Measurements of Lambda Cold Dark Matter (ΛCDM) parameters with Markov chain Monte Carlo method using Cosmic Microwave Background Radiation data

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The cosmic microwave background (CMB) is an important source of data on the early universe. They can fit to get many important cosmological parameters. In this project, we fit it to the standard model of the Lambda Cold Dark Matter (ΛCDM) to get the basic parameters, especially the density parameters, and the Hubble's constant H_0. We also apply a constrain on H_0 from the redshift data of type Ia supernovae(SNe Ia). The density parameters from CMB data are $\Omega_m = 0.3039 \pm 0.0090, \Omega_\Lambda = 0.6961 \pm 0.0090$, and the Hubble's constant $H_0 = 67.7 \pm 1.1$. From SNe Ia, we get $H_0 = 69.97 \pm 0.45, \Omega_m = 0.275 \pm 0.071, \Omega_\Lambda = 0.71 \pm 0.12$. The density parameters from the two models agree with each other. While combining H_0 of these two, we get an $H_0 = 69.4$.

I. INTRODUCTION

Lambda Cold Dark Matter (ΛCDM) [1] is currently widely used as a standard model of Big Bang, dark matter, and dark energy cosmology. It attempts to explain the cosmic microwave background radiation(CMB), the large-scale structure of the universe, and supernova observations of the accelerating expansion of the universe.

The simple ΛCDM model is based on six parameters: physical baryon density parameter $\Omega_b h^2$; physical dark matter density parameter $\Omega_c h^2$; the age of the universe t_0; scalar spectral index n_s; curvature fluctuation amplitude A_s; and reionization optical depth τ. These parameters are generally not computed by existing theories, but are usually obtained through data fitting, like the cosmic microwave background (CMB) map. Since now, data from the Planck spacecraft[2] is the most accurate source for some of the critical parameters.

With known $t_0 = 13.799 \times 10^9 years$ in model, we can fit the data using model from PICO [3] with Markov chain Monte Carlo (MCMC).[4] With these parameters, the Hubble constant H_0 and the dark energy density Ω_m can be readily calculated. These two parameters can be checked for consistency with the redshift relation for supernovae. Here we fit the distance modulus data of Type Ia Supernovae(SNe Ia) [5] with model `LambdaCDM` from `astropy` still with MCMC.

A. Theoretical Background

Hubble constant H_0 is the constant of proportionality in the relation between the velocities of remote galaxies and their distances.[6] It expresses the rate at which the universe is expanding. Measurements of H_0 derived from observations of supernovae (SNe) calibrated by the cosmic distance ladder, are currently in tension with the value of H_0 inferred from Planck satellite observations of anisotropies in the CMB assuming a flat ΛCDM cosmological model.

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In the case of the CMB, for instance, the calibration of H_0 is derived directly from the measurement of the angular scale subtended by rs. CMB power spectrum measurements constrain the physical energy densities of baryons $\Omega_b h^2$ and matter $\Omega_c h^2$, where $h = \frac{H_0}{100Km/s/Mpc}$. Then we can infer the dark energy density:

$$\Omega_m = \Omega_b + \Omega_c = \frac{\Omega_b h^2 + \Omega_c h^2}{(\frac{H_0}{100Km/s/Mpc})^2}. \quad (1)$$

From the definitions of the cosmological parameters:

$$1 = \Omega_m + \Omega_\Lambda + \Omega_k, \quad (2)$$

Where Ω_Λ is the Dark energy density parameter, Ω_k is the curvature density. These density parameters are the ratios of the average density of matter and energy in the universe to the critical density. In the model of CMB, we are mainly assuming a flat universe, which means that this universe has no spatial curvature. thus we set $\Omega_k = 0$. In this case, we can obtain Ω_Λ from

$$\Omega_\Lambda = 1 - \Omega_m. \quad (3)$$

To give further constraints on these parameters, we can fit the luminosity distance D_L of supernovae with redshift z. We are using specifically the type Ia supernovae, as they are standardizable candles.

