distribution of the sample we got is in Fig. 1.

We discard the first 1000 sample data and thin it by 15 and obtain the flat sample data. We worked out its means and standard deviations as our parameters and their errors, listed above:

$$\begin{aligned} A_s &= (2.19 \pm 0.12) \times 10^{-9}, \\ N_s &= 0.9713 \pm 0.0059, \\ \tau &= 0.079 \pm 0.029, \\ \Omega_b h^2 &= 0.02230 \pm 0.00023, \\ \Omega_c h^2 &= 0.1170 \pm 0.0024, \\ H_0 &= 67.7 \pm 1.1. \end{aligned}$$

The plot of the model with these parameters is like Fig. 2. Then we can use eqn. 1 to get

$$\Omega_m = 0.3039 \pm 0.0090.$$

Plug into eqn. 3, we can get

$$\Omega_\Lambda = 0.6961 \pm 0.0090.$$

#### B. Parameters from SNe Ia

The data we use mainly include redshift  $z$  from  $10^{-2}$  to 1.5. We applied precisely the same approaches as CMB data to do the MCMC and flat the result sample. The distribution of samples we get from MCMC is in Fig. 3.

The results parameters we get are:

$$\begin{aligned} H_0 &= 69.97 \pm 0.45, \\ \Omega_m &= 0.275 \pm 0.071, \\ \Omega_\Lambda &= 0.71 \pm 0.12. \end{aligned}$$

By comparison, we can conclude that the error ranges of  $H_0$  from SNe and CMB do not coincide, but the error range of  $\Omega_m$  and  $\Omega_\Lambda$  overlap. The  $H_0$ 's that didn't overlap may be due to the overfitting of SNe data, thus too narrow error bar. The result from SNe is supposed to have a larger error bar than CMB.

Also the  $\Omega_k$  might not be 0 here, as we didn't use the `FlatLambdaCDM` model here. According to the eqn. 2,  $\Omega_k = 0.015 \pm 0.191$ , which is very approaching 0. So we consider it reasonable for the flat  $\Lambda$ CDM model. To improve, `FlatLambdaCDM` can be used for further experiments.

The plot of the model with these parameters is like Fig. 4.

#### C. The accuracy level of $H_0$ and data size

In order for people to get their desired percentage error level of  $H_0$  from the CMB, we did MCMC for different data volumes to get the following Fig. 5. It shows how accurate  $H_0$  can be obtained with the first few lines of the data if people use the same method as us. We observe that the percentage error decrease to 5% at about the first 1000 rows of data. For data less than 500 rows (first 20% of the whole data), the error rapidly increases. This might be due to the lack of peaks; as we can see from the fig. 2, there is only one peak before  $L < 500$ .

#### D. Combination of $H_0$ from two datasets

In order to get  $H_0$  after combining two sets of data, we plot the  $P(H_0|data)$  from both to the same axis in Fig. 6. The combined posterior of  $H_0$  is:

$$p(H_0|twodatasets) \propto p(H_0|CMB) \times p(H_0|SNe). \quad (4)$$

This function reaches its highest point at  $H_0 = 69.4$ .

### IV. CONCLUSION

In this project, we fit the data of the cosmic microwave background radiation (CMB) from Planck spacecraft with the model of `pico`, and the data of Type Ia Supernovae (SNe Ia) with model `LambdaCDM` from `astropy`. Both using the method of Markov chain Monte Carlo (MCMC). We aim to study the basic parameters of Lambda Cold Dark Matter ( $\Lambda$ CDM) and the Hubble's constant  $H_0$ . From the CMB model, we obtain  $A_s = (2.19 \pm 0.12) \times 10^{-9}$ ,  $N_s = 0.9713 \pm 0.0059$ ,  $\tau = 0.079 \pm 0.029$ ,  $\Omega_b h^2 = 0.02230 \pm 0.00023$ ,  $\Omega_c h^2 = 0.1170 \pm 0.0024$ ,  $H_0 = 67.7 \pm 1.1$ . From the SNe we got  $H_0 = 69.97 \pm 0.45$ ,  $\Omega_m = 0.275 \pm 0.071$ ,  $\Omega_\Lambda = 0.71 \pm 0.12$ . All the results are considered reasonable compare to the official data, except the first sigma ranges of  $H_0$  didn't overlap. But we combine the two  $H_0$ 's posterior together

and get a final  $H_0 = 69.4$ . One thing in our project

that we can improve next time is using `FlatLambdaCDM` instead of `LambdaCDM` as the model of SNe data.

