# Finals Review

1) $\frac{x^2}{4} + \frac{y^2}{9} + 2y - x + 9 = 0$

Goal : Get this into ellipse form $\left(\frac{(x-c_1)^2}{a^2} + \frac{(y-c_2)^2}{b^2} = 1\right)$

① Bring all like terms together

$\frac{x^2}{4} - x + \frac{y^2}{9} + 2y + 9 = 0$ → $\frac{x^2-4x}{4} + \frac{y^2+18y}{9} + 9 = 0$

or : $\frac{x^2-4x}{4} + \frac{y^2+18y}{9} = -9$

② Complete the square

$\frac{x^2-4x}{4} \Rightarrow x^2-4x+4 \rightarrow (x-2)^2$ (added $\frac{4}{4}=1$)

$a^2+2ab+b^2$

$a=x,\ b=-2$

$\frac{y^2+18y}{9} \Rightarrow y^2+18y+81 \rightarrow (y+9)^2$ (added $\frac{81}{9}=9$)

$a^2+2ab+b^2$

$a=y,\ b=9$

③ Bring everything together, balance it out.

$\frac{(x-2)^2}{4} + \frac{(y+9)^2}{9} = -9 + \underbrace{1+9}$ added from completing the square

$\frac{(x-2)^2}{4} + \frac{(y+9)^2}{9} = 1$ ✓ ellipse form!

↳ minor axis = 4 (B)

center $(2,-9)$

2) focus = $(1,4)$, directrix = $y=2$ Goal: find equation

① establish hypothetical point → $(x,y)$ some point on parabola

distance$_{\text{point to focus}}$ = distance$_{\text{point to directrix}}$

② find distances (to focus) : $d_f = \sqrt{(x-1)^2+(y-4)^2}$ (distance formula) $d_d = \sqrt{(y-2)^2}$

③ set them equal → $\sqrt{(y-2)^2} = \sqrt{(x-1)^2+(y-4)^2}$

$(y-2)^2 = (x-1)^2+(y-4)^2$

expand → $y^2-4y+4 = (x-1)^2+(y^2-8y+16)$

$-y^2+8y-4 \qquad\qquad -y^2+8y-4$

$= \quad 4y = (x-1)^2+12$

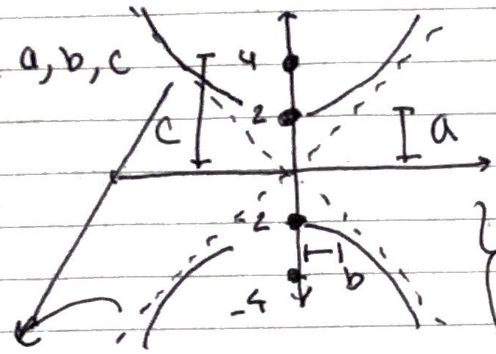
$4y-12 = (x-1)^2 \rightarrow (x-1)^2 = 4(y-3)$ (C)

Hyperbola axes : $a \to$ distance from center to vertex
$b \to$ distance from vertex to asymptote
$c \to$ distance from center to focus

$[c^2 = a^2 + b^2]$

3) hyperbola w/ foci $(0, \pm 4)$, vertices $(0, \pm 2)$
Goal: find equation

① find a, b, c




$\to$ since vertices are $\pm$, and only on y axis, center must be $(0,0)$

$[c = 4]$ $[a = 2]$

$b^2 = c^2 - a^2 = \overset{16}{4^2} - \overset{4}{2^2} = 12$

$[b = \sqrt{12}]$

Asymptotes (every hyperbola has 2)

② Equation $\to$ is it $y^2$ first or $x^2$ ?
$\hookrightarrow$ $y^2$ b/c the hyperbola stretches out vertically

$\cancel{\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1}$ $\quad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

vertical horizontal

③ Plug in $\quad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \quad = \quad \frac{y^2}{4} - \frac{x^2}{12} = 1 \quad$ (B)

4) $(y-2)^2 + 20(x+1) = 0$ Goal: find focus-directrix

① This is called the focus-directrix form of a parabola.

