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Centre number	Candidate number
Surname	
Forename(s)	
Candidate signature	I declare this is my own work.

A-level **MATHEMATICS**

Paper 3

Tuesday 20 June 2023

Afternoon

Time allowed: 2 hours

Materials

- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- You must answer each question in the space provided for that question.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet
- You do not necessarily need to use all the space provided.

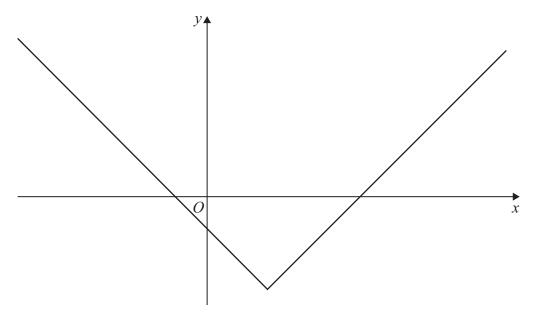
For Examiner's Use			
Question	Mark		
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Section A

Answer all questions in the spaces provided.

1 The graph of y = f(x) is shown below.



One of the four equations listed below is the equation of the graph y = f(x) Identify which one is the correct equation of the graph.

Tick (\checkmark) one box.

[1 mark]

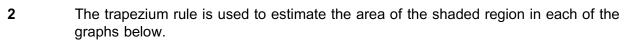
$$y = |x + 2| + 3$$

$$y = |x + 2| - 3$$

$$y = |x - 2| + 3$$

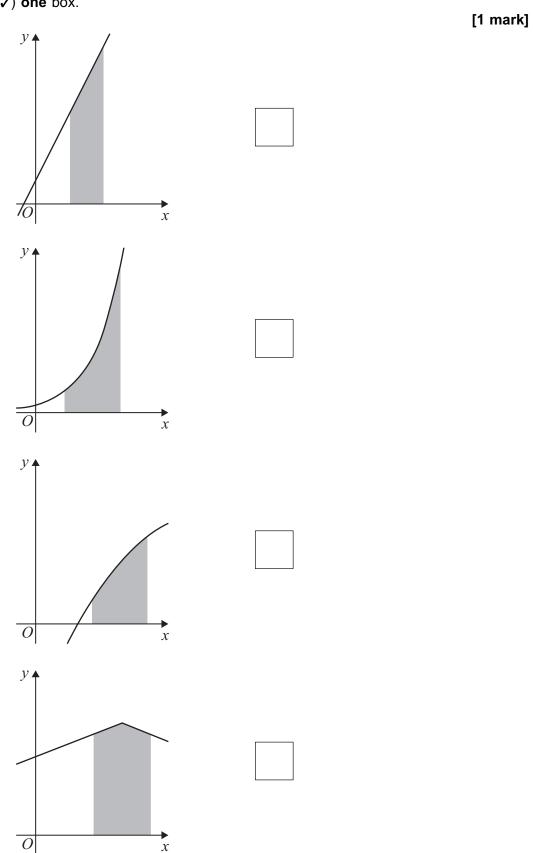
$$y = |x - 2| - 3$$





Identify the graph for which the trapezium rule produces an overestimate.

Tick (✓) one box.





Turn over ▶

3	A curve with equation $y = f(x)$ passes through the point (3, 7)

Given that f'(3) = 0 find the equation of the normal to the curve at (3, 7)

Circle your answer.

[1 mark]

$$y = \frac{7}{3}x \qquad \qquad y = 0 \qquad \qquad x = 3 \qquad \qquad x = 7$$

$$y = 0$$

$$x = 3$$

$$x = 7$$

$$\frac{5-\sqrt[3]{x}}{x^2}$$

in the form

$$5x^p - x^q$$

where p and q are constants.



