Lecture 17 - Fork Parallelism Part 2

DSE 512

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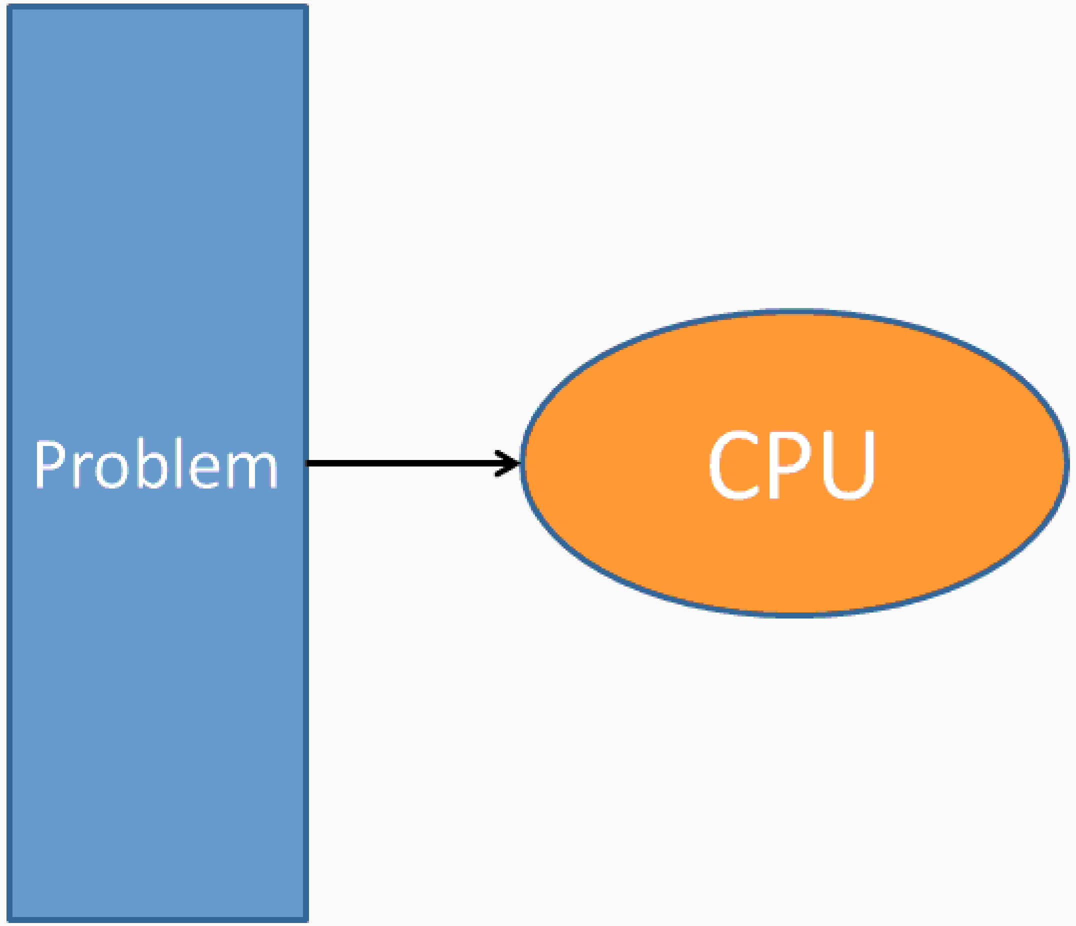
From Last Time