The form (template) is $(y-k)^2 = 4p(x-h)$ (if vertical)
$(x-h)^2 = 4p(y-k)$ (if horizontal)

How to read

Vertex $\to (h, k)$

Focus $\to (h, k+p)$

Directrix $\to$ ($y = k - p$ vertical)
($x = k - p$ if horizontal)

☆ just look @ whichever one is squared

③ Bring equation to form
$\to$ we want variables on opposite sides

$(y-2)^2 + 20(x+1) = 0 \Longrightarrow (y-2)^2 = -20(x+1)$

☆ watch the signs (+/-) $\quad (y-k)^2 = 4p(x-h)$

$h = -1, \; k = 2, \; p = -5$

Vertex $\to (-1, 2)$

Focus $\to (-1, 2-5) \to (-1, -3)$

Directrix $\to$ $y = 2 - (-5) = 7$ $\quad$ (C)

5) Given ellipse on graph. Goal: Find equation.

① Find attributes → 1) center = (0,0)
2) major axis (x in this case) $a = 7$
3) minor axis $b = 7/2$

② plug values into equation:

$$\frac{x^2}{7^2} + \frac{y^2}{(7/2)^2} = 1 \Rightarrow \frac{x^2}{49} + \frac{y^2}{(49/4)} = 1$$

(Take reciprocal of 2nd fraction denominator)

$$\frac{x^2}{49} + \left(\frac{4}{49}\right)\left(\frac{y^2}{1}\right) = 1$$

$$\frac{x^2}{49} + \frac{4y^2}{49} = 1 \qquad \underline{(D)}$$

6) $\log_8(x-3) + \log_8(x+4) = 1$ Goal: Solve

Add prop for logs:

$$\log_8(x-3) + \log_8(x+4) = \log_8((x-3)(x+4)) = 1$$

IGNORE ALL

~~★ if you get 1 on the other side, $\log_{any} 0 = 1$~~

~~so:~~

$$\cancel{\log_8((x-3)(x+4)) = \log_8(0)}$$

$$\cancel{(x-3)(x+4) = 0 \rightarrow x = 3, \text{ or } -4}$$

OF THIS

$$\log_8(\underbrace{(x-3)(x+4)}) = 1$$

$\log_8 8 = 1$ since $8^1 = 8$

$(x-3)(x+4) = 8$ FOIL!

$x^2 + 4x - 3x - 12 = 8 \rightarrow x^2 + x - 12 = 8$

SET TO 0 $\rightarrow x^2 + x - 12 - 8 = 0$

$x^2 + x - 20 = 0$

add to 1, mult to -20 → different signs 1+, 1-

$(5, -4)$

$\underbrace{(x+5)(x-4)} = 0$

$x = 4, \underline{-5}$

extraneous, log can't be (-)

$x = 4$ $\underline{(B)}$

7) $4^{3\log_4 x} = x^3$ Goal: True or false?

① Simplify left side

② $4^{3\log_4 x} = x^3$

reverse power rule : $4^{\log_4 x^3} = x^3$

③ $a^{\log_a x} = x$ rule : $\cancel{4^{\log_4}}\ 4^{\log_4 x^3} = x^3 = x^3$ ✓

True (A)

8) $\log_2(x^3) - \log_2(x) = \log_2(x^2)$ Goal: True or false?

Subtraction Log Rule → $\log_2\left(\frac{x^3}{x}\right) = \log_2(x^2)$

Subtract = divide $\log_2(x^2) = \log_2(x^2)$ → True (A)

9) $4\sqrt{128} = 4^{3x}$ Goal: Solve

① Know the powers of 2 up to at least 8

$2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$, $2^6 = 64$, $2^7 = 128$,

$2^8 = 256$ ~ like be able to recognize, not memorize

② 128 is $2^7$ → $\sqrt{128} = (128)^{1/2} = (2^7)^{1/2} = 2^{7/2}$

4 is $2^2$ → $4\sqrt{128} = 2^2 \cdot 2^{7/2} = 2^{4/2 + 7/2} = 2^{11/2}$

$4^{3x}$ is $(2^2)^{3x} = 2^{6x}$

$2^{11/2} = 2^{6x}$ → $6x = 11/2$ → $x = \frac{11}{2} \cdot \frac{1}{6} = \frac{11}{12}$ (D)