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- [1] A. Maeder, An alternative to the `lambdacdm` model: The case of scale invariance, *The Astrophysical Journal* **834**, 194 (2017).
  - [2] Planck Collaboration, Aghanim, N., Akrami, Y., Ashdown, M., and Aumont, J., Planck 2018 results - v. cmb power spectra and likelihoods, *A&A* **641**, A5 (2020).
  - [3] W. A. Fendt and B. D. Wandelt, Pico: Parameters for the impatient cosmologist, *The Astrophysical Journal* **654**, 2 (2007).
  - [4] J. S. Speagle, P. L. Capak, D. J. Eisenstein, D. C. Masters, and C. L. Steinhardt, Exploring photometric redshifts as an optimization problem: an ensemble MCMC and simulated annealing-driven template-fitting approach, *Monthly Notices of the Royal Astronomical Society* **461**, 3432 (2016).
  - [5] N. Suzuki, D. Rubin, C. Lidman, G. Aldering, R. Amanullah, K. Barbary, L. F. Barrientos, J. Botyanszki, and M. Brodwin, The hubble space telescope cluster supernova survey: V. improving the dark energy constraints above  $z_{cl}$  and building an early-type-hosted supernova sample, *The Astrophysical Journal* **746**, 85 (2012).
  - [6] O. H. Philcox, B. D. Sherwin, G. S. Farren, and E. J. Baxter, Determining the hubble constant without the sound horizon: Measurements from galaxy surveys, *Physical Review D* **103**, 10.1103/physrevd.103.023538 (2021).
  - [7] D. Foreman-Mackey, D. W. Hogg, D. Lang, and J. Goodman, emcee: The mcmc hammer, *Publications of the Astronomical Society of the Pacific* **125**, 306 (2013).

