Project 3 - True Cost of a Mortgage

Group B12

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1 Problem Statement

Find the true cost of a mortgage, taking in to account a range of factors such as inflation, cost of living and others. There are many types of interest that you may pay on a mortgage, all of them affecting which is the best way of getting the lowest price. Investigate this.

2 Introduction

The true cost of a mortgage contains a variety of factors, ranging from the type of mortgage and interest rate a customer chooses, to personal problems that can effect such circumstances. Using various variables such as number of repayments and annual interest rates, a formula can be made to allow individuals to work out their own personal mortgage. This figure however may need to be adapted based on outside circumstances such as losing a job or changing house prices. All of this being highlighted in the report. Some variables have been controlled to enable a simple equation to be made, such as the interest rate being fixed over the whole mortgage, and payments are made in monthly installments. Using this, the equation can be used by anyone wanting to find the true cost of their mortgage.

3 Variables

We will use the following variables throughout our analysis.

Variable	**Symbol**
Number of Repayments	N
Monthly Mortgage Payment	M
Annual Interest Rate	I
Capital remaining after N payments	P_N
Total Interest over whole loan	T
Total cost over whole loan	C

4 Background

4.1 Types of mortgage

Fixed rate mortgages These mortgages stay the same throughout the term length that is decided between you and your mortgage provider. They usually have a two or five year fix which means that the interest charged is the same throughout that period, and then reassessed at the end of that fix.

Variable rate mortgages On the other hand, variable rate mortgages have interest rates that can change at any time. Most likely this will be a standard variable rate (SVR) in which the interest rate rises or falls alongside the base rate set by the Bank of England.

Normally with these mortgages you can overpay or leave anytime whereas you cant do that with fixed rate.

Tracker mortgages These mortgages change in line with another interest rate, usually the Bank of England base rate, plus an extra percentage. This usually is a two or five year fix though some can last until you switch to another deal.

Interest-only mortgages In these mortgages, you only pay off the interest generated for that month each month, meaning that you will be paying off less each month, compared to other mortgages. However, at the end of your term, you will have to pay off the remaining capital, which will be the initial size of the mortgage. The idea is that the borrower will be investing money elsewhere to generate enough to pay this off later.

4.2 Types of interest rates

Fixed interest rate This rate is unaffected by changes in the market rate and so borrowers have a fixed percentage of their loan to repay over an agreed term length.

Variable interest rate This interest rate can be changed at any point during the agreed term length, by the lender, with the changes often being a consequence of an increase or decrease in the base rate.

Annual percentage rate APR is calculated using any interest charges as well as other fees and is meant to show a borrower the overall impact/cost of their loan, meaning APR is used to compare different loans.

Annual equivalent rate AER is the interest rate for savings, rather than loans like APR, but again can be used to as a comparison tool.

5 Fixed interest rate mortgage

5.1 Deriving formula for repayments

Let N be the number of repayments, M be the monthly mortgage payment, I be the annual interest rate, P_0 be the total loan amount initially where P_N is the amount owing after N payments. After the first month we reduce the amount owing by

$$M - P_0 r$$

where $r = \frac{I}{12}$, so the amount owing after the first payment will be

$$P_1 = P_0(1+r) - M$$

with the general recursive formula

$$P_{N+1} = P_N(1+r) - M. \tag{1}$$

Next we show this is equivalent to saying

$$P_N = P_0(1+r)^N - [M + M(1+r) + M(1+r)^2 + \ldots + M(1+r)^{N-1}]$$

where we use the principle of mathematical induction.

Proof: Let $P(N)$ be the statement "$P_N = P_0(1+r)^N - [M + M(1+r) + M(1+r)^2 + \ldots + M(1+r)^{N-1}]$". For $P(1)$ we have

$$P_1 = P_0(1+r) - M$$

which we showed was the formula for the first payment above. Thus we have $P(1)$ holds trivially. Next assume the statement holds for some $N \in \mathbb{N}$. Now for $P(N+1)$ we have

$$P_{N+1} = P_N(1+r) - M$$

(from equation (1)) so by applying the induction hypothesis we have

$$\begin{aligned} P_{N+1} &= [P_0(1+r)^N - (M + M(1+r) + \ldots + M(1+r)^{N-1})](1+r) - M \\ &= P_0(1+r)^{N+1} - [M + M(1+r) + \ldots + M(1+r)^N] \end{aligned}$$

which means $P(N) \Rightarrow P(N+1)$. Hence, the statement holds for all $N \in \mathbb{N}$ by the principle of mathematical induction.

