The Neutral Competitive Exclusion model

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Outline

- Motivations and Background
- Neutral Competitive Exclusion Model
- Simplified Toy Neutral Competitive exclusion Model
- Summary

Dynamical Process on Complex Networks

Walking and searching:

Diffusion process, random walks

Epidemic Spreading:

SIS, SIR, …

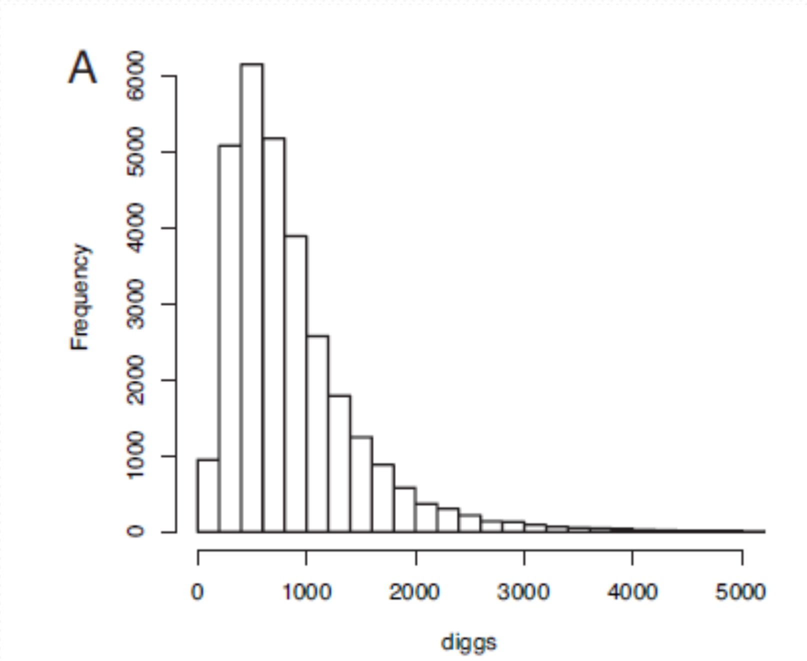
Social networks and collective behavior:

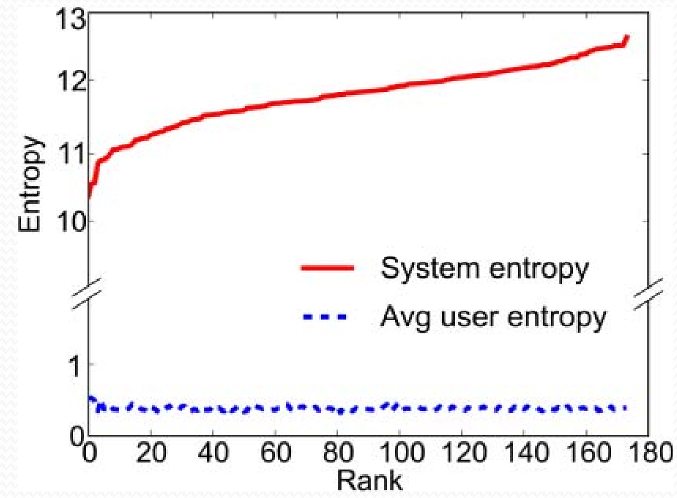
Opinion formation, Rumor and information spreading, Axelrod model, Prisoner's dilemma…

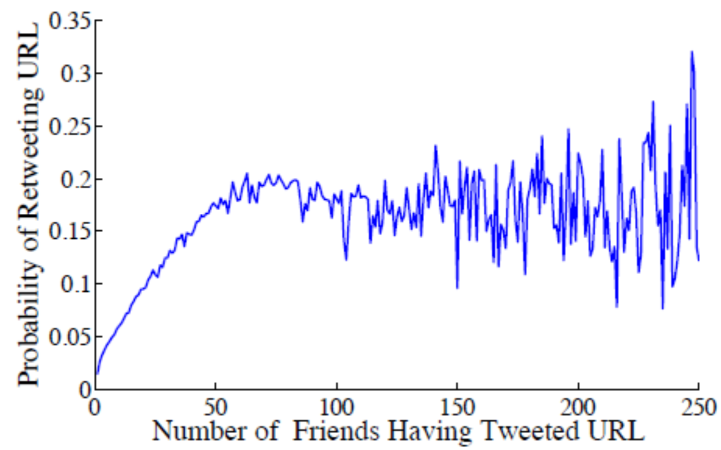
Traffic on complex network:

traffic and congestion

Motivations





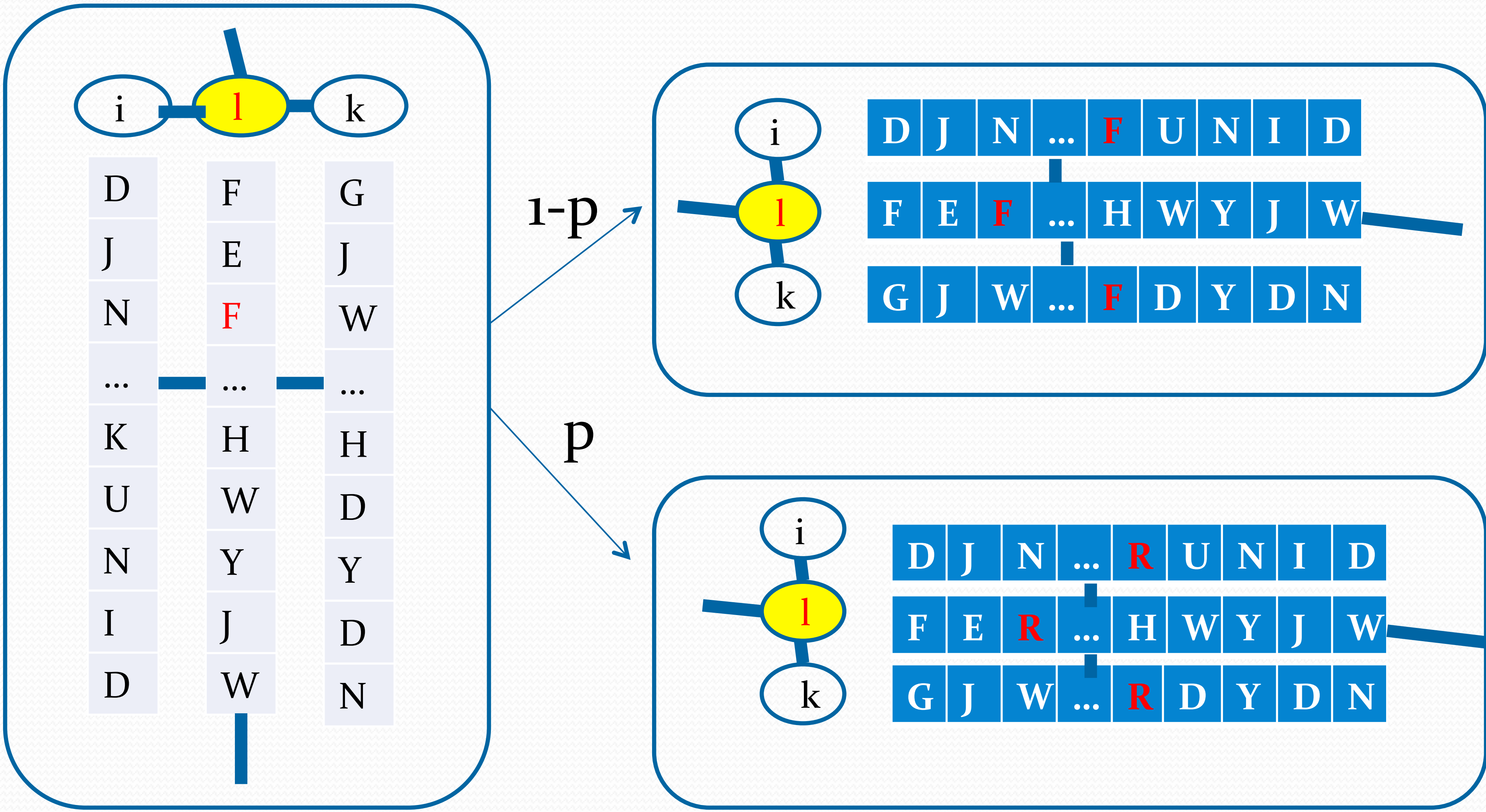


(a) Friend response function, averaged over all users

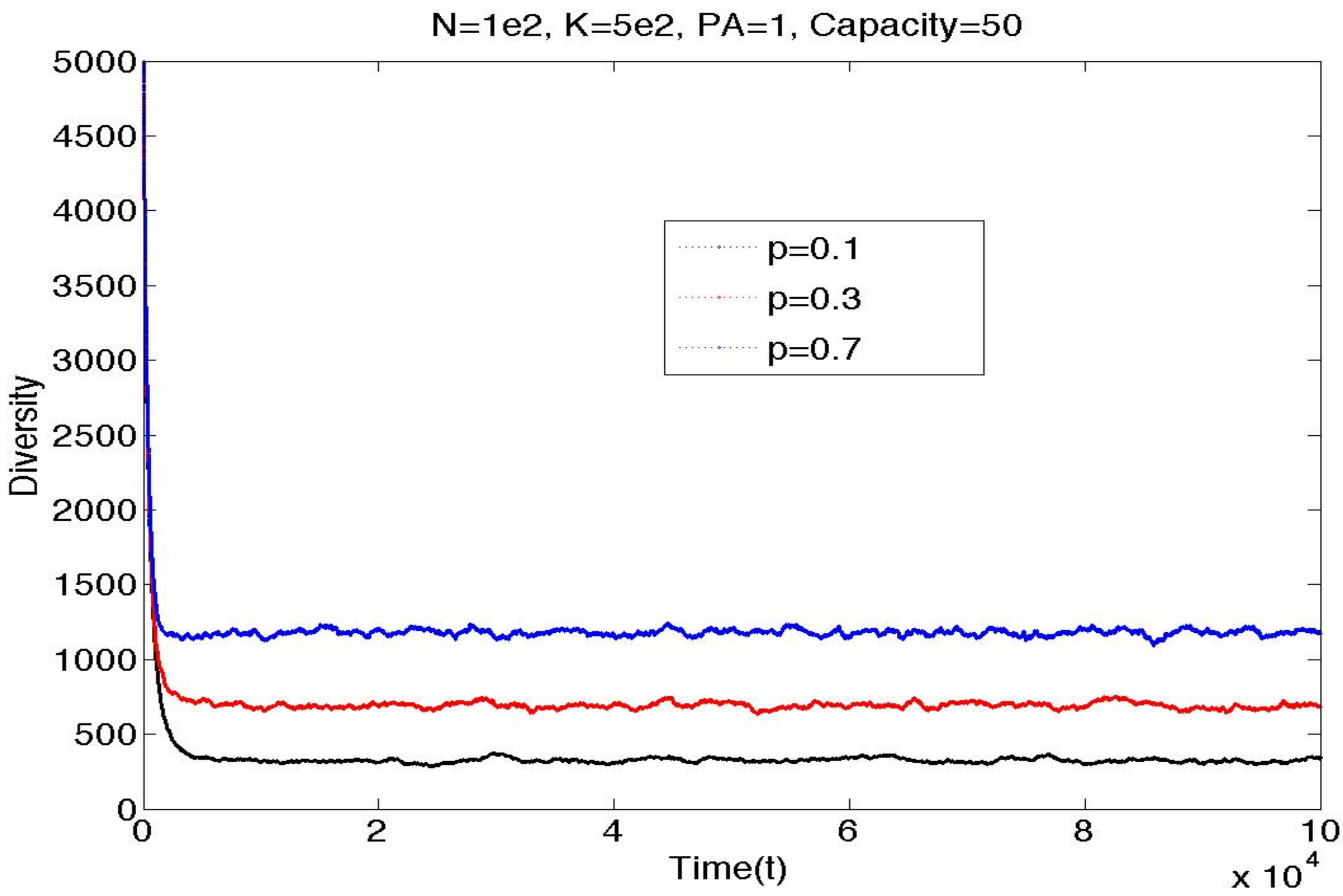
Nathan Oken Hodas and Kristina Lerman, How visibility and divided attention constrain social contagion, In ASE/IEEE International Conference on Social Computing (SocialCom-2012)

Fang Wu and Bernardo A. Huberman, Novelty and Collective attention. PNAS, 104 (45),2007

