Data analysis - gamma distribution

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TASK 1

Generate 3 samples from the Gamma distribution with parameters of your choice (different ones) for k and lambda. Compare the obtained means and variances in the samples with theoretical values. Plot histograms for each sample and compare them with the theoretical density. Also, draw theoretical cumulative distribution functions (preferably all 3 on one plot for comparison).

*Create box plots for the samples (preferably all 3 on one plot).

```
#define parameter values
k_1 <- 4
k_2 <- 6
k_3 <- 7
lambda_1 <- 2
lambda_2 <- 3.3
lambda_3 <- 3.5

#generate 3 random samples from the Gamma distribution
sample_1 <- rgamma(1000, k_1, lambda_1)
sample_2 <- rgamma(1000, k_2, lambda_2)
sample_3 <- rgamma(1000, k_3, lambda_3)

#determine the shape of the curve by calculating mean values
mean_1 <- k_1 / lambda_1
mean_2 <- k_2 / lambda_2
mean_3 <- k_3 / lambda_3

#determine the dispersion of data around the mean by calculating variance values
variance_1 <- k_1 / (lambda_1^2)
variance_2 <- k_2 / (lambda_2^2)
variance_3 <- k_3 / (lambda_3^2)
```

```
#display mean values
cat(paste0("Theoretical mean value 1 = ", mean_1, "\nSample mean 1 = ",
           mean(sample_1)), '\n')
```

```
## Theoretical mean value 1 = 2
## Sample mean 1 = 1.97331196502283
```

```
cat("\n")
```

```r
cat(paste0("Theoretical mean value 2 = ", mean_2, "\nSample mean 2 = ",
           mean(sample_2)), '\n')
```

```
## Theoretical mean value 2 = 1.81818181818182
## Sample mean 2 = 1.8155259723778
```

```r
cat("\n")
```

```r
cat(paste0("Theoretical mean value 3 = ", mean_3, "\nSample mean 3 = ",
           mean(sample_3)), '\n')
```

```
## Theoretical mean value 3 = 2
## Sample mean 3 = 2.00438878733142
```

```r
#display variance values
cat(paste0("Theoretical variance value 1 = ", variance_1, "\nSample variance 1 = ",
           var(sample_1)), '\n')
```

```
## Theoretical variance value 1 = 1
## Sample variance 1 = 0.950735657915041
```

```r
cat("\n")
```

```r
cat(paste0("Theoretical variance value 2 = ", variance_2, "\nSample variance 2 = ",
           var(sample_2)), '\n')
```

```
## Theoretical variance value 2 = 0.550964187327824
## Sample variance 2 = 0.539718041558136
```

```r
cat("\n")
```

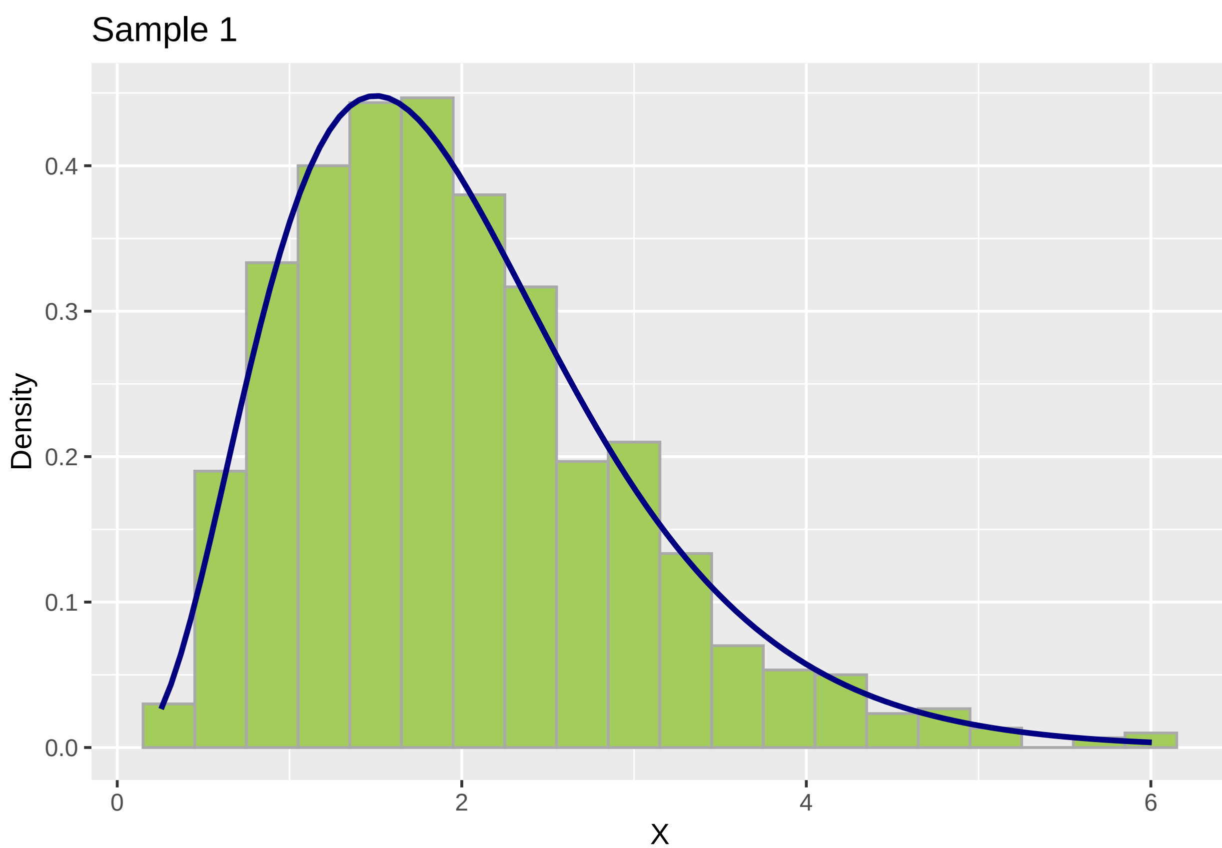
```r
cat(paste0("Theoretical variance value 3 = ", variance_3, "\nSample variance 3 = ",
           var(sample_3)), '\n')
```

```
## Theoretical variance value 3 = 0.571428571428571
## Sample variance 3 = 0.592090944992454
```