□

Next we use the formula for the sum of a geometric progression

$$S_N = a\frac{(1-r^N)}{1-r}$$

to calculate

$$P_N = P_0(1+r)^N - M\left(\frac{1-(1+r)^N}{1-(1+r)}\right),$$

where we have first term M and common ratio $1+r$. Hence when our mortgage has been repaid (i.e. when $P_N = 0$) we have

$$M\frac{(1+r)^N - 1}{r} = P_0(1+r)^N$$

then dividing through to make M the subject of our formula to obtain

$$\begin{aligned} M &= \frac{rP_0(1+r)^N}{(1+r)^N - 1} \\ &= \frac{rP_0}{1 - \frac{1}{(1+r)^N}}. \end{aligned} \tag{2}$$

We can solve for N to find the number of repayments for a given loan amount. We rearrange (2) as follows

$$\begin{aligned} & & M &= \frac{rP_0}{1 - \frac{1}{(1+r)^N}} \\ \iff & & \frac{1}{(1+r)^N} &= \frac{M - rP_0}{M} \\ \iff & & (1+r)^N &= \frac{M}{M - rP_0} \\ \iff & & N\log(1+r) &= \log\left(\frac{M}{M - rP_0}\right) \\ \iff & & N &= \frac{\log(\frac{M}{M-rP_0})}{\log(1+r)} \end{aligned} \tag{3}$$

to obtain a formula for calculating the number of repayments. We also have that

$$T = NM - P_0,$$

where T is the total interest incurred on the loan.

6 Amortisation Schedule

We can apply the formula derived above to create an *amortisation schedule* [3], whereby the whole loan (including interest) is paid off in regular payments by the end of the term. For example, suppose we take out a £150,000 loan paid over 25 years, at a fixed nominal rate of 3.5%[1]. We assume interest is calculated and charged monthly.

In this case, the loan is paid over 25 years = 300 months, so, $N = 300$. The initial principal $P_0 = 150000$, and $I = 0.035 \Rightarrow r = 0.002917$. Using (2) we obtain a monthly payment M that will allow the loan to be paid off within the term limit.

$$\begin{aligned} M &= \frac{rP_0(1+r)^N}{(1+r)^N - 1} \\ &= \frac{0.002917 \cdot 150000 \cdot (1.002917)^{300}}{(1.002917)^{300} - 1} \\ &= 750.94. \end{aligned}$$

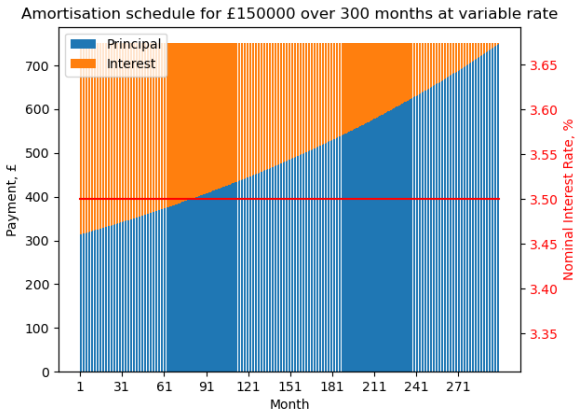


Figure 1: Monthly amortisation schedule, fixed rate. (Ignore the white horizontal lines.)

Figure 1 shows a graph of these monthly payments, and the proportion of each that is interest. At the start of the loan term, the principal is high, so a lot of interest is charged. As the principal is paid off over the course of the term, the amount of interest charged decreases.