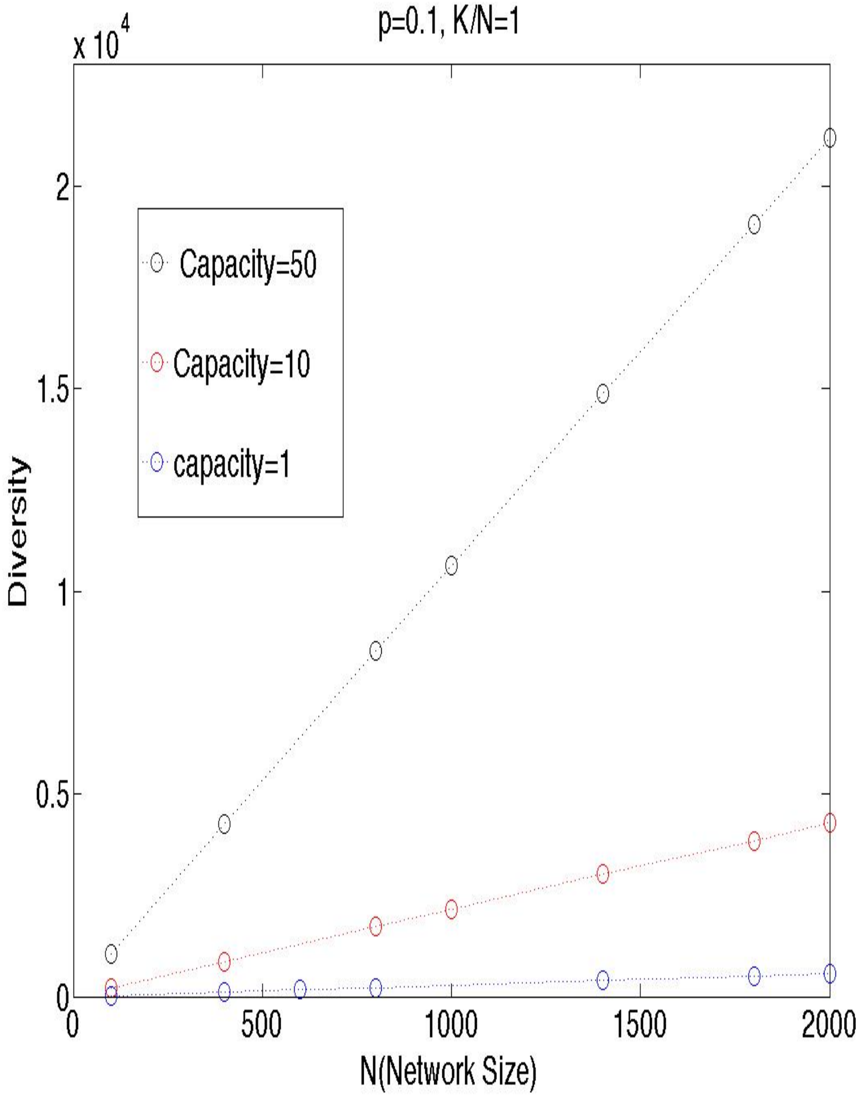
Neutral Competitive Exclusion Model

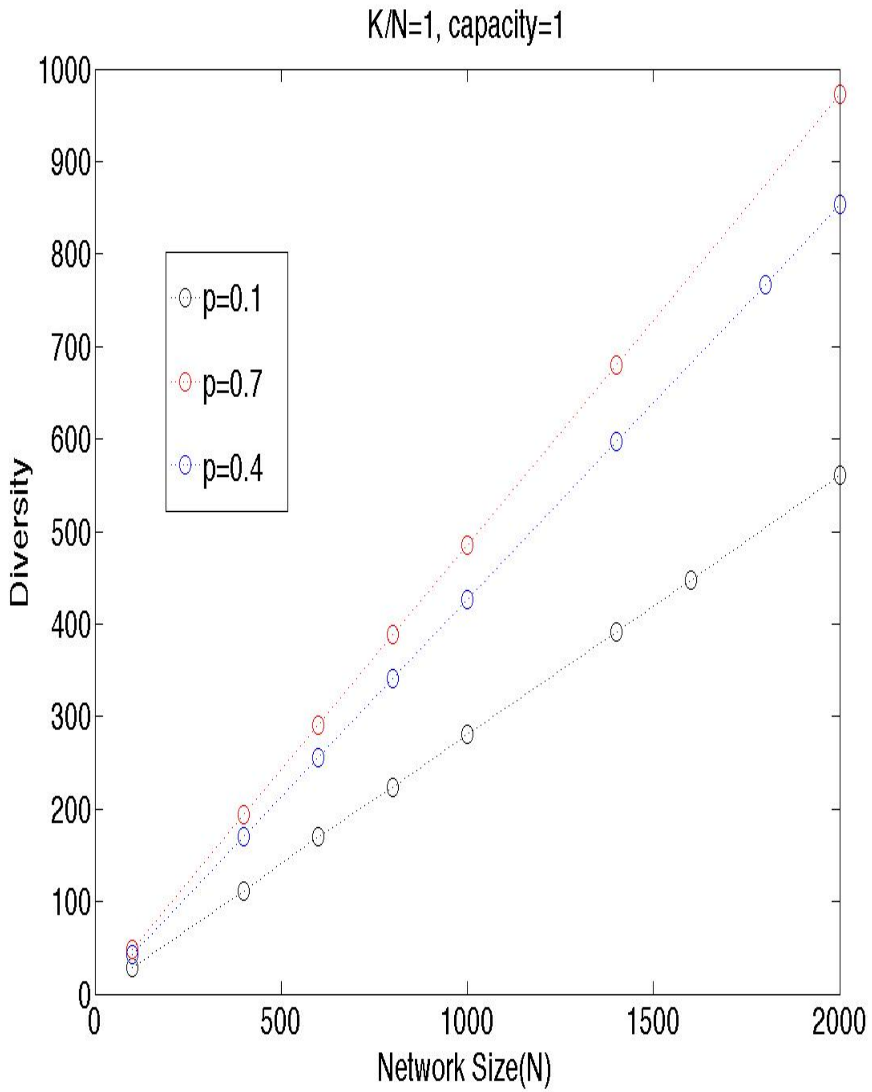


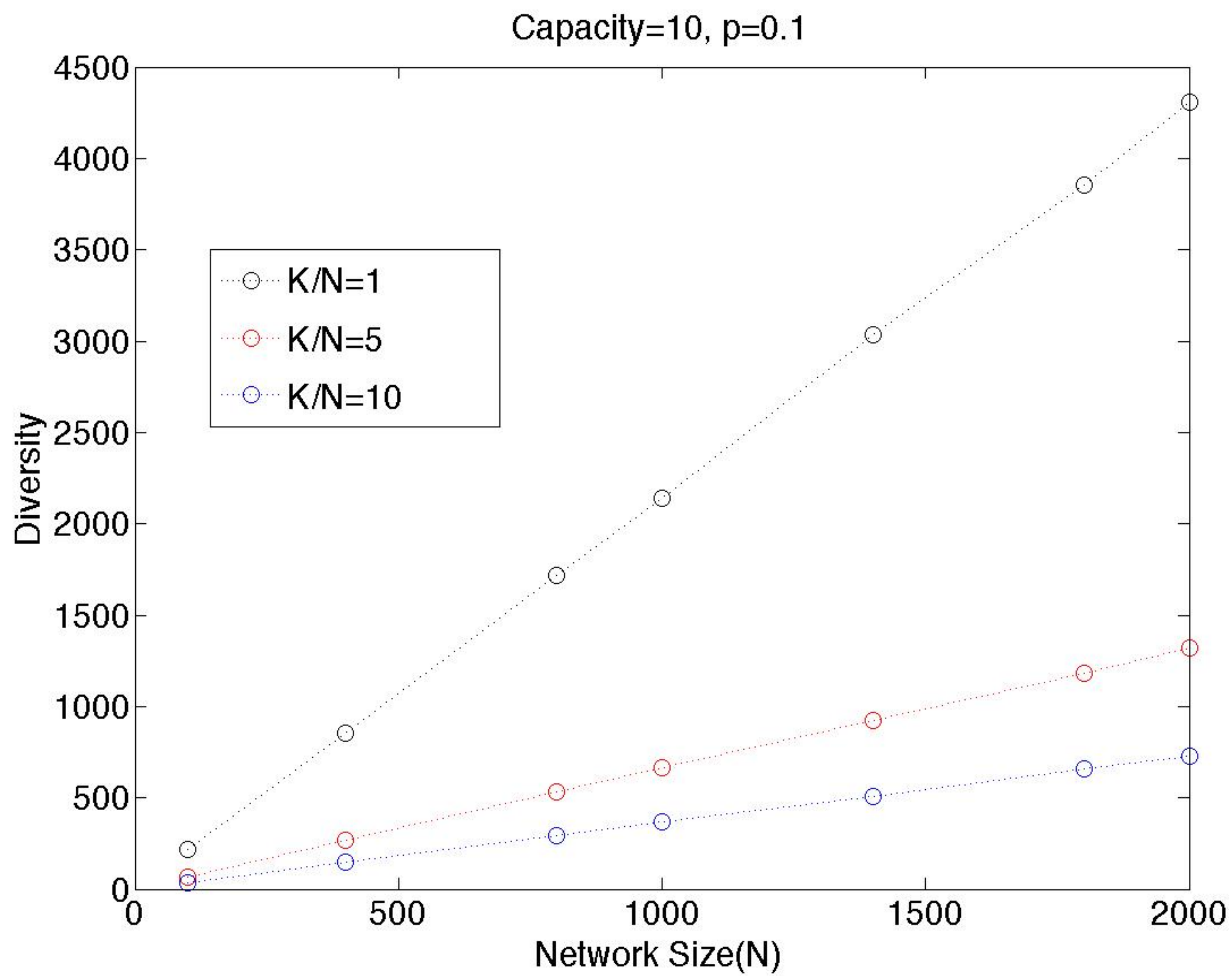
Neutral Competitive Exclusion Model on the Random Network