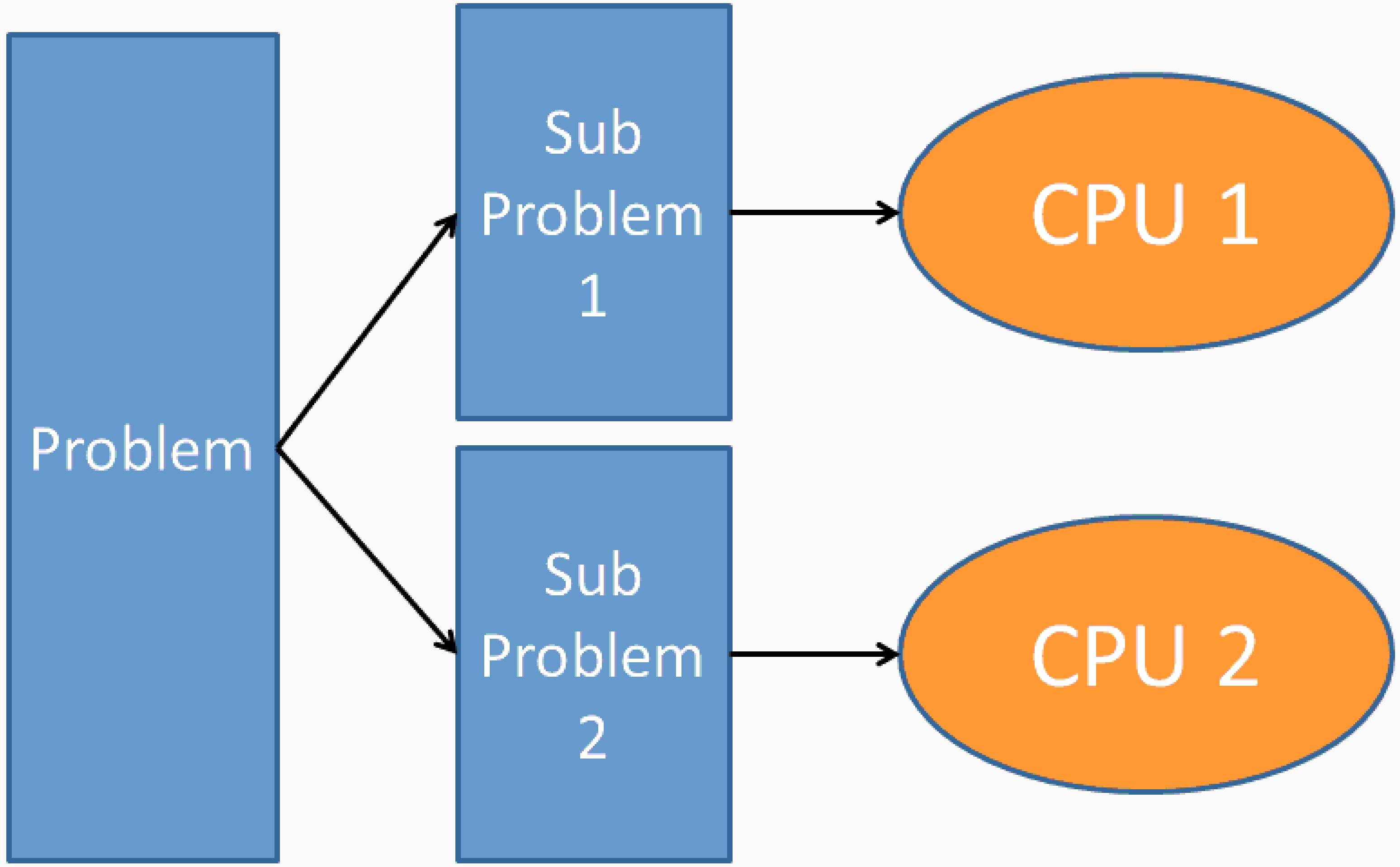
- New homework "soon"
- Questions?