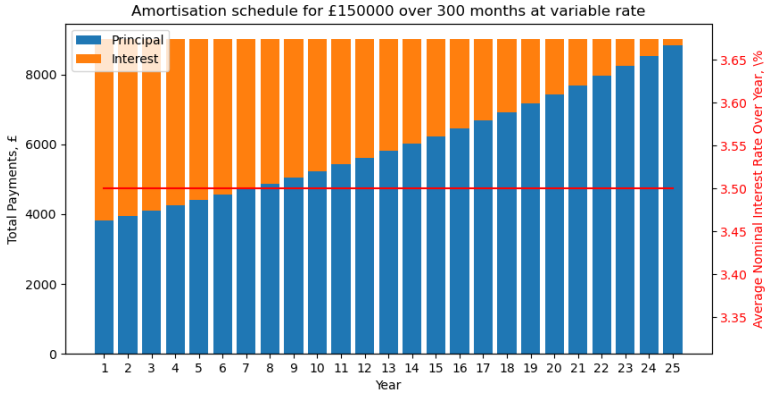
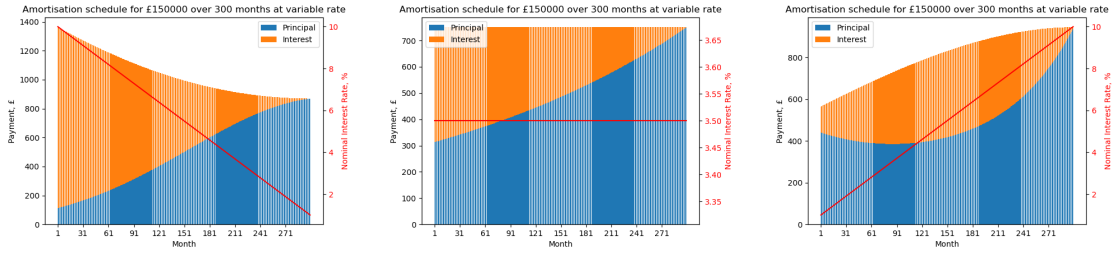


Figure 2: Monthly amortisation schedule, fixed rate. (Ignore the white horizontal lines.)

[1]These are the actual terms of a mortgage offered by Nationwide Building Society, which we found using MoneySuperMarket's comparison tool [5], with the exception that the fixed rate only lasted for 10 years.



(a) Decreasing rate.
Total paid: £310,820

(b) Constant rate.
Total Paid: £225,280

(c) Increasing rate.
Total paid: £241,627

Figure 3: Monthly amortisation schedule, variable rates

In Figure 2 shows the same data but added up over each year. See Table 1 in Appendix A for the data shown in this figure.

6.1 Variable Rates

In practice, completely fixed rates are rare. Instead, the rate is regularly changed by the bank, banks will often offer an initial period during which the rate is fixed.

The exact variation in the rate of interest throughout the term has a huge impact on the total amount paid. If the interest rate is high at the start of the term, a lot of interest will be charged since the principal has not been paid reduced by much. If the interest rate is high at the *end* of the term, it will not cause as high an increase in interest since the principal will have been mostly paid off.

We built a program to simulate how different changes in rate may affect the total amount paid. After each month, the program calculates the amount to be paid using the formula we derived in Section 5.1, as if that rate were to continue for the rest of the term. This ensures that the full loan is paid by the end of the term regardless of any variation in interest rate.

We analysed the effect of three *linear* rate variations on the mortgage example from Figure 1. Of course, in reality the rate would not change linearly, but this model gives an idea of how different changes affect the total amount paid.

In Figure 3(a), the interest rate decreases from 10% nominal to 0.2% nominal. In Figure 3(c), it increases from 0.2% nominal to 10% nominal. Figure 3(b) displays Figure 1 again for comparison. Note the difference in y-axis scale across the three diagrams; vastly more interest is paid in the decreasing rate example (as expected) whereas the increasing rate had little effect on the total amount paid (since it was lower than 3.5% at at the start of the term).

This shows that the total amount paid is very sensitive to the interest rate at the start of the mortgage. This is why lenders often offer fixed rates for a period at the start of the term. However, the borrower should be aware of the risk that if their rate goes up substantially at the start of the term, the total amount they pay towards the mortgage will increase substantially regardless of how low the rate is set during the rest of the term.

