MODULE *EP3*

EXTENDS *Integers*, *Sequences*, *TLC*, *FiniteSets*, *Naturals*

CONSTANTS *IntermediateProviders*, *Quota*, *SupplyRecords*, *Requesters*

VARIABLES *sr*, *quota*, *AC*, *target*, *requesters*, *total*

Variable defs:
1. *sr* = Supply Records
2. *quota* = *Quota*
3. *AC* = Asking Chain
4. *target* = Currently focused target
5. *requesters* = Exit Rooms
6. *total* = the list numbers that every requester can get

$$vars \triangleq \langle sr, quota, AC, target, requesters, total \rangle$$

Check for valid suppliers, quota has to be greater than 0 & not in the asker chain

$$GetDPs(a, chain) \triangleq SelectSeq(IntermediateProviders, \text{LAMBDA } p : (quota[p][a] > 0) \land (p \notin chain))$$

$$GetStatus(p, akr, a\_asked, a\_chain) \triangleq \left[\begin{array}{rl} name & \mapsto p, \\ DSA & \mapsto \text{IF } sr[p] \geq a\_asked \text{ THEN } a\_asked \text{ ELSE } sr[p], \\ cur\_hold & \mapsto \text{IF } sr[p] \geq a\_asked \text{ THEN } a\_asked \text{ ELSE } sr[p], \\ DPs & \mapsto GetDPs(p, a\_chain), \\ akr & \mapsto akr, \\ requested & \mapsto a\_asked, \\ chain & \mapsto \{p\} \cup a\_chain \end{array} \right]$$

Making sure ask no more than the quota

$$\begin{array}{l} AskAmount(pdr, akr, asked) \triangleq \\ \qquad \text{IF } quota[pdr][akr] \leq asked \\ \qquad \text{THEN } quota[pdr][akr] \\ \qquad \text{ELSE } asked \end{array}$$

$$GetRequesterDP \triangleq GetStatus(requesters[1], -1, 10000, \{-1\})$$

$$\begin{array}{l} GetIP \triangleq GetStatus(\\ \qquad\qquad target.DPs[1], \\ \qquad \text{LET } t \triangleq [target \text{ EXCEPT } ![\text{“DPs”}] = Tail(target[\text{“DPs”}])] \\ \qquad \text{IN } \quad t, \\ \qquad\qquad AskAmount(target.DPs[1], target.name, target.requested), \\ \qquad\qquad target.chain \quad) \end{array}$$

Update supply recodrds as soon as the new *DP* is created

$$UpdateSR(Ip, IpHold) \triangleq sr' = [sr \text{ EXCEPT } ![Ip] = sr[Ip] - IpHold]$$

Update the quota after the *cur_hold* is given

$$UpdateQuota(p, a, p\_hold) \triangleq quota' = [quota \text{ EXCEPT } ![p][a] = quota[p][a] - p\_hold]$$

$$\begin{array}{l}
\mathit{GiveAndUpdateQuota} \triangleq \\
\quad \land\ \mathit{target}' = [\mathit{target}\ \textsc{except}\ \begin{array}[t]{l} ![\text{“akr”}][\text{“cur\_hold”}] = \mathit{target}[\text{“akr”}][\text{“cur\_hold”}] + \mathit{target}[\text{“cur\_hold”}], \\ ![\text{“cur\_hold”}] = 0, \\ ![\text{“requested”}] = \mathit{target}[\text{“requested”}] - \mathit{target}[\text{“cur\_hold”}]] \end{array} \\
\quad \land\ \mathit{UpdateQuota}(\mathit{target}.\mathit{name},\ \mathit{target}[\text{“akr”}].\mathit{name},\ \mathit{target}[\text{“cur\_hold”}])
\end{array}$$

$$\mathit{CheckQuota} \triangleq \mathit{quota}[\mathit{target}.\mathit{name}][\mathit{target}[\text{“akr”}].\mathit{name}]$$

Condition 1: If target has nothing to give
Condiiton 2: and quota for its asker is 0 or it has no one to ask Actions:

1. Make switch the target to the previous one
2. Remove the last one in AC
3. Update the second last one

$$\begin{array}{l}
\mathit{CanRemove} \triangleq \\
\qquad \land\ \mathit{target}[\text{“cur\_hold”}] = 0 \\
\qquad \land\ \lor\ \mathit{CheckQuota} = 0 \\
\qquad \ \lor\ \mathit{target}[\text{“DPs”}] = \langle\rangle
\end{array}$$

Remove the last one in AC and update the the second last one

$$\begin{array}{l}
\mathit{Remove} \triangleq \\
\quad \land\ \mathit{CanRemove} \\
\quad \land\ AC' = [i \in 1 \mathrel{..} (\mathit{Len}(AC) - 1) \mapsto \textsc{if}\ i = (\mathit{Len}(AC) - 1)\ \textsc{then}\ \mathit{target}[\text{“akr”}]\ \textsc{else}\ AC[i]] \\
\quad \land\ \mathit{target}' = \mathit{target}[\text{“akr”}] \\
\quad \land\ \textsc{unchanged}\ \langle \mathit{requesters},\ \mathit{sr},\ \mathit{quota},\ \mathit{total} \rangle
\end{array}$$

Condition 1: If now target is not exit room
Condition 2: *Quota* from target to its asker is not 0
Condition 3: Target still have some *DPs*
Condition 4: Target has noting to give Actions:

1. Add the new IP to AC
2. Update supply records

$$\begin{array}{l}
\mathit{AddIPAndSwitchT} \triangleq \\
\quad \land\ AC' = \textsc{let}\ \mathit{IP} \triangleq \mathit{GetIP} \\
\quad \phantom{\land\ AC' = {}} \textsc{in}\ \ \mathit{Append}(AC,\ \mathit{IP}) \quad \text{Add } \mathit{IP} \text{ to } AC \\
\quad \land\ \mathit{target}' = AC'[\mathit{Len}(AC')] \qquad \text{switch target} \\
\quad \land\ \mathit{UpdateSR}(\mathit{target}'.\mathit{name},\ \mathit{target}'.\mathit{DSA})
\end{array}$$

$$\begin{array}{l}
\mathit{CanAskDP} \triangleq \\
\quad \land\ \mathit{target}[\text{“cur\_hold”}] = 0 \\
\quad \land\ \mathit{CheckQuota} > 0 \\
\quad \land\ \mathit{target}[\text{“DPs”}] \neq \langle\rangle \\
\quad \land\ \mathit{AddIPAndSwitchT} \\
\quad \land\ \textsc{unchanged}\ \langle \mathit{quota},\ \mathit{requesters},\ \mathit{total} \rangle
\end{array}$$

Condition 1: target's current hold is greater than 0 Actions:

1. Update asker and quota
2. Clear target's hold
3. No change for these variables

$$
\begin{aligned}
&\mathit{CanGiveAkr} \triangleq \\
&\qquad \land\ \mathit{target}[\text{"cur\_hold"}] > 0 \\
&\qquad \land\ \mathit{GiveAndUpdateQuota} \\
&\qquad \land\ \textsc{unchanged}\ \langle \mathit{sr},\ \mathit{AC},\ \mathit{requesters},\ \mathit{total} \rangle
\end{aligned}
$$

Condition 1: And it has no DP $\mathit{left}(\mathit{target}[\text{"DPs"}] = \langle\rangle)$ Actions:

1. Give the its *cur_hold* to total list
2. Remove it from AC
3. No change for these $\mathit{variables}(\mathit{sr},\ \mathit{quota},\ \mathit{requesters})$

$$
\begin{aligned}
&\mathit{UpdateTotal} \triangleq \\
&\qquad \land\ \mathit{target}[\text{"DPs"}] = \langle\rangle \\
&\qquad \land\ \mathit{total}' = \mathit{Append}(\mathit{total},\ \mathit{target}[\text{"cur\_hold"}]) \\
&\qquad \land\ AC' = \mathit{Tail}(AC) \\
&\qquad \land\ \textsc{unchanged}\ \langle \mathit{sr},\ \mathit{quota},\ \mathit{requesters},\ \mathit{target} \rangle
\end{aligned}
$$