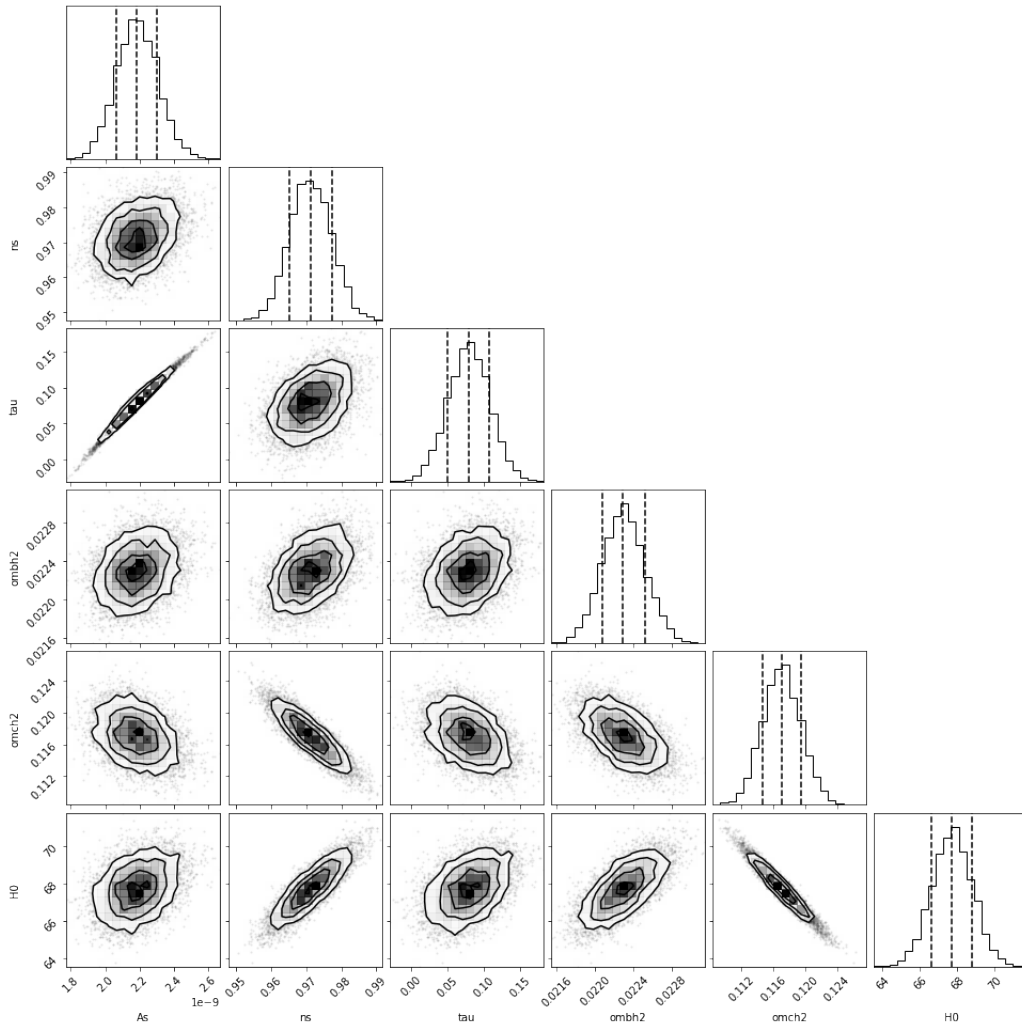


FIG. 1. Corner plot of CMB parameters using `corner` package.

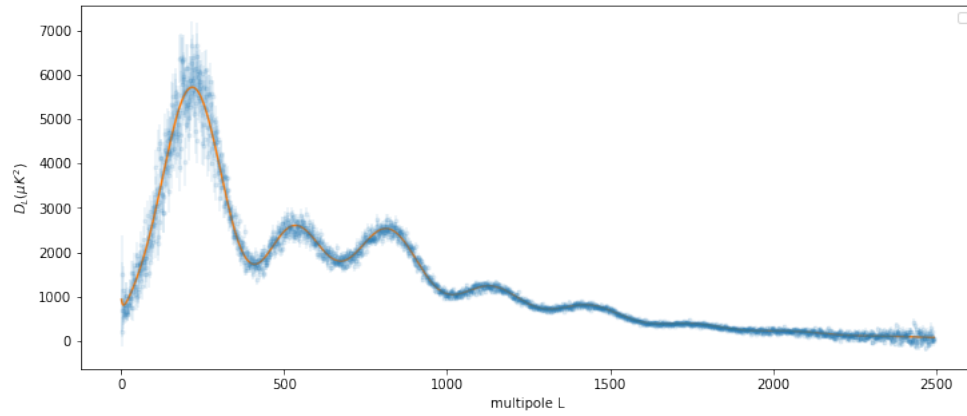


FIG. 2. Plot of Planck CMB data with error bars and our result parameters fitting in the pico model.

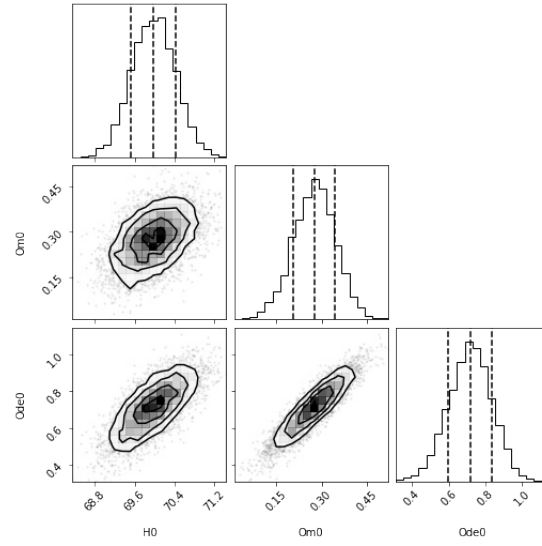


FIG. 3. Corner plot of SNe Ia parameters using `corner` package.

