Part 1

We have a lamp with $P = 25\text{W}$ and $\varepsilon = 0.20$. This yields the radiant flux:

$$\Phi = P\varepsilon = 25\text{W} \cdot 0.20 = 5\text{W}$$

In 1 second, we can calculate the amount of energy emitted:

$$Q = \Phi t = 5\text{W} \cdot 1\text{s} = 5\text{J}$$

We then calculate the amount of energy per photon with $\lambda = 500$:

$$E = \frac{hc}{\lambda} = \frac{6.626 \cdot 10^{-34}\ \text{Js} \cdot 2.9976 \cdot 10^{8}\ \text{m/s}}{500 \cdot 10^{-9}\text{m}} \approx 3.97 \cdot 10^{-19}\text{J}$$

We can then calculate the number of photons:

$$\#\text{Photons} = \frac{Q}{E} = \frac{5\text{J}}{3.97 \cdot 10^{-19}\text{J}} \approx 1.26 \cdot 10^{19}$$

Part 2

Since we assume ideal conditions, $\varepsilon = 1$. With voltage and current we can calculate the power of the bulb:

$$P = UV = 2.4\text{V} \cdot 0.7\text{A} = 1.68\text{W}$$

Since $\varepsilon = 1$ this is equal to the radiant flux:

$$\Phi = \varepsilon P = P = 1.68\text{W}$$

Since the light source is isotropic, we can calculate the radiant intensity since the light source is a perfect sphere:

$$I = \frac{\text{d}\Phi}{\text{d}\omega} = \frac{\Phi}{\Omega} = \frac{1.68\text{W}}{4\pi} \approx 0.134\text{W}$$

To determine the radiant exitance, we need the area of the bulb:

$$A = 4\pi r^2 = 4\pi(1\ \text{cm})^2 = 4\pi\text{cm}^2$$

Since the light is emitted equally over all areas of the sphere, the radiant exitance is:

$$M = \frac{\text{d}\Phi}{\text{d}A} = \frac{\Phi}{A} = \frac{1.68\text{W}}{4\pi\text{cm}^2} \approx 0.1337\frac{\text{W}}{\text{cm}^2} = 1337\frac{\text{W}}{\text{m}^2}$$

We can calculate the amount of energy emitted in 5 minutes using the radiant flux as it is constant:

$$E = \Phi t = 1.68\ \text{J/s} \cdot 300\text{s} = 504\text{J}$$

Part 3

We assume the pupil to be a perfect disc looking directly at the bulb.

We want to project the pupil onto the bulb to find the solid angle of the bulb which light hits the pupil. We determine the angle from the center of the bulb to the top of the pupil.

$$\theta_1 = \tan^{-1}\left(\frac{\frac{1}{2}d}{r}\right) = \tan^{-1}\left(\frac{\frac{1}{2} \cdot 6 \cdot 10^{-3}\text{m}}{1\text{m} + 0.005\text{m}}\right) \approx 0.0030$$

We can then integrate to find the solid angle:

$$\begin{aligned}
\omega &= \int_0^{2\pi} \int_0^{\theta_1} \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\varphi \\
&= \int_0^{2\pi} \mathrm{d}\varphi \int_{\cos\theta_1}^{\cos 0} \mathrm{d}(\cos\theta) \\
&= 2\pi(\cos 0 - \cos\theta_1) \\
&= 2\pi(1 - \cos(0.0030)) \\
&= 0.000028
\end{aligned}$$

Using this solid angle and the intensity found earlier, we can calculate the flux hitting the pupil:

$$\Phi = I \cdot \omega = 0.134\mathrm{W} \cdot 0.000028 = 0.000003752\mathrm{W}$$

Using the area of the pupil we can get the irradiance:

$$E = \frac{\Phi}{A} = \frac{0.000003752\mathrm{W}}{\pi(3 \cdot 10^{-3}\mathrm{m})^2} \approx 0.1326\frac{\mathrm{W}}{\mathrm{m}^2}$$

