(Maximum Score After Applying Operations on a Tree)

Q) Given an undirected tree with n nodes labelled from 0 to $(n-1)$, and rooted at node 0. You will be given edges $[a, b]$, where there is an undirected edge from $(a - b)$. There is a values array, where values[i] represent value associated with the i^{th} node.

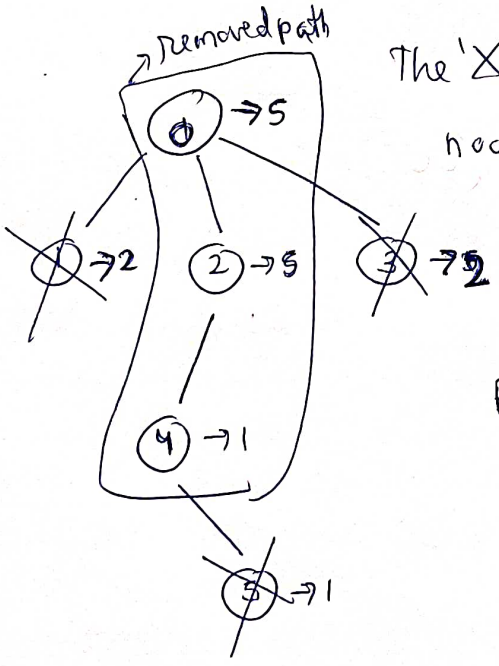
In 1 operation

- Pick any node
- Add values[i] to your score
- Set values[i] to 0.

In this operation we have to make sure that the tree stays healthy.

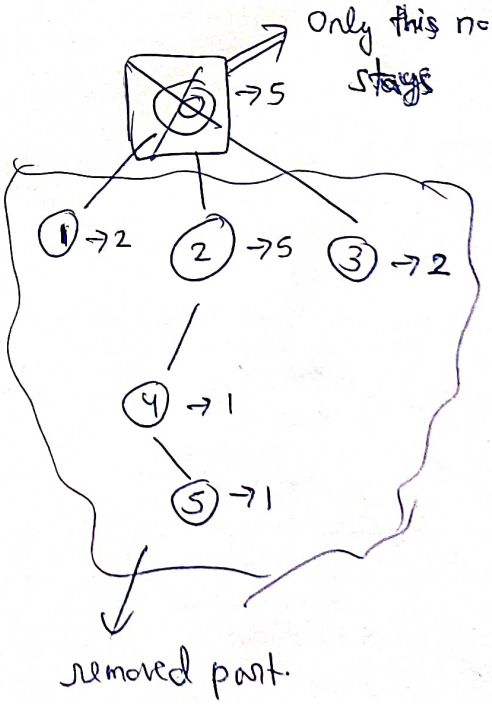
[A tree is healthy if the sum of values on the path from the root to every leaf is different than zero].

Return maximum score obtained.



The 'X' marked nodes stay, rest nodes are removed. (0)

Every path has non-zero sum

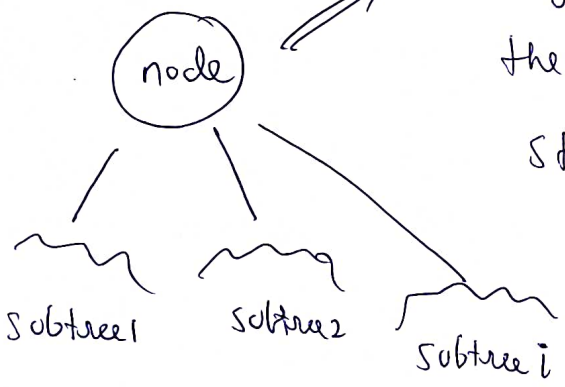


(Ans) = 11

Solution

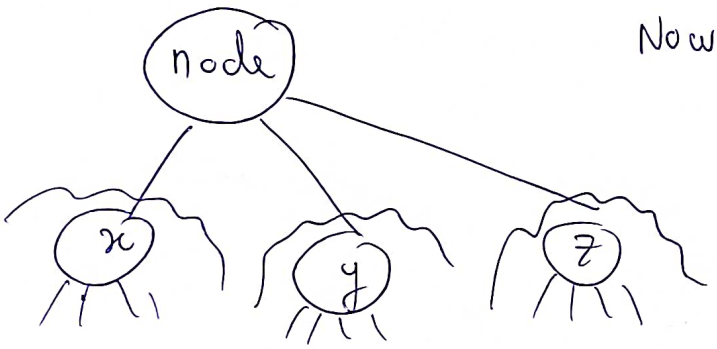
The catch of the solution is to solve it using dp.

Lets take two variables



Lets say the node as root of its all i subtrees. the maximum score that it can have, let it be stored in dp[node],

And let sum of all the nodes of the subtree be in sum[node].



Now there a 2 ways of getting maximum score.

1st option: Lets stay the root, (node itself). If we remove node, we get to keep all other nodes of its subtree. Because from root remove all paths below are healthy.

$$\left(dp[node] = \sum_{j=x}^{z} sum[j]\right)$$

2nd option: Let the node(root) be removed and considered in maximum score calculation.

If we remove the node, then we have to ensure all the subtrees [x, y, z] are healthy.

And dp[x], dp[y], dp[z] contains max score kept for them and they are still healthy

Choose the max of both.

$$\left(dp[node] = values[node] + \sum_{j=x}^{z} dp[j]\right)$$