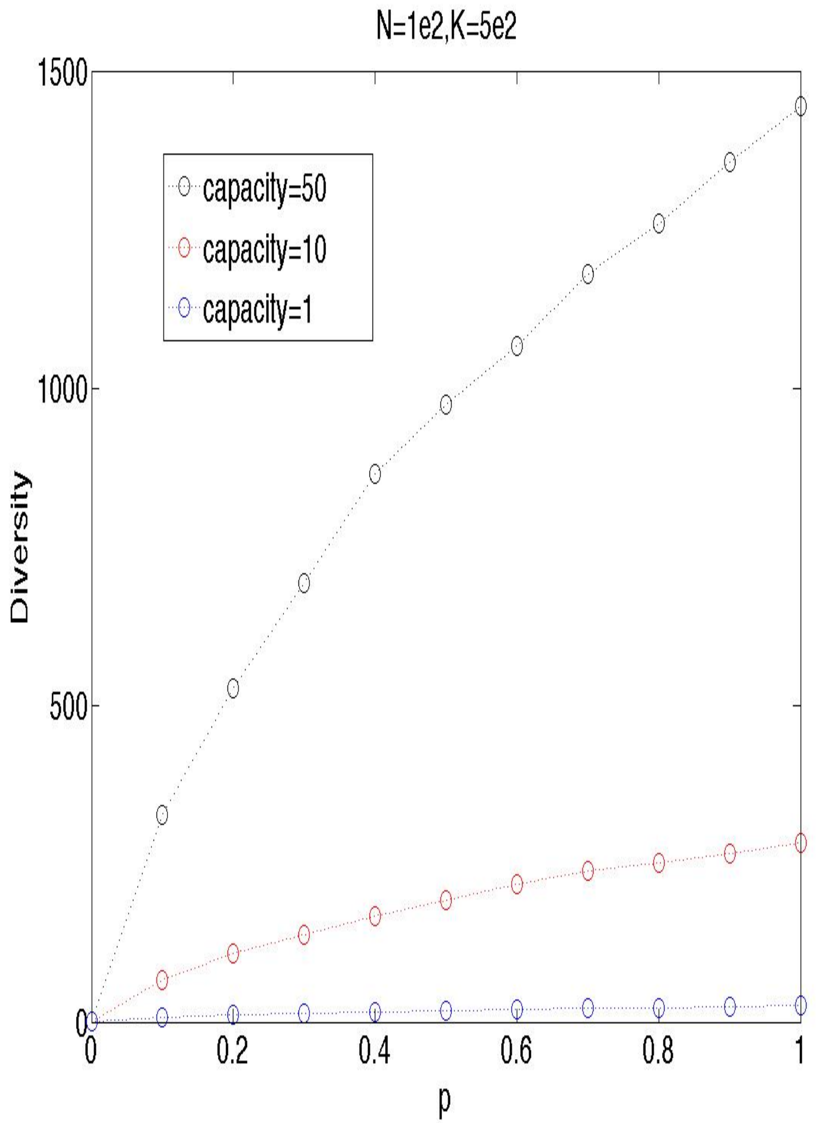


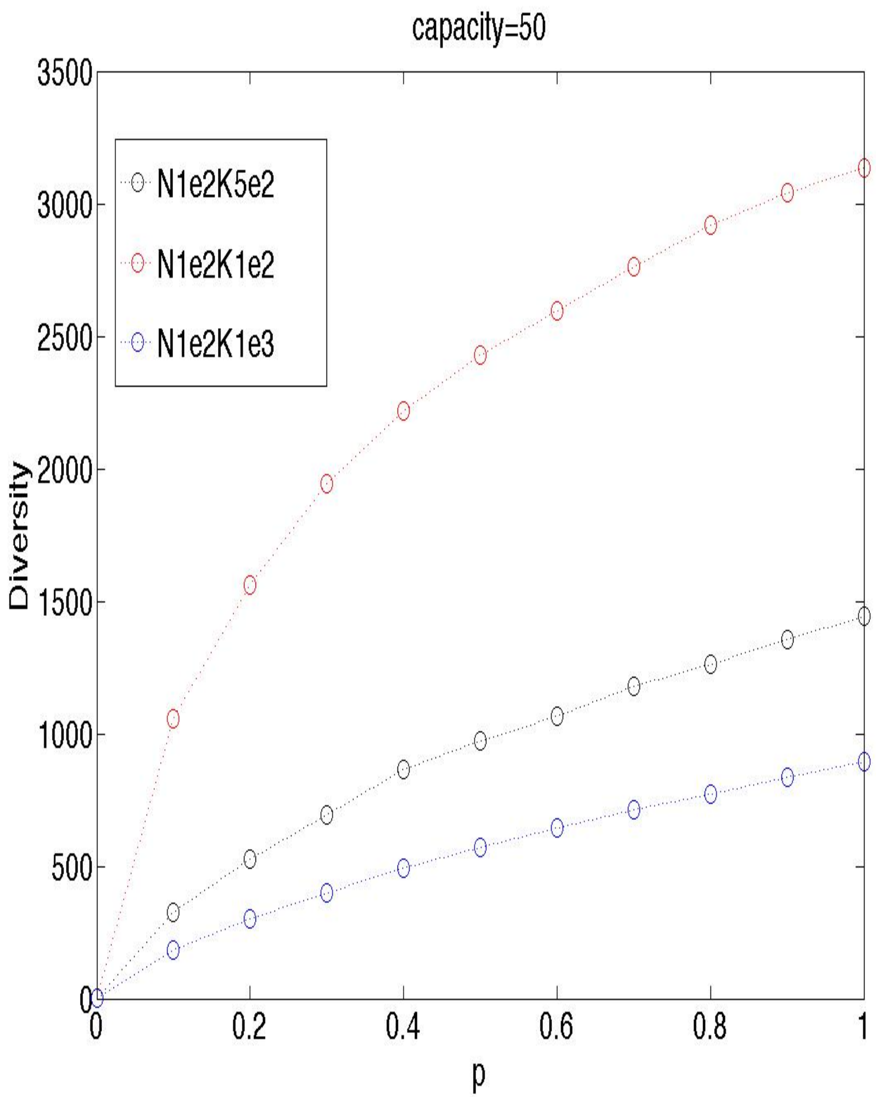
Diversity


p=0.1, K/N=1
x 10^4
Diversity
N(Network Size)
Capacity=50
Capacity=10
capacity=1
0
0.5
1
1.5
2
500
1000
1500
2000

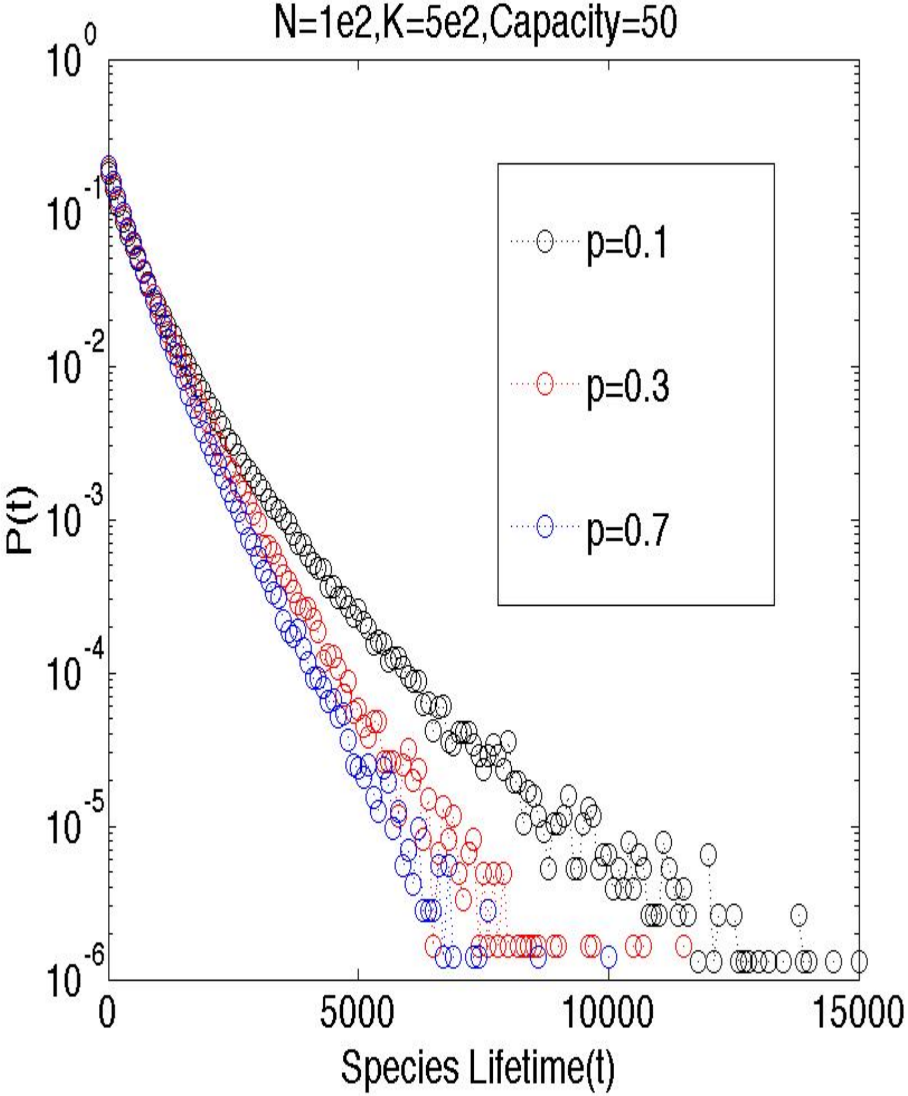

K/N=1, capacity=1
Diversity
Network Size(N)
p=0.1
p=0.7
p=0.4
0
100
200
300
400
500
600
700
800
900
1000
500
1000
1500
2000

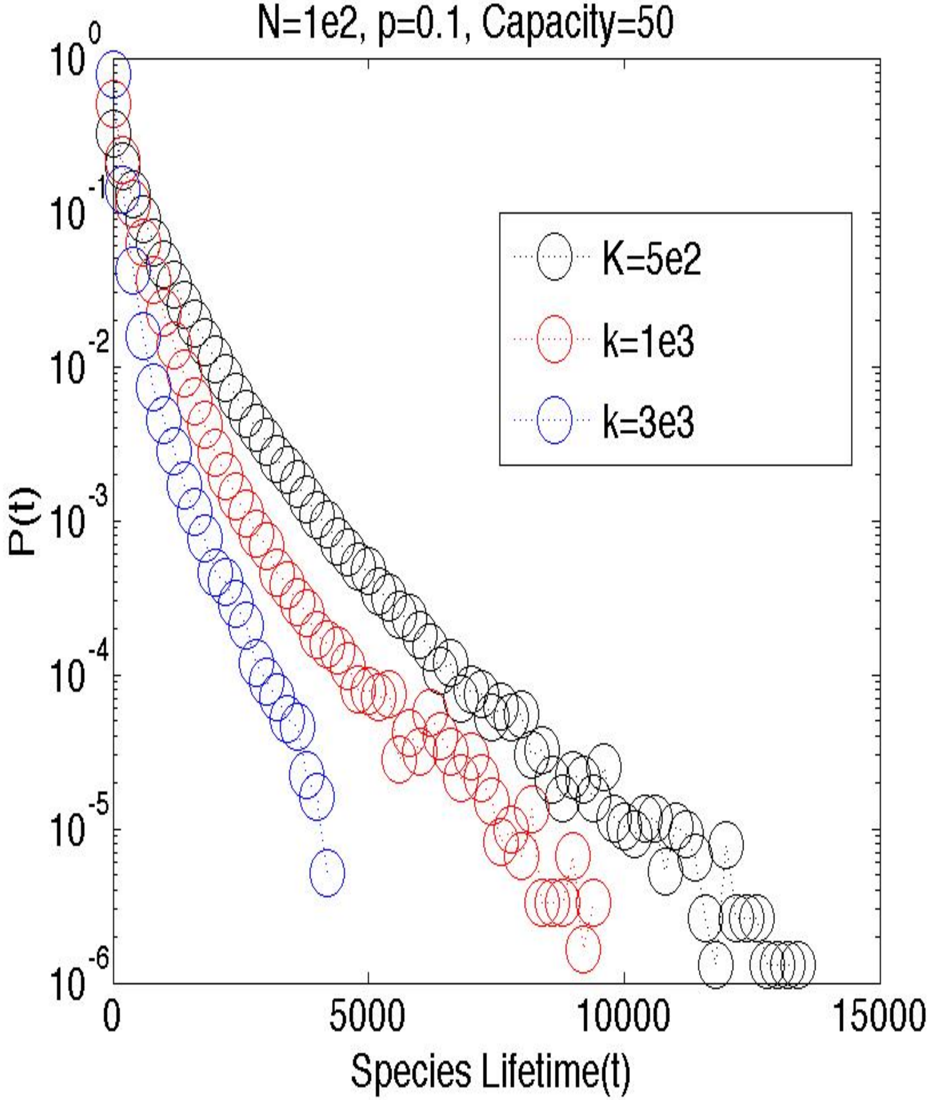

Capacity=10, p=0.1
Diversity
Network Size(N)
K/N=1
K/N=5
K/N=10
0
500
1000
1500
2000
2500
3000
3500
4000
4500
0
500
1000
1500
2000

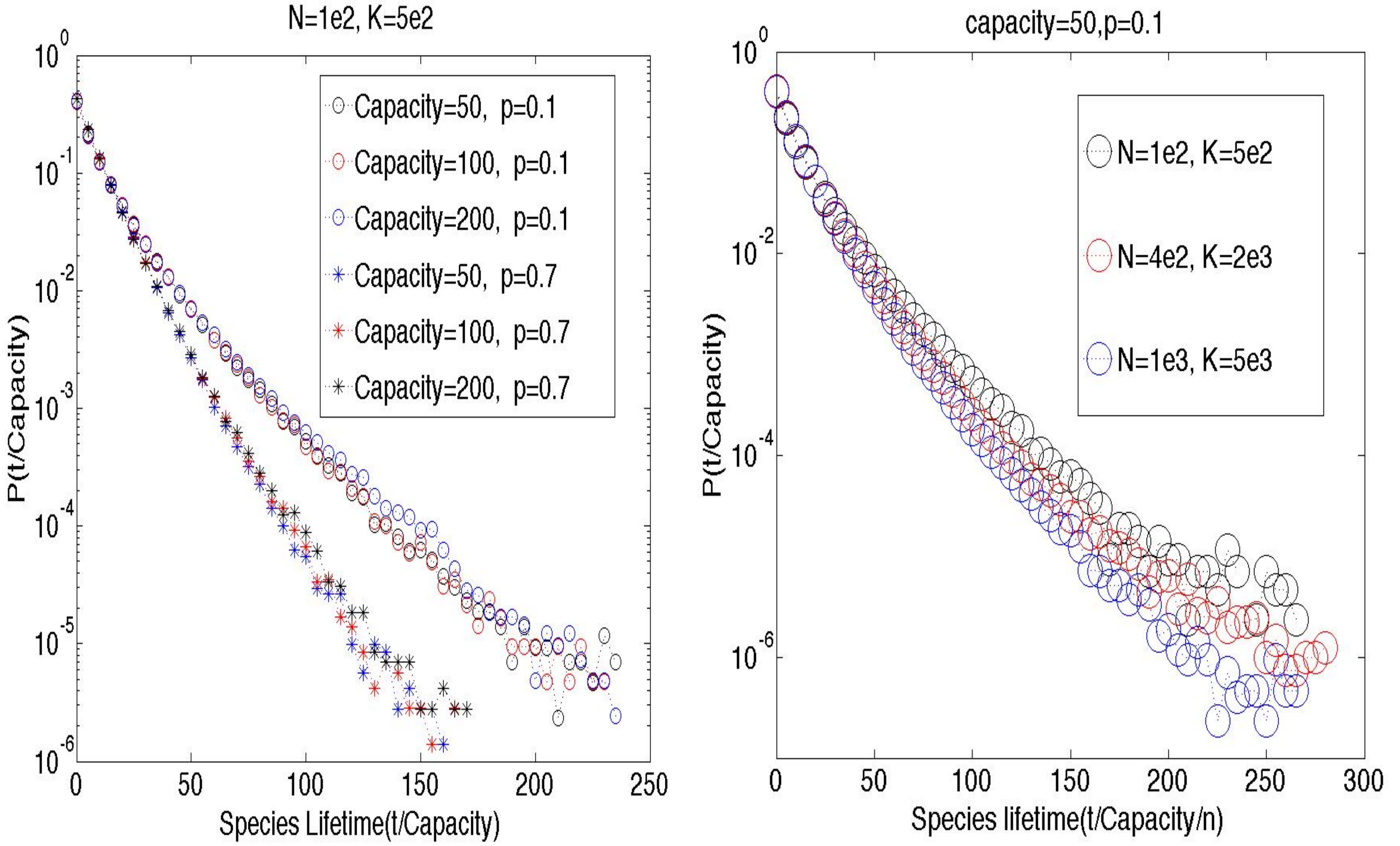

N=1e2,K=5e2
capacity=50
capacity=10
capacity=1
Diversity
p
0
500
1000
1500
0.2
0.4
0.6
0.8
1


capacity=50
N1e2K5e2
N1e2K1e2
N1e2K1e3
Diversity
p
0
500
1000
1500
2000
2500
3000
3500
0.2
0.4
0.6
0.8
1

