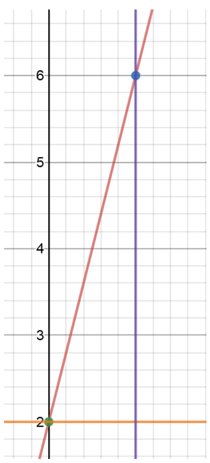
# MATH 138 LECTURE 1

## 1 Area Under Curves

Problem: Find the area between the graph of $y = f(x)$ and the x-axis over the interval $[a, b]$.

**Example 1:** Suppose $f(x) = 4x + 2$ and $a = 0$, $b = 1$.




Recall that the area of a trapezoid is:
Area = Area of a Triangle + Area of a Rectangle
Area = 2 + 2
Area = 4

**Example 2:** Suppose $f(x) = 2x^2 + 1$ where $a = 0$ and $b = 6$.

Approximation 1: Construct a rectangle A over the interval $[0, 6]$ with height equal to $f(6) = 73$. You will find that the area is 438.

Approximation 2: Construct two rectangles A and B. A is over the interval $[0, 3]$ with height equal to $f(3) = 19$ and B is over the interval $[3, 6]$ with height equal to $f(6) = 73$.

Area = 57 + 219 = 276

Approximation 3: Split into 3 rectangles. You will find that the area is 230.

## 2 Definitions and Formal Procedure

Suppose $y = f(x)$ is a bounded function on $[a, b]$.

**Partition of an Interval [a, b]**: A finite set $p = \{t_0, t_1, ..., t_{n-1}, t_n\}$ with $a = t_0 < t_1 < t_2 < ... < t_{n-1} < t_n = b$.

i) $p = \{0, 2, 4, 6\}$ is a partition of $[0, 6]$.
ii) $p = \{0, 0.8, 1.2, 1.6, 2\}$ is a partition of $[0, 2]$.

**REMARK:** The points in a partition do not have to be evenly spaced.

In general, the partition $p$ defines $n$ subintervals $[t_0, t_1], [t_1, t_2], ..., [t_{n-1}, t_n]$ where $[t_{i-1}, t_i]$ is the $i^{th}$ subinterval of $p$.

$\Delta t = t_i - t_{i-1}$ denotes the length of the $i^{th}$ subinterval of $p$.

Notation: $|P|$ is the norm of the partition $[a, b]$.
$|P| = max\{\Delta t, \Delta t_2, ..., \Delta t_n\}$

E.g. $P = \{0, 0.8, 1.2, 1.6, 2\}$
$\Delta t_1 = 0.8, \Delta t_2 = 0.4, \Delta t_3 = 0.4, \Delta t_4 = 0.4$
Thus, $|P| = 0.8$.

**Regular $n$-partition of** $[a, b]$**:** $p^{(n)} = \{t_0, t_1, ..., t_n\}$ where $t_i = \dfrac{i(b-a)}{n}$.

**REMARK:** $\Delta t_i = \dfrac{b-a}{n}$ for each $i = 1, 2, ..., n$.

## 3 Summary

To approximate areas under curves, we can take its rectangular area. When we take smaller and smaller areas, we get a better approximation of the area.

**Procedure:**
1. We take the partition of the interval $[a, b]$ into $n$ sections where $n \in \mathbb{N}$, or we can partition them irregularly.
2. We take the length of the subinterval and multiply it by the value of the left/mid/right endpoint.
3. Repeat for all subintervals and add the area together to obtain the approximation.