II. TECHNICAL APPROACH

The Markov Chain Mont Carlo (MCMC) algorithm is a useful tool for performing Bayesian inference, especially when we got little knowledge about the prior but had to work with a large amount of data. To generate the posterior function for the MCMC, We set a uniform prior to limit the scope of our parameters. We modify the scope according to the result of the corner plot we generate with the MCMC simulation data. All data outside the scope is set to 0, or say $-\infty$ by the log form.

The final parameter range we use for CMB is:

$$\begin{aligned} 1e-9 < A_s &< 5e-9, \\ 0.5 < n_s &< 1.5, \\ -0.1 < \Omega_b h^2 &< .2, \\ 0 < \Omega_b h^2 &< 0.5, \\ -0.5 < \tau &< 0.5, \\ 0 < H_0 &< 150. \end{aligned}$$

The final parameter range we use for CMB is:

$$\begin{aligned} 50 < H_0 &< 100, \\ 0.0 < \Omega_m &< 0.6, \\ 0.1 < \Omega_\Lambda &< 1.2. \end{aligned}$$

We use the likelihood function:

$$p(data|theta) = \frac{1}{\sigma\sqrt{2\pi}} exp(-\frac{1}{2}\frac{(D_{L,data} - D_{L,model})^2}{\sigma^2}).$$

Then add up the log function above to get the posterior. With the posterior function, we can run MCMC on some randomly generated Gaussian distributed data using the `emcee` package to get the sample.[7] The mean and standard deviation of the sample were calculated as our parameter result.

III. RESULT

A. Parameters from CMB

The raw data of the CMB map from Planck gives data from multipole moment L from 0 to 2508. We ran it with MCMC for 4500 iterations with 30 runners. The distribution of the sample we got is in Fig. 1.

We discard the first 1000 sample data and thin it by 15 and obtain the flat sample data. We worked out its means and standard deviations as our parameters and their errors, listed above:

$$\begin{aligned} A_s &= (2.19 \pm 0.12) \times 10^{-9}, \\ N_s &= 0.9713 \pm 0.0059, \\ \tau &= 0.079 \pm 0.029, \\ \Omega_b h^2 &= 0.02230 \pm 0.00023, \\ \Omega_c h^2 &= 0.1170 \pm 0.0024, \\ H_0 &= 67.7 \pm 1.1. \end{aligned}$$

The plot of the model with these parameters is like Fig. 2. Then we can use eqn. 1 to get

$$\Omega_m = 0.3039 \pm 0.0090.$$

Plug into eqn. 3, we can get

$$\Omega_\Lambda = 0.6961 \pm 0.0090.$$

These two data are considered reasonable comparing to the official parameter from Planck 2015.[2]

B. Parameters from SNe Ia

The data we use mainly include redshift z from 10^{-2} to 1.5. We applied precisely the same approaches as CMB data to do the MCMC and flat the result sample. The distribution of samples we get from MCMC is in Fig. 3.

The results parameters we get are:

$$\begin{aligned} H_0 &= 69.97 \pm 0.45, \\ \Omega_m &= 0.275 \pm 0.071, \\ \Omega_\Lambda &= 0.71 \pm 0.12. \end{aligned}$$

By comparison, we can conclude that the error ranges of H_0 from SNe and CMB do not coincide, but the error range of Ω_m and Ω_Λ overlap. The H_0's that didn't overlap may be due to the overfitting of SNe data, thus too narrow error bar. The result from SNe is supposed to have a larger error bar than CMB.

Also the Ω_k might not be 0 here, as we didn't use the `FlatLambdaCDM` model here. According to the eqn. 2, $\Omega_k = 0.015 \pm 0.191$, which is very approaching 0. So we consider it reasonable for the flat ΛCDM model. To improve, `FlatLambdaCDM` can be used for further experiments.

The plot of the model with these parameters is like Fig. 4.

C. The accuracy level of H_0 and data size

In order for people to get their desired percentage error level of H0 from the CMB, we did MCMC for different data volumes to get the following Fig. 5. It shows how accurate h0 can be obtained with the first few lines of the data if people use the same method as us. We observe that the percentage error decrease to 5% at about the first 1000 rows of data. For data less than 500 rows (first 20% of the whole data), the error rapidly increases. This might be due to the lack of peaks; as we can see from the fig. 2, there is only one peak before $L < 500$.