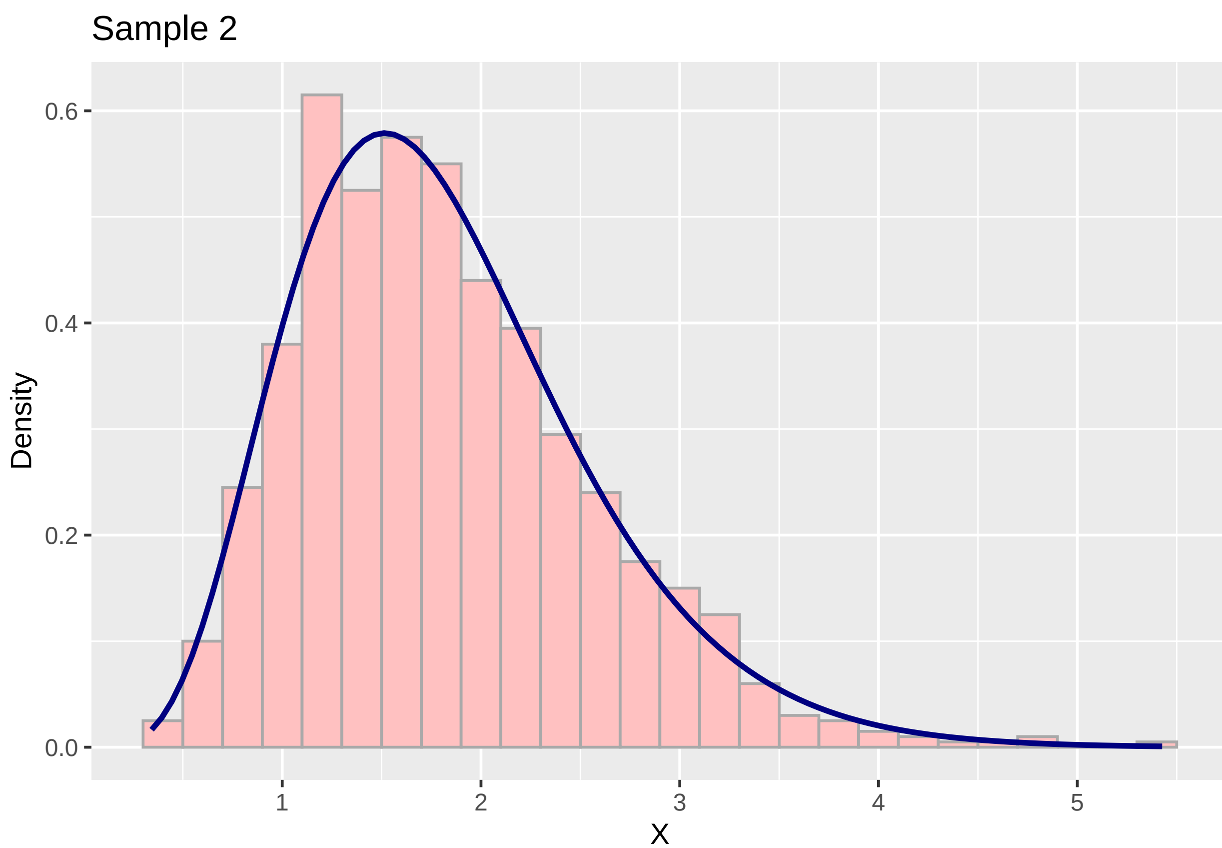
As we can see, the obtained mean values are very close to their theoretical counterparts. The situation with variances is similar to that of means. While the obtained values are not identical to the theoretical ones, they are very close to each other. Therefore, we can infer that the mean is an appropriately chosen estimator of the expected value.

```r
#create a histogram for the first sample
#create a dataframe where the column is the sample
frame_1 <- data.frame(x = sample_1)

#plot the histogram
plot_1 <- ggplot(frame_1, aes(x)) +
         geom_histogram(binwidth = 0.3, aes(y = after_stat(density)),
```