Reviewing Fork Parallelism

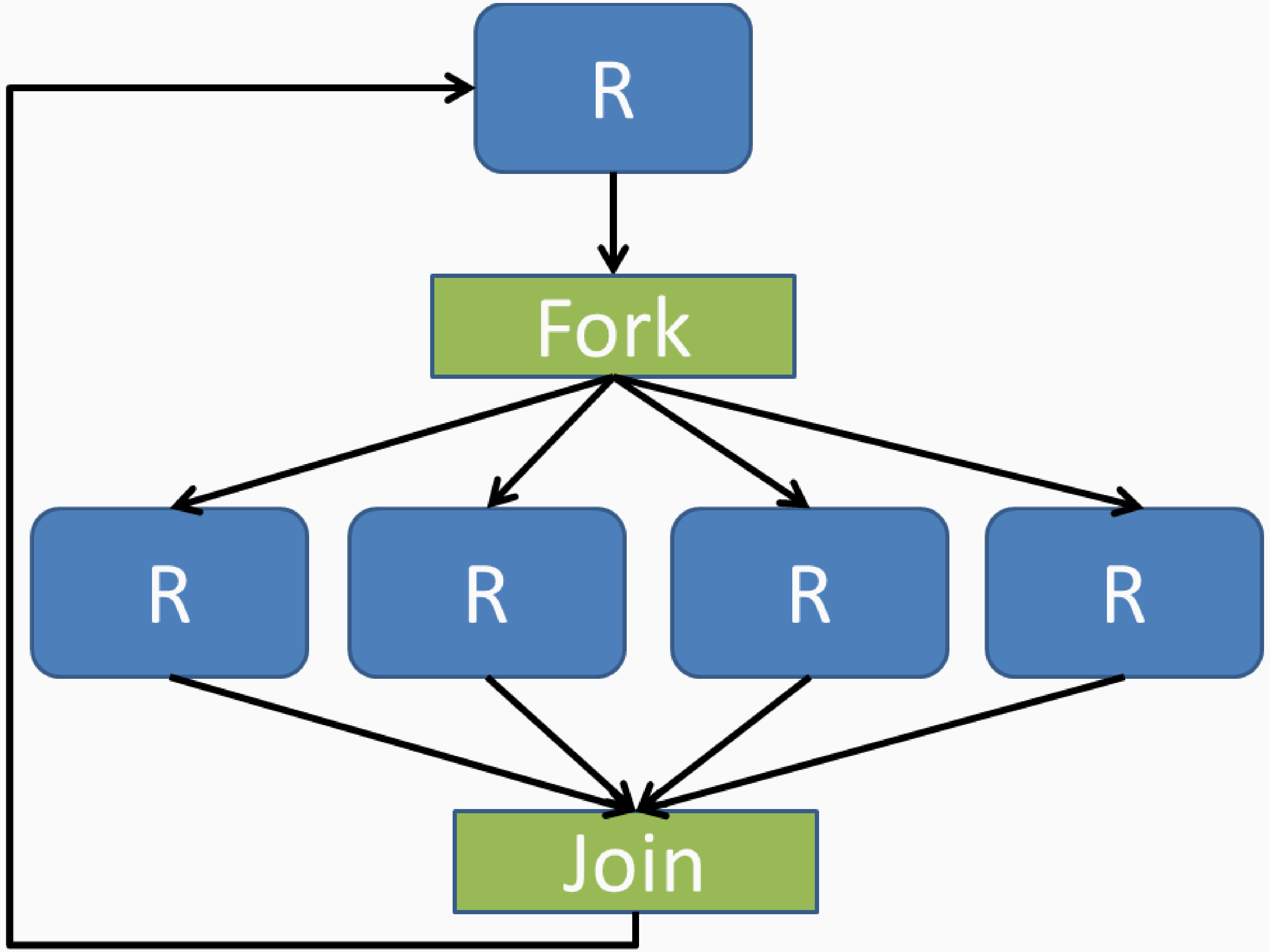
Serial Programming



Parallel Programming



Fork/Join Parallelism



Parallelism in Theory

Parallelism in Practice

Improving k-Means Clustering Outcomes in Parallel

Parallelism in k-Means Clustering

- Parallelize the algorithm itself
  - Good for very large datasets
  - Tricky...
- Parallelize on the number of random starts
  - Easy to do!
  - May not be that valuable
- Parallelize on the number of clusters
  - Also easy to do!
  - Turn your brain off! Machine your learnings!!!

Measuring Cluster Performance

- Would like an analogue of classification accuracy
- How do we compare two sets of cluster labels?
- Rand Measure / Rand Index https://en.wikipedia.org/wiki/Rand_index
    - Given a set of n elements S and two partitions of S into r-length subsets $X = \{X_1, \ldots, X_r\}$ and $Y = \{Y_1, \ldots, Y_r\}$

