International Journal of Modern Physics C
Vol. 19, No. 12 (2008) 1939–1947

THE STRONG CONSENSUS OPINION DYNAMICS ON ADAPTIVE NETWORKS

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Received 18 March 2008
Accepted 27 September 2008

Many real-world networks are characterized by adaptive changes in their topology depending on the state of their nodes. Here we explore the continuous Sznajd opinion model on an adaptive network, where the link between agents with far apart opinion will get rewired. Our investigations reveal that the adaptation of the network topology fosters cluster formation by enhancing communication between agents, though the Sznajd rule is most effective to achieve a full synchronization of the agents. The interplay between dynamics and topology can have important consequences for the spreading of the opinions.

Keywords: Sociophysics; Mote Carlo simulations.

PACS Nos.: 89.75.-k, 87.23.Ge.

1. Introduction

Over the last few years, the study of statistical and complex systems has been proved extremely valuable in providing insight into emerging interdisciplinary fields of science in many natural, technological, and social systems.[1–3] In particular, many efforts have focused recently on the mathematical modeling of a rich variety of social phenomena, such as social influence and self-organization, cooperation, opinion formation and spreading, evolution of social structures, etc,[4–7] while opinion dynamics modeling represents a challenging field which can be possibly applied to understand the emergence of collective behaviors, like consensus or polarization in social groups.[4]

The interest in opinion dynamics so far can be directed in two distinct directions. On the one hand, attention has been paid to the dynamics of the models, revealing that simple dynamic rules, such as "United we Stand, Divided we

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fall",[8] majority rule,[9] or bounded confidence,[10] can be used to generate different opinion models. On the other hand, research has focused on large ensembles of dynamical systems, where the interaction between individual agents is described by a complex graph.[1,2] These studies have shown that the network topology can have a strong impact on the dynamics of the models. Until now, few studies have considered the case that nodes are endowed with quenched attributes and links are formed depending on such fixed node properties.[11–15] In fact, real systems are mostly in between these two extreme cases: both intrinsic properties of nodes (like opinion) and connections among them vary with time over comparable temporal scales. The interplay of the two evolutions is then a natural issue to be investigated. Moreover, the evolution of the topology and the dynamical processes can drive each other with complex feedback effects. The topology may indeed have an impact on the evolution of the united states, which in its turn determines how the topology can be modified: the network becomes adaptive.

In this paper, we explore how the coevolution of an adaptive network of interacting agents and of the agents' opinion influence each other, and how the final state of the system depends on this coevolution.

2. The Model

The model we consider is based on the Sznajd model for the continuous opinions.[16] In the original Sznajd model, each agent is represented by a spin state variable $s_i = \pm 1$ and each randomly selected pair of nearest neighbors convinces all its neighbors of the pair opinion, only if the pair shares the same opinion: "United we stand, divided we fall." However, the opinions, being continuous, can never be equal, as required by the Sznajd rule, but we have to soften this condition. As a matter of fact, instead of equality, we can demand "closeness," i.e., the two opinions must differ from each other by a number less than some real number d. This immediately recalls the principle of Bounded Confidence which characterizes both the model of Deffuant and that of KH.[17,18] There, the parameter d is called confidence-bound and, if $|o_i - o_j| < d$, the two agents are compatible, which means that they agree with each other. The dynamics of our model are defined as follows: the network of N $(i = 1, \ldots, N)$ agents with M edges are initially connected with each other at random, and opinions o are assigned to nodes which can vary between 0 and 1 and is initially uniform at random. We then study by computer simulation the dynamics in which on each step of the simulation we either break an edge between two agents whose opinions differ more than the bound, or we change the opinions of all its neighbors of the pair closer. To be specific, on each step we pick a node i, and one of its neighbors j is chosen at random. If the degree k_i of that node is zero, we do nothing. Otherwise, we do the following:

(1) With probability p, an attempt to break the connection between i and j is made: if $|o(i,t) - o(j,t)| > d$, a new node k is chosen at random and the link (i,j) is rewired to (i,k).
(2) With probability $1 - p$ on the other hand, the node pair influences to all of its neighbors: if $|o(i,t) - o(j,t)| < d$, the opinions of the neighbors of the pair i and j evolve according to

$$o(\Gamma(i), t+1) = \frac{(o(i,t) + o(j,t))}{2}\,,$$
$$o(\Gamma(j), t+1) = \frac{(o(i,t) + o(j,t))}{2}\,, \tag{1}$$

where $\Gamma(i)$ is the set of neighbor node i. Step 1 represents the terminating relationship between agents who disagree and step 2 represents the outward influence of acquaintances of the pair to the others, opinions becoming similar as a result of acquaintance.