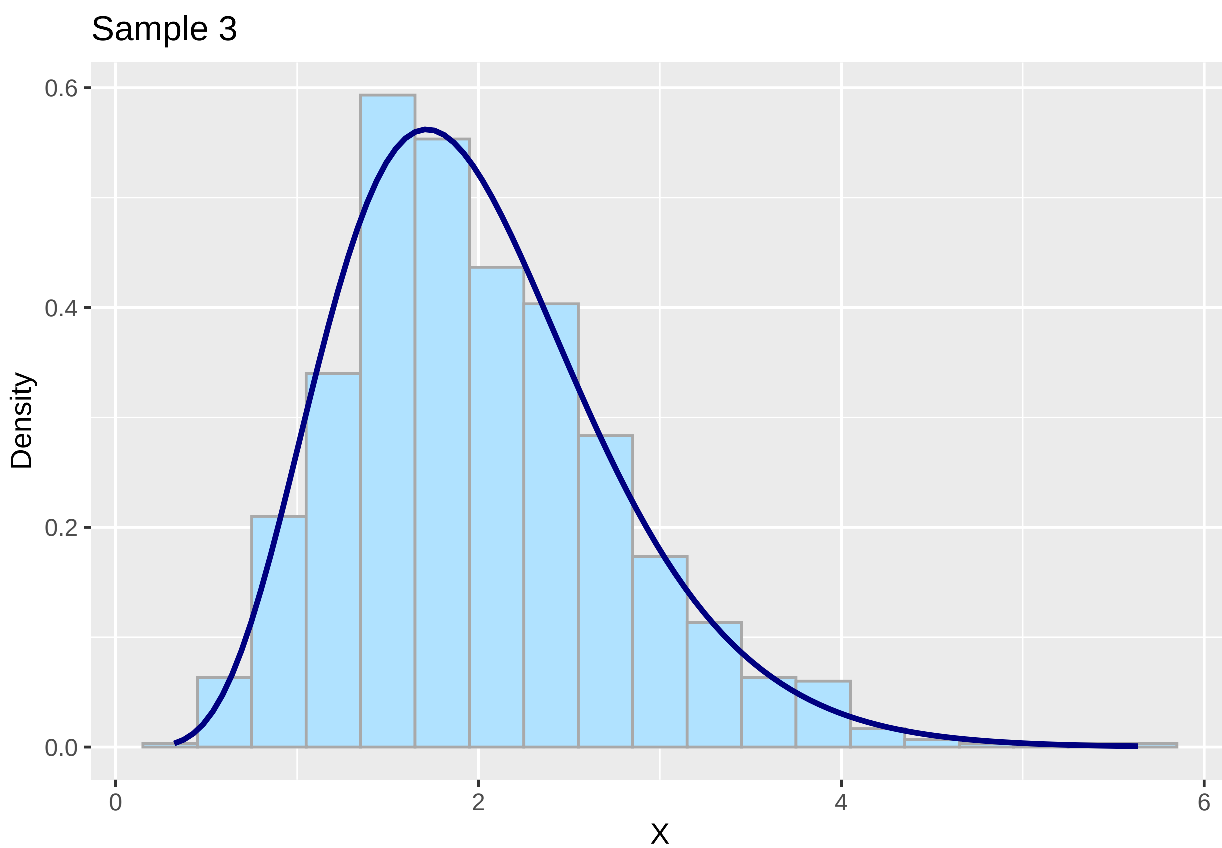
```
                     fill = "darkolivegreen3",color = "darkgrey") +
         stat_function(fun = dgamma, args = list(k_1, lambda_1),
                      color = "navy", lwd=1) +
         ggtitle("Sample 1") + xlab("X") + ylab("Density")

plot_1
```




```
#create a histogram for the second sample
#create a dataframe where the column is the sample
frame_2 <- data.frame(x = sample_2)

#plot the histogram
plot_2 <- ggplot(frame_2, aes(x)) +
         geom_histogram(binwidth = 0.2, aes(y = after_stat(density)),
                       fill = "rosybrown1",color = "darkgrey") +
         stat_function(fun = dgamma, args = list(k_2, lambda_2),
                      color = "navy", lwd=1) +
         ggtitle("Sample 2") + xlab("X") + ylab("Density")

plot_2
```