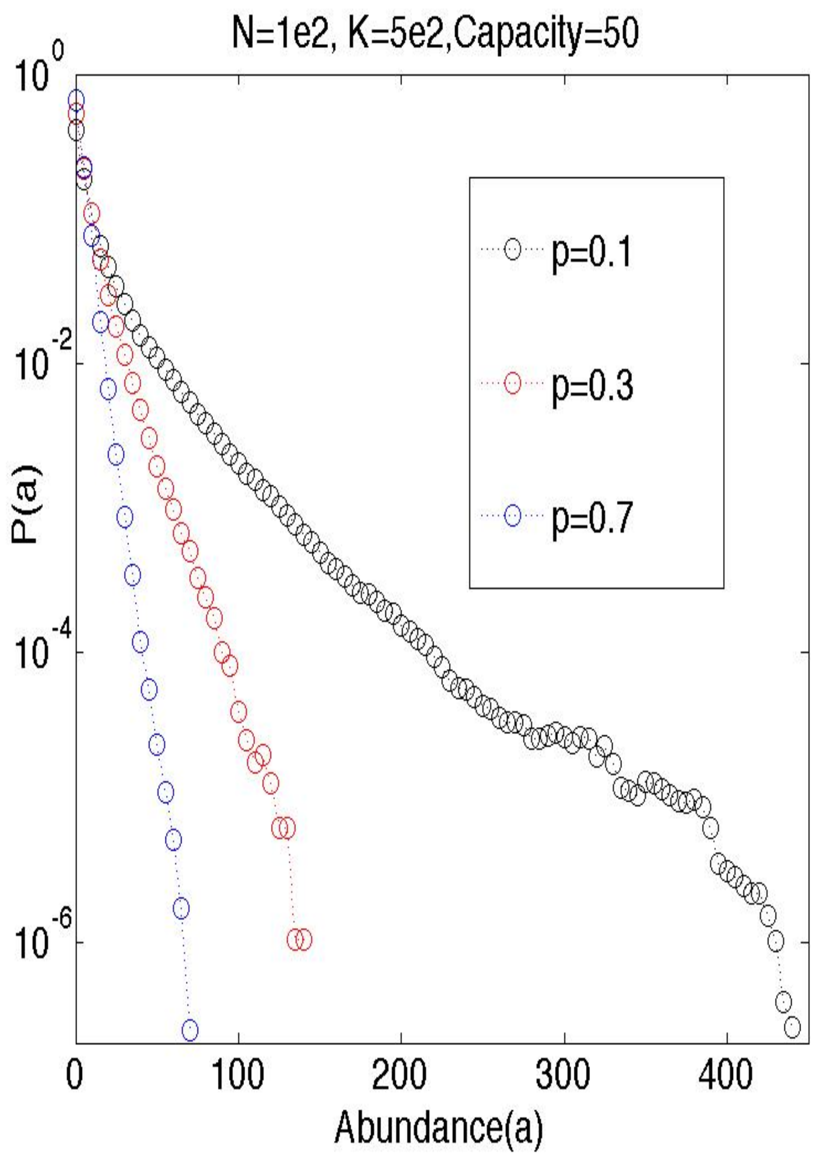
Lifetime

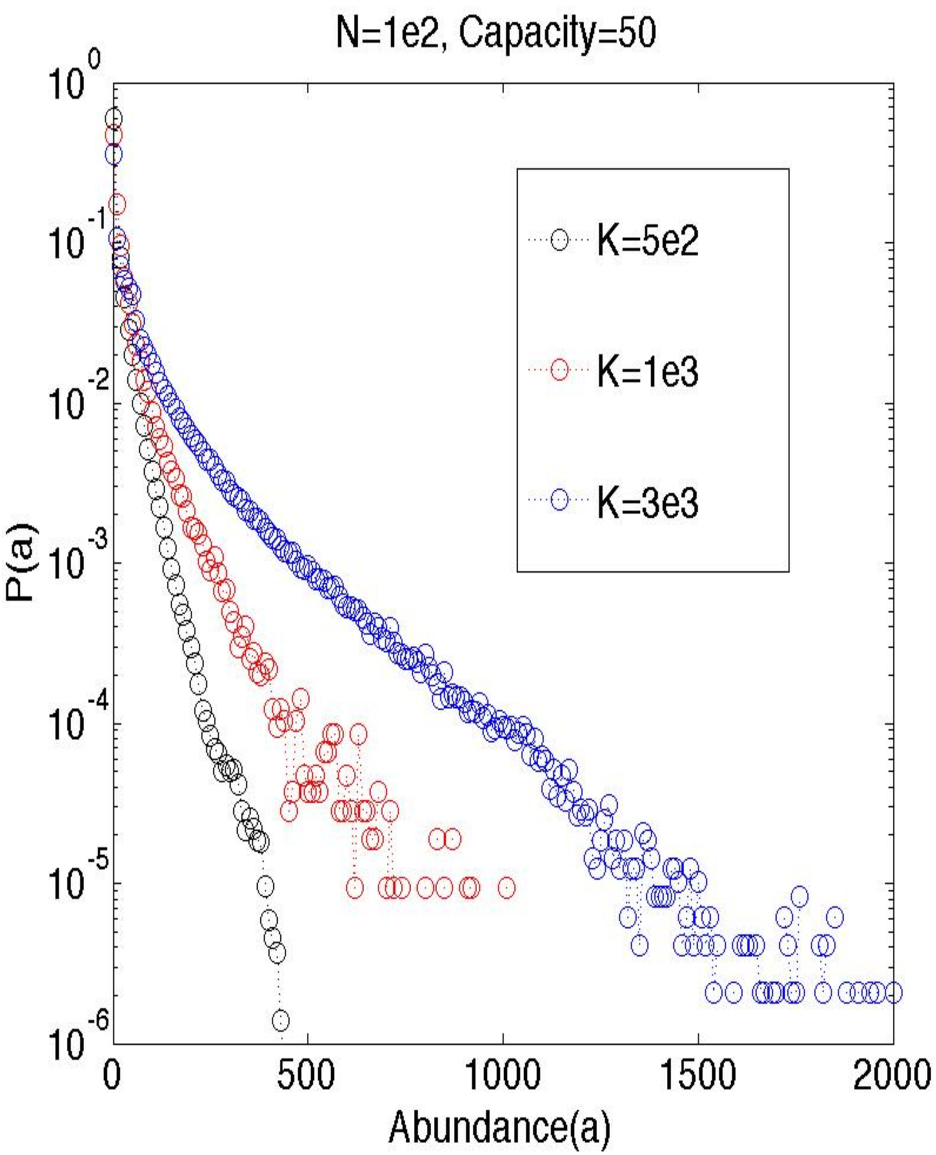

N=1e2,K=5e2,Capacity=50
P(t)
Species Lifetime(t)
p=0.1
p=0.3
p=0.7
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
5000
10000
15000

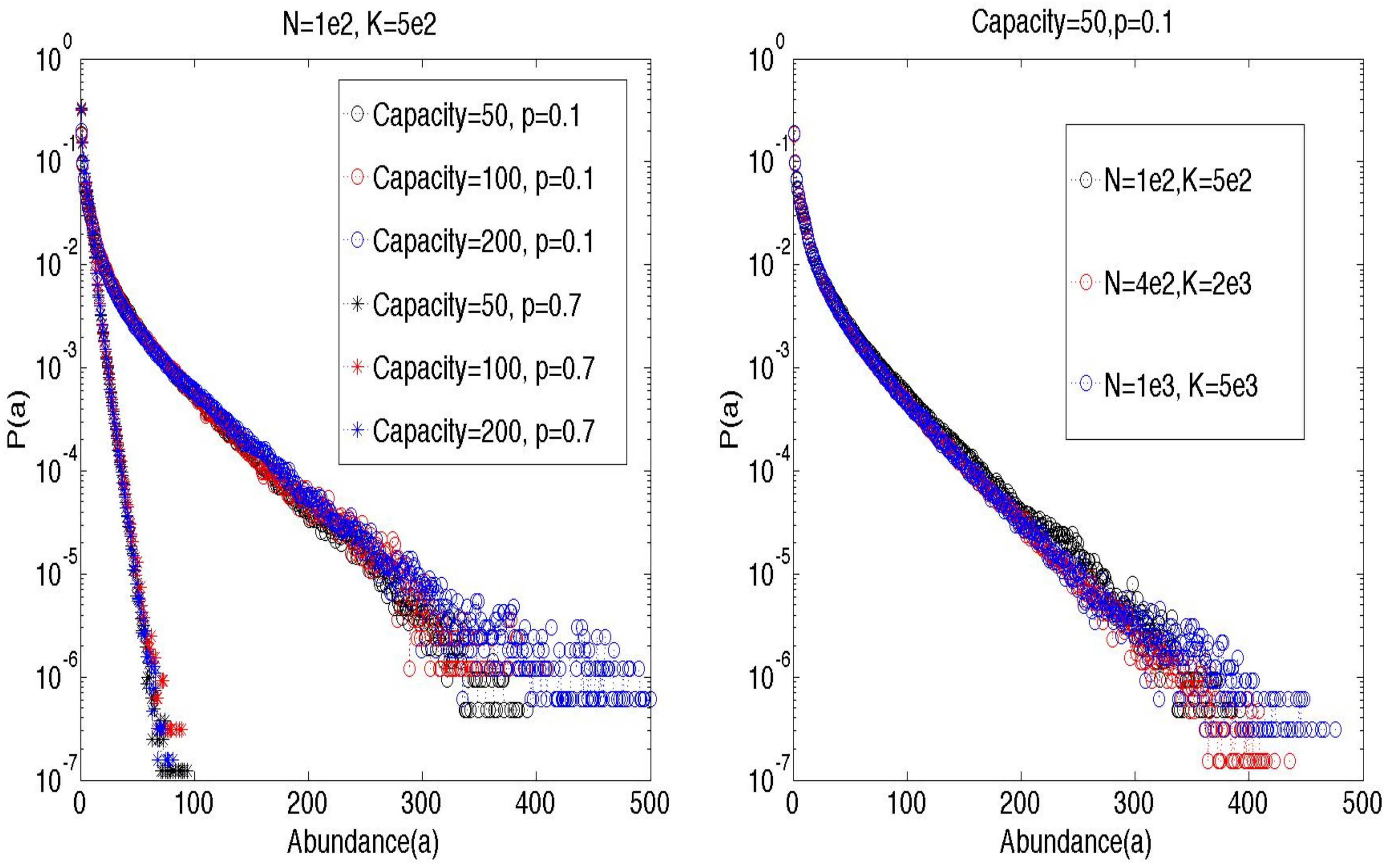

N=1e2, p=0.1, Capacity=50
P(t)
Species Lifetime(t)
K=5e2
k=1e3
k=3e3
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
5000
10000
15000


N=1e2, K=5e2
Capacity=50, p=0.1
Capacity=100, p=0.1
Capacity=200, p=0.1
Capacity=50, p=0.7
Capacity=100, p=0.7
Capacity=200, p=0.7
P(t/Capacity)
Species Lifetime(t/Capacity)
capacity=50,p=0.1
N=1e2, K=5e2
N=4e2, K=2e3
N=1e3, K=5e3
P(t/Capacity)
Species lifetime(t/Capacity/n)

