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**Two** **Notes** **on** **Dynamics** **of** **Fixed** **Capital**

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ABSTRACT. This notes present two propositions on ixed capital. The irst one concerns extension of the so-called Cambridge-equation. We start from the sys- tem of commodity production, in which aged ixed capital is jointly produced. By eliminating aged ixed capital, we can establish equilibria of production prices and the uniform growth path. With respect to gross proit, *i.e.* the internal reserve, and the accumulation of capital from gross proit, we establish the Cambridge equa- tion. The second concerns accelerated depreciation of ixed capital. If depreciation of ixed capital is made in a period shorter than its duarability and depreciation is reinvested for ixed capital at once, then the oscillation of capacity of the pro- duction process concerned wil have a larger spike, and thus the whole economic process will be disturbed harder than otherwise.

**1.** **Introduction**

Fixed capital is the central core of materialistic capital. Hence, the centre of capital accumulation is the accumulation of ixed capital.

One of most important features of ixed capital is that it resides in the production process for more than one cycle of reproduction. Hence, evaluation of ixed capital needs a special consideration in accounting. That is, amortisation of ixed capital should be made.

Amortisation is a procedure to calculate a part of value of advanced ixed cap- ital which should be recovered in the current cycle of production. In this sense, amortisation is a part of cost of production, irst of all.

As opposed to liquid cost, however, the amount of depreciation is not usually kept at the hands of producers until ixed capital should be replaced. Hence, there is a posibility that the amount of depreciation can be reinvested for additional units

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of ixed capital. In this case, the amount of depreciation plays a role, as if it were a part of the source for investment.

The plan of the article is as follows. Section 2 is for the equilibrium analysis of the economy *à* *la* Marx-Sraffa. Section 3 will focus on depreciation of ixed capital and its reinvestment, which has been known as the Marx-Engels effect or the Ruchti- Lohmann effect. It is conirmed that reinvestment gives a not small impact on the economy. In Section 4, further problems of accelerated depreciation, will be taken up. Section 5 will give short, tentative conclusions and future problems.

**2.** **Marx-Sraffa’s** **Price** **Basis**

Price and quantity equilibria of joint production systems will be summarised here.

A general joint-production system is not discussed, however. In what follows, it is assumed that aged ixed capital is jointly produced; there is no process that produces only aged ixed capital.1

Let B and M stand for the output and the input matrices, respectively. M in- cludes indirect inputs of wage goods. Assume that B and M are of the same dimen- sion, say m 人 n; the economy consists of m types of commodities and n processes. Let p stand for an m-dim price vector, and the uniform proit rate equilibrium is expressed by p such that

pB D 入pM;

where 入 denotes the proit factor.

Sraffa(1960) showed that the system of equations in the above can be reduced to a system with only brand-new commodities. Okishio-Nakatani(1975) applied the reduction procedure to the Marxian system with *ex* *ante* wages. Asada(1982) named these models as SON. A formal summary of this reduction is given in Li- Fujimori(2013).

We consider here a small scale, simpliied linear model of this kind. Put

0 k1 0 0 k2 0 0 1 0 1 1 1 0 0 01

B 0 k1 0 0 k2 0 C Bk1 0 0 k2 0 0 C

M D ; B D 

B 0 0 k1 0 0 k2 C B 0 k1 0 0 k2 0 C

(fl1 fl1 fl1 fl2 fl2 fl2 A ( 0 0 0 1 1 1A

1 This kind of economies was named as the plain economy by Shiozawa. See Fujimori(1982, p.36.)

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and the full price vector is expressed by: p D (p1 p p p2 ), where p indi- cates ixed capital of age i.

In the reduced system, to which only brand-new commodities belong, the sys- tem will be represented by a set of the following:

K D ! ; b.r; T/ D . ; T/ ! ; F D ! ; L D (l1 l2)

1

with the *rate* *of* *depreciation* .r; T/ D 1 C .1 C r/ C . . . C .1 C r/C一1 ; T D 3, for durability T. One obtains

 D .rI C b.r; T//K C .1 C r/FL

for an equilibrium price vector  D (p1 p2).

**3.** **Pasinetti’s** **Formula**

If ixed capital is applied in the production process, the Pasinetti Formula (*Cf.* Pasinetti, 1962) should be reformulated. It is Koshimura(1968) that irst tried to reestablish the Pasinetti formula with ixed capital. Koshimura built a model in the value terms, so that his extension turned out to be a rather complicated one.

Li(2012) extended the Pasinetti formula to the case with ixed capital. On the quantity side, one has

(3.1) Bx D .1 C g/Mx C u;

where u stands for consumption. A similar reduction of the system with aged ixed capital to the system with only brand-new commodities is possible.