The evolution behavior and its final state of the model are clear. Since both of the two steps tend to decrease the number of node pairs with different opinions. The system will ultimately evolve into a state which can be characterized by the investigation of the opinion clusters of agents. In the final state, these clusters are made of agents sharing the opinion within the confidence bound. In the case of the static networks, the whole agents consensus to a one finial opinion cluster with all nodes taking the same opinion, while for the coevolving networks, we also keep track of the topology clusters which correspond simply to the various connected components of the network, and in this case we find that the topological and opinion cluster naturally coincide finally. Note that in our model, we keep the average degree $\bar{k} = 2M/N$ fixed when increasing the system size N, and in this condition we ask what happens as we vary the rewiring rate p under the different confidence bounds d.

3. Simulation Results

Before turning to a detailed analysis of the model, we illustrate in Figs. 1 and 2 the different behaviors observed for static and adaptive networks. The figures show the evolution of the opinions of 200 from $N = 1000$ agents as a function of time, in each case for one single realization of the dynamics with $d = 0.15$. The opinions are initially randomly distributed on the interval [0, 1]. When the interaction network is static, local convergence processes take place and lead to a one-opinion cluster in the final state with all agents sharing the same opinion. This is greatly different from the Deffuant or KH models. Figure 2, which corresponds to an adaptive network with $p = 0.7$, is strikingly in contrast with the static case: one macroscopic size opinion cluster and a few small size groups are observed. We will investigate these differences in more detail in the next subsections. In particular, the whole cluster-size distribution gives a complete description of the system. Interesting summaries

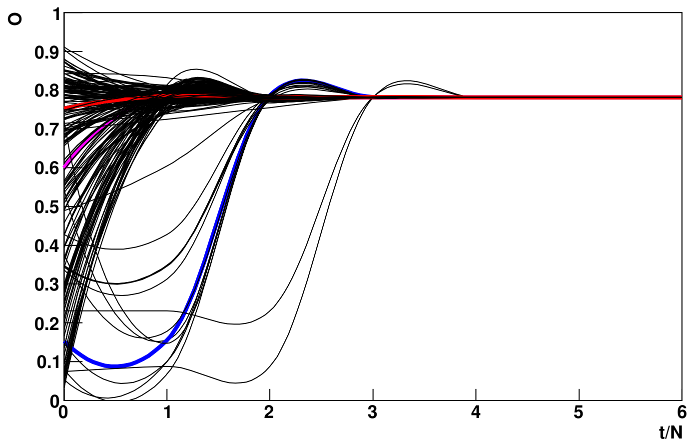

Fig. 1. Evolution of the opinions of 20% of the population, denoted by lines, for a system of 10^3 agents with tolerance $d = 0.15$ and average degree $\bar{k} = 5$, on a static network for a single run. The evolution of the opinion of a few individuals is highlighted with color.

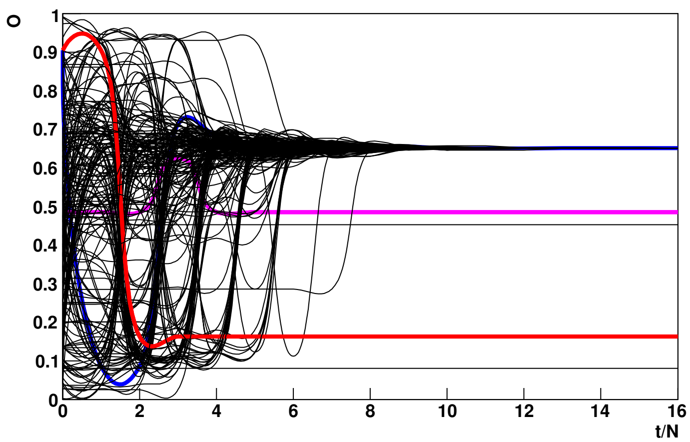

Fig. 2. Same plot as for Fig. 1 for an adaptive network when the rate of update is $p = 0.7$ ($N = 10^3$, $\bar{k} = 5$, and $d = 0.15$).

are given by the number of clusters, the size of the largest opinion-cluster which will tell us about the behavior of the clusters with macroscopic size (because of the evolution that eventually leads to the final large one cluster with few small clusters) and the relaxation time for the whole system.