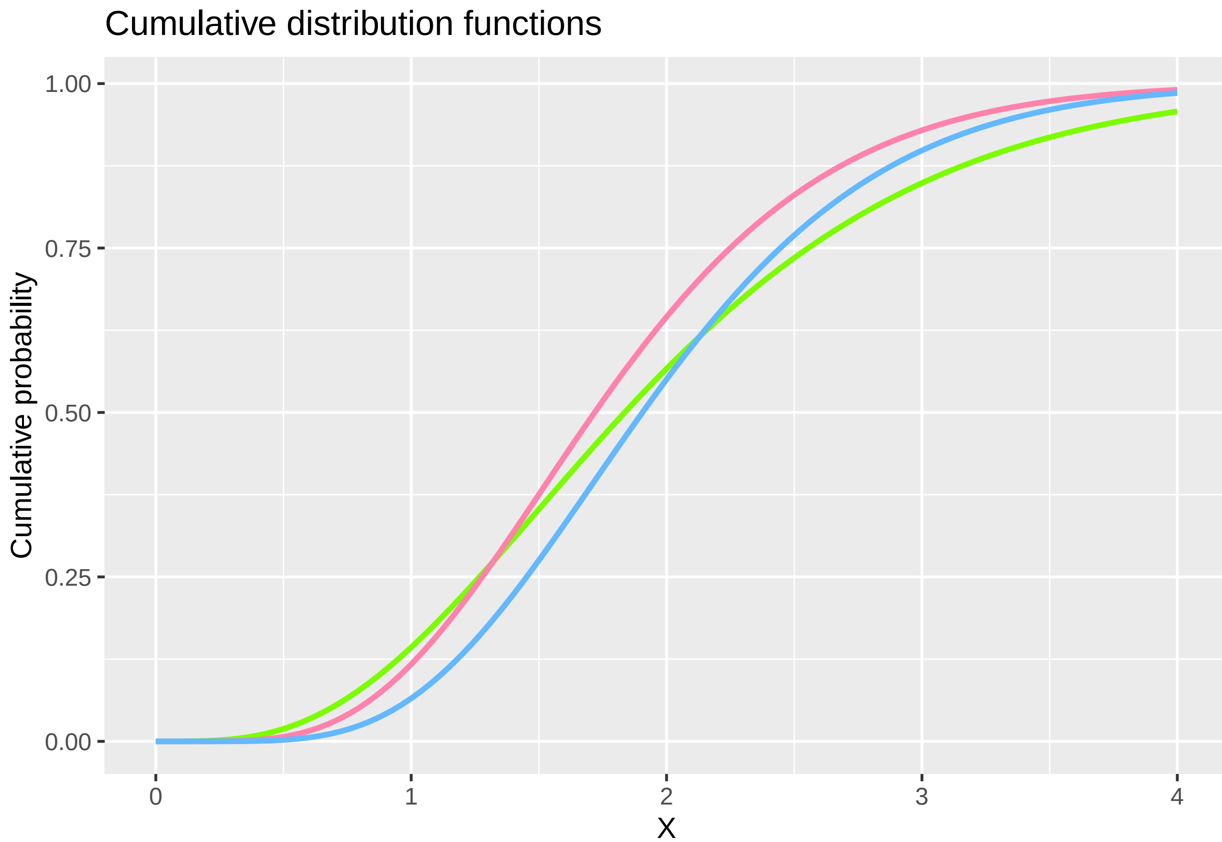



```r
#create a histogram for the third sample
#create a dataframe where the column is the sample
frame_3 <- data.frame(x = sample_3)

#plot the histogram
plot_3 <- ggplot(frame_3, aes(x)) +
         geom_histogram(binwidth = 0.3, aes(y = after_stat(density)),
                        fill = "lightskyblue1",color = "darkgrey") +
         stat_function(fun = dgamma, args = list(k_3, lambda_3),
                       color = "navy", lwd=1) +
         ggtitle("Sample 3") + xlab("X") + ylab("Density")

plot_3
```



On the plots, it is noticeable that when smaller parameters are provided, the histogram closely resembles the theoretical density (as seen in the plot for the first sample). As larger parameters are used, the plots begin to diverge more from each other (comparing, for instance, the plot of the first sample, where smaller parameters were used, to the plot of the last sample, where larger parameters were used).

```
#plot theoretical cumulative distribution functions for individual samples on one plot
sequence = seq(0, 4, 1)
cumulative <- ggplot(data.frame(sequence), aes(x=sequence)) +
             stat_function(fun = pgamma, args = list(k_1, lambda_1),
                           color = "lawngreen", lwd=1) +
             stat_function(fun = pgamma, args = list(k_2, lambda_2),
                           color = "palevioletred1", lwd=1) +
             stat_function(fun = pgamma, args = list(k_3, lambda_3),
                           color = "steelblue1", lwd=1) +
             xlab("X") + ylab("Cumulative probability") +
  ggtitle("Cumulative distribution functions")

cumulative
```

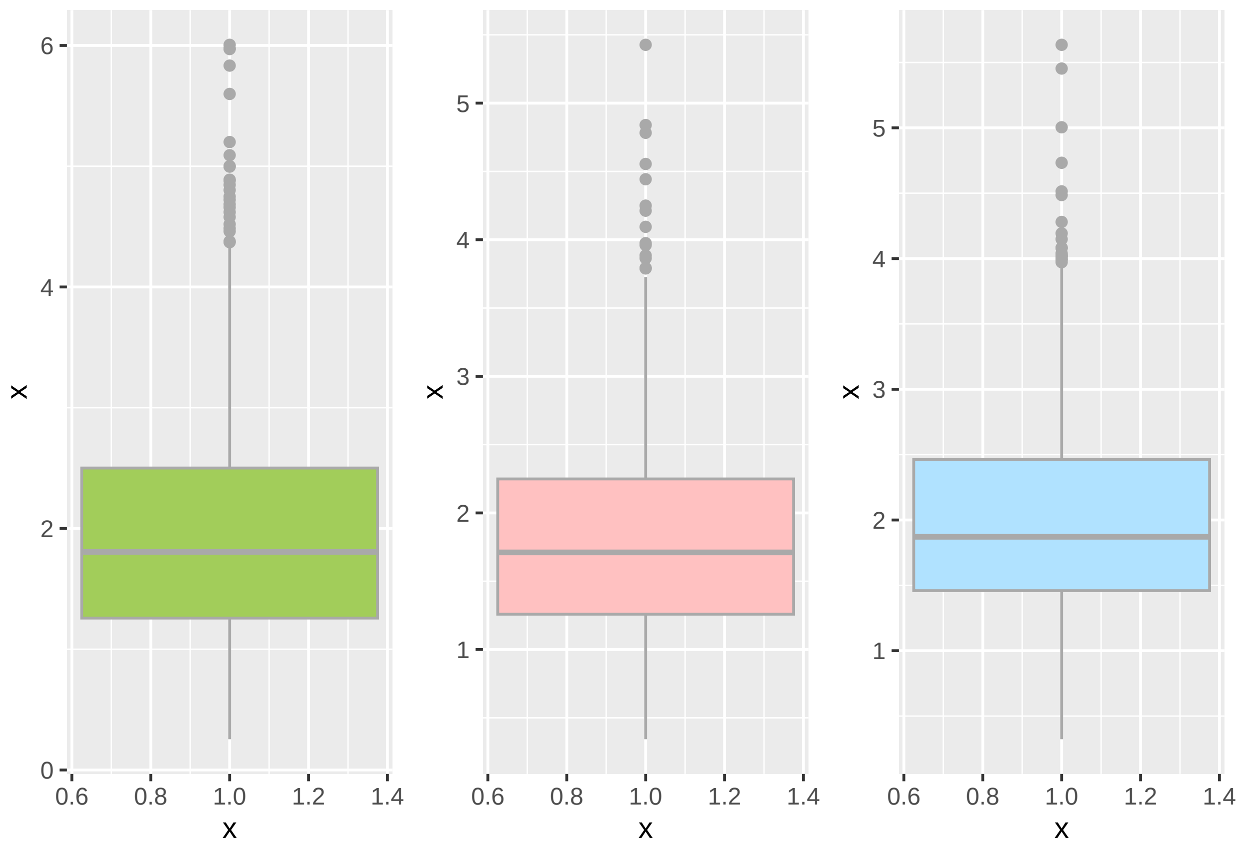



```
#the green curve corresponds to the first sample (as the histogram color is green),
#the pink curve corresponds to the second sample
#and the blue curve corresponds to the third sample.
```

