Homework 1

Problem 1.1 ADEMP Structure

Answer the following questions:

- How many simulation scenarios will you be running?

18 scenarios.

- What are the estimand(s)

The average treatment effect ($\beta_{treatment}$)

- What method(s) are being evaluated/compared?

The study evaluates the multiple linear regression model and compares three methods for constructing confidence intervals: 1) Wald confidence intervals; 2) nonparametric bootstrap percentile intervals; 3) nonparametric bootstrap-t intervals.

- What are the performance measure(s)?
    - Bias of $\hat{\beta}$
    - Coverage of $\hat{\beta}$
    - Distribution of $se(\hat{\beta})$
    - Computation time across methods

Problem 1.2 nSim

Based on desired coverage of 95% with Monte Carlo error of no more than 1%, how many simulations (n_{sim}) should we perform for each simulation scenario? Implement this number of simulations throughout your simulation study.

We should perform 475 simulations for each scenario.

Problem 1.3 Implementation

Problem 1.4 Results summary

Bias of $\hat{\beta}$

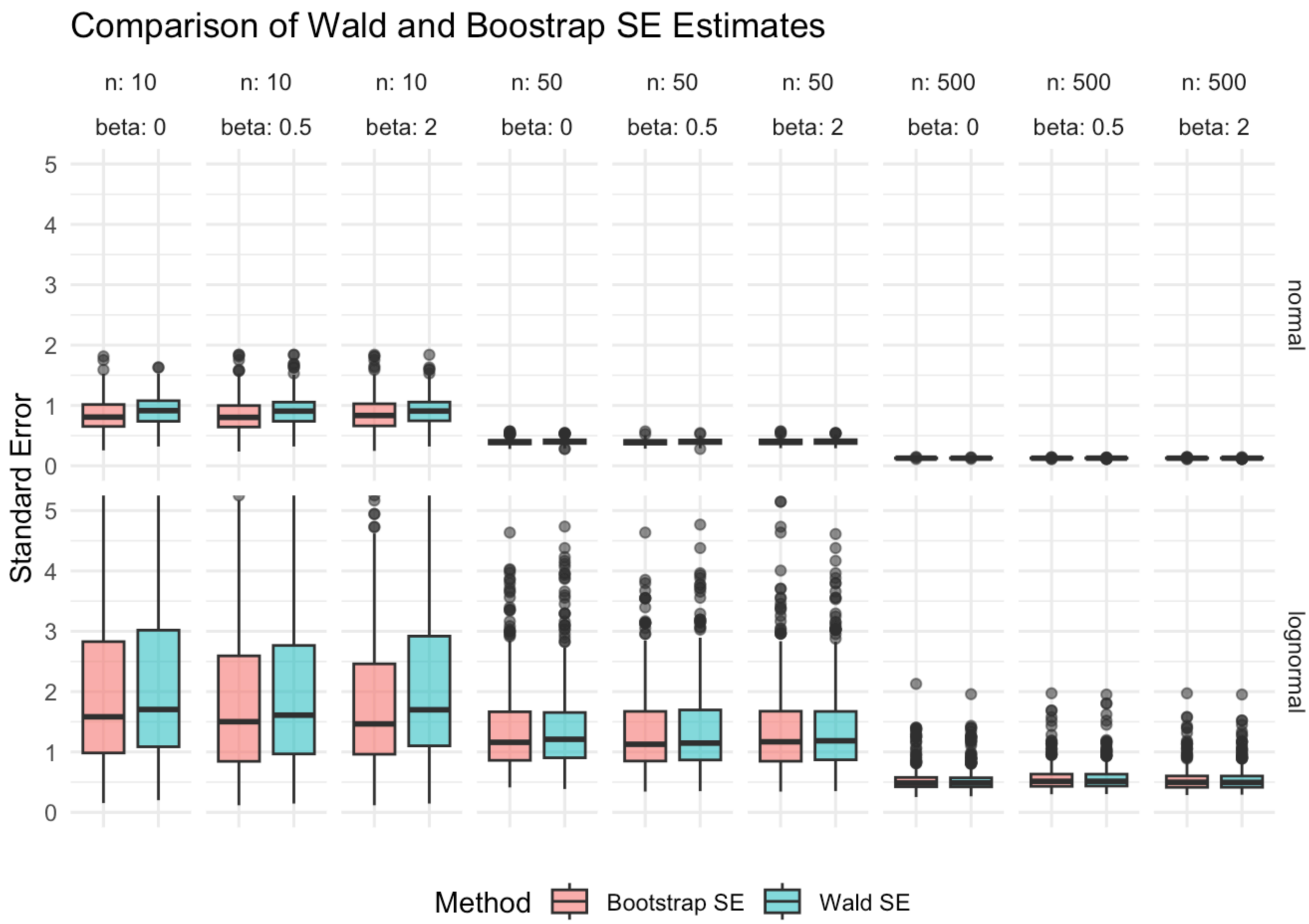
Average Bias of $\hat{\beta}$ Across 475 Simulations								
n = 10			n = 50			n = 500		
β = 0	β = 0.5	β = 2	β = 0	β = 0.5	β = 2	β = 0	β = 0.5	β = 2
normal								
0.015	0.040	0.011	0.021	−0.007	−0.013	0.002	−0.006	−0.004