3.1. *Consensus formation on static networks*

Let us first consider the Continuous Sznajd model on a static network. In all the simulations we have carried out, the system converges to a configuration where all agents have one and the same opinion (complete consensus), for any value of d. This

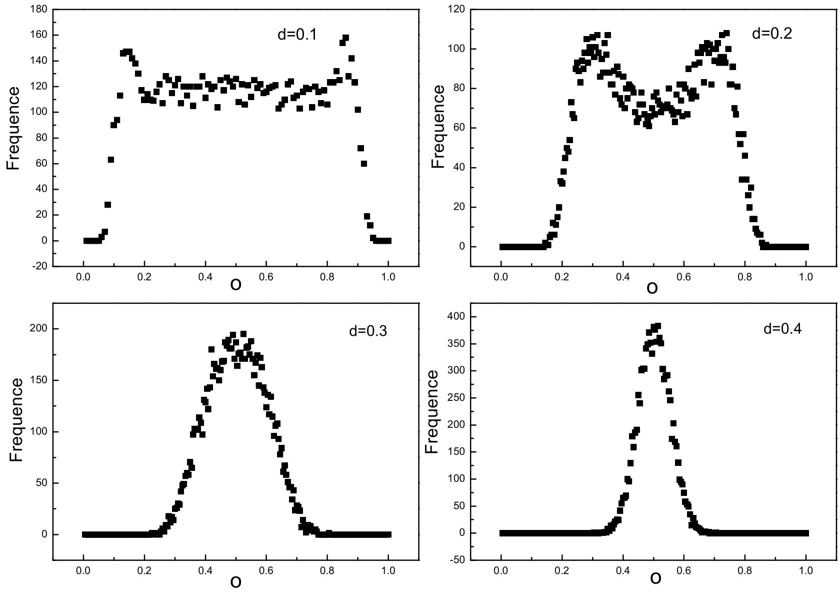

Fig. 3. Probability distribution of the final surviving opinion for continuous Sznajd model on network with $N = 1000$ and average degree $\bar{k} = 5$ for the 10 000 different initial samples.

result, which matches that of the original discrete version, shows that the Sznajd dynamics is most effective to achieve a full synchronization of the agents. Moreover, we find that the result holds independently of the initial distribution of opinions, which need not be uniform, and the value of the final opinion o_f of the agents is not 1/2, as in the models of the Deffuant and KH, but it can take any value in a range centered at 1/2. The width of the range and the probability distribution of o_f depend on d. In Fig. 3, we show the probability distribution of o_f for the random networks and four values of d, obtained from 10 000 different initial samples. As one can see, the histograms are all symmetric with respect to the center opinion 1/2, as expected, but their shapes vary with d. We distinguish three characteristic profiles, flat, double-peaked, and single-peaked for low, intermediate, and high values of d, respectively. These results are totally different from the varied Deffuant model in which there exists small fragment clusters and present the polarized-to-fragmented transition on static networks. However, these differences come out as the result of different persuading powers defined by the dynamics rules. In the Sznajd model, all the neighbor nodes of the pair are convinced to the same opinion, while in the Deffuant model, only the opinions of the pair are changed.

3.2. *Consensus formation on adaptive networks*

Let us now turn to the case of adaptive network in which either pair of agents with far apart opinions can break their connection and rebuild new one with another agent randomly chosen, or the pair of agents can influence all its neighbors. The

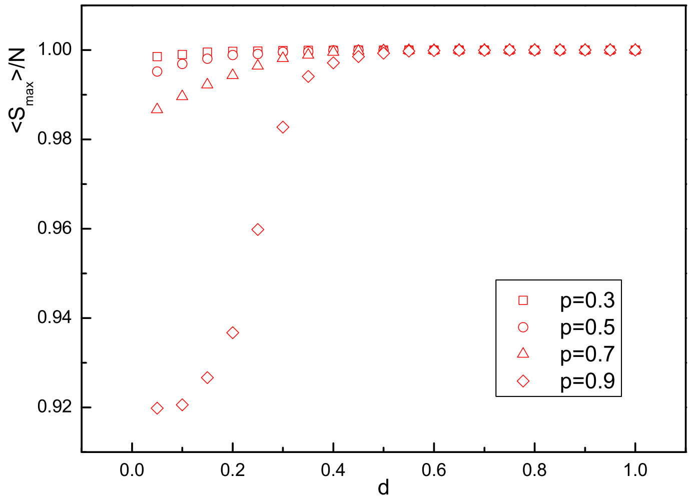

Fig. 4. Size of the largest clusters in the final state as a function of the tolerance value on the adaptive network for different rewiring rates on adaptive network with average degree $\bar{k} = 5$ and $N = 1000$ for averaged 100 samples.