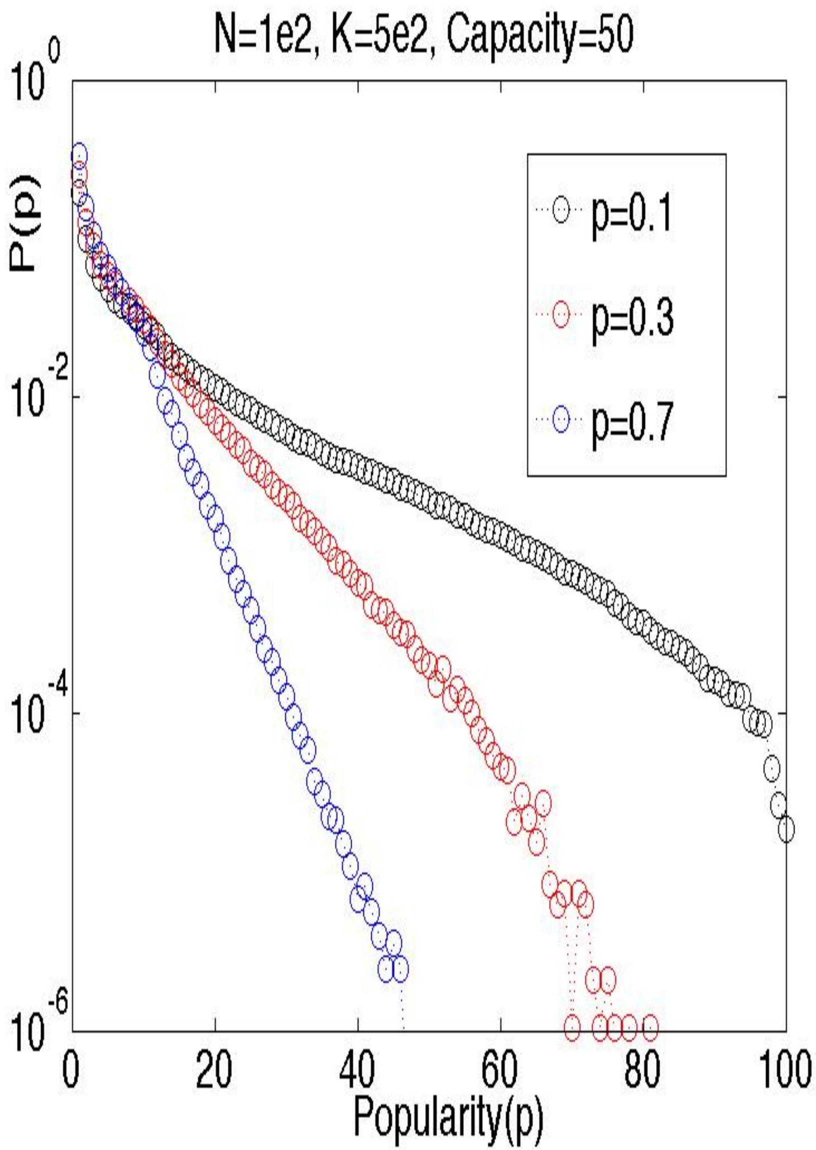
Abundance

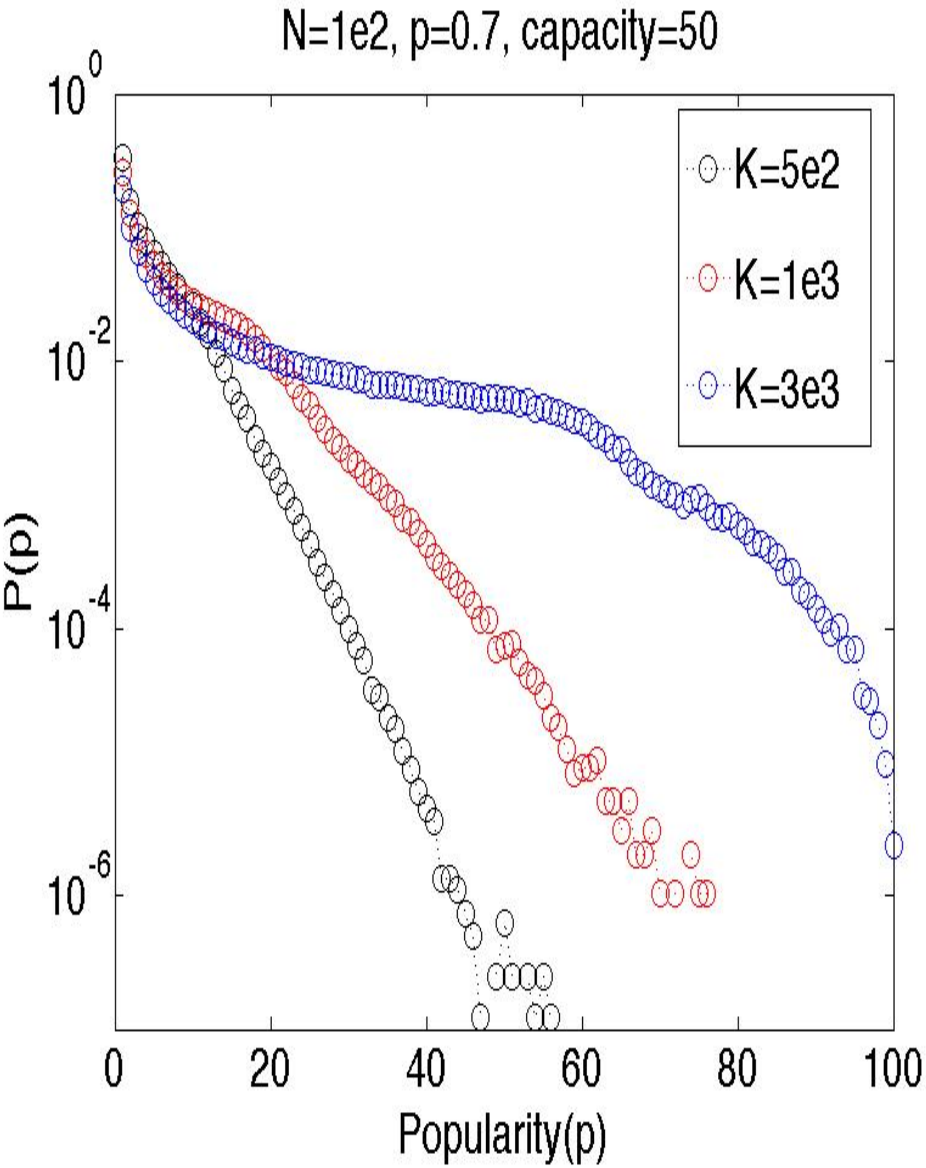

N=1e2, K=5e2,Capacity=50
p=0.1
p=0.3
p=0.7
P(a)
Abundance(a)
10^{0}
10^{-2}
10^{-4}
10^{-6}
0
100
200
300
400


N=1e2, Capacity=50
K=5e2
K=1e3
K=3e3
P(a)
Abundance(a)
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
500
1000
1500
2000

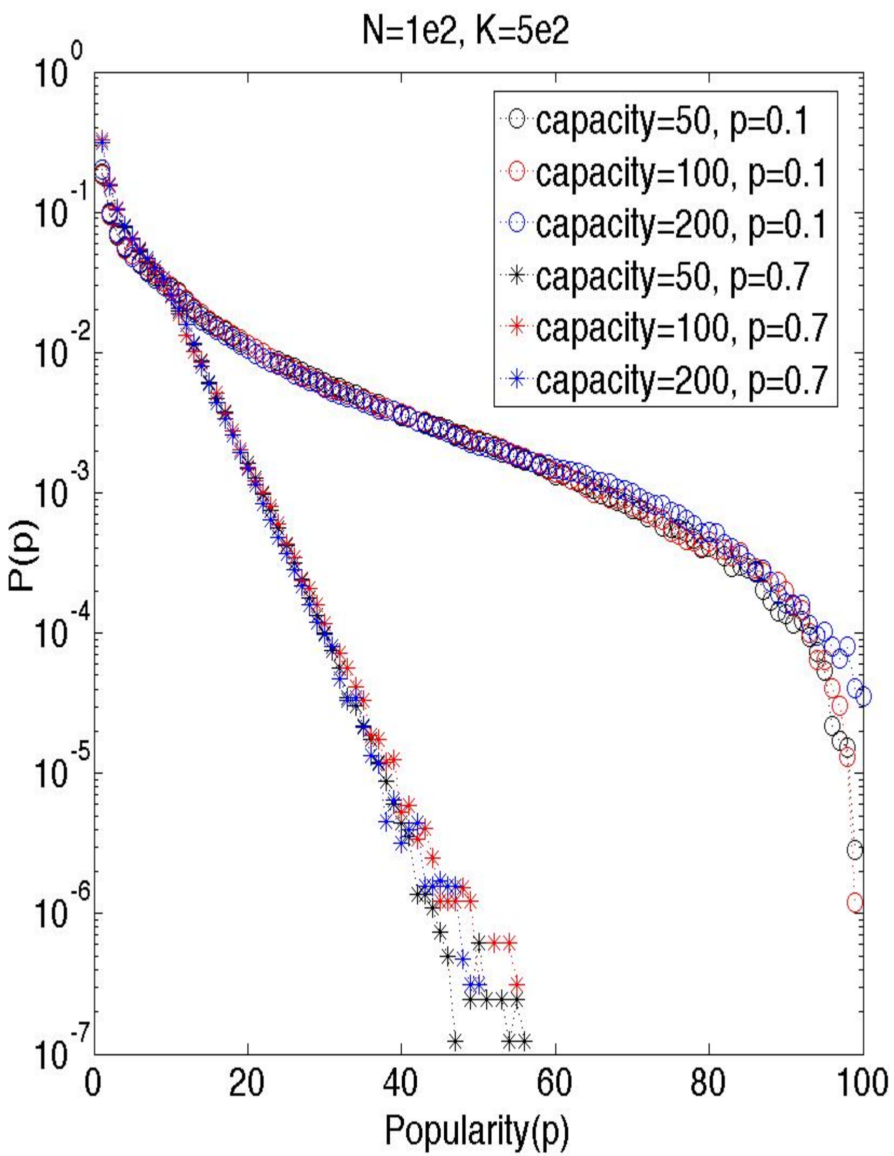

N=1e2, K=5e2
Capacity=50, p=0.1
Capacity=100, p=0.1
Capacity=200, p=0.1
Capacity=50, p=0.7
Capacity=100, p=0.7
Capacity=200, p=0.7
P(a)
Abundance(a)
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
100
200
300
400
500
Capacity=50,p=0.1
N=1e2,K=5e2
N=4e2,K=2e3
N=1e3, K=5e3

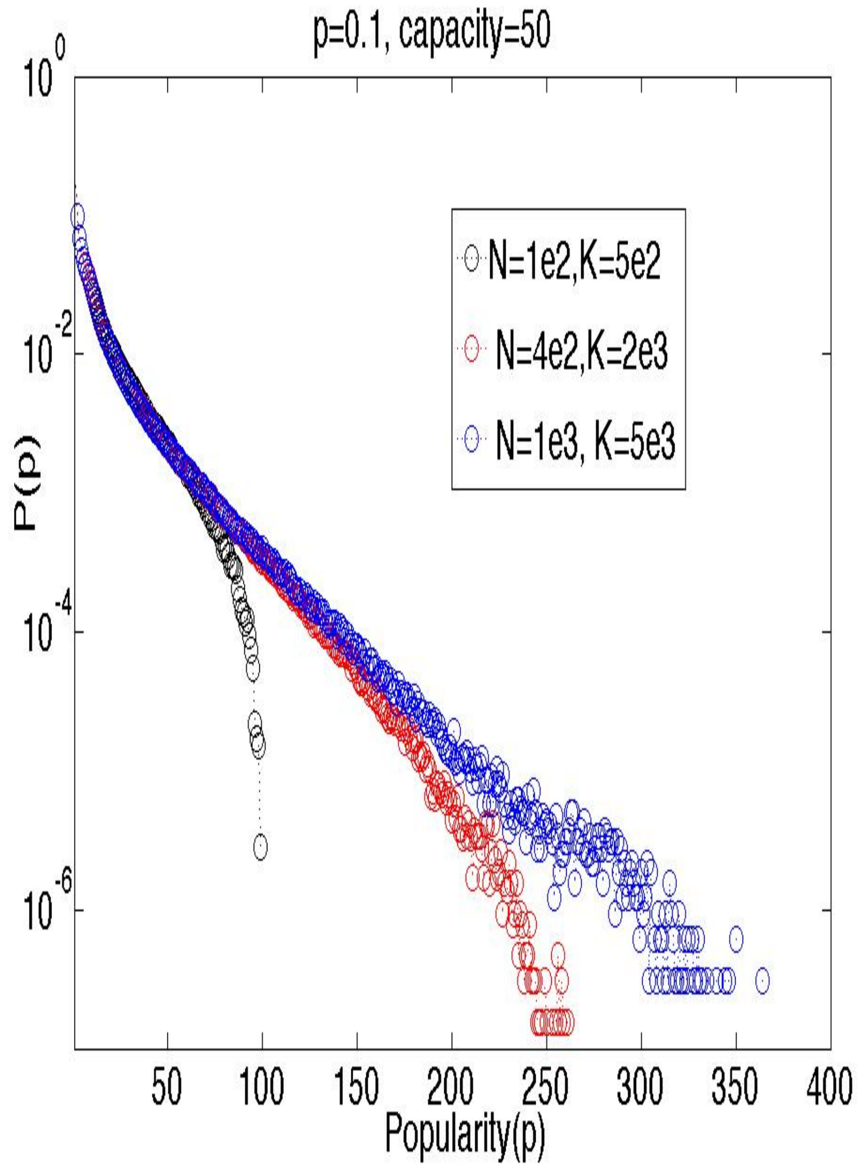
Populartiy


N=1e2, K=5e2, Capacity=50
P(p)
Popularity(p)
p=0.1
p=0.3
p=0.7
10^0
10^{-2}
10^{-4}
10^{-6}
0
20
40
60
80
100


N=1e2, p=0.7, capacity=50
P(p)
Popularity(p)
K=5e2
K=1e3
K=3e3
10^0
10^{-2}
10^{-4}
10^{-6}
0
20
40
60
80
100

CNetS | Center for Complex Networks and Systems Research





Short Summary

- Diversity in the system first decreases and then reaches to 1(when p=0) or a quasistable state with fluctuation(when $p \neq 0$) when the system reaches to steady state, and is linear growth with system size, and nonlinear growth with p.

- The lifetime is exponential distribution, can be rescaled by capacity and system size. It decreases with increasing p and average degree.