The sum of brand-new commodities produced throughout the economy is given by the following:

3

(3.2) qi D Xx:

hD1

If the economy is in the balanced growth state, one has

(3.3) ki x D ki x D ki xi1 :

(3.4) q D ( .b.g; T/ C gI/K C .1 C g/FL)q C C;

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TABLE 1. Input-Output Relationships

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | |  | inputs | | outputs | | |
| process | | 1 | 2 | 3 | 1 | 2 | 3 |
| t | 0  1  2 | k1x |  |  | x k1x |  |  |
| 0  t C 1 1  2 | | k1x | k1x |  | x k1x | x  k1x |  |
| 0  t C 2 1  2 | | k1x | k1x | k1x | x k1x | x  k1x | x |

q D ! ; C D ! ; b .g; t/ D

In the brand-new commodity world, one has

.g; t/

0

! :

(3.5) q D b .r; t/Kq C r Kq C .1 C r/FLq;

(3.6) q D b .g; t/Kq C g Kq C .1 C g/FLq C C;

so that

This implies that *gross* *proht* or *internal* *reserve* is expressed by

(3.7) r .K C FL/q C b .r; t/Kq D g .K C FL/q C b .g; t/Kq C C:

net proit C depreciation D net investment C replacement C consumption:

Hence, we deine

gross proit

gross proit rate D initial capital ;

gross investment

rate of accumulation D :

gross proit

Let S.t/ denote the nominal amount of ixed capital in period t. One has

(3.8) 」S.t/ D S.t C 1/ — S.t/ D gS.t/:

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This can be rewritten as

(3.9)

(3.10)

From (3.8),

」S.t/ D k1x C k2x C g.k1x C k2x/

D  (k1q1 C k2q2 ):

S.t/ D 」S.t/ D  (k1q1 C k2q2 ):

In the production-price term, one has

 (p1k1q1 C p1k2q2 ) D  (g Kq C .g; 3/Kq):

Hence,

(3.11)

(3.12)

Therefore,

D

r

D

a

|  |
| --- |
| r Kq C .r; 3/Kq |
| (g Kq C .g; 3/Kq); : |

(3.13) a 人 r D  人  D g:

Thus, Pasinetti’s formula holds in this case, if depreciation is evaluated by the so- called pension method.

Note that the above formula does not hold for the case of r D 0.

One sees that depreciation of ixed capital is a part of cost of production on one side, but on the other side it is a source for capital accumulation and appears as if it were a part of proit. Depreciation of ixed capital has two faces.

**3.1.** **A** **small** **note** **on** **technologies** **with** **greater** k**.** By way of excurssion, we examine the sensisivity of gross proit to durability and the magnitude of k.

If we consider a point of equilibrium, it is clear that an increase in the input coeficient k will decrease the rate of proit.

In fact, this can be illustrated as follows. Suppose the simplest case that there is only one tye of brandnew commodities, that is, the other remaining commodities are all aged ixed capital. Assume that there is only one type of ixed capital of zero-age.

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The model is characterised by the output- and input-matrices, such as

0 1 1 11 0k 0 01

B D Bk 0 0 C ; A D B0 k 0 C ;

( A ( A

0 k 0 0 0 k

respectively. Since A and B are square matrices, one can compute the eigensystem of BA一1 . The reciprocal of eigenvalues, except zeros, gives the spectrum of pB D 入pA. It is evident that an inclrease in k will decrease 入 with a constant level of the

real wage rate, so that a greater ixed capital coeficient will pull down the rate of

proit: < 0.

On the other hand, investment in heavier ixed capital will bring back a greater amount of depreciation, so that a greater amount of gross proit will be gained by capital-deeping investment.

In capitalists’economies, capitalists will avoid employing less eficient system of technologies. In the class struggle situation, howver, in which there is a possi- bility to decrease the real wage rate or labour inputs by further investment in ixed capital, capitalists will resort to capital-deepening technologies with greater gross proit and a larger k in the short-run.

In fact, put p D 1 and q D 1, and the internal reserve z D r.k C !l/ C '.r/k. We have dz D dr..1 Cd'/k C!l/ C .r C'.r//dk Crld! Cr!dl: Hence, there is a possible combination of dz > 0; dk > 0; dr 下 0. (! denotes wages.)

**4.** **Marx-Engels-Ruchti-Lohmann-Kalecki-Steindl** **Effect**

**4.1.** **Introduction.** Before discussing in depth the above two-sidedness of depre- ciation of ixed capital, we take up reinvestment of depreciation of ixed capital.