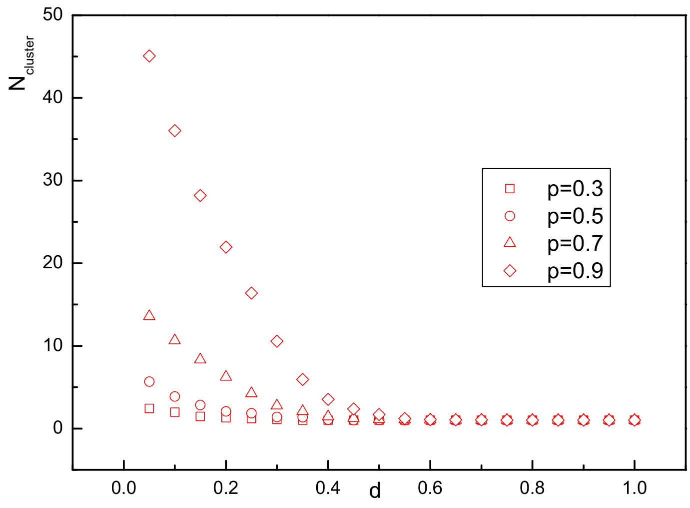

Fig. 5. Number of clusters in the final state as a function of the tolerance of the agents for different rewiring rates on adaptive network with average degree $\bar{k} = 5$ and $N = 1000$ for averaged 100 samples.

rate of attempts to rewire connections is given by p: the larger the p and the faster rewiring can occur. Figures 4 and 5 display the size of the largest clusters and the total number of the clusters in the final state with system size $N = 1000$ and $\bar{k} = 5$; all of these results are averaged for 100 samples.

As we can see from Fig. 4, the size of the largest cluster increased when the parameter d approaches 1 and this trend is more evident when the rewiring rate gets larger. This result is easy to understand for that the larger reconnecting rate and the lager confidence bound between the node pairs lead to the more possibility of reforming a new opinion component. When $d > 0.5$, only one final cluster containing all nodes is obtained. As d decreases, $\langle S_{\max}\rangle/N$ decreases very slowly and the number of the clusters N_{cluster} increases: a few number of small clusters of finite size appears. For the small rate p, the maximum number of clusters is less than 10 such as $p = 0.3$, while the larger rewiring rate results in the larger maximum number of clusters. Comparing these results with the varied Deffuant model on adaptive network, the number of clusters in our model with the same system size is much more less than that in Ref. 12, which proves that Sznajd rule is more effective in reaching synchronization than the other rules.

For the different system sizes with fixed average degree, we get the largest cluster size $\langle S_{\max}\rangle/N$ and the percent of clusters $\langle N_{\text{cluster}}\rangle/N$ shown as in Figs. 6 and 7, which also averaged for 100 samples. The properties of the largest cluster size and the number of clusters seldom changes under the fixed average degree with different system sizes, which indicate that the property of the system segment is not affected by the system size. This result is more or less the same as in Refs. 12 and 19 that strongly suggested the mechanism behind the adaptive rule.[20]

Finally, we get the relaxation time that the system needed to reach the final state with $N = 1000$, as shown in Fig. 8 for averaged 100 samples. The relaxation time increased as the rewiring rate p increases, and when $d = 0.2$, there is a turning point which becomes distinguished with the increased rewiring rate p.

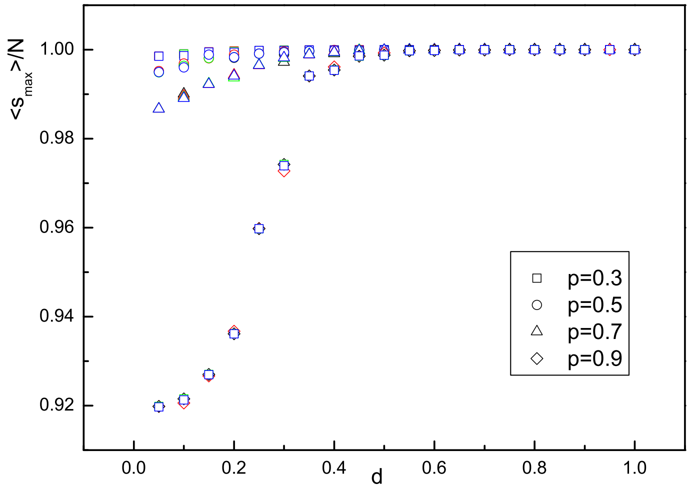

Fig. 6. Size of the largest clusters in the final state as a function of the tolerance value on the adaptive network for different rewiring rates on adaptive network with average degree $\bar{k} = 5$ and $N = 500$ (black), 1000 (red), 2000 (green), 5000 (blue) for averaged 100 samples.