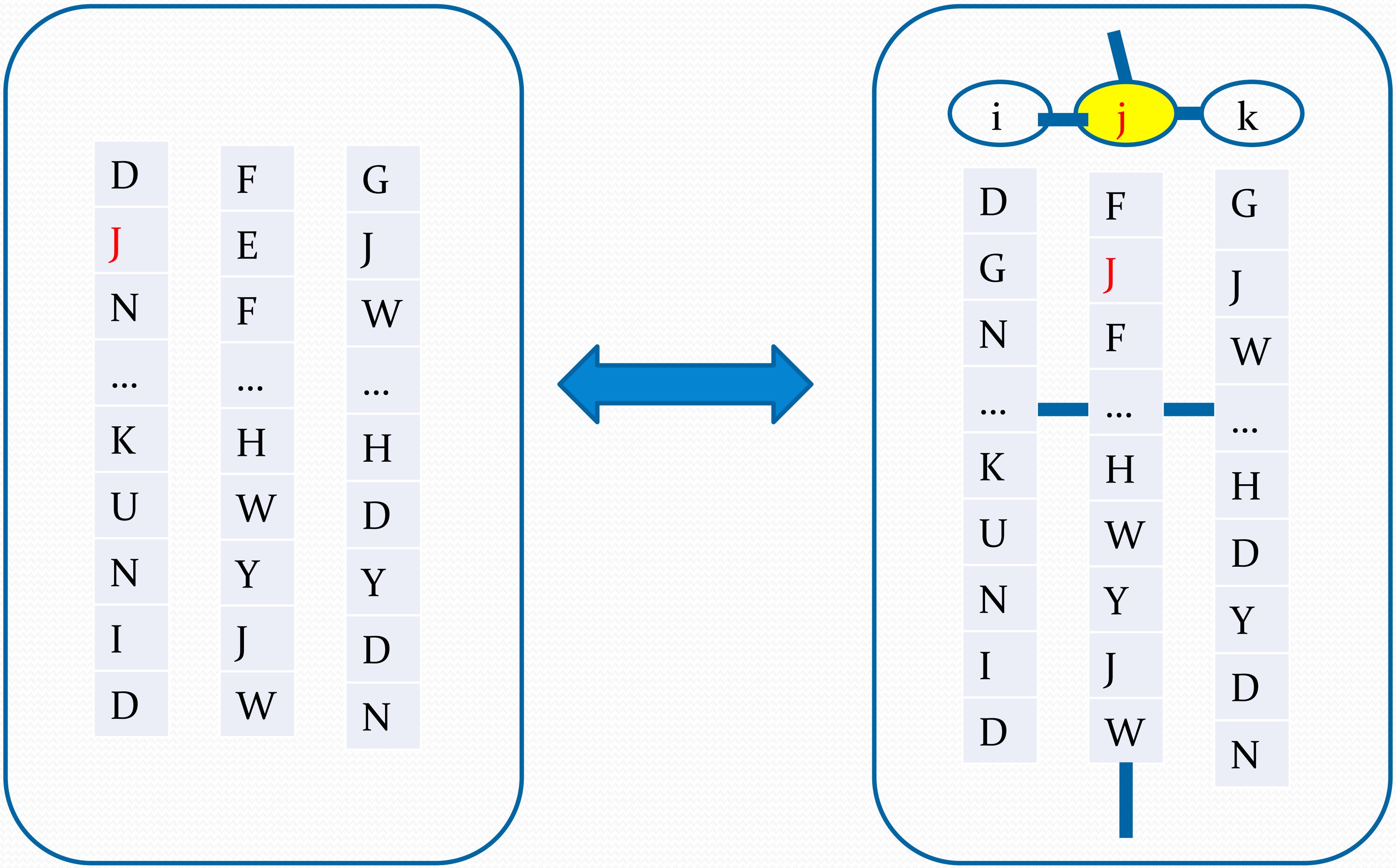
- The abundance is exponential distribution, and independent of capacity and system size. It decreases with increasing p, and increases with increasing average degree

- The popularity is exponential distribution, and independent of capacity and system size. It is decrease with increasing p, increase with increasing average degree.

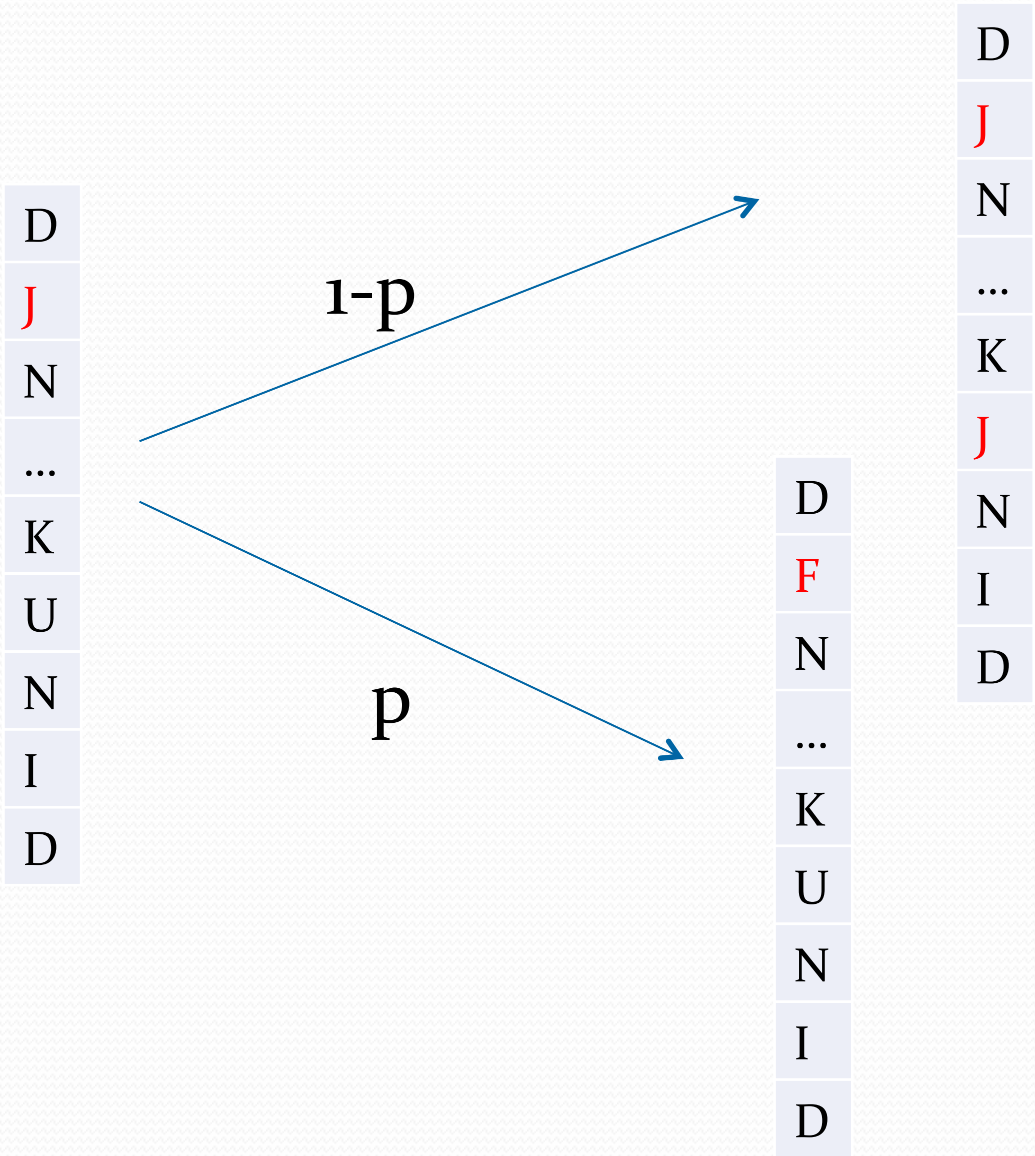
Neutral competitive exclusion model generates robust diversity, exponential distribution of lifetime, abundance and popularity. The result are main influenced by p and average degree.

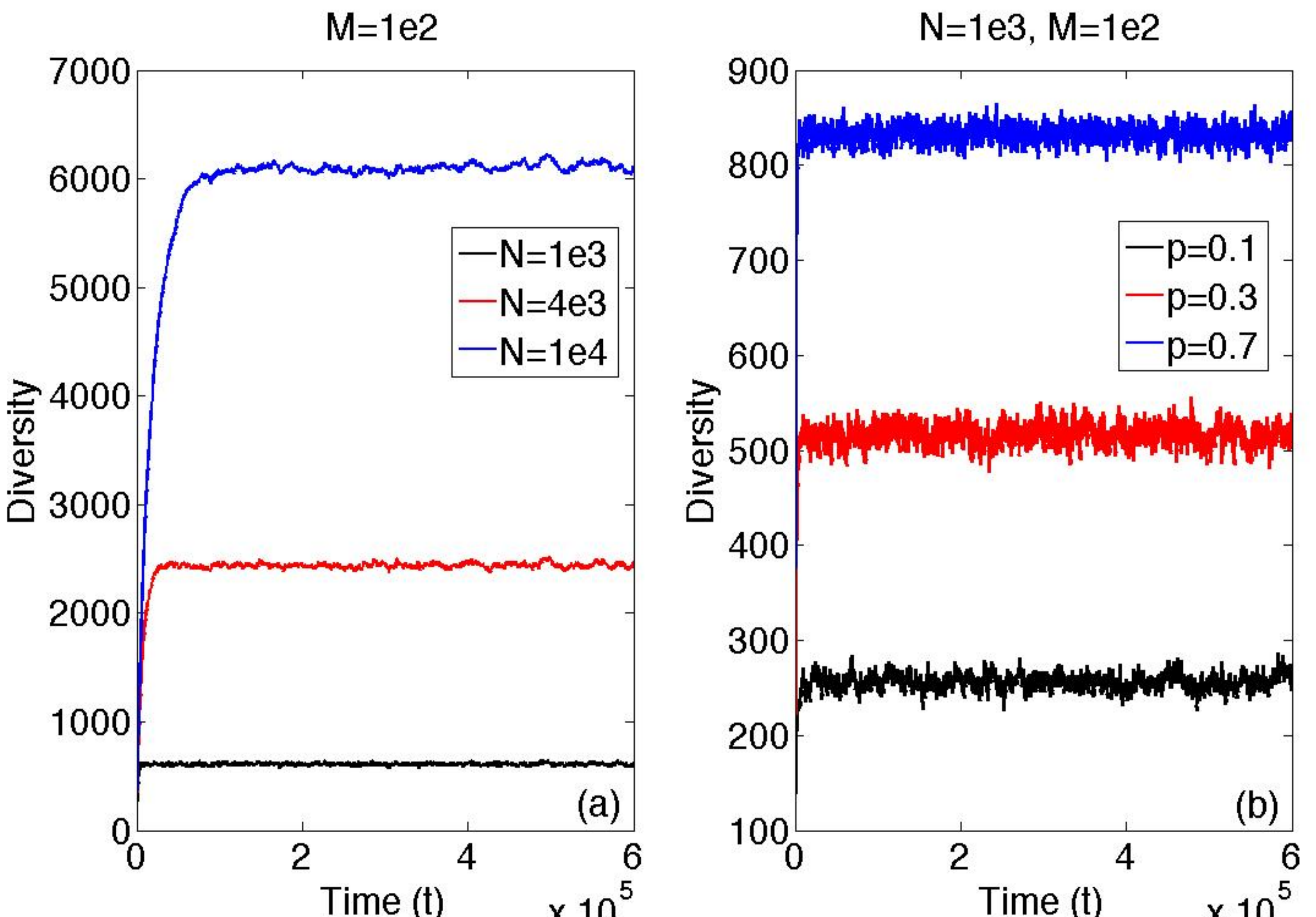
Simplified Toy Neutral Competitive exclusion Model

Neutral Competitive Exclusion Model



Simplified Toy Neutral Competitive exclusion Model



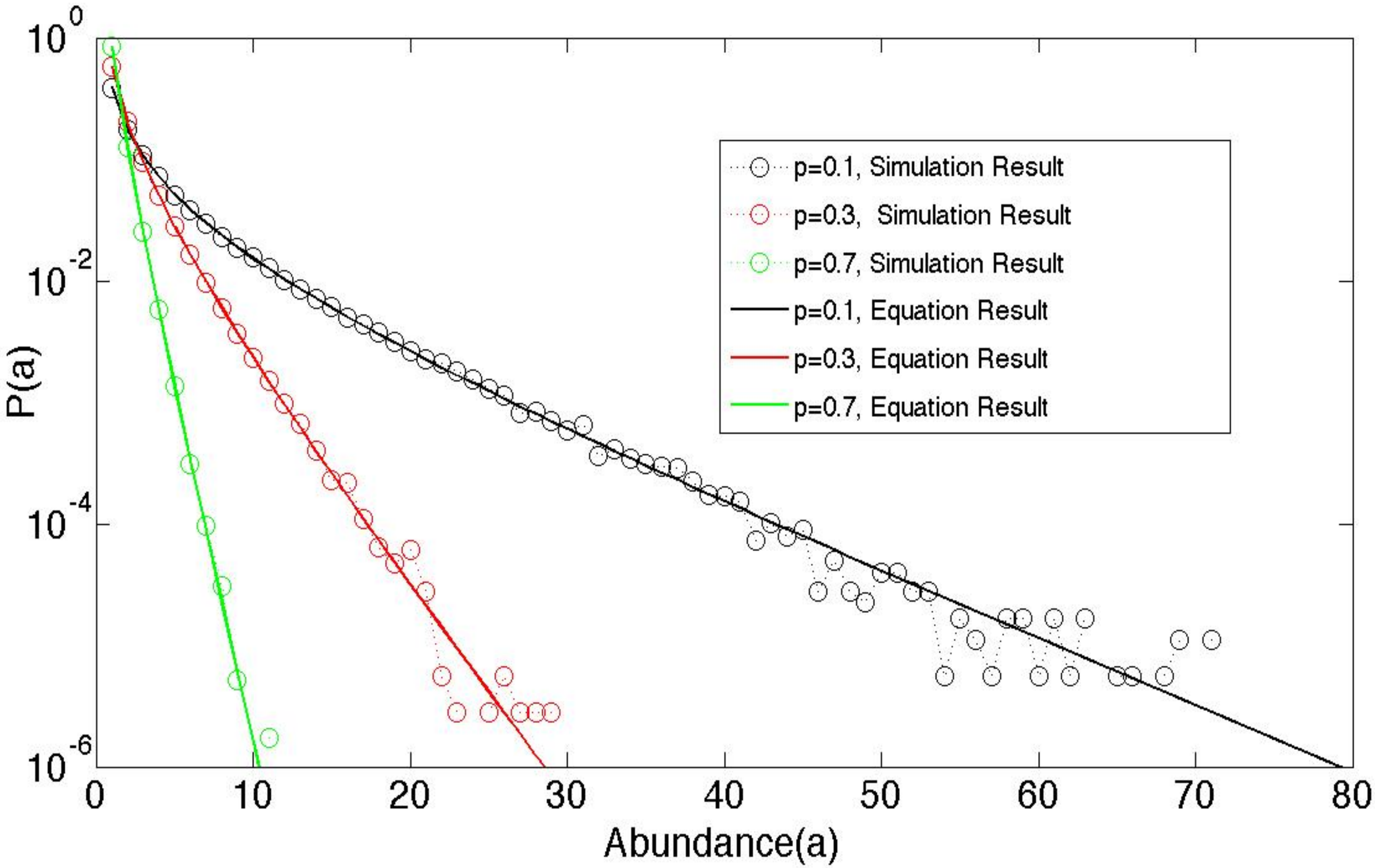

M=1e2
7000
6000
5000
4000
3000
2000
1000
0
Diversity
N=1e3
N=4e3
N=1e4
(a)
0
2
4
6
Time (t)
x 10^5
N=1e3, M=1e2
900
800
700
600
500
400
300
200
100
Diversity
p=0.1
p=0.3
p=0.7
(b)
0
2
4
6
Time (t)
x 10^5