Part 4

$$\Phi = P\varepsilon = 200\mathrm{W} \cdot 0.20 = 40\mathrm{W}$$

We assume that light is emitted equally in all directions. Then the intensity of the light becomes:

$$I = \frac{\mathrm{d}\Phi}{\mathrm{d}\omega} = \frac{\Phi}{\Omega} = \frac{40\mathrm{W}}{4\pi} \approx 3.183\mathrm{W}$$

Using the intensity we can calculate the irradiance 2 meters away. We look at the point directly beneath the light, such that $\theta = 0$ and $\cos\theta = 1$:

$$E = I\frac{\cos\theta}{r^2} = \frac{I}{r^2} = \frac{3.183\mathrm{W}}{(2\mathrm{m})^2} = \frac{3.183\mathrm{W}}{4\mathrm{m}^2} = 0.796\frac{\mathrm{W}}{\mathrm{m}^2}$$

We then calculate the illuminance:

$$\text{Illuminance} = \text{Irradiance} \cdot 685\frac{\mathrm{lm}}{\mathrm{W}} \cdot V(650\text{ nm}) = 0.796\frac{\mathrm{W}}{\mathrm{m}^2} \cdot 685\frac{\mathrm{lm}}{\mathrm{W}} \cdot 0.1 = 54.526\frac{\mathrm{lm}}{\mathrm{m}^2}$$

Part 5

The irradiance/illuminance at the screen is equal on either side:

$$E_1 = E_2$$

Assuming each can be considered a point light, we can calculate the irradiance:

$$E_1 = \frac{I_1}{r_1^2}$$

$$E_2 = \frac{I_2}{r_2^2}$$

Therefore: $\frac{I_1}{r_1^2} = \frac{I_2}{r_2^2} \Rightarrow I_2 = I_1\frac{r_2^2}{r_1^2}$

Since the mapping from irradiance to illuminance is linear (given some λ) this also holds for the illuminance values:

$$I_2 = I_1 \frac{r_2^2}{r_1^2} = 40 \text{ lm/sr } \frac{(65 \text{ cm})^2}{(35 \text{ cm})^2} = 137.96 \text{ lm/sr}$$

Part 6

$$B = L\pi = 5000\pi \frac{\text{W}}{\text{m}^2}$$

$$\Phi = B * A = 5000\pi \frac{\text{W}}{\text{m}^2} \cdot (0.10\text{m})^2 = 50\pi\text{W}$$

Part 7

$$\begin{aligned}
B &= \int_{2\pi} L \cos\theta \, \mathrm{d}\omega \\
&= \int_{2\pi} 6000 \cos\theta \text{ W/(m}^2\text{ sr)} \cos\theta \, \mathrm{d}\omega \\
&= 6000 \text{ W/(m}^2\text{ sr)} \int_{2\pi} \cos\theta \cos\theta \, \mathrm{d}\omega \\
&= 6000 \text{ W/(m}^2\text{ sr)} \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \cos\theta \cos\theta \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\varphi \\
&= 6000 \text{ W/(m}^2\text{ sr)} \ 2\pi \int_0^{\frac{\pi}{2}} \cos\theta \cos\theta \sin\theta \, \mathrm{d}\theta \\
&= 6000 \text{ W/(m}^2\text{ sr)} \ 2\pi \int_{\cos 0}^{\cos\frac{\pi}{2}} \cos\theta (-\cos\theta) \, \mathrm{d}(\cos\theta) \\
&= 6000 \text{ W/(m}^2\text{ sr)} \ 2\pi \int_{\cos 0}^{\cos\frac{\pi}{2}} -(\cos\theta)^2 \, \mathrm{d}(\cos\theta) \\
&= 6000 \text{ W/(m}^2\text{ sr)} \ 2\pi \int_1^0 -x^2 \, \mathrm{d}x \\
&= 6000 \text{ W/(m}^2\text{ sr)} \ 2\pi \left[-\frac{1}{3}x^3\right]_1^0 \\
&= 6000 \text{ W/(m}^2\text{ sr)} \ 2\pi \left(0 - \left(-\frac{1}{3}\right)\right) \\
&= 6000 \text{ W/(m}^2\text{ sr)} \ 2\pi \frac{1}{3} \\
&= 4000\pi \text{ W/(m}^2\text{)}
\end{aligned}$$

Part 8 (optional)

Part 9 (optional)