$$R = \frac{a+b}{\binom{n}{2}}$$

    Where

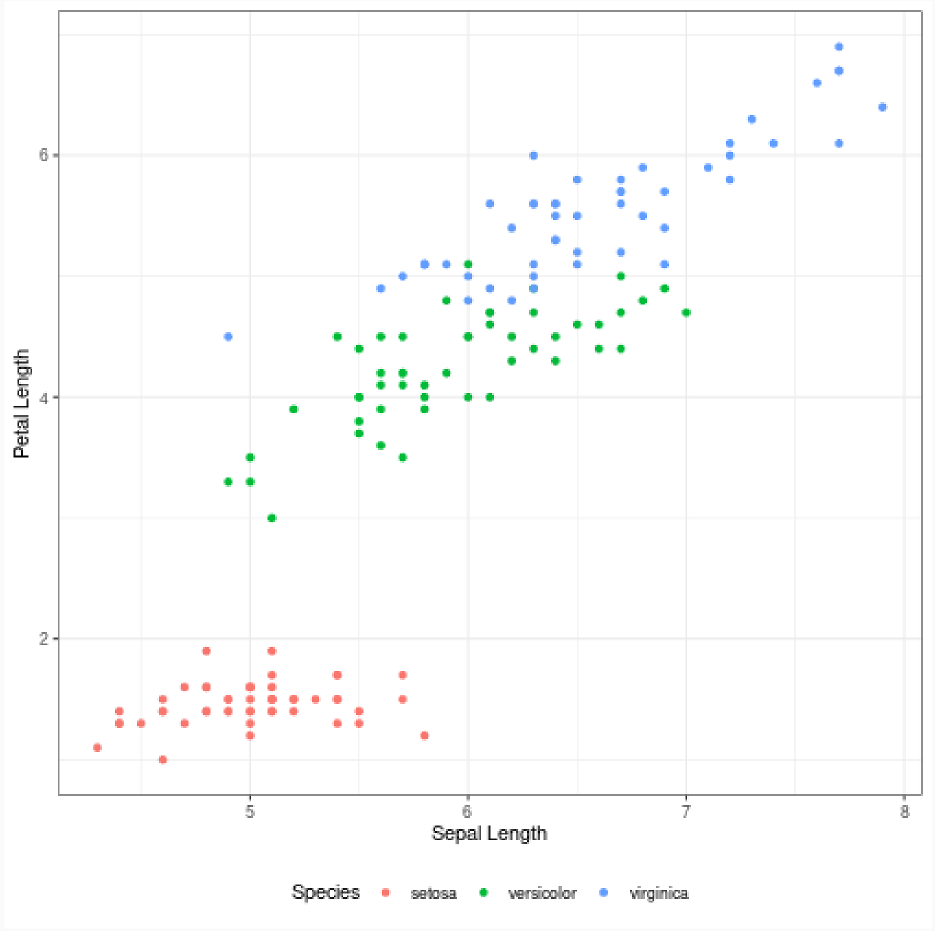
        - a is the # of pairs of elements of S that agree across X and Y
        - b is the # of pairs of elements of S that disagree across X and Y

Rand Measure

```r
rand_measure = function(l1, l2){
  n = length(l1)
  a = b = 0L

  for (i in 1:n){
    for (j in (i+1L):n){
      if (j > n) # R indexing is stupid
        break

      same1 = (l1[i] == l1[j])
      same2 = (l2[i] == l2[j])

      if (same1 && same2)
        a = a + 1L
      else if (!same1 && !same2)
        b = b + 1L
    }
  }

  (a + b) / choose(n, 2)
}
```