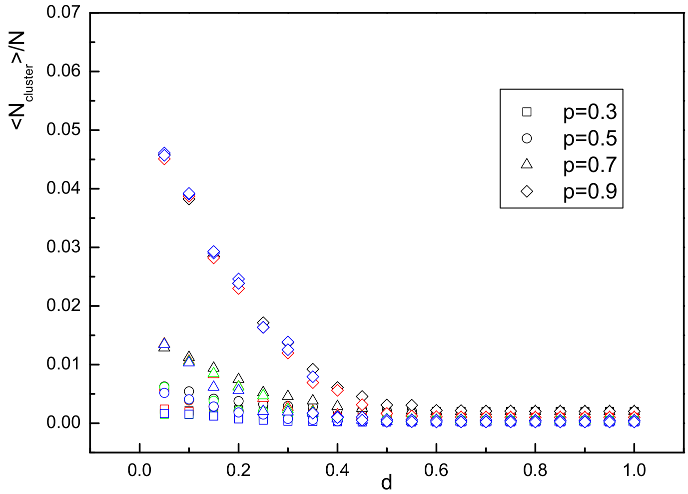

Fig. 7. Number of clusters in the final state as a function of the tolerance of the agents for different rewiring rates on adaptive network with average degree $\bar{k} = 5$ and $N = 500$ (black), 1000 (red), 2000 (green), 5000 (blue) for averaged 100 samples.

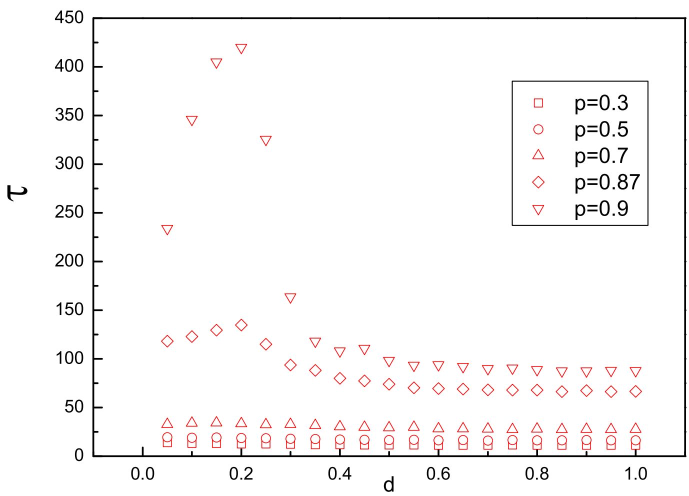

Fig. 8. The relaxation time as a function of the tolerance of the agents for different rewiring rates on adaptive network with average degree $\bar{k} = 5$ and $N = 1000$ for averaged 100 samples.

4. Conclusion

In this paper, we have studied consensus formation on static and adaptive networks through the investigation of the continuous Sznajd opinion model with confidence bound: agents with close enough opinion reach an agreement, while they cannot communicate if their opinions are too far apart. In the static network, the dynamic will evolve into the state of final one cluster with all agents sharing the same opinion, and the final opinion o_f distributed differently with varied confidence bound d. The shape takes the form of flat, double-peaked, and single-peaked for low, intermediate, and high values of d with the center at $d = 1/2$, respectively. While on the adaptive network, the link between two agents who disagree with each other can break and rebuild a new one with another agent. In this adaptive network, the opinion cluster coincides with the topology cluster and the final state is a large cluster with a few small clusters. The largest size of clusters and the number of clusters reach the extremes as the confidence bound d approaches the maximum, and they change rapidly under larger rewiring rate p. The relaxation time lasts longer when rewiring rate p increased and there is a turning point at $d = 0.2$ when $p = 0.9$. Above all, both the results on static and adaptive networks show that the Sznajd rule is strongly favorable in the consensus state.

Acknowledgments

The authors acknowledge the discussions with Balazs Kozma and Tu Yu-Song. This work is supported in part by the National Natural Science Foundation of China under Grant Nos. 70571027 and 10635020, and the Ministry of Education of China under Grant No. 306022 foundation.

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