5	A curve has equation $y = 3e^{2x}$	
	Find the gradient of the curve at the point where $y = 10$	[3 marks]

Turn over for the next question

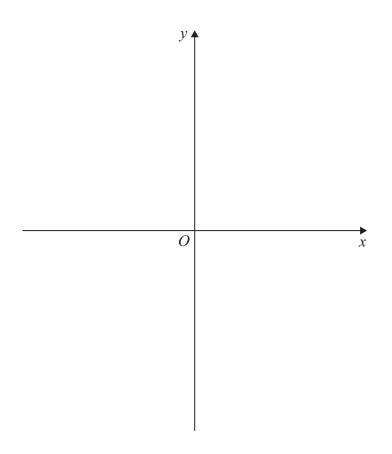


6 (a) Sketch the curve with equation

$$y = x^2(2x + a)$$

where a > 0

[3 marks]



6 (b) The polynomial p(x) is given by

$$p(x) = x^2(2x + a) + 36$$

6 (b) (i) It is given that x + 3 is a factor of p(x)

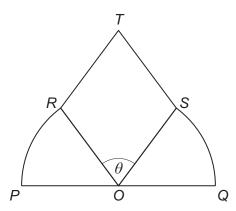
Use the factor theorem to show a = 2

[2 marks]

6 (b) (ii)	State the transformation which maps the curve with equation
	$y = x^2(2x+2)$
	onto the curve with equation
	$y = x^2(2x+2) + 36$
	[2 marks]
	·
6 (b) (iii)	The polynomial $x^2(2x+2) + 36$ can be written as $(x+3)(2x^2+bx+c)$
	Without finding the values of b and c , use your answers to parts (a) and (b)(ii) to explain why
	$b^2 < 8c$
	[2 marks]
	Turn over for the next question



A new design for a company logo is to be made from two sectors of a circle, *ORP* and *OQS*, and a rhombus *OSTR*, as shown in the diagram below.



The points P, O and Q lie on a straight line and the angle ROS is θ radians.

A large copy of the logo, with PQ = 5 metres, is to be put on a wall.

7 (a) Show that the area of the logo, A square metres, is given by

$$A = \frac{25}{8}(\pi - \theta + 2\sin\theta)$$

·	[4 marks]



7 (b) (i)	Show that the maximum value of A occurs when $\theta = \frac{\pi}{3}$		
	Fully justify your answer.		
	[6	6 marks]	
	Question 7 continues on the next page		



7 (b) (ii)	Find the exact maximum value of A	
. , . ,		marks
7 (c)	Without further calculation, state how your answers to parts (b)(i) and (b)(ii) wo	uld
- (-)	change if <i>PQ</i> were increased to 10 metres.	
	[2]	marks
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8	Use the substitution $u = x^5 +$	2 to show that	
		$\int_0^1 \frac{x^9}{(x^5+2)^3} \mathrm{d}x = \frac{1}{180}$	[7 marks]



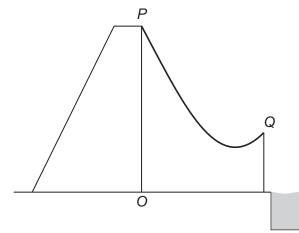
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9 A water slide is the shape of a curve *PQ* as shown in **Figure 1** below.

Figure 1



The curve can be modelled by the parametric equations

$$x = t - \frac{1}{t} + 4.8$$

$$y = t + \frac{2}{t}$$

where $0.2 \le t \le 3$

The horizontal distance from O is x metres.

The vertical distance above the point O at ground level is y metres.

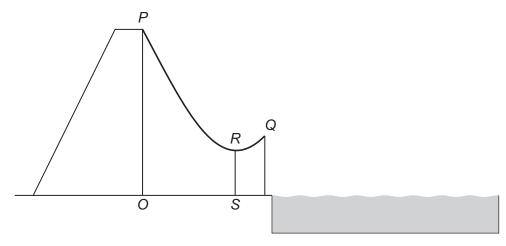
P is the point where t = 0.2 and Q is the point where t = 3

9 (a)	To make sure speeds are safe at Q , the difference in height between P and be less than 7 metres.	Q must
	Show that the slide meets this safety requirement.	[3 marks]
9 (b) (i)	Find an expression for $\frac{dy}{dx}$ in terms of t	
	C.V	[3 marks]
	Question 9 continues on the next page	



9 (b) (ii) A vertical support, *RS*, is to be added between the ground and the lowest point on the slide as shown in **Figure 2** below.