7 General discussion

7.1 Assumptions

- The mortgage is taken out in the UK.
- People are able to pay up to 10% more than the monthly payment towards the mortgage until a fee is required to pay more.
- Payments are made in monthly installments and this is when the interest rate is calculated.

- Type of interest quoted - AER, nominal, APR, etc.
- (later) ignore fees, moving costs, estate agent fees, etc.
- Assume fixed interest rate stays at the same level for the entire mortgage repayment.
- Assume the borrower doesn't want to take out an interest-only mortgage, as they are very uncommon.

7.2 Different Interest Rates

Fixed interest rates allow people to know the exact amount they must pay each month without worrying whether it will suddenly increase or decrease depending on inflation. Whilst people may lose out if suddenly interest rates drop, they are protected against having interest rates suddenly increase. This can cause fixed interest rates to be higher than variable interest rates. It is a safer option for most people and could be considered more convenient. However, it must be noted that the set interest rates do not last for the complete duration of the loan repayment. The interest rates you pay only last for the term of lending which is typically a few years but can vary in length. Then you must change the mortgage product, otherwise the interest rate will revert to standard APR which is typically a lot higher. However, as more of your loan gets paid off, the bank's investment into you becomes safer and you are seen as more reliable, hence the deals will typically get better the more money you have invested. However, the interest rates given still depend on inflation rate.

Variable interest rates are more unpredictable. This means that they can often give better deals than fixed interest rates. The interest rate changes monthly and is centred around inflation. This can cause issues with being able to afford the repayment costs if inflation causes the interest rates to rise higher than expected.

7.3 Personal Circumstance

Personal income and house price will determine the interest rate and total cost of a mortgage. The cost of living for a household should also be considered before taking out a mortgage. It is imperative to work out how much money per month someone can afford to put towards paying off the mortgage as this will impact the type of mortgage they get. If there is limited supplementary income, it is more likely that they will be given a high interest rate to compensate for the lower monthly repayments. Whilst it would end up costing more overall, it can be more sustainable and manageable in the long run.

Should someone come into an inheritance or suddenly gain a substantial amount of money, it would be beneficial to dedicate some of that to paying off the mortgage. This would overall reduce the amount they have to pay back because the interest rate will be calculated from a lower sum of money. If the individual had access to this money before taking out the mortgage, then the initial deposit could be a higher percentage of the overall house price, meaning that the bank would most likely issue a lower interest rate for the whole duration of the contract.

7.4 Outstanding Circumstance

It may be relevant to discuss the impact of Covid-19 on the true cost of mortgages. Due to a significant decrease in income for most households over the past year, there has been a substantial quantity of mortgage holidays; over 1.6 million homeowners suspended their mortgage repayment as they were unable to afford it. This is approximately 1 in 7 households [2]. As a result of this, additional interest gets added to each mortgage meaning either a longer repayment time or higher monthly repayments. Interest rates have also decreased dramatically, with the Bank of England changing the base rate to 0.1% (the lowest ever in UK history [4]). As of such, currently is a good time to be getting a mortgage loan. However, many lenders require higher and increased deposits to have better security over the properties.

7.5 Interest Only

With an interest-only mortgage, you only pay the interest of the loan, then once the term has ended you are left with the original amount that you borrowed still to pay off using a separate account. It's very difficult nowadays to get an interest-only mortgage; since the 2008 financial crisis, when these types of mortgage repayments became very popular, it soon became apparent that hundreds of thousands of customers would struggle to pay off their loan once the term had ended [6]. This is since most people did not have a secure plan for how to repay the rest of their mortgage once they had finished paying off the interest - due to the high risk and the global economic crash in 2008, the return from most savings vehicles were significantly lower than people were expecting. Because of this unpopularity of interest-only mortgages with lenders, we have assumed that the borrower instead will be taking out a repayment mortgage, where you repay both the interest and the loan at the same time.

7.6 Overpayments

Typically, overpaying will be favourable as we will be saving on interest that would have been incurred otherwise. For example, using (3) with $M = 1000$, $r = 0.0025$ and $P_0 = 200000$ gives a mortgage term of 278 months (rounded to the nearest month). By increasing the monthly payments by 10%, the term reduces by 35 months (to the nearest month). However, in this case we see approximately a 13% decrease in the number of repayments.