From the cumulative distribution function plot, we can observe that the larger the obtained mean was, the slower the cumulative distribution function grows.

More precisely, the pink sample had the smallest mean, so the cumulative distribution function on the plot increases the fastest. On the other hand, the mean of the green sample was the largest, hence the cumulative distribution function increases the slowest.

```
#plot boxplots

plot_box_1 <- ggplot(frame_1, aes(x = 1, y = x)) +
  geom_boxplot(fill = "darkolivegreen3", color = "darkgrey")

plot_box_2 <- ggplot(frame_2, aes(x = 1, y = x)) +
  geom_boxplot(fill = "rosybrown1", color = "darkgrey")

plot_box_3 <- ggplot(frame_3, aes(x = 1, y = x)) +
  geom_boxplot(fill = "lightskyblue1", color = "darkgrey")

grid.arrange(plot_box_1, plot_box_2, plot_box_3, ncol=3)
```



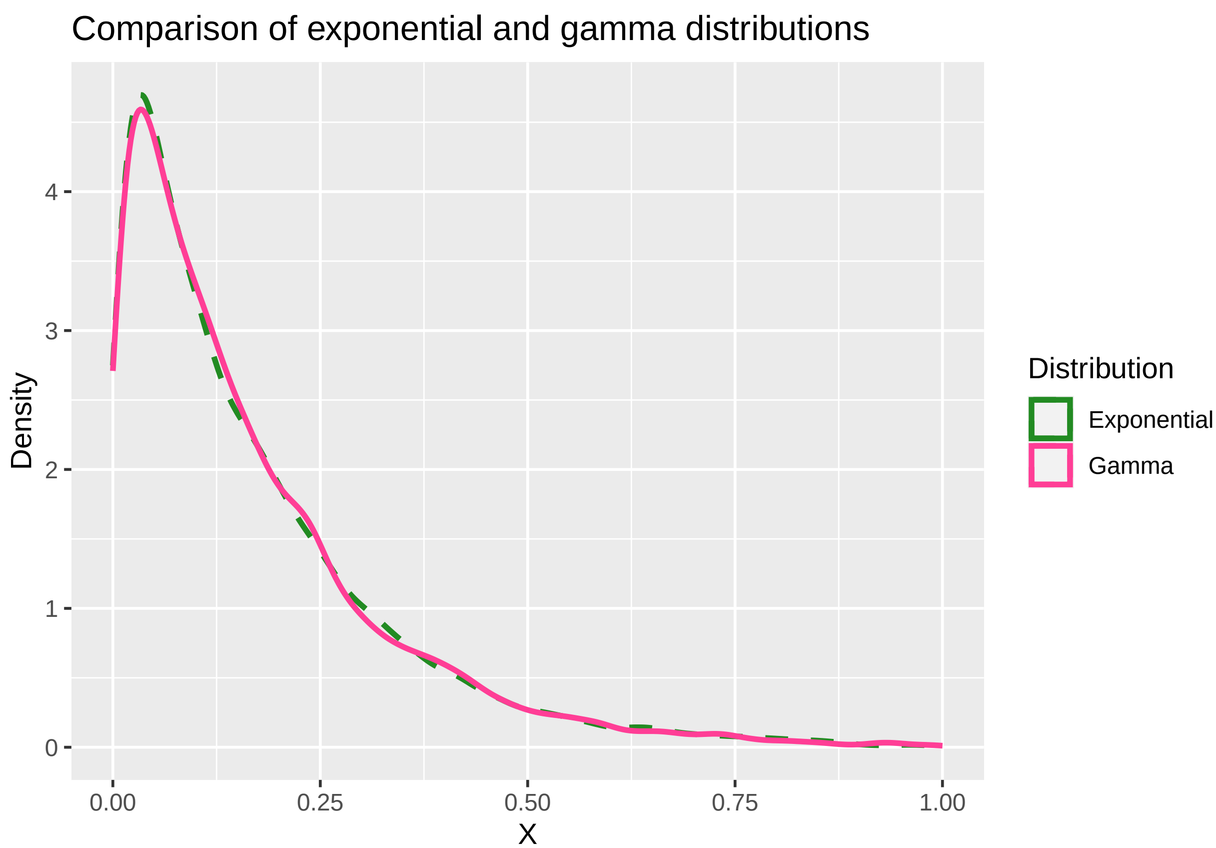
TASK 2

Empirically verify (e.g. using a density plot) that the exponential distribution with parameter lambda is a special case of the Gamma distribution with parameters lambda and k.

```r
#define parameter values
lambda <- 6
p <- 10000
k <- 1

#generate a random sample 'p' from an exponential distribution with parameter 'lambda'
exp_sample <- rexp(p, lambda)

#generate a sample with the same parameter 'lambda' and parameter 'k' for the Gamma distribution