D. Combination of H_0 from two datasets

In order to get H_0 after combining two sets of data, we plot the $P(H_0|data)$ from both to the same axis in Fig. 6. The combined posterior of H_0 is:

$$p(H_0|twodatasets) \propto p(H_0|CMB) \times p(H_0|SNe). \quad (4)$$

This function reaches its highest point at $H_0 = 69.4$.

IV. CONCLUSION

In this project, we fit the data of the cosmic microwave background radiation (CMB) from Planck spacecraft with the model of `pico`, and the data of Type

Ia Supernovae (SNe Ia) with model `LambdaCDM` from `astropy`, Both using the method of Markov chain Monte Carlo (MCMC). We aim to study the basic parameters of Lambda Cold Dark Matter (ΛCDM) and the Hubble's constant H_0. From the CMB model, we obtain $A_s = (2.19 \pm 0.12) \times 10^{-9}$, $N_s = 0.9713 \pm 0.0059$, $\tau = 0.079 \pm 0.029$, $\Omega_b h^2 = 0.02230 \pm 0.00023$, $\Omega_c h^2 = 0.1170 \pm 0.0024$, $H_0 = 67.7 \pm 1.1$. From the SNe we got $H_0 = 69.97 \pm 0.45$, $\Omega_m = 0.275 \pm 0.071$, $\Omega_\Lambda = 0.71 \pm 0.12$. All the results are considered reasonable compare to the official data, except the first sigma ranges of H_0 didn't overlap. But we combine the two H_0's posterior together and get a fianl $H_0 = 69.4$. One thing in our project that we can improve next time is using `FlatLambdeCDM` instead of `LambdaCDM` as the model of SNe data.

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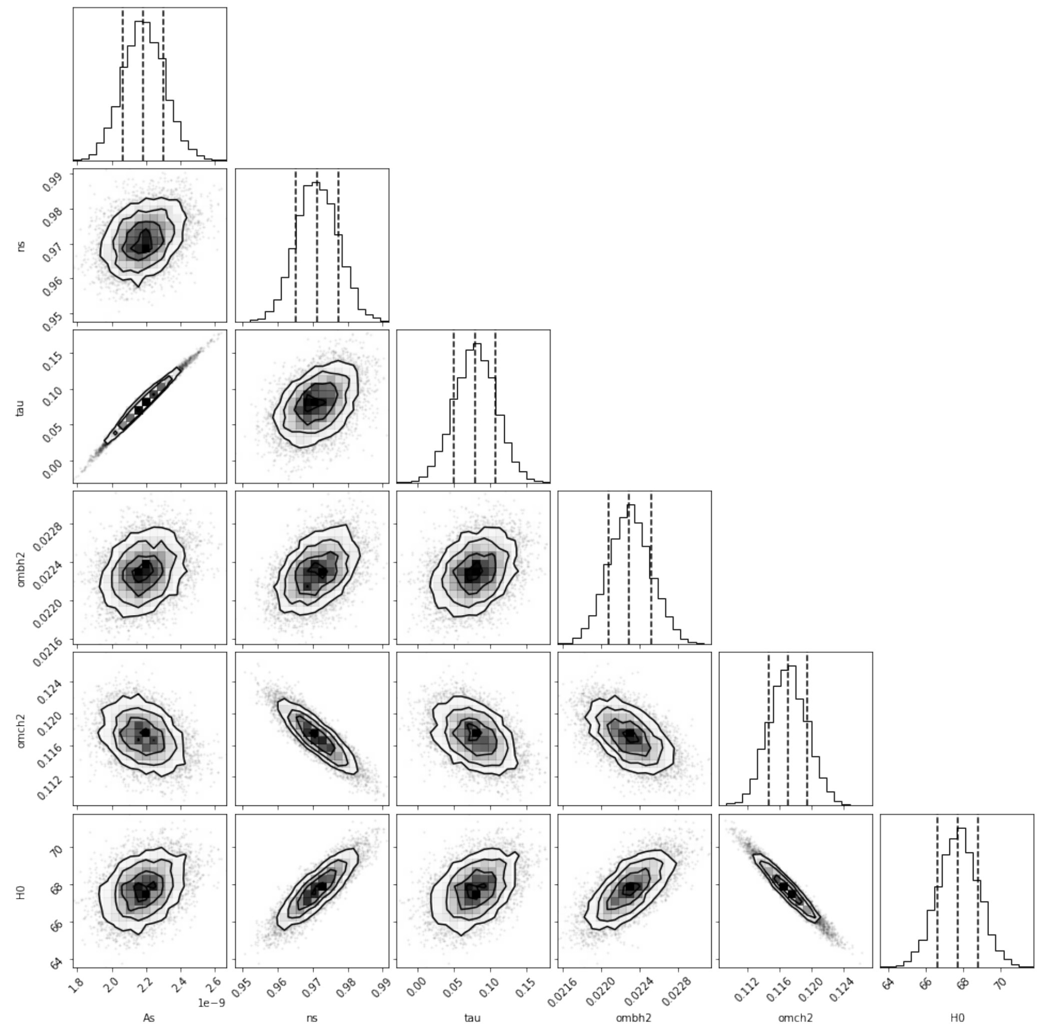