Condition 1: There's still some DP to ask Actions:

1. Create an IP from the first DP of taret
2. Append it to AC
3. Remove the first DP from the target's DP
4. No change for these $\mathit{variables}(\mathit{sr},\ \mathit{quota},\ \mathit{total})$

$$
\begin{aligned}
&\mathit{RequesterAddIP} \triangleq \\
&\qquad \land\ \mathit{target}[\text{"DPs"}] \neq \langle\rangle \\
&\qquad \land\ \mathit{AddIPAndSwitchT} \\
&\qquad \land\ \textsc{unchanged}\ \langle \mathit{quota},\ \mathit{total},\ \mathit{requesters} \rangle
\end{aligned}
$$

Condition 1: If there's no IP in AC

Condition 2: IF there's still requesters in the $\mathit{List}(\mathit{requester} \neq \langle\rangle)$ Actions:

1. Select the first requester and create an IP
2. Add the IP to AC
3. Change the target to the AC's first IP
4. Remove the first one in $\mathit{requesters}(\mathit{requesters}' = \mathit{Tail}(\mathit{requesters}))$
5. No change to sr, quota, target, total

$$
\begin{aligned}
&\mathit{SelectRequesterAndAddToAC} \triangleq \\
&\qquad \land\ \mathit{requesters}\ \neq \langle\rangle \\
&\qquad \land\ \qquad AC' = \textsc{let}\ DP \triangleq \mathit{GetRequesterDP} \\
&\qquad \qquad \qquad \qquad \textsc{in}\ \ \mathit{Append}(AC,\ DP) \\
&\qquad \land\ \mathit{target}' = AC'[1] \\
&\qquad \land\ \mathit{requesters}' = \mathit{Tail}(\mathit{requesters}) \\
&\qquad \land\ \textsc{unchanged}\ \langle \mathit{sr},\ \mathit{quota},\ \mathit{total} \rangle
\end{aligned}
$$

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$Init \triangleq$
    $\land \quad sr = SupplyRecords$
    $\land \quad quota = Quota$
    $\land \quad target = GetStatus(1, \ -1, 10000, \{-1\})$
    $\land \quad requesters = Requesters$
    $\land \quad AC = \langle target \rangle$
    $\land \quad total = \langle \rangle$

$Next \triangleq$
  $\lor \land Len(AC) = 0$
     $\land SelectRequesterAndAddToAC$  Has nothing in $AC$, pick one in requesters and remove it from requesters
  $\lor \land Len(AC) = 1$
     $\land \lor RequesterAddIP$  When $AC$ only has requester, and it have $DPs$ to ask, add one to $AC$
        $\lor UpdateTotal$  When the requester has nothing to ask, append its hold to total list, and remove itself
  $\lor \land Len(AC) > 1$
     $\land \lor CanGiveAkr$  Once have something to give and has quota, give
        $\lor CanAskDP$  Havs nothing to give, and still has quota, and still has $DPs$
        $\lor Remove$  Has nothing above, remove and switch target

$SUM \triangleq \text{LET } Sum[i \in 1 \,..\, Len(total)] \triangleq \text{IF } i = 1 \text{ THEN } total[i] \text{ ELSE } total[i] + Sum[i-1]$
           $\text{IN} \quad \text{IF } total = \langle \rangle \text{ THEN } 0 \text{ ELSE } Sum[Len(total)]$
$Expectation \triangleq \Diamond\Box(SUM = 160)$
$Spec \triangleq Init \land \Box[Next]_{vars}$

$FairSpec \triangleq Spec \land \mathrm{WF}_{vars}(Next)$

$\text{THEOREM } FairSpec \Rightarrow Expectation$
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* Modification History
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