gamma_sample <- rgamma(p, k, lambda)

#generate density plots for these distributions
exp_density <- data.frame(x = exp_sample, y = dexp(exp_sample, lambda))
gamma_density <- data.frame(x = gamma_sample, y = dgamma(gamma_sample, k, lambda))

sequence = seq(0, 4, 1)
distributions <- ggplot(data.frame(sequence), aes(x=sequence)) +
        geom_density(data = exp_density, aes(x = x, color = "Exponential"),
                     alpha = 0.5, linewidth=1, linetype = "dashed") +
```

```
        geom_density(data = gamma_density, aes(x = x, color = "Gamma"),
                     alpha = 0.5, linewidth=1) +
        scale_color_manual(name="Distribution",
                           values = c("forestgreen", "violetred1")) +
        xlim(0, 1) + xlab("X") + ylab("Density") +
  ggtitle("Comparison of exponential and gamma distributions")

distributions
```

```
## Warning: Removed 25 rows containing non-finite values (`stat_density()`).

## Warning: Removed 30 rows containing non-finite values (`stat_density()`).
```

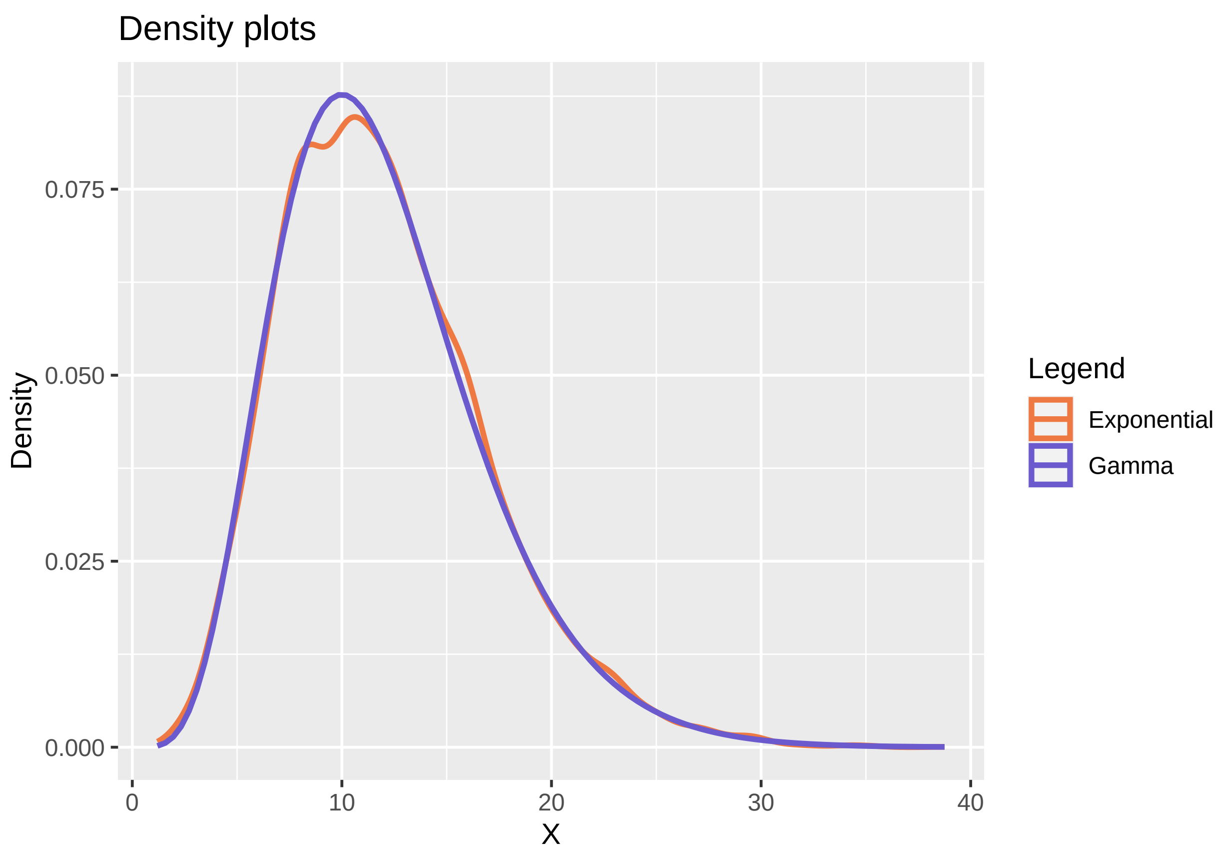


The plot clearly shows that the distributions have very similar densities, thus it can be concluded that the exponential distribution with parameter lambda is a special case of the Gamma distribution with parameters lambda and k.