10) $\log_2\left(\frac{\sqrt{3}x^2}{y^4 z^7}\right)$ Goal: write as sum/diff

① Reverse Division rule → $\log_2\left(\frac{\sqrt{3}x^2}{y^4 z^7}\right) = \log_2(\sqrt{3}x^2) - \log_2(y^2 z^7)$

distribute

② reverse mult rule → $(\log_2\sqrt{3} + \log_2 x^2) - (\log_2 y^4 + \log_2 z^7)$

③ power rule → $\log_2\sqrt{3} + 2\log_2 x - 4\log_2 y - 7\log_2 z$

$\sqrt{3} = (3^{1/2})$

power rule : $\frac{1}{2}\log_2 3$ $\frac{1}{2}\log_2 3 + 2\log_2 x - 4\log_2 y - 7\log_2 z$ (B)

11) $\frac{\log(y)}{3} + 9\log x - 2\log 2$ → Goal: make 1 log

① ↓ figure this out

② reverse power → $9\log x - 2\log 2 = \log x^9 - \log 2^2$

③ reverse division → $\log\left(\frac{x^9}{2^2}\right)$

$\frac{\log(y)}{3} = \frac{1}{3}\log(y) = \log(y^{1/3}) = \log(\sqrt[3]{y})$

(reverse power)

$\log(\sqrt[3]{y}) + \log\left(\frac{x^9}{2^2}\right)$

④ Addition rule

$\log\left(\frac{x^9\sqrt[3]{y}}{2^2}\right)$ (B)

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12) $7^{3x+2} = 5$ Goal: Solve (hypothetically)

① $\log_7(7^{3x+2}) = 3x+2$ (Definition of log)

$\log_7(5) = \log_7 5$

② isolate x → $3x+2 = \log_7 5$ → $3x = \log_7 5 - 2$ → $x = \frac{\log_7 5 - 2}{3}$ (C)

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13) $\log_{14}(x) + \log_{14}(x+5) = 1$ Goal: Solve

① Addition rule:

$\log_{14}(x) + \log_{14}(x+5) = \log_{14}((x)(x+5))$

② Right side → $\log_{14} 14$ is 1; so right side $= \log_{14} 14$

③ take away logs → $\log_{14}((x)(x+5)) = \log_{14}(14)$

distribute

$x^2 + 5x = 14$ → $x^2 + 5x - 14 = 0$

7, −2

$(x+7)(x-2) = 0$

$x = -7, 2$ (D)

↳ extraneous

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14) $\frac{1}{4}(\log_7(x^8) - \log_7(y^2)) + \log_7(9)$ Goal: Condense.

① subtraction rule → $\frac{1}{4}(\log_7\left(\frac{x^8}{y^2}\right)$ → ② reverse power

$\log_7\left(\sqrt[4]{\frac{x^8}{y^2}}\right)$

② Addition rule: $\log_7\left(\sqrt[4]{\frac{x^8}{y^2}}\right) + \log_7(9)$

$= \log_7\left(9\sqrt[4]{\frac{x^8}{y^2}}\right) = \log_7\left(9 \cdot \frac{x^2}{\sqrt[4]{y^2}}\right)$ (A)

$(\sqrt[4]{x^8} = (x^8)^{1/4} = x^2)$

15) $R = \frac{1}{10^{12}} e^{-t/8233}$, $R = \frac{1}{13^{11}}$ Goal: Solve for t

① multiply $10^{12}$

$\frac{1}{13^{11}} = \frac{1}{10^{12}} e^{-t/8233} \longrightarrow \frac{10^{12}}{13^{11}} = e^{-t/8233}$

② Take ln of both sides ($\ln e^x = x$ property)

8223, not 8233

$\ln\left(\frac{10^{12}}{13^{11}}\right) = \ln\left(e^{-t/8223}\right) = \frac{-t}{8233}$