Average Bias of $\hat{\beta}$ Across 475 Simulations								
n = 10			n = 50			n = 500		
β = 0	β = 0.5	β = 2	β = 0	β = 0.5	β = 2	β = 0	β = 0.5	β = 2
lognormal								
0.176	−0.209	−0.195	0.090	−0.002	0.155	−0.011	−0.040	−0.006

Coverage of $\hat{\beta}$

Average Coverage of 95% Wald, Percentile & Bootstrap-t CIs					
family	n	beta_true	coverage_wald	coverage_percentile	coverage_t
normal	10	0.0	0.926	0.848	0.945
normal	10	0.5	0.951	0.869	0.976
normal	10	2.0	0.957	0.906	0.974
normal	50	0.0	0.926	0.926	0.931
normal	50	0.5	0.945	0.933	0.941
normal	50	2.0	0.922	0.912	0.926
normal	500	0.0	0.954	0.954	0.945
normal	500	0.5	0.939	0.943	0.937
normal	500	2.0	0.935	0.933	0.926
lognormal	10	0.0	0.977	0.861	0.936
lognormal	10	0.5	0.951	0.844	0.960
lognormal	10	2.0	0.964	0.864	0.943
lognormal	50	0.0	0.964	0.924	0.897
lognormal	50	0.5	0.964	0.912	0.884
lognormal	50	2.0	0.966	0.924	0.891
lognormal	500	0.0	0.973	0.949	0.924
lognormal	500	0.5	0.966	0.949	0.933
lognormal	500	2.0	0.985	0.966	0.941

Distribution of $se(\hat{\beta})$



Computation time across methods

Average Computation Time for Wald, Percentile & Bootstrap-t CIs

family	n	beta_true	time_wald	time_percentile	time_t
normal	10	0.0	0.007	0.505	169.902
normal	10	0.5	0.007	0.509	166.921
normal	10	2.0	0.008	0.529	168.568
normal	50	0.0	0.007	0.517	173.953
normal	50	0.5	0.007	0.515	173.625
normal	50	2.0	0.007	0.516	174.564
normal	500	0.0	0.007	0.544	200.597
normal	500	0.5	0.007	0.553	199.982
normal	500	2.0	0.007	0.554	199.987
lognormal	10	0.0	0.007	0.496	168.355

Average Computation Time for Wald, Percentile & Bootstrap-t CIs					
family	n	beta_true	time_wald	time_percentile	time_t
lognormal	10	0.5	0.006	0.411	143.714
lognormal	10	2.0	0.006	0.441	151.516
lognormal	50	0.0	0.006	0.444	155.849
lognormal	50	0.5	0.006	0.456	156.557
lognormal	50	2.0	0.006	0.446	156.140
lognormal	500	0.0	0.006	0.489	180.460
lognormal	500	0.5	0.006	0.476	179.873
lognormal	500	2.0	0.006	0.479	180.365

Problem 1.5 Discussion

Summary of main findings:

The bias of $\hat{\beta}$ decreases as the sample size (n) increases. In small samples (n = 10 or 50), misspecifying the error distribution leads to biased estimates when the true error distribution is lognormal, but this bias diminishes in larger samples (n = 500). The Wald confidence interval maintains coverage near its nominal level (95%) when the error distribution is normal and slightly exceeds it when the true distribution is lognormal, likely due to conservative standard error (SE) estimates under misspecification. The bootstrap percentile interval performs poorly in small samples (n = 10) but improves with increasing n. In contrast, the bootstrap-t interval performs well in small samples but does not improve with larger n. SE estimates from the Wald and bootstrap methods are similar, with bootstrap estimates slightly higher. Under model misspecification, SE estimates exhibit greater variance and right skewness. The variance of SE estimates decreases as n increases. Regarding computation time, the Wald method is the fastest, followed by the percentile method, while the bootstrap-t method is significantly more computationally intensive. No clear trend is observed in bias, coverage, SE, or computation time across different true β values.

- Regarding computation time, the Wald method is the fastest, followed by the percentile method, while the bootstrap-t method is significantly more computationally intensive.
- The bootstrap-t method for constructing confidence intervals provides the best coverage when $\epsilon_i \sim N(0, 2)$.
- The Wald method for constructing confidence intervals provides the best coverage when $\epsilon_i \sim logNormal(0, \log(2))$.

GitHub repository: https://github.com/wqian22/bios731_hw1_qian.git (https://github.com/wqian22/bios731_hw1_qian.git)