FIG. 1. Corner plot of CMB parameters using `corner` package.

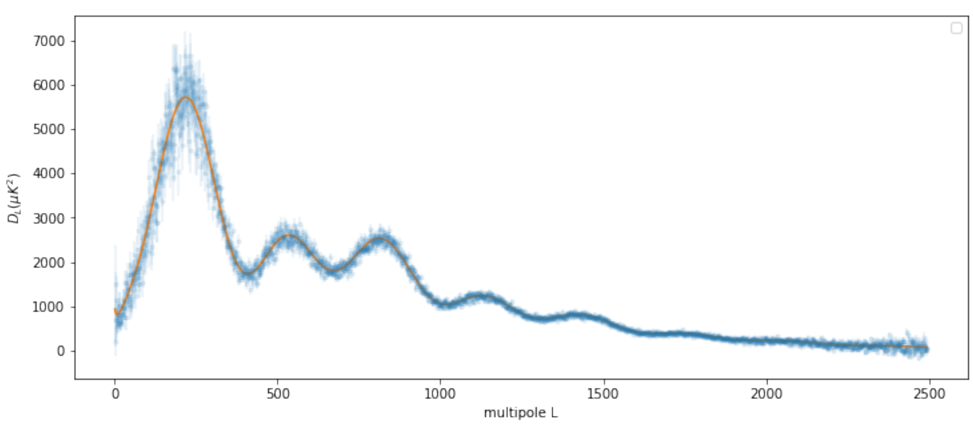

FIG. 2. Plot of Planck CMB data with error bars and our result parameters fitting in the pico model.

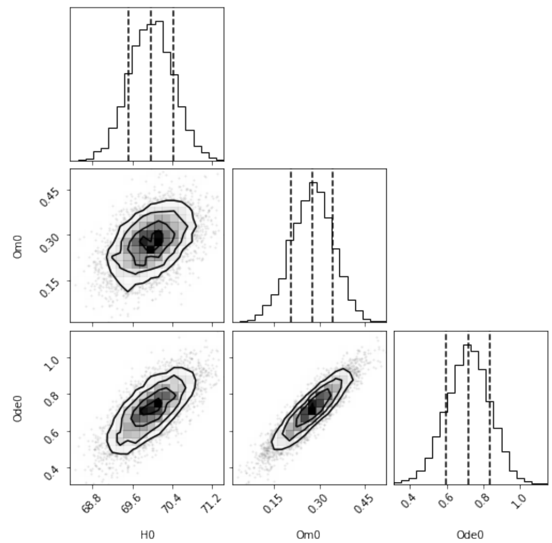

FIG. 3. Corner plot of SNe Ia parameters using `corner` package.