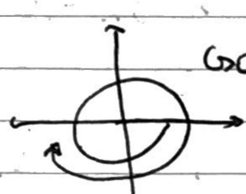
③ isolate t → $\frac{-t}{8223} = \ln\left(\frac{10^{12}}{13^{11}}\right) \rightarrow t = -8223 \ln\left(\frac{10^{12}}{13^{11}}\right)$

④ Calculator! → $t \approx 4797$ (B)

16) $-858°$ Goal: find terminal ray

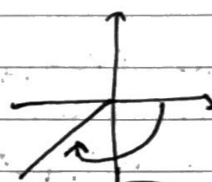
① Divide by 360° and get remainder

-858 remainder 360
= -138
↓



Goes that way

② Chart remainder angle →

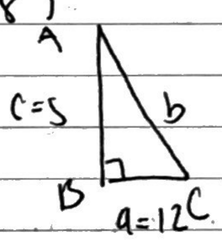


Quadrant 3 ✓ (A)

17) lower bound → ~~diff~~ $x = \pi$

since $\cos(\pi) = 0$, $\cos^{-1}(\pi)$ → undefined.

18)




$b^2 = a^2 + c^2 \rightarrow b^2 = 12^2 + 5^2$

$b^2 = 169 \rightarrow b = 13$

~~1/2~~ $\frac{13}{\sin(90)} = \frac{5}{\sin(\angle C)}$

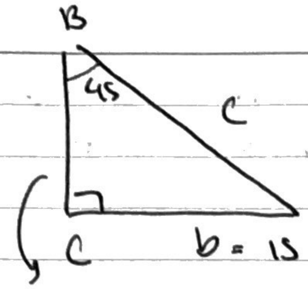
$\sin(\angle C) = \frac{5 \sin 90}{13}$

$\sin(\angle C) = 0.38$

$\angle C = \sin^{-1}(0.38) \approx 23°$ (D)

$\frac{1}{\sin(\theta)} = \csc(\theta)$, $\frac{1}{\cos(\theta)} = \sec(\theta)$, $\frac{1}{\tan(\theta)} = \cot(\theta)$

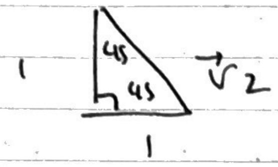
19) a




① find $\angle B =$ $180 - 45 - 90 = 45$

Goal: Solve for all sides & angles.

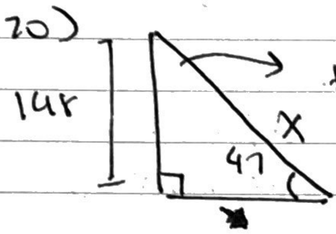
② recognize




special right triangle 1-1-2 sides, 45-45-90 angles

③ since b = 15, a must be 15, $c = 15\sqrt{2}$ (A)

20)




① top angle $= 180 - 90 - 47 = 43$

Goal: find horizontal distance

② Law of sines

$\frac{148}{\sin(47)} = \frac{x}{\sin 90} \rightarrow x = \frac{148 \sin 90}{\sin 47}$

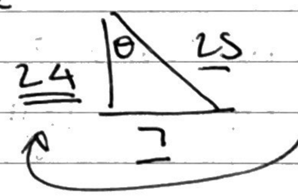
$\approx 202$ (C)

21) $\sin(\theta) = \frac{7}{25}$ $\cot(\theta) = \frac{1}{\tan(\theta)}$

Goal: find $\cot(\theta)$

↳ ① Draw triangle

$\sin(\theta) = \frac{O}{H} \rightarrow$




24, 7, 25 pythagorean triple

$24^2 + 7^2 = 25^2$

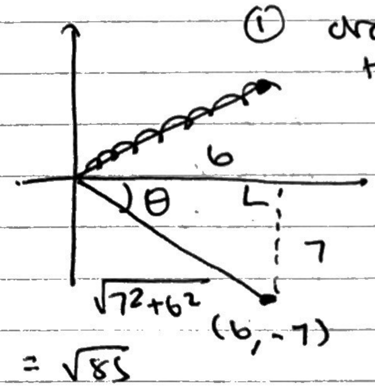
② find $\tan\theta$: $\frac{O}{A} = \frac{7}{24}$

$\cot(\theta) = \frac{1}{\frac{O}{A}} = \frac{A}{O} = \frac{24}{7}$ ~~(B)~~

for $\frac{\pi}{2} < \theta < \pi$

2nd quadrant, tan is negative, so is cot → (B)

22)




① draw point, draw triangle around point.