TASK 3

Empirically verify (e.g. using density plots) that the sum of k independent variables with an exponential distribution with the same parameter lambda has a Gamma distribution with parameters lambda and k.

```r
#define parameter values
k <- 6
lambda <- 0.5

#generate independent variables with exponential distribution
variable_1 <- rexp(10000, lambda)
variable_2 <- rexp(10000, lambda)
variable_3 <- rexp(10000, lambda)
variable_4 <- rexp(10000, lambda)
variable_5 <- rexp(10000, lambda)
variable_6 <- rexp(10000, lambda)

#sum the independent variables
variables_sum <- variable_1 + variable_2 + variable_3 + variable_4 + variable_5 + variable_6

#plot density plots for the sum of exponential distributions and the Gamma distribution
plot_variables <- ggplot(data.frame(variables_sum),
                         aes(x=variables_sum)) +
           geom_density(aes(x = variables_sum,
                            color = "Exponential"), alpha = 0.8, lwd = 1) +
           stat_function(fun=dgamma, args=list(k,lambda),
                         aes(color = "Gamma"), lwd = 1)+
           scale_color_manual(name="Legend",
                              values = c("Exponential" = "sienna2", "Gamma" = "slateblue")) +
           xlab("X") + ylab("Density") +
  ggtitle("Density plots")

plot_variables
```



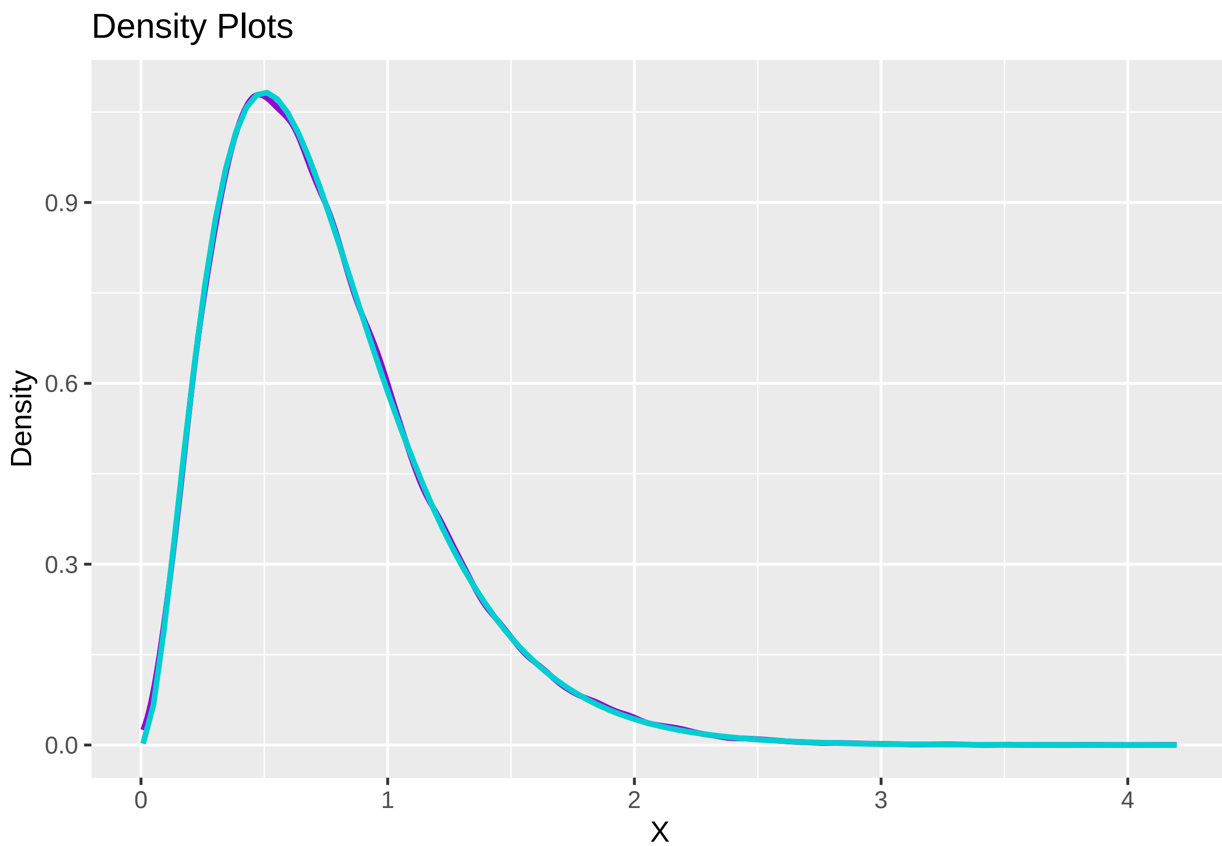
Since the density plots of the distributions have the same shape on the graph, it can be concluded that the sum of k independent variables with an exponential distribution with the same parameter lambda follows a Gamma distribution with parameters lambda and k.