Abundance: Simultaneous equations

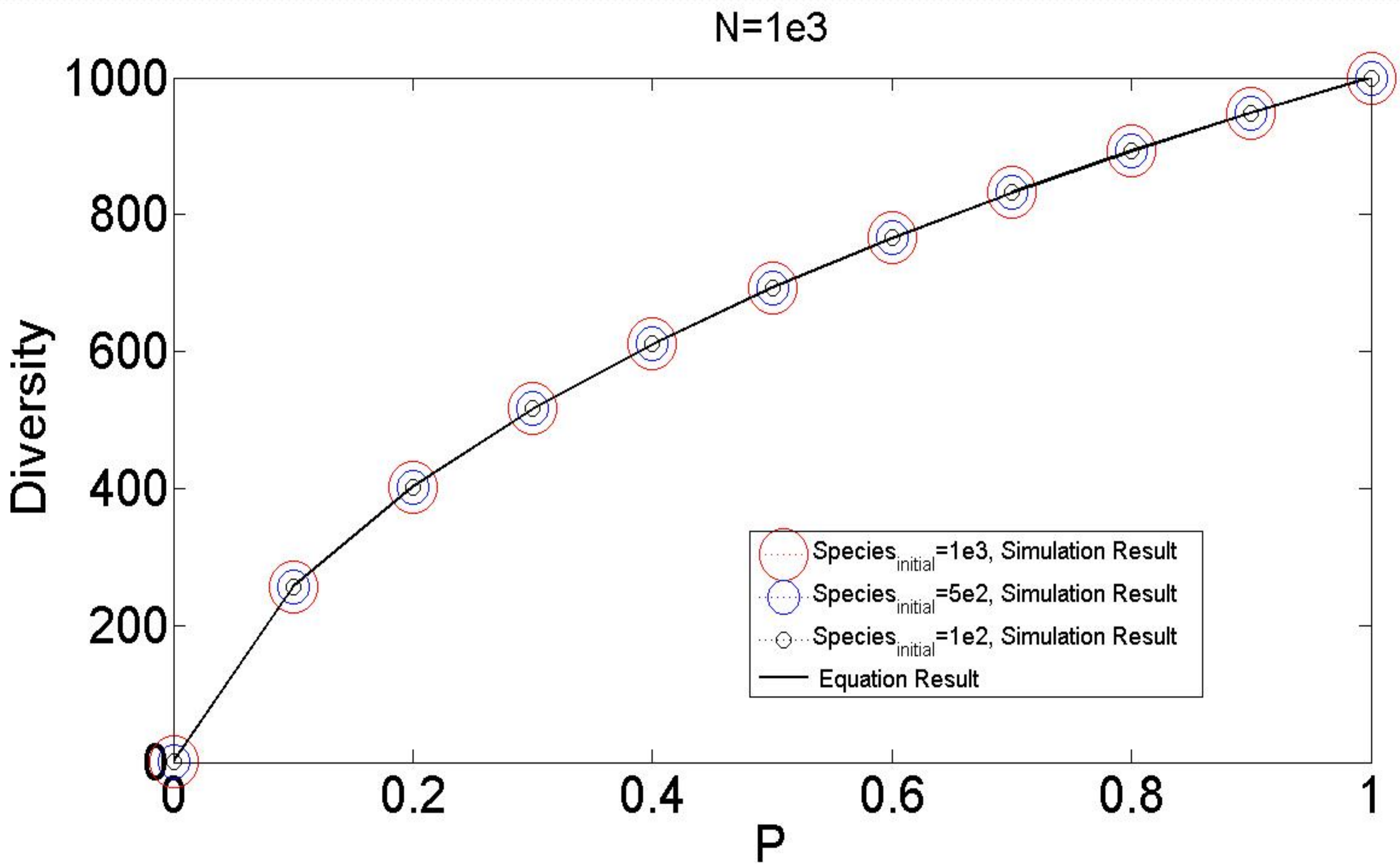
$f_1 \leftrightarrow f_2 \leftrightarrow f_3 \leftrightarrow f_4 \leftrightarrow f_5 \leftrightarrow \cdots \leftrightarrow f_k \leftrightarrow \cdots \leftrightarrow f_{N-1} \leftrightarrow f_N$

$$
\begin{cases}
f_1 \dfrac{1}{N}[p+(1-p)(1-\dfrac{1}{N})] = p \\
f_2 \dfrac{2}{N}[p+(1-p)(1-\dfrac{2}{N})] = f_1 \dfrac{1}{N}(1-\dfrac{1}{N})(1-p) \\
\cdots \\
f_k \dfrac{k}{N}[p+(1-p)(1-\dfrac{k}{N})] = f_{k-1} \dfrac{k-1}{N}(1-\dfrac{k-1}{N})(1-p) \\
\cdots \\
f_{N-1} \dfrac{N-1}{N}[p+(1-p)(1-\dfrac{N-1}{N})] = f_{N-2} \dfrac{N-2}{N}(1-\dfrac{N-2}{N})(1-p) \\
f_N \dfrac{N}{N}[p] = f_{N-1} \dfrac{N-1}{N}(1-\dfrac{N-1}{N})(1-p)
\end{cases}
$$

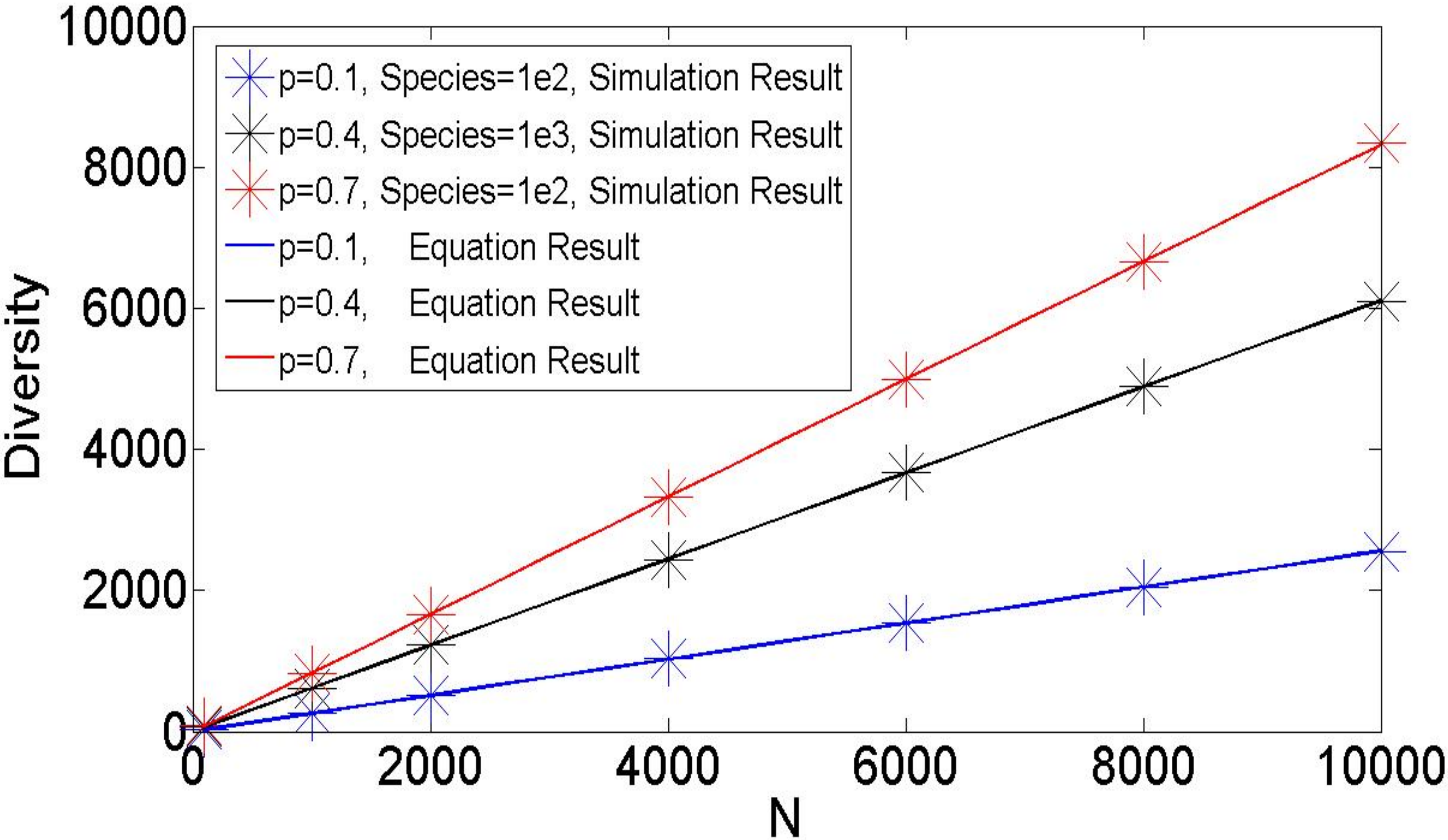
$$f_k = \frac{Np}{k} \frac{N!}{(N-k)!} \frac{(1-p)^{k-1}}{\prod_{i=N-k}^{N-1} [Np + i(1-p)]} \approx p(1-p)^{k-1}$$



$$D=\sum_{k=1}^{N} f_k=\sum_{k=1}^{N}\frac{Np}{k}\frac{N!}{(N-k)!}\frac{(1-p)^{k-1}}{\prod_{i=N-k}^{N-1}[Np+i(1-p)]}\approx Np\sum_{k=1}^{N}\frac{(1-p)^{k-1}}{k}\approx\frac{Np}{1-p}Ei$$



$$D=\sum_{k=1}^{N} f_k=\sum_{k=1}^{N}\frac{Np}{k}\frac{N!}{(N-k)!}\frac{(1-p)^{k-1}}{\prod_{i=N-k}^{N-1}[Np+i(1-p)]}\approx Np\sum_{k=1}^{N}\frac{(1-p)^{k-1}}{k}\approx\frac{Np}{1-p}Ei$$