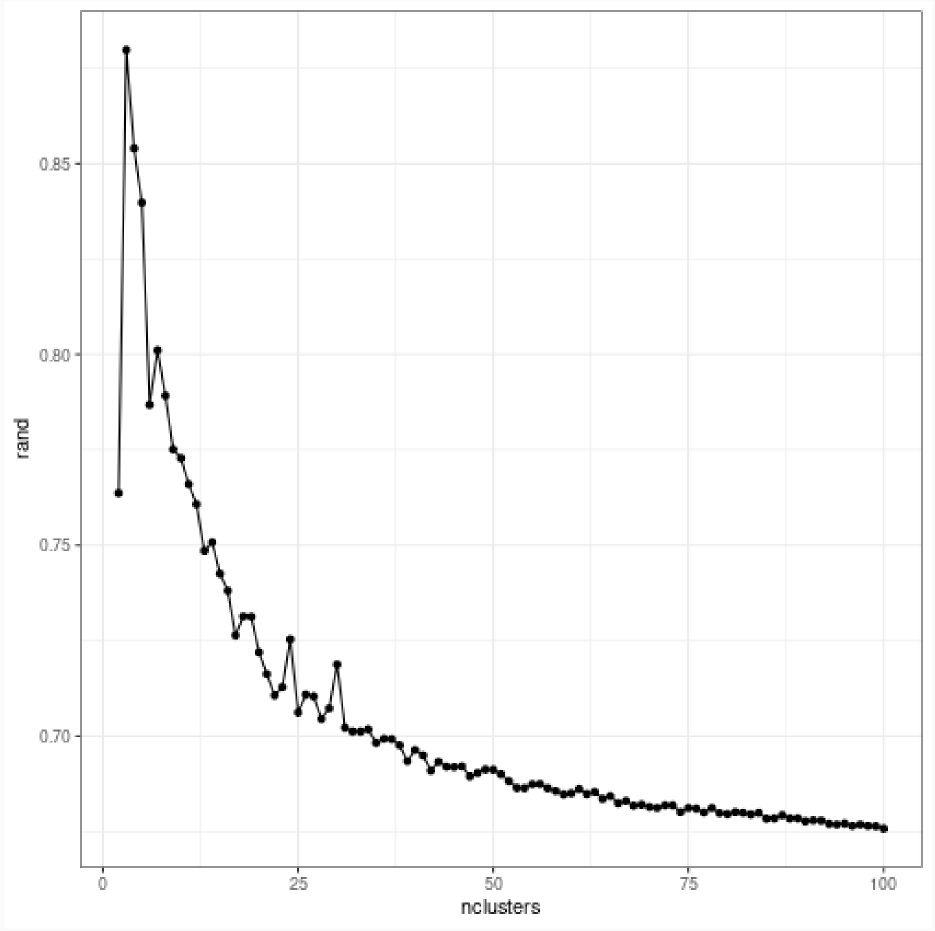
Using the Iris Dataset

- Fisher's famous iris dataset
- Required by law in clustering lectures



Finding the k in k-Means

```r
library(parallel)
labels = as.numeric(iris[, 5])
x = iris[, -5]

options(mc.cores=8)
nclusters = 2:100
clusters = mclapply(
  nclusters,
  function(i) kmeans(x, i, nstart=10)$clu
)
rand = mclapply(
  clusters,
  rand_measure,
  l2 = labels
) |> unlist()
```



Leave-One-Out Cross Validation

Our Model

- Motor Trend Car Road Tests (mtcars) dataset
- Predict
    - Miles per gallon
- Predictors:
    - Weight (1000's of pounds)
    - Number of cylinders
    - Gross horsepower

Estimating Model MSE

- Determine the leave-one-out (LOO) cross-validation (CV) MSE
  - Compute each LOO model MSE
  - Overall MSE is the mean of the model MSE's
- Why
  - less biased estimator
  - *expensive* to calculate
  - good teaching example

Quick note

- *Many* packages for R and Python do CV well
- Some of those offer implicit parallelism!
- We're doing it by hand to learn
- Some software engineering principles have been compromised for educational purposes

Estimating MSE with LOOCV

```r
model_mse = function(mdl){
  mean(mdl$residuals^2)
}

mse_loo = function(i){
  data = mtcars[-i, ]
  mdl = lm(mpg ~ wt + cyl + hp, data=data)
  model_mse(mdl)
}

mse = function(ncores){
  parallel::mclapply(1:nrow(mtcars), mse_loo, mc.cores = ncores) |>
    unlist() |>
    mean()
}
```