TASK 4

Empirically verify (e.g. using density plots) the statement that if the variable X follows a Gamma distribution with parameters lambda and k, then the variable cX (for some c>0) follows a Gamma distribution with parameters lambda/c and k.

```
#assign values to parameters
lambda <- 2
k <- 3

#assign a value for c
c <- 0.5

#define Gamma distribution and c*Gamma distribution
gamma_distribution = rgamma(100000, k, lambda)
cgamma_distribution = c * gamma_distribution

#plot density plots for the Gamma distribution and c*Gamma distribution
density_plot <- ggplot(data.frame(cgamma_distribution),
                       aes(x=cgamma_distribution)) +
          geom_density(color = "darkviolet", lwd=1) +
```

```
        stat_function(fun = dgamma, args = list(k, lambda/c),
                      color="darkturquoise", lwd=1) +
        xlab("X") + ylab("Density") + ggtitle("Density Plots")

density_plot
```



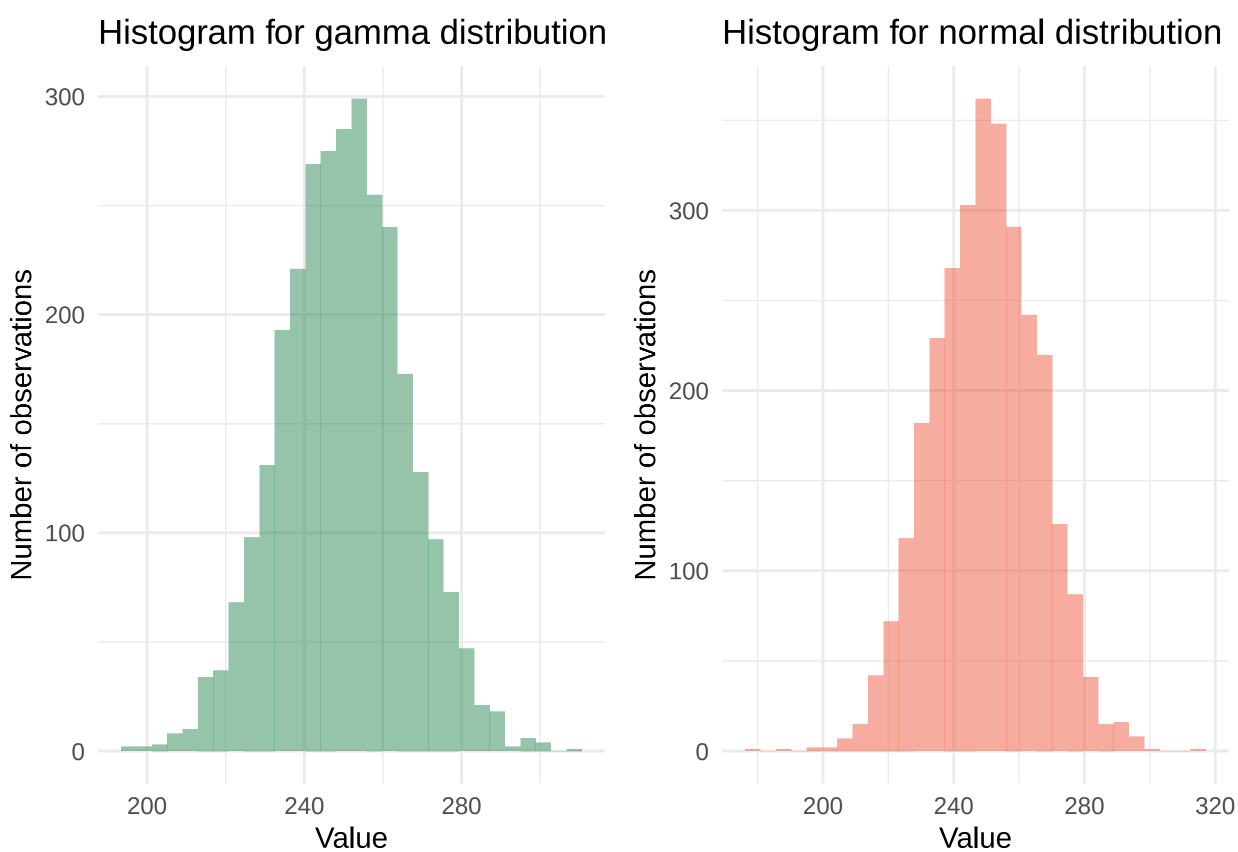
The plot shows that the Gamma distribution (purple color) and the cGamma distribution (turquoise color) have the same shape and overlap each other. Therefore, it can be inferred that the cGamma distribution is also a Gamma distribution.

TASK 5

Compare samples from a Gamma distribution with a fixed parameter lambda and large parameters k (suggested k > 50) and samples from a normal distribution with mean k/lambda and variance k/lambda^2.

```
#describe parameters
lambda <- 1
k <- 250
n <- 3000

#generate samples from Gamma and Normal distributions
gamma_sample <- rgamma(n, k, lambda)
normal_sample <- rnorm(n, k/lambda, sqrt(k)/lambda)
```

```
#create histogram for the Gamma distribution
hist_gamma <- ggplot() +
           geom_histogram(aes(gamma_sample), bins = 30,
                          alpha = 0.5, fill = "seagreen") +
           labs(x = "Value", y = "Number of observations",
                title = "Histogram for gamma distribution") +
  theme_minimal()

#create histogram for the normal distribution
hist_normal <- ggplot() +
          geom_histogram(aes(normal_sample), bins = 30,
                         alpha = 0.5, fill = "tomato2") +
          labs(x = "Value", y = "Number of observations",
               title = "Histogram for normal distribution") +
  theme_minimal()

grid.arrange(hist_gamma, hist_normal, ncol = 2)
```



As the value of k increases, the gamma distribution becomes more similar to the normal distribution.