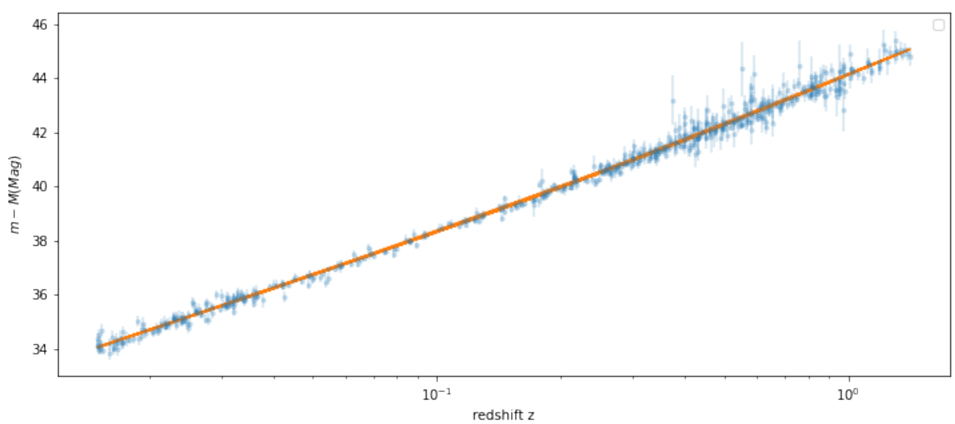

FIG. 4. Plot of SNe Ia data with error bars and our result parameters fitting in the `LambdaCDN` model from `astropy` package.

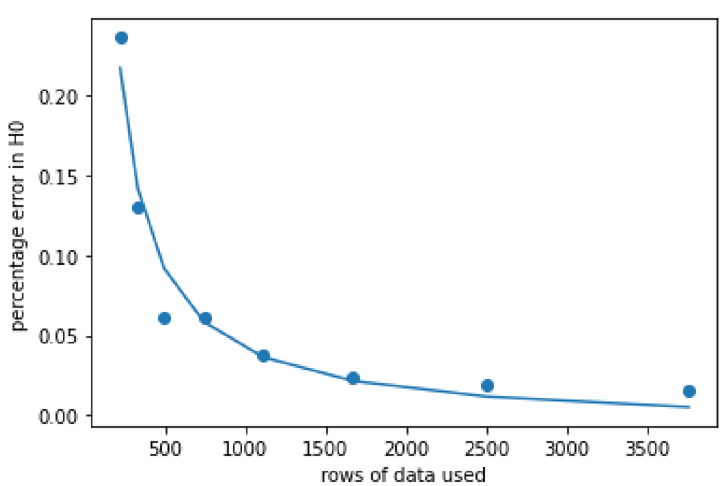

FIG. 5. Plot of level of percentage error vs. number of columns of CMB data used for MCMC.

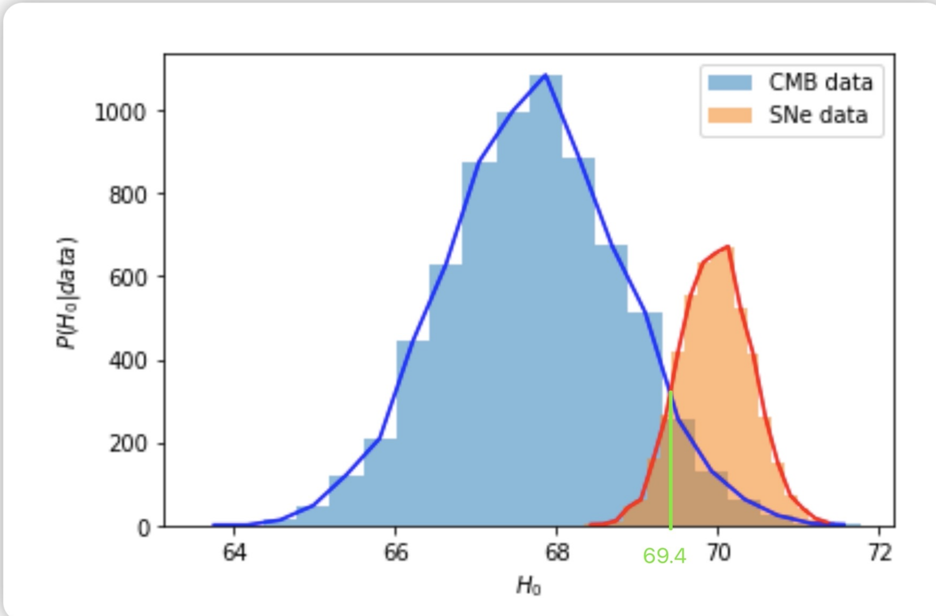

FIG. 6. $P(H_0|CMB)$ and $P(H_0|SNe)$ in the same axis.