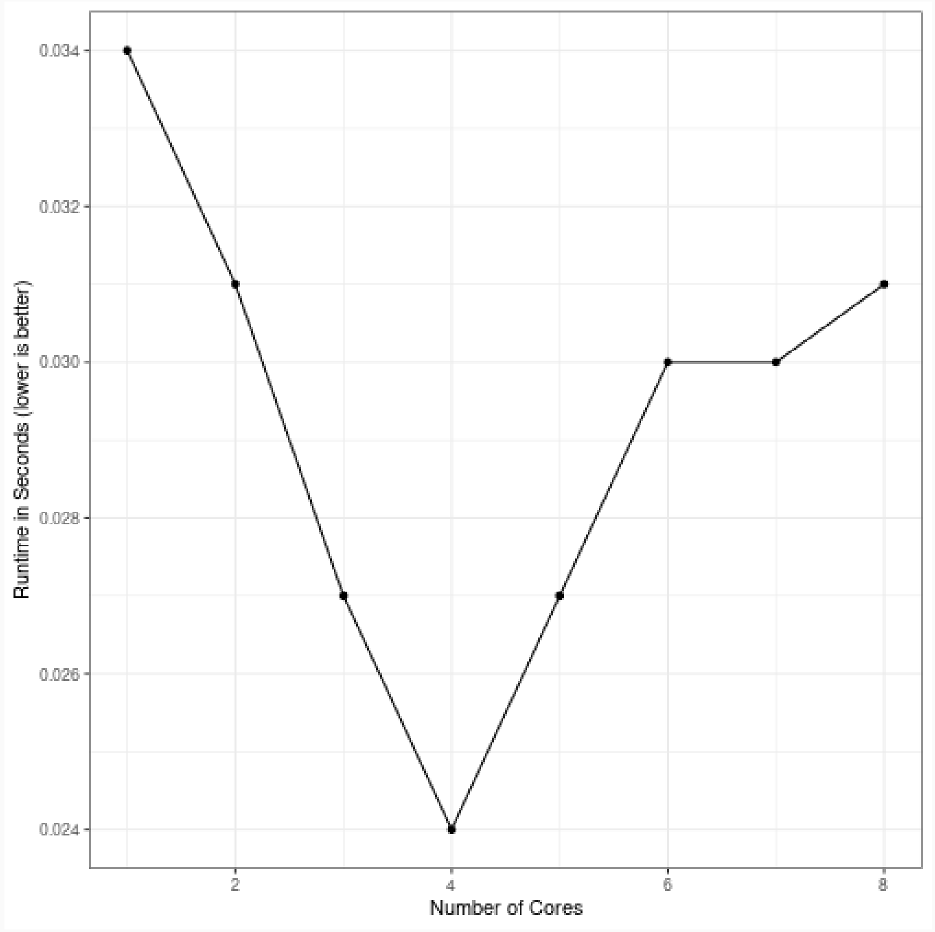
Values and Performance

```
lm(mpg ~ wt + cyl + hp, data=mtcars) |>
  model_mse()
```

```
## [1] 5.519391
```

```
mse(1)
```

```
## [1] 5.494924
```



Parallel SVD

Data Parallelism Challenges

- Rarely done with fork
- Generally better suited to *persistent* workers
- Why?

Connection to Eigendecomposition

$$\begin{aligned} A^T A &= \left(U\Sigma V^T\right)^T \left(U\Sigma V^T\right) \\ &= V\Sigma U^T U\Sigma V^T \\ &= V\Sigma^2 V^T \end{aligned}$$

Computing the "Normal Equations" Matrix

Choose $b > 0$ and split A into b blocks of rows:

$$A = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_b \end{bmatrix}$$

Then

$$A^T A = \sum_{i=1}^{b} A_i^T A_i$$

Recall: SVD with SQL

```
svd(big_tbl[, -1])$v
```

```
##              [,1]       [,2]
## [1,] 0.0495752  0.9987704
## [2,] 0.9987704 -0.0495752
```

```
B = matrix(0, 2, 2)
rpb = 7
num_blocks = get_num_blocks(n, rpb)
for (blockid in 1:num_blocks){
  ids = blockid_to_indices(blockid, n, rp
  query = paste(
    "SELECT * FROM big_tbl WHERE ind >=",
    ids$ind_low,
    "AND ind <=",
    ids$ind_high
  )
  A_i = dbGetQuery(db, query)
  B = B + crossprod(as.matrix(A_i[, -1]))
}
svd(B)$vt
```

```
##              [,1]       [,2]
## [1,] 0.0495752  0.9987704
## [2,] 0.9987704 -0.0495752
```

Crossproduct-Based SVD Algorithms

Out-of-core

- Inputs
    - $A_{m \times n}$
    - Number of blocks b
- Procedure
    - Initialize $B_{n \times n} = 0$
    - For each $1 \leq i \leq b$
        - Read block of rows A_i
        - Compute $B = B + A_i^T A_i$
    - Factor $B = \Lambda \Delta \Lambda$

Parallel

- Inputs
    - $A_{m \times n}$
    - Number of cores c
- Procedure
    - For each $1 \leq i \leq c$ in parallel
        - Read block of rows A_i
        - Return $B_c = A_i^T A_i$
    - Sum $B = \sum_{i=1}^{c} B_c$
    - Factor $B = \Lambda \Delta \Lambda$

Distributing Work

- Need to be able to distribute "work" (say tasks) among workers
- Similar concept/different implementation to out-of-core work distribution
- Function that takes:
  - total number of tasks
  - total number of workers
  - worker index/rank
- Sounds a lot like task parallelism…

But I thought we were doing data parallelism?