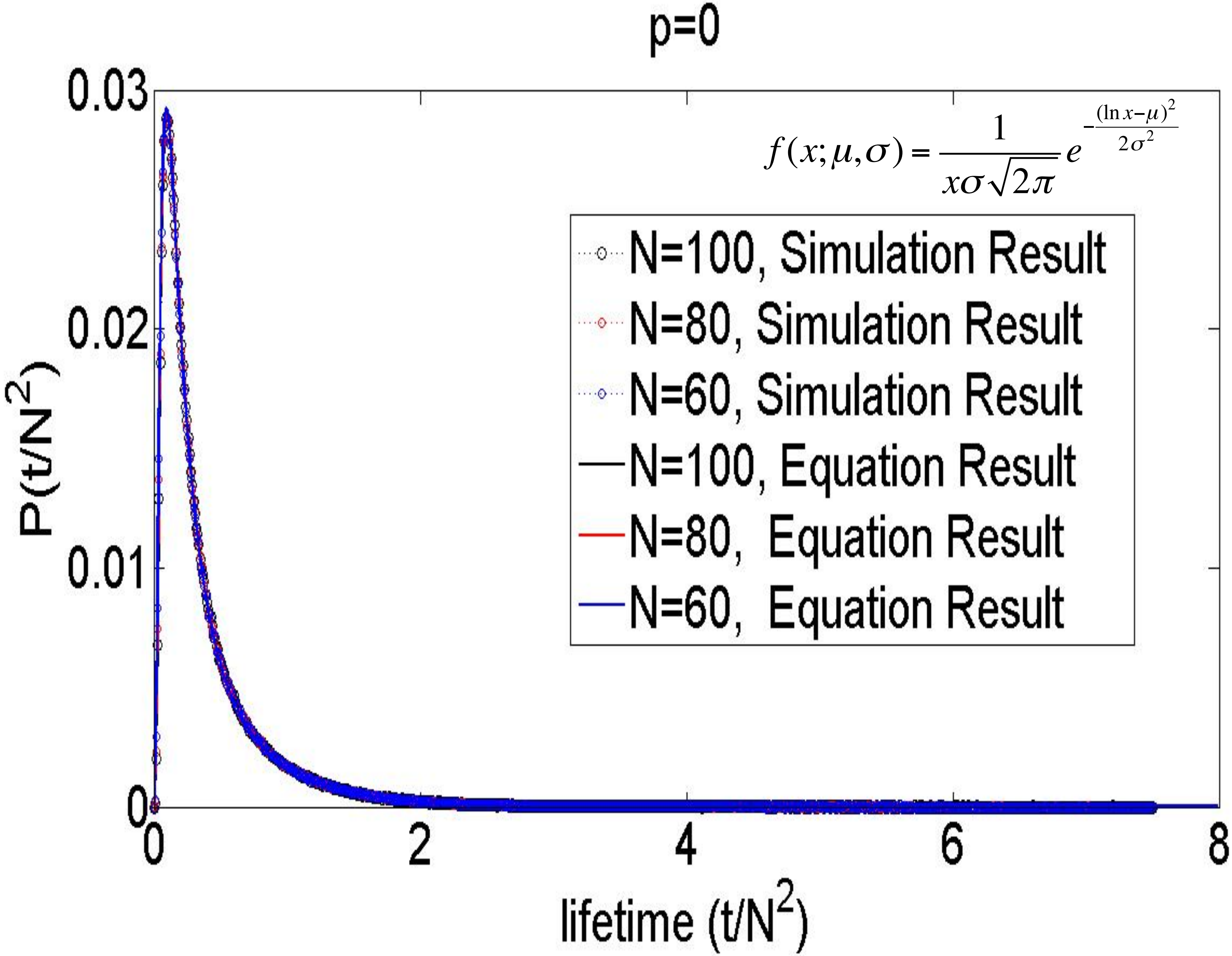


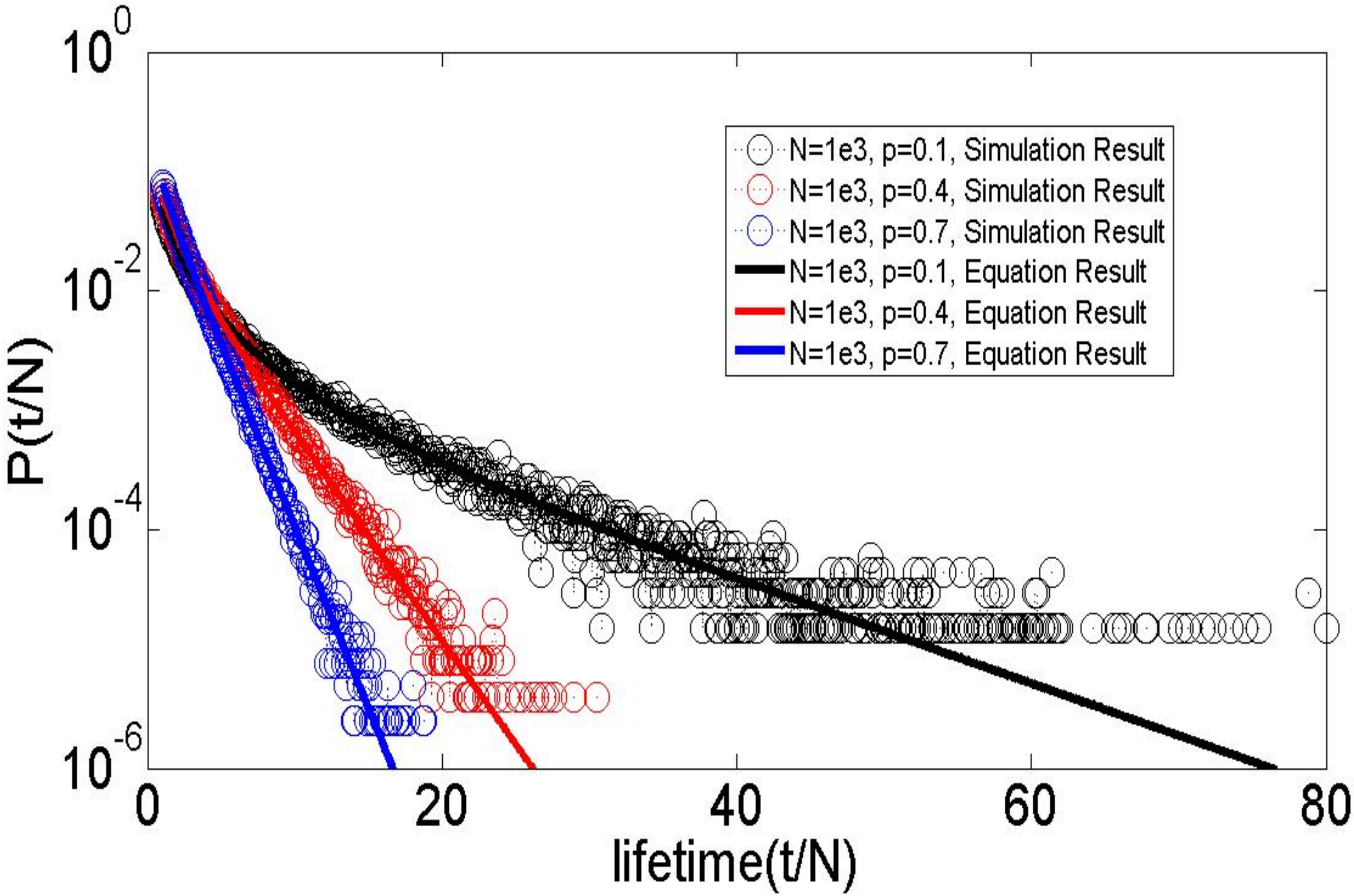
Lifetime : evolution Equation

$$p(n,t+1) = p(n-1,t)\frac{N-(n-1)}{N}\frac{n-1}{N}(1-p)$$

$$+\,p(n,t)\frac{N-n}{N}[\frac{N-n}{N}(1-p)+p]+\frac{n}{N}\frac{n}{N}(1-p)$$

$$+\,p(n+1,t)\frac{n+1}{N}[\frac{N-(n+1)}{N}(1-p)+p]$$

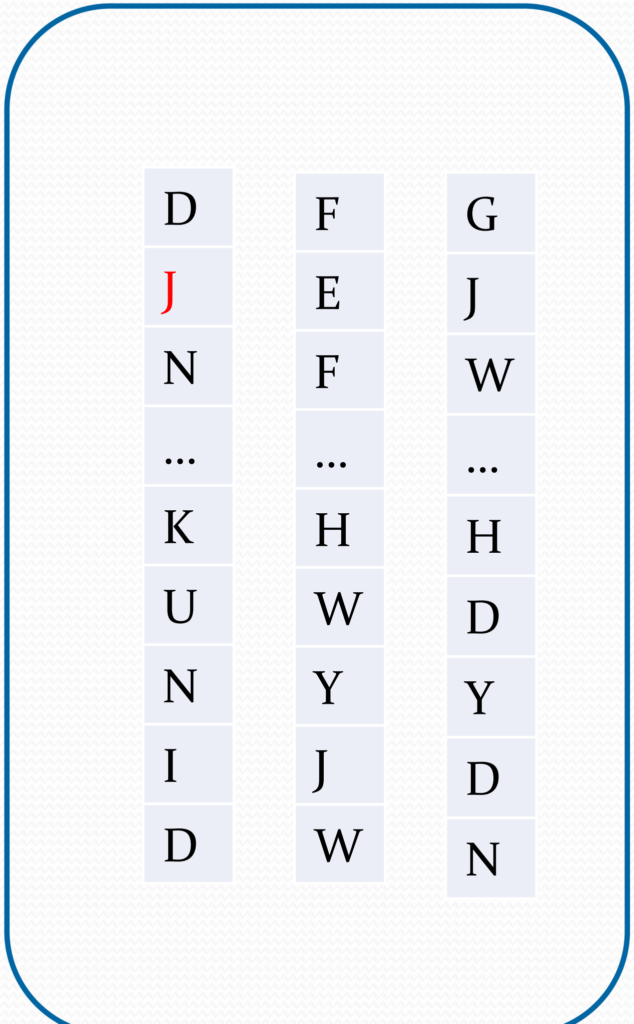

p=0
$$f(x;\mu,\sigma)=\frac{1}{x\sigma\sqrt{2\pi}}e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}$$
N=100, Simulation Result
N=80, Simulation Result
N=60, Simulation Result
N=100, Equation Result
N=80, Equation Result
N=60, Equation Result
0.03
0.02
0.01
0
0
2
4
6
8
$P(t/N^2)$
lifetime (t/N^2)


N=1e3, p=0.1, Simulation Result
N=1e3, p=0.4, Simulation Result
N=1e3, p=0.7, Simulation Result
N=1e3, p=0.1, Equation Result
N=1e3, p=0.4, Equation Result
N=1e3, p=0.7, Equation Result
P(t/N)
lifetime(t/N)
10^{0}
10^{-2}
10^{-4}
10^{-6}
0
20
40
60
80

Short Summary

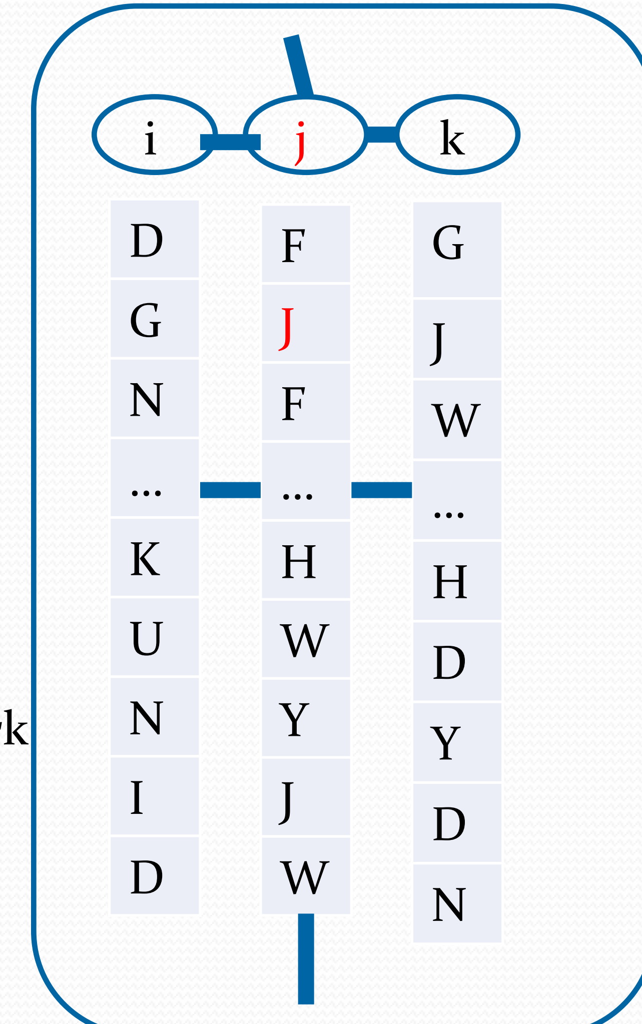
- Diversity decreases to 1(when p=0) or increase to N(p=1), or a quasistable state with fluctuation(when $0<p<1$) when system reaches to steady state.
- Diversity linear growth with system size, and nonlinear growth with p
- The lifetime is log-normal distribution (when p=0) or exponential distribution(when $p \neq 0$), and decrease with increasing p.
- The abundance is exponential distribution and decrease with increasing p

Simplified Toy Neutral Competitive exclusion Model



theoretical framework

Neutral Competitive Exclusion Model



Evolution Equation for lifetime:

$$p(n,t+1) = p(n,t)\frac{N-(n-1)}{N}\frac{n-1}{N}(1-p)$$
$$+ p(n,t)\frac{N-n}{N}[\frac{N-n}{N}(1-p)+p] + \frac{n}{N}\frac{n}{N}(1-p)$$
$$+ p(n+1,t)\frac{n+1}{N}[\frac{N-(n+1)}{N}(1-p)+p]$$

$$\begin{cases}
f_1\frac{1}{N}[p+(1-p)(1-\frac{1}{N})] = p \\
f_2\frac{2}{N}[p+(1-p)(1-\frac{2}{N})] = f_1\frac{1}{N}(1-\frac{1}{N})(1-p) \\
\ldots \\
f_k\frac{k}{N}[p+(1-p)(1-\frac{k}{N})] = f_{k-1}\frac{k-1}{N}(1-\frac{k-1}{N})(1-p) \\
\ldots \\
f_{N-1}\frac{N-1}{N}[p+(1-p)(1-\frac{N-1}{N})] = f_{N-2}\frac{N-2}{N}(1-\frac{N-2}{N})(1-p) \\
f_N\frac{N}{N}[p] = f_{N-1}\frac{N-1}{N}(1-\frac{N-1}{N})(1-p)
\end{cases}$$

Simultaneous Equations for abundance

Thank you !