Figure 2



Find the length of RS	
	[4 marks]



9 (b) (iii) Find the acute angle the slide makes with the horizontal at Q					
	Give your answer to the nearest degree. [2 marks]				

END OF SECTION A TURN OVER FOR SECTION B



Section B

Answer all questions in the spaces provided.

Which of the following is **not** a possible value for a product moment correlation coefficient?

Circle your answer.

[1 mark]

$$-\frac{6}{5}$$

$$-\frac{3}{5}$$

1

11 A and B are mutually exclusive events.

Which one of the following statements must be correct?

Tick (✓) one box.

[1 mark]

$$P(A \cup B) = P(A) \times P(B)$$

$$P(A \cup B) = P(A) - P(B)$$

$$P(A \cap B) = 0$$

$$P(A \cap B) = 1$$

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12	It is known that, on average, 40% of the drivers who take their driving test at a local test centre pass their driving test.						
	Each day 32 drivers take their driving test at this centre.						
	The number of drivers who pass their test on a particular day can be modelled by the distribution B $(32,0.4)$						
12 (a)	State one assumption, in context, required for this distribution to be used. [1 mark]						
12 (b)	Find the probability that exactly 7 of the drivers on a particular day pass their test. [1 mark]						
12 (c)	Find the probability that, at most, 16 of the drivers on a particular day pass their test. [1 mark]						
12 (d)	Find the probability that more than 12 of the drivers on a particular day pass their test. [2 marks]						



12 (e)	Find the mean number of drivers per day who pass their test.	
	[1 mark	(]
		-
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		-
12 (f)	Find the standard deviation of the number of drivers per day who pass their test. [2 marks	≱]
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13	There are two types of coins in a money box:	
	• 20% are bronze coins	
	• 80% are silver coins	
	Craig takes out a coin at random and places it back in the money box.	
	Craig then takes out a second coin at random.	
13 (a)	Find the probability that both coins were of the same type. [2 marks]]
		-
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13 (b)	Find the probability that both coins are bronze, given that at least one of the coins	
	is bronze.	
	is bronze. [2 marks]
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14	The mass of aluminium cans recycled each day in a city may be modelled by a normal distribution with mean 24 500 kg and standard deviation 5 200 kg.
14 (a)	State the probability that the mass of aluminium cans recycled on any given day is not equal to 24 500 kg. [1 mark]
14 (b)	To reduce costs, the city's council decides to collect aluminium cans for recycling less frequently.
	Following the decision, it was found that over a 24-day period a total mass of 641 520 kg of aluminium cans was recycled.
	It can be assumed that the distribution of the mass of aluminium cans recycled is still normal with standard deviation 5 200 kg, and that the 24-day period can be regarded as a random sample.
	Investigate, at the 5% level of significance, whether the mean daily mass of aluminium cans recycled has changed . [7 marks]



c)	A member of the council claims that if a different sample of 24 days had been used the hypothesis test in part (b) would have given the same result.
	Comment on the validity of this claim. [2 mai
	Turn over for the next question



15 (a)	A random	samp	le of eight cars	was selecte	ed from the	Large Data	Set.	
	The mass	ses of	these cars, in k	ilograms, we	ere as follov	vs.		
	950	989	1247	1415	1506	1680	1833	2040
	It is given	that,	for the population	on of cars in	the Large I	Data Set:		
				lower quar	tile = 1167			
				med	lian = 1393			
				upper quar	tile = 1570			
15 (a) (i)	It was ded	cided t	to remove any o	of the masse	es which fall	outside the	e following	interval.
	media	n – 1.	5× interquartile	range ≤ ma	$\operatorname{ass} \leq media$	an $+$ 1.5 \times	interquartile	e range
	Show that	t only	one of the eight	t masses in	the sample	should be	removed.	[3 marks]
15 (a) (ii)	Write dow	n the	statistical name	for the mas	ss that shou	ıld be remo	ved in part	(a)(i). [1 mark]



15 (b) The table shows the probability distribution of the number of previous owners, N, for a sample of cars taken from the Large Data Set.