Distributing Work: R Implementation

```r
get_my_tasks = function(num_tasks, num_workers, my_id){
  if (num_tasks == num_workers)
    my_id + 1
  else if (num_tasks > num_workers){
    local = as.integer(num_tasks / num_workers)
    rem = num_tasks %% num_workers
    if (rem == 0 || (my_id < (num_workers - rem))){
      start = my_id * local
      start:(start + local - 1) + 1
    } else {
      start = my_id*(local + 1) - (num_workers - rem)
      start:(start + local) + 1
    }
  } else {
    if (num_tasks > my_id)
      my_id + 1
    else
      integer(0)
  }
}
```

Distributing Work: Python Implementation

```python
def get_my_tasks(num_tasks, num_workers, my_id):
    if num_tasks == num_workers:
        return range(my_id, my_id+1)
    elif num_tasks > num_workers:
        local = int(num_tasks / num_workers)
        rem = num_tasks % num_workers

        if rem == 0 or (my_id < (num_workers - rem)):
            start = my_id*local
            return range(start, start + local)
        else:
            start = my_id*(local + 1) - (num_workers - rem)
            return range(start, start + local + 1)
    else:
        if num_tasks > my_id:
            return [my_id]
        else:
            return range(0)
```

Distributing Work: Implementation Details

- chops up `num_tasks` tasks roughly evenly across `num_workers` workers
- `num_tasks` - number of tasks to distribute among workers
- `num_workers` total number of worker processes
- `my_id` 0-based worker index
- Return
    - R: 1-based index
    - Python: 0-based index

Parallel SVD at a High Level

- Chop up the input matrix
    - We can split by rows; other data layouts possible, but complicated
    - Each worker gets a chunk of the matrix
    - The chunks are non-overlapping
- Each worker computes its local crossproduct (normal equations matrix)
- The manager (calling process) sums the local worker matrices
- The manager computes the SVD on the (now global) crossproduct matrix

Parallel SVD

```r
crossprod_local = function(x, i, ncores)
{
  my_rows = get_my_tasks(nrow(x), ncores, i)
  x_i = x[my_rows, ]
  crossprod_local = crossprod(x_i)
  crossprod_local
}

crossprod_parallel = function(x, ncores)
{
  ret = parallel::mclapply(
    (1:ncores) - 1,
    crossprod_local,
    x=x,
    ncores=ncores
  )

  Reduce('+', ret)
}
```

Comparison

```r
set.seed(1234)
m = 1000
n = 3
x = matrix(rnorm(m*n), m, n)

crossprod_parallel(x, ncores=4)
```

```
##             [,1]      [,2]       [,3]
## [1,] 994.39527  54.97576   14.77759
## [2,]  54.97576 961.98697  -34.08700
## [3,]  14.77759 -34.08700 1024.64272
```

```r
crossprod(x)
```

```
##             [,1]      [,2]       [,3]
## [1,] 994.39527  54.97576   14.77759
## [2,]  54.97576 961.98697  -34.08700
## [3,]  14.77759 -34.08700 1024.64272
```

Benchmark

```r
set.seed(1234)
m = 100000
n = 250
x = matrix(rnorm(m*n), m, n)

openblasctl::openblas_set_num_threads(1)
system.time(crossprod_parallel(x, ncores=8))[3]
```

```
## elapsed
##   0.124
```

```r
openblasctl::openblas_set_num_threads(8)
system.time(crossprod(x))[3]
```

```
## elapsed
##   0.154
```

Wrapup

Wrapup

- Parallelism
  - Exploits independence
  - Can save time; can *waste* time
- Fork
  - (relatively) cheap mechanism for parallelism
  - great for task parallelism
  - less useful for data parallelism
- Be careful with RNG's in parallel!
- Next time: intro to MPI

Questions?