In their correspondence Marx and Engles discussed whether reinvesting depre- ciation increases the amount products of the production process. Later, the same problem was disscussed by Ruchti and Lohmann in 1940s . Hence, this effect was named as the Marx-Engles effect or the Ruchti-Lohamann effect. Stendl(1952) paid attention to this effect in discussing the prosperity and decline of the post- war American capitalism. Therefore, we name this effect, icluding Kalecki(1954), as the Marx-Engles-Ruchti-Lohmann-Kalecki-Steindl effect, in short MERLKS, in what follows. We discuss the working mechanism of this effect with some simple examples.

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**4.2.** **Normal** **depreciation.** In the simplest case with a constant ixed rate of depre- ciation, say the reciprocal of the durability of ixed capital, it is known that the total capacity of production lines will be increased, that this magnitude of increases in capacity is determined by durability alone, and that the top peek of increase comes in exactly the same with durability.

It is assumed, for the sake of simplicity, that ixed capital is divisable *ad* *inihni-* *tum*.

The following is a simple example of MERLKS with durability 5.

TABLE 2. MERLKS—Normal depreciation

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| period | | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| capacity | | 1000 | 1200 | 1440 | 1728.0 | 2073.60 | 1488.320 | 1585.984 |
| age | 0  1  2  3  4 | 1000 | 200  1000 | 240  200  1000 | 288.0 240.0 200.0 1000.0 | 345.60 288.00 240.00 200.00  1000.00 | 414.720 345.600 288.000 240.000  200.000 | 297.664 414.720 345.600 288.000  240.000 |
| depreciation | | 200 | 240 | 288 | 345.6 | 414.72 | 297.664 | 317.197 |

Reinvestment of depreciation of ixed capital will create uctuations in the economy, and the cycle of this uctuation will not be shorter than the twice of

durability, although dynamics of this renewal process is converging to the stationary

state. The stationary state of MERLKS is given by its multiplier p D , where m denotes durability. In the above example case, it is  人 1000 D 1666:66 : : :.

As simple calculation shows, the effect indicates that the amount of ixed capital will converge to the stationary value, when depreciation of ixed capital of each year is just invested.

The irst apex comes in more or less the same year length with its durability. This means that the reinvestment of depreciation of ixed capital brings about the upward thrust or spike of the total capacity of ixed capital installed in the produc- tion process.

The length of periods in which the stationary state is reached is usually much longer than durability of ixed capital.

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**4.3.** **Yamada-Yamada’s** **renewal** **dynamics.** In Li-Fujimori(2013), some formal results of MERLKS are reviewed in view of Yamada-Yamada(1960) . Yamada- Yamada’s system of equations is given by the next three:

(4.1) D.t/ D K.t — 1/;

(4.2) H.t/ D F.t — m/ C D.t — m/;

(4.3) K.t/ D K.t — 1/ C F.t/ C D.t/ — H.t/:

In what follows, we ignore net investment: F.t/ D 0. Then, one obtains

Hence,

(4.4)

K.t/ — ( 1 C )K.t — 1/ C K.t — m — 1/ D 0:

K.t C m C 1/ — ( 1 C )K.t C m/ C K.t/ D 0:

The characteristic equation of the above is expressed by

(4.5) 入mC1 — ( 1 C )入m C  D 0:

After factorisation, one obtains:

.入 — 1/2 (入m一1 C 入m一2 C . . . C ) D 0:

Therefore, characteristic roots other than duplicated 1 are derived from the follow- ing:

(4.6) 入m一1 C 入m一2 C . . . C  D 0:

By applying Eneström-Kakeya’s theorem on polynomial equations to (4.6), Yamada- Yamada proved that the dynamic path of ixed capital converges to a steady state, with diminishing oscillation.2; 3

2As for Eneström-Kakeya’s theorem, refer to *e.g.* Anderson-Saff-Varga(1979).

3The characteristic equation of the original Yamada-Yamada model has duplicated root 1. Hence,

the Jordan form of the companion matrix of (4.5) includes a Jordan block !; namely, the

dynamics of the system is not free from the possibility of resonance.

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Yamada-Yamada(1960) found that the characteristic equation of renewal dy- namics of ixed capital has duplicated dominant roots, but did not refer to possibil- ity of resonance caused by duplicate roots.4 Although renewal dynamics itself is converging, the problem is how soon it converges.

**4.4.** **Accelerated** **depreciation.** This subsection will see how the situation will be changed if accelerated depreciation takes place.