n	0	1	2	3	4	5	6 or more
P(N = n)	0.14	0.37	0.9k	0.25	0.4k	1.7k	0

Question 15 continues on the next page

15 (c)	An expert team is investigating whether there have been any changes in ${\rm CO}_2$ emissions from all cars taken from the Large Data Set.
	The team decided to collect a quota sample of 200 cars to reflect the different years and the different makes of cars in the Large Data Set.
15 (c) (i)	Using your knowledge of the Large Data Set, explain how the team can collect this sample.
	[2 marks]
15 (c) (ii)	Describe one disadvantage of quota sampling.
	[1 mark]



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16	A farm supplies apples to a supermarket.
	The diameters of the apples, ${\cal D}$ centimetres, are normally distributed with mean 6.5 and standard deviation 0.73
16 (a) (i)	Find $P(D < 5.2)$ [1 mark]
	[i mark]
16 (a) (ii)	Find $P(D > 7)$ [1 mark]
16 (a) (iii)	The supermarket only accepts apples with diameters between 5 cm and 8 cm.
	Find the proportion of apples that the supermarket accepts. [1 mark]

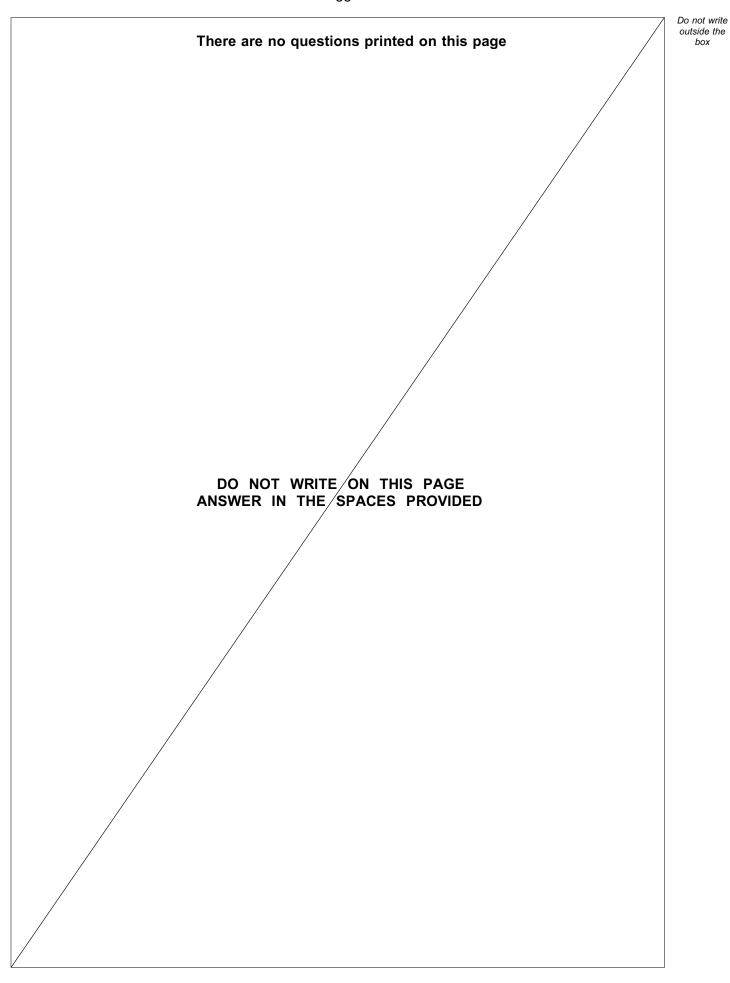


(b)	The farm also supplies plums to the supermarket.
	These plums have diameters that are normally distributed.
	It is found that 60% of these plums have a diameter less than 5.9 cm.
	It is found that 20% of these plums have a diameter greater than 6.1 cm.
	Find the mean and standard deviation of the diameter, in centimetres, of the plums
	supplied by the farm. [6 marks]
	
	·



17	A council found that 70% of its new local businesses made a profit in their first year.
	The council introduced an incentive scheme for its residents to encourage the use of new local businesses.
	At the end of the scheme, a random sample of 25 new local businesses was selected and it was found that 21 of them had made a profit in their first year.
	Using a binomial distribution, investigate, at the 2.5% level of significance, whether there is evidence of an increase in the proportion of new local businesses making a
	profit in their first year. [6 marks]
	·
	END OF QUESTIONS







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