1 movie:225

film:236

3

(a) Answer: (n+1)/2n

1-1/k^2=(k^2-1)/k^2=(k-1)(k+1)/k^2=(1*3/2^2)(2*4/3^2)(3*5/4^2)…(n-1)(n+1)=(n+1)/2n

(b) Answer:4

3^1mod7=3,3^2mod7=2, 3^3mod7=6, and then 3^6mod7=6^2or(-1)^2;3^1000=(3^6)^166(3^4),

3^1000mod7=(-1*3)(-1^2)^166=-3or4

(c) Answer:1

½+1/4+1/8+1/16+…=1

(d) Answer: 2

(lg81/lg7)/(lg7/lg9)=(lg81/lg9)=log9 81=2

(e) Answer: 4n

Log2 2^(2*2n)=log2 2^4n=4n

(f) Answer:1

Log17 (221/13)=1

4

Answer:an=(n+1)!

A1=2=2!,a2=6=3!,a3=24=4!...an=(n+1)!

Proof:

Basic case: for n=0, A0=(0+1)!=1!

Induced case: As an=(n+1)! We need to prove a(n+1)=(n+2)! By the definition formula, we have a(n+1)=an+(n+1)!(n+1) Thus an=(n+1)!

An+(n+1)!(n+1)=(n+1)!+(n+1)!(n+1)=(n+1)!(1+n+1)=(n+1)!(n+2)=(n+2)!, so a(n+1)=(n+2)!, provided an=(n+1)!

5

(a) Answer: f(n)=theta(g(n))
Proof: As f(n)=n and g(n)=2n+1 are both linear, there exists a c such c*g(n) could be both the upper bound and the lower bound of f(n) for larger n. Thus f(n)=theta(g(n)).

(b) Answer: f(n)=omega(g(n))
Proof: g(n)=sqrt(n)=n^1/2.Because g(n) has lower dimension than f(n), for large n, c*g(n) could only be the lower bound of f(n). Thus f(n)=omega(g(n)).

(c)Answer:f(n)=O(g(n))

(d)Answer:f(n)=O(g(n))

6 (a) Answer: T(n)=1+5(log2 n)

As T(2)=T(1)+5=6, T(4)=T(2)+5=11, T(8)=T(4)+5=16. T(16)=T(8)+5=21, so T(n)=1+5(log2 n)

(b)

Answer: T(n)=sum of 1/n(n from 1 to n)

T(1)=T(0)+1=1,T(2)=T(1)+1/2,T(3)=T(2)+1/3 Thus as the answer showed.

Hn(harmonic series up to n)

(c) Answer: Basic case: For n=1, T(n)=1 and T(1)=1+5(log2 1)=1 as showed.
Induced case: As T(n)=1+5(log2 n), we need to prove T(2n)=1+5(log 2 (2n)).
T(2n)=1+5(log2 (2n))=1+(5(log2 2)+5(log2 n)=1+5+5(log2 n)=6+5(log2 n)
By the definition, T(2n)=T(n)+5=1+5(log2 n)+5=6+5(log2 n)
Thus we get T(2n)=1+5(log2 n)
Q.E.D

7 (a) formula: T(n)=T(n/2)+c2

Base case T(1)=c1

Solution: c(log2 n)+c1

(b) formula: T(n)=2T(n/2)+c2n+c3

Base case: T(1)=c1

Solution: nlgn

8 (a) Answer: process: (2,12)-(4,6)-(16,3)-16(256,1)-256(256^2,0)-1

So the result is 256*16*1.

(b) Try to find the nth power of x. For instance, if x=2, n=12, it will find out 2^12.

(c) T(0)=d

T(n)=T([n/2])+c

(d)T(n)=c*(log2 n)+d