



Data Structure and Algorithm Training Program

**Week 1 : Pseudo Code Solution Part 2**

### Problem 1 : Recursive program for finding max value in an array

- Recurrence Relation :  $T(n)=T(n-1) + c$
- Time Complexity :  $O(n)$
- Space Complexity :  $O(n)$   
[Recursion call stack]

```
int recursive_FindMax(A[], n)
{
    if (n == 1)
        return A[0]

    max = recursive_FindMax(A, n-1)

    if(max < A[n-1])
        max = A[n-1]
    return max
}
```

### Problem 3 : Iterative program for binary search

- Time Complexity :  $O(\log n)$
- Space Complexity :  $O(1)$
- At every step of iteration, decreasing the input size by half

```
int binarySearch( A[], l, r, key )
{
    while (l <= r)
    {
        mid = l+(r-l) / 2

        if (A[mid] == key)
            return mid

        if (A[mid] < key)
            l = mid + 1
        else
            r = mid - 1
    }
    return -1
}
```

### Problem 8 : Recursive program for Insertion sort

- Recurrence Relation :  $T(n)=T(n-1) + cn$
- Time Complexity :  $O(n^2)$
- Space Complexity :  $O(n)$   
[Recursion call stack]

```
recursive_InsertionSort( A[], n )
{
    if (n <= 1)
        return

    recursive_InsertionSort ( A, n-1 )
    key = A[n-1]
    i = n-2
    while (i >= 0 && A[i] > key)
    {
        A[i+1] = A[i]
        i = i-1
    }
    A[i+1] = key
}
```

## Problem 9 : Write a Program to find the median of two sorted arrays A and B of size n after merging both the array

### Important details related to the problem

- Both the arrays are sorted
- Size of both the arrays are equal
- median of a sorted array - A value which divides the whole array into two equal size.  
If n is odd, median = Value at  $n/2$  position.  
If n is even, median = Average of Value at  $n/2-1$  and  $n/2$  position
- Size of the sequence after merging both the array would be  $2n$  which is even. So output should be average of value at  $(n-1)$ th and  $(n)$ th position

### Brute force Solution

- Time Complexity :  $O(n)$
- Space Complexity :  $O(n)$
- Using the merge approach of merge sort

#### Solution Steps

- Take Extra memory  $C[]$  of size  $2n$
- Merge A and B into a larger array  $C[]$
- Return  $(C[n-1]+C[n])/2$

### Improved Version of Brute force solution

- Time Complexity :  $O(n)$
- Space Complexity :  $O(1)$  [In place]
- Track  $n-1$ th and  $n$ th value via  $k$ ,  $m1$  and  $m2$

```
int arrayMedian(A[], B[], n)
{
    if ( A[n-1] < B[0] )
        return ( A[n-1] + B[0] ) / 2
    else if ( B[n-1] < A[0] )
        return ( A[0] + B[n-1] ) / 2
    else
    {
        i=0, j=0, k=0
        while(k < n-1)
        {
            if (A[i]<B[j])
            {
                i = i+1
                k = k+1
            }
            else
            {
                j = j+1
                k = k+1
            }
        }

        if (A[i]>B[j])
        {
            m1=B[j]
            j = j+1
        }
        else
        {
            m1 = A[i]
            i = i+1
        }

        if (A[i] > B[j])
            m2 = B[j]
        else
            m2 = A[i]

        return (m1+m2)/2
    }
}
```

*Corner case (Think!)*

*Customized loop of merging two sorted array where Incrementing the loop till k reach n-1*

*find first middle value m1*

*find second middle value m2*

## Continued...

### Efficient Pseudo Code : Divide and Conquer Approach of binary search

- Time Complexity :  $O(\log n)$
- Space Complexity :  $O(\log n)$   
[Recursion Call Stack]
- After every comparison with median of both the arrays, decreasing the input size by half
- Boundary conditions : Odd and even values of  $n$
- After comparison, include the medians of both the array in recursive call because both can be part of the solution!
- Base case :  $n=2$ ,  $n=1$  and  $n=0$

Base Cases

```
int arrayMedian(A[], B[], n)
{
    if (n <= 0)
        return -1
    if (n == 1)
        return ( A[0] + B[0] ) / 2
    if (n == 2)
        return (max( A[0], B[0] )+
                min( A[1], B[1] ))/ 2
    m1 = findMedian( A , n )
    m2 = findMedian( B , n )
    if (m1 == m2)
        return m1
    if (m1 < m2)
    {
        if (n%2 == 0)
            return arrayMedian
                (A+n/2-1, B , n/2+1)
        else
            return  arrayMedian
                (A+n/2, B , n/2)
    }
}
```

Divide

Conquer

```
else
{
    if (n%2 == 0)
        return arrayMedian
            (A, B+n/2-1, n/2+1)
    else
        return arrayMedian
            (A, B+n/2 , n/2)
}
}
```

```
int findMedian(C[],n)
{
    if (n%2 == 0)
        return (C[n/2] + C[n/2-1])/ 2
    else
        return C[n/2]
}
```

**Similar Question** : median of two sorted arrays A and B of different size  $m$  and  $n$  after merging both the array

## Problem 10: Find position of an element in a sorted array of infinite numbers

Brute force approach : Linear search from the start

- if  $A[i] == \text{key}$ , return  $i$
- Time Complexity :  $O(i)$
- Space Complexity :  $O(1)$

Improved Version of Brute force solution : increment the interval size by some constant  $c$

- Track the interval where the key would be present
- Increment the interval size by some constant  $c = O(1)$
- At every iteration, compare the value present at right end with key. If  $\text{key} > A[r]$ , then increment the interval size by  $c$
- After finding the interval, apply the binary search in interval
- Time complexity = interval search + Binary Search =  $O(i/c) + O(\log c) = O(i) + O(1) = O(i)$   
[Because  $c$  is some constant Value]
- Space Complexity =  $O(1)$

`int search_InfiniteArray(int A[], key)`

```
{
    l = 0
    r = c
    while (A[r] < key)
    {
        l = r
        r = r + c
    }
    return binarySearch(A, l, r, key)
}
```

*Incrementing the loop by some constant order*

Efficient Solution : Approach of exponential search

- At every step, increment the interval size by double
- After finding the interval range after  $k$ th step  
Lower bound of interval =  $2^{k-1}$   
Upper bound of interval =  $2^k$   
Interval Size/ Total Values in Interval =  $2^{k-1}$
- Time complexity = interval search + Binary Search =  $O(\log i) + O(\log i) = O(\log i)$   
[Here  $i$  is the position of key in sorted array]
- Space Complexity =  $O(1)$

`int search_InfiniteArray(A[], key)`

```
{
    l = 0
    r = 1
    while (A[r] < key)
    {
        l = r
        r = 2 * r
    }
    return binarySearch(A, l, r, key)
}
```

*Incrementing the loop by exponential order*

### Similar Problems

- 1] Find the occurrence of first 1 in an infinite sorted array of 0s and 1s
- 2] Find the point where a monotonically increasing function becomes positive first time

Enjoy Algorithms!

Thank You.