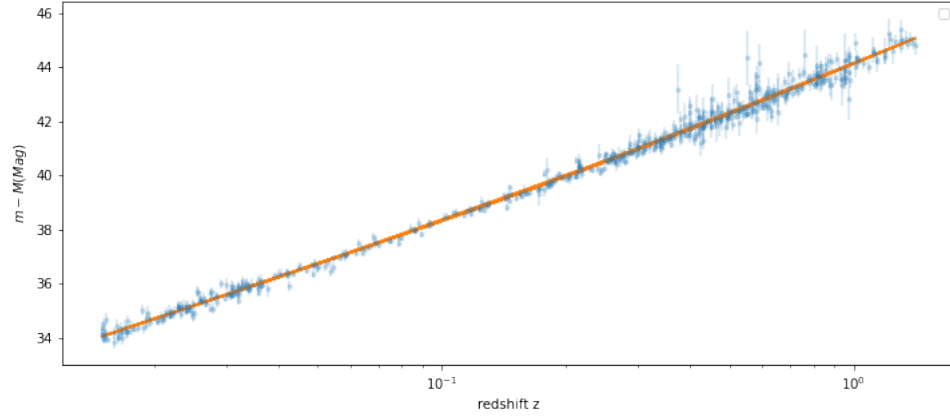


FIG. 4. Plot of SNe Ia data with error bars and our result parameters fitting in the `LambdaCDM` model from `astropy` package.

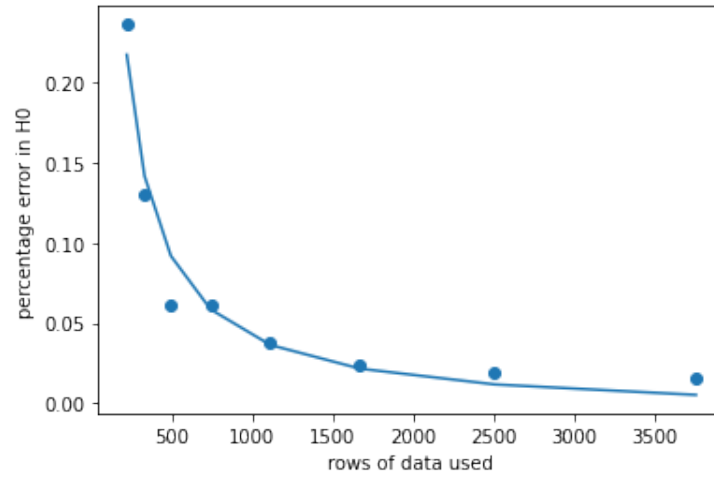


FIG. 5. Plot of level of percentage error vs. number of columns of CMB data used for MCMC.

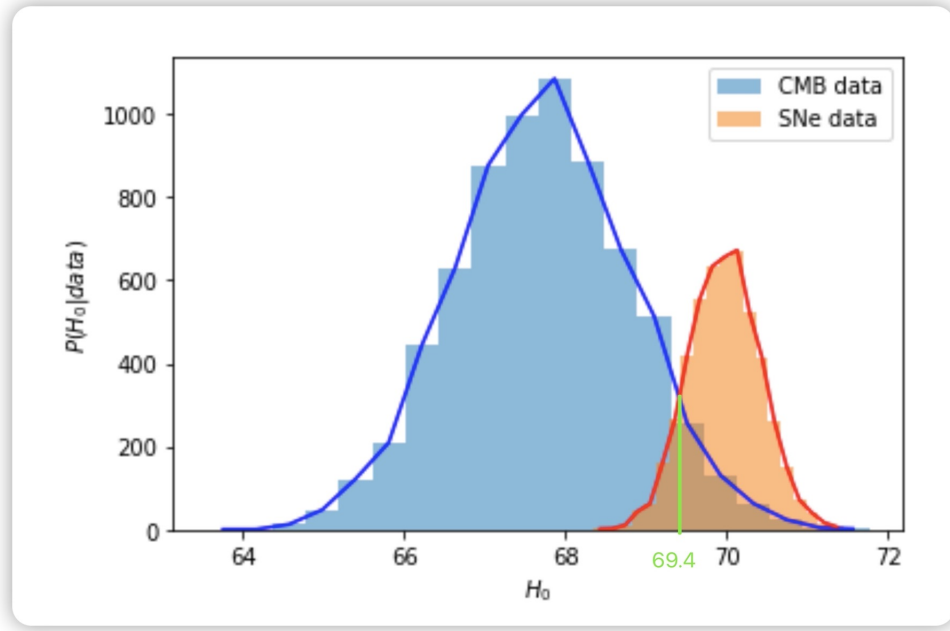


FIG. 6.  $P(H_0|CMB)$  and  $P(H_0|SNe)$  in the same axis.