Goal: find $\csc\theta$

② $\csc(\theta) = \frac{1}{\sin(\theta)}$

$\sin(\theta) = O/H \rightarrow \frac{7}{\sqrt{85}}$

$\csc(\theta) = \sqrt{85}/7$

↳ add (−) → $-\frac{\sqrt{85}}{7}$ (D)

23) $y = \frac{1}{8}\cos\left(3x - \frac{\pi}{2}\right)$ — Goal: find pd, phase shift

① cos function template:

$\frac{1}{8}\cos(3x - \pi/2) + 0$

② $f(x) = A\cos(Bx - C) + D$

period $= 2\pi/B = \frac{2\pi}{3}$

phase shift $= C/B = \frac{\pi/2}{3} = \frac{\pi}{6}$ (C)

24) $y = a\sin(b(x-h)) + k$ — Goal: find equation

Graph given

① Get rid of C, D (phase shift vertically is down not up)

② It's not B since $6(x+3) = 6x + 18$ meaning that 18/3 is his phase shift (not true)

(A)

25) $\arcsin\left(\sin\left(\frac{13\pi}{6}\right)\right)$ — Goal: Break down.

① $\sin^{-1}(\sin(x)) = x$

② $\arcsin(x) = \sin^{-1}(x) \rightarrow \sin^{-1}\left(\sin\left(\frac{13\pi}{6}\right)\right) = \frac{13\pi}{6}$ ~~(B)~~

③ correct for extra rotation → $\frac{13\pi}{6} - 2\pi = \frac{13\pi}{6} - \frac{12\pi}{6} = \frac{\pi}{6} \rightarrow$ (C)

if it's over 360° or $2\pi$, you have to subtract that value from it.

26) $\frac{1}{2}\sin(2(\theta)) = \frac{\tan(\theta)}{\csc^2(\theta)}$ — Goal: True or False

① double angle → $\sin 2\theta = 2\sin\theta\cos\theta$

$\frac{1}{2}\sin 2\theta = \sin\theta\cos\theta$

②

$\csc\theta = \frac{1}{\sin\theta}$ $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$

Right side: $\frac{\cancel{\sin\theta}}{\cos\theta} \cdot \frac{1}{\cancel{\sin^2\theta}\ \sin\theta} = \frac{1}{\cos\theta\sin\theta}$.

$\sin\theta\cos\theta \neq \frac{1}{\sin\theta\cos\theta} \rightarrow$ False (B)

27) $7\cos\left(\frac{\pi}{4}+\theta\right)$ Goal: Simplify

① Sum formula.

$= 7\left(\cos\frac{\pi}{4}\cos\theta - \sin\frac{\pi}{4}\sin\theta\right)$

② (plug in $\cos \pi/4$, $\sin \pi/4$)

$\left\{ \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \quad \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \right.$

$7\left(\frac{\sqrt{2}}{2}\cos\theta - \frac{\sqrt{2}}{2}\sin\theta\right)$

$= \frac{7\sqrt{2}}{2}(\cos\theta - \sin\theta)$ (D)

28) $4\sin(\pi+\theta) - 4\cos\left(\frac{\pi}{2}-\theta\right)$ Goal: Simplify

① Sum ② Difference

$4(\sin\pi\cos\theta + \cos\pi\sin\theta)$ $\quad 4\left(\cos\frac{\pi}{2}\cos\theta + \sin\frac{\pi}{2}\sin\theta\right)$