Most of analyses of ixed capital depend on the assumption that the production processes proceed normally as expected *ex* *ante*. That is, ixed capital will be used until it reaches its technical durability.

Let us supposethat durability of ixed capital is m years. Its depreciation, how- ever, is done in the irst C D m — 1 years. That is, ixed capital is operated without depreciation in the period of its inal age. This is a simple case of accelerated de- preciation.

Yamada-Yamada’s equation will be replaced by the following:

(4.7) H.t/ D D.t — m/;

(4.8) K.t/ D K.t — 1/ C D.t/ — H.t/;

(4.9) D.t/ D  .K.t/ — D.t — m — 1// :

From these, one gets

D.t/ — C D.t — 1/ C 1 D.t — m/ C 1 D.t — m — 1/

(4. 10) C — 1 C — 1 C — 1

— D.t — m — 2/ D 0:

The characteristic equation of this is given by

(4.11) 入mC2 — 入mC1 C 入2 — 入 —  D 0:

This can be factorised as follows:

(4.12) .入 — 1/2 (入m C 入m一1 C 入m一2 C . . . C ) D 0:

The next example is a modiication of the above Table 2 to show accelerated depreciation. We assume that ixed capital operates for 5 periods, but depreciation

4 The matrix for renewal dynamics of ixed capital is similar to that of population dynamics,

known as the Leslie matrix.

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4. Hence, every equipment is operated in the last period of its life without keeping any economic value. Besides, we ignorethe tax system.

will be made in 4 periods. The rate of depreciation is set to a constant, *i.e.* 1

TABLE 3. MERLKS—Accelerated depreciation

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| period | | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| capacity | | 1000 | 1250 | 1562.5 | 1953.125 | 2441.406 | 1488.32 | 1939.697 |
| age | 0  1  2  3  4 | 1000 | 250  1000 | 312.5 250.0 1000.0 | 390.625 312.500 250.000 1000.000 | 488.281 390.625 312.500 250.000  1000.000 | 360.352 488.281 390.625 312.500  250.000 | 387.939 360.352 488.281 390.625  312.500 |
| depreciation | | 250 | 312.5 | 390.625 | 488.281 | 360.352 | 387.939 | 406.799 |

Remark that ixed capital of age 4 is counted for as a part of capacity, but it has no value, and hence never enters into the object of depreciation any longer.

As is easily seen from tables, accelerated depreciation yields a larger increase in capacity for longer periods than otherwise. Accelerated depreciation reveals to be something like *positive* *externalities*.

It seems that all of irms know, however, that their equipments will sooner or later confront with new equipments with new technology. Hence, the accelerated amortisation method is often employed by irms, because most of irms are threat- ened by the advent of new technology.

Remark that the multiplier of MERLKS p depends on the period of deprecia-

tion, and not on durability. In the above case one has p D  .  D 2.

**5.** **Concluding** **Remarks**

We dealt with depreciation of foxed capital from the angle of its inuence on growth and uctuation of the economy.

In 1960s, many Japanese Marxian economists discussed MERLKS in relation to the theory of crisis. Many of them were then based on value-ridden scheme of production by Marx, so that they seem to deny the inuence of MERLKS to economic uctuations in the end. They seem to regard depreciation only as cost.

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They relied on the fact that the amount of ixed capital converges in the long-run. They did not notice that the true problem is how soon it converges.

If we take into account that the convergence time is greater than durability, MERLKS should not be ignored in considering the dynamic process, the part of which is a crisis.

In this section, we summarise the points which were taken up with respect to ixed capital in this short article.

As we saw in the above, depreciation has two faces. Is depreciation cost or proit? The answer might be ‘both.’Fixed capital possesses two faces (Paton- Littleton, 1940); in one side, depreciation is a cost factor, and in the other, depreci- ation becomes a part of the source for investment.

Now, the relationship among the rate of proit, the rate of growth and the rate of accumulation is an important point in discussing economic theory. Our analysis shows that depreciation should be treated as a part of the source for investment, and we can reestablish the so-called Cambridge equation. The Cambrdige equation is considered in the brandnew commodity world seems to give a good explanation of economies, say China’s economy in the past 20 years. We will elaborate on this point in near future.

If we focus on reinvestment of depreciation, we immeadiately see that it will shift the stationary state of the economy upward, to the one with higher level of production capacity from the initial one, which is called the MERLKS effect. This movement is a converging oscillation. Its convergence is slow, accompanied by spikes of capacity. Hence, one should pay attention to this nature of ixed capital; ixed capital is an indispencible basis of production, but, at the same time, invest- ment of ixed capital brings about disequilibrium in the economy.

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