A possible issue which may arise as a result of overpaying are early repayment charges (ERCs). The mortgage provider will generally offer an overpayment allowance of 10% per annum of the original loan, with fees typically between 1% and 5% on any amount paid in excess of this.

The timing of mortgage overpayments is also crucial. Depending on how often the interest is calculated by the mortgage provider, with most providers using daily interest, it may be more beneficial to save the money that would have gone towards the overpayments. In particular, if your mortgage provider is calculating interest annually and the interest rate is set in April but your overpayment is made a few months later then the extra payment wont affect the outstanding balance for a year.

8 Conclusion

In summary, the total cost of the mortgage can be calculated by subbing in the suitable values into:

$$C = \frac{NrP_0}{1 - \frac{1}{(1+r)^N}}$$

This is assuming that you are using a fixed rate mortgage, if you are however using a variable rate mortgage you can only estimate the true cost as it requires predicting interest rates going into the future. If you have sufficient reason to believe that interest rates will decline over the course of your term then it will be beneficial to choose a variable rate, to estimate this cost you can alter the value of I, and in turn r, to what would be the average interest rate over your term.

References

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[5] MoneySuperMarket. *Compare Our Best Mortgage Rates*. URL: https://www.moneysupermarket.com/mortgages/. (accessed: 19.04.2021).

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[7] Jitse Niesen. *MATH1510: Financial Mathematics I*. URL: http://www1.maths.leeds.ac.uk/~jitse/math1510/notes-all.pdf. (accessed: 26.03.2021).

[8] The Money Advice Service. *Understanding Different Types of Mortgages*. URL: https://www.moneyadviceservice.org.uk/en/articles/mortgage-interest-rate-options#discount-mortgages. (accessed: 16.04.2021).

Appendix A Further Amortisation Data

Table 1: Annual amortisation schedule data for constant rate example shown in Figure 2. The payments are in GBP (£), and should have been rounded to 2 decimal places.

Year	Principle	Principle Paid	Rate p.a.	Interest Paid	Total Interest Paid	Total Paid
0	150000	0	0	0	0	0
1	146177.9	3822.151	0.035	5189.073	5189.073	9011.224
2	142219.8	3958.093	0.035	5053.131	10242.2	18022.45
3	138120.9	4098.871	0.035	4912.354	15154.56	27033.67
4	133876.2	4244.655	0.035	4766.569	19921.13	36044.9
5	129480.6	4395.624	0.035	4615.6	24536.73	45056.12
6	124928.7	4551.963	0.035	4459.261	28995.99	54067.35
7	120214.8	4713.863	0.035	4297.361	33293.35	63078.57
8	115333.3	4881.521	0.035	4129.704	37423.05	72089.79
9	110278.1	5055.141	0.035	3956.083	41379.14	81101.02
10	105043.2	5234.937	0.035	3776.287	45155.42	90112.24
11	99622.07	5421.128	0.035	3590.096	48745.52	99123.47
12	94008.13	5613.941	0.035	3397.283	52142.8	108134.7
13	88194.52	5813.612	0.035	3197.612	55340.41	117145.9
14	82174.14	6020.384	0.035	2990.84	58331.25	126157.1
15	75939.63	6234.511	0.035	2776.713	61107.97	135168.4
16	69483.38	6456.254	0.035	2554.971	63662.94	144179.6
17	62797.5	6685.883	0.035	2325.341	65988.28	153190.8
18	55873.82	6923.679	0.035	2087.545	68075.82	162202
19	48703.89	7169.934	0.035	1841.291	69917.11	171213.3
20	41278.94	7424.946	0.035	1586.278	71503.39	180224.5
21	33589.91	7689.029	0.035	1322.195	72825.59	189235.7
22	25627.41	7962.504	0.035	1048.72	73874.31	198246.9
23	17381.7	8245.706	0.035	765.5179	74639.83	207258.2
24	8842.72	8538.981	0.035	472.2432	75112.07	216269.4
25	0	8842.687	0.035	168.5377	75280.61	225280.6