$= 4(0 + (-1)\sin\theta)$ $\quad = 4(0 + 1(\sin\theta))$

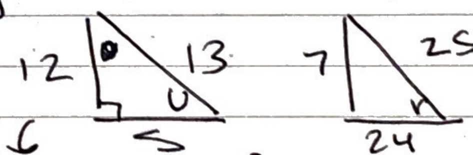
$= -4\sin\theta$

③ put together

$-4\sin\theta - 4\sin\theta = -8\sin\theta$ (A)

29) $\cos(u+v)$ $\quad \cos u = -\frac{5}{13}$, $\sin v = -\frac{7}{25}$ Goal: simplify

① find $\sin u$, $\cos v$ by drawing triangles




pythag triple 5, 12, 13

$\sin u = -\frac{12}{13}$ (sin is negative in Q3)

Another pythag triple 7, 24, 25

$\cos v = -\frac{24}{25}$

② Plug into formula

$\cos u\cos v - \sin u\sin v$

$= \left(-\frac{5}{13}\right)\left(-\frac{24}{25}\right) - \left(-\frac{12}{13}\right)\left(-\frac{7}{25}\right) = \frac{36}{325}$ (D)

① double angle

30) $\sin 2u = 2\sin u\cos u$ Goal: Solve

② plug in

$= 2\left(-\frac{12}{13}\right)\left(-\frac{5}{13}\right) = \frac{120}{169}$

(A)

31) $\tan\left(\frac{v}{2}\right)$ Goal: Solve.

(1) Half-angle formula (2) (plug in)

$$= \frac{1-\cos v}{\sin v} = \frac{1-(-24/25)}{(-7/25)} = \frac{49/25}{-7/25} = \frac{49}{25} \cdot -\frac{25}{7} \to -7 \text{ (C)}$$

32) $\cot(\sin^{-1}(3y))$ ~~$6~~ Goal: Simplify

(1) FORMULA → cot to $\sin^{-1}$ → $\cot(\sin^{-1}(x)) = \frac{\sqrt{1-x^2}}{x}$

(2) plug in → $\cot(\sin^{-1}(3y)) = \frac{\sqrt{1-9y^2}}{3y}$ (B)

33) $\cos\left(\frac{\pi}{6}\right)\cos\left(\frac{7\pi}{6}\right) + \sin\left(\frac{5\pi}{3}\right)$ Goal: Solve

(1) Derive from unit circle → $\frac{\sqrt{3}}{2}$ → $\frac{\sqrt{3}}{2}$ → $-\frac{\sqrt{3}}{2}$

(2) Put together → $\left(\frac{\sqrt{3}}{2}\right)^2 - \frac{\sqrt{3}}{2} = \frac{3}{4} - \frac{\sqrt{3}}{2} = \frac{3-2\sqrt{3}}{4} = \frac{-3+2\sqrt{3}}{4}$ (A)

34) $\frac{\csc(x)}{\sin(x)} - \csc(x)\sin(x)$ Goal: Simplify

(1) We know: $\csc(x) = \frac{1}{\sin(x)}$

$$\frac{\frac{1}{\sin(x)}}{\sin(x)} - \frac{1}{\cancel{\sin(x)}}\cancel{\sin(x)} \to \frac{1}{\sin^2(x)} - 1$$

$$= \frac{1-\sin^2 x}{\sin^2 x}$$

(2) Reverse pythagorean property

$$\frac{\cos^2 x}{\sin^2 x} = \left(\frac{\cos x}{\sin x}\right)^2 = (\cot x)^2 = \cot^2 x \text{ (B)}$$

35) $\frac{\sin(2\theta)}{2\tan(\theta)}$ Goal: Simplify

① double angle:

$2\sin\theta\cos\theta$

$= 2\cdot\frac{\sin\theta}{\cos\theta}$ $\frac{2\sin\theta\cos\theta}{\left(\frac{2\sin\theta}{\cos\theta}\right)}$ ② $2\cancel{\sin\theta}\cos\theta\cdot\frac{\cos\theta}{2\cancel{\sin\theta}}$

(since $\tan\theta = \sin\theta/\cos\theta$)

$\cos^2\theta$ (C)

36)

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