

Sampling-based Path Finding

■ Lecture 3



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Acknowledgements: Slides based on a previous version, courtesy of Dr. Chaoqun Wang.

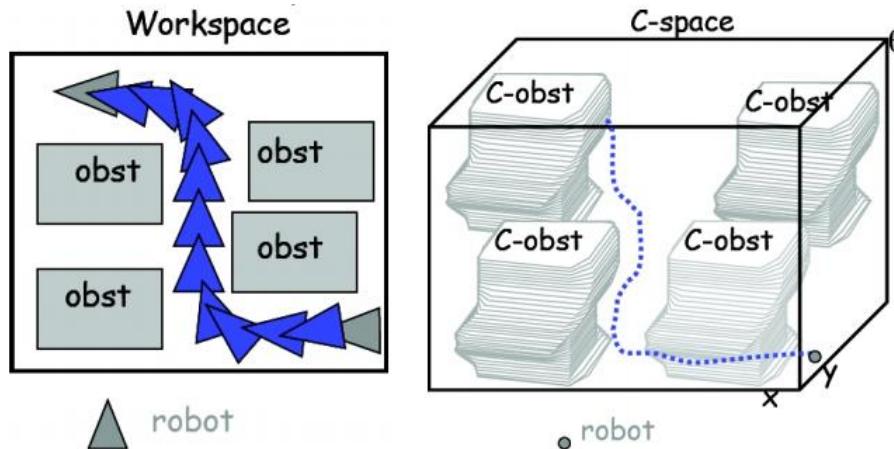




Preliminaries

From Workspace to Configuration Space

- Robot transformed into a point in configuration space.
- Paths transformed from swept volume into 1 dimensional curves.





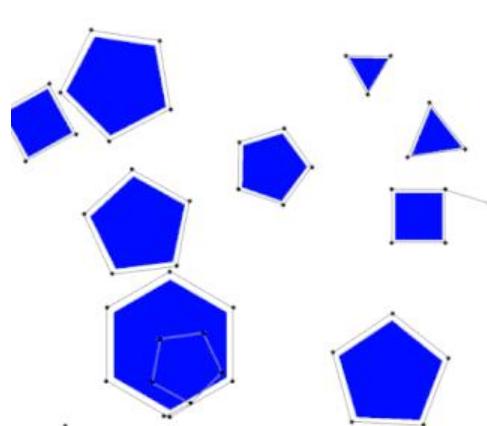
Preliminaries

Sampling-based Planners (or *Probabilistic Methods*)

capture the connectivity of the solution space to bring a feasible or optimal solution

by constructing Trees or Graphs connected by discrete C-space samples sampled incrementally or in batch
in the continuous C-space without explicitly constructing it.

Their performance mostly depends on *sampling-strategy*, *collision-checking*, and *neighbor-search*.



Animation from Wikipedia



Preliminaries

Two Fundamental Tasks

- **Exploration**

Acquires information about the topology of the search space, i.e., how subsets of the space are connected.

- **Exploitation**

Incrementally improves the solution by processing the available information computed by the exploration task.



Terminology

- **Probabilistic Completeness**

A **feasible** solution will be found, if one exists, with probability one as the number of samples approaches infinity.

- **Asymptotical (Near) Optimality**

The cost of the returned solution converges almost surely to **the optimum** as the number of samples approaches infinity.

- **Anytime**

Quickly find a feasible but not necessarily optimal solution, then incrementally improve it over time.



Brief History

- PRM 1996 TRA
- RRT 1998
- RRT-connect 2000 ICRA
- Visibility PRM 2000
- PRM* 2011 IJRR
- RRG 2011 IJRR
- **RRT* 2011 IJRR**
- RRT*-Smart 2012 ICMA
- **RRT# 2013 ICRA**
- FMT* 2013 ISRR
- Informed-RRT* 2014 IROS
- RRTx 2014 WAFR 2015 IJRR
- BIT* 2015 ICRA
- LBT-RRT 2016 TRO
- SST* 2016 IJRR
- RABIT* 2016 ICRA
- DRRT 2017 RSS
- AIT* 2020 ICRA
- **GuILD 2021 Arxiv**
- EIT* 2022 IJRR
- ...



Content

1. Feasible path planning methods :

Probabilistic Road Map (PRM),

Rapidly-exploring Random Tree (RRT)

2. Optimal path planning methods:

RRT*

3. Accelerate convergence:

RRT#, Informed-RRT*, and GuILD

Feasible Path Planning Methods



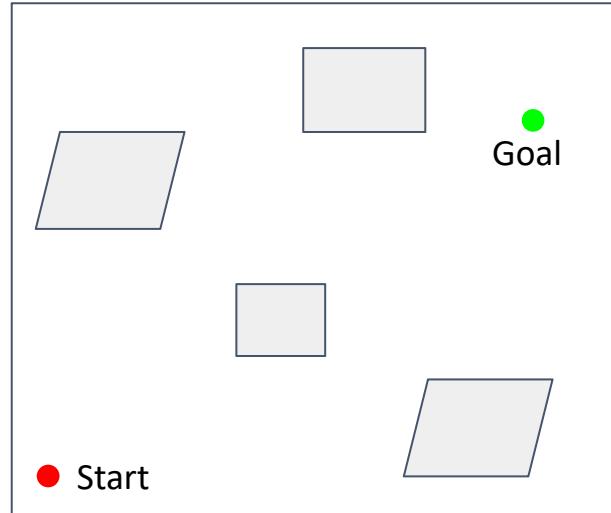
Probabilistic Road Map

What is PRM?

- Probe space connectivity in a graph structure;
- Fine coverage of free space;
- Divide planning into two phases:

Learning phase

Query phase



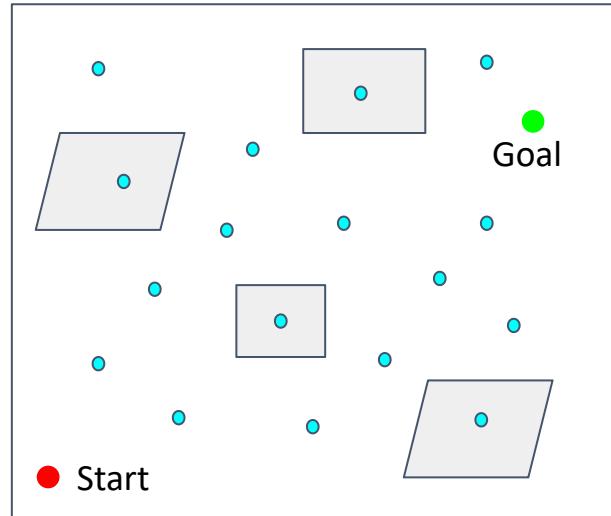


Probabilistic Road Map

Learning phase:

Detect the C-space with random points and construct a graph that represents the connectivity of the environment.

- Sample N points in C-space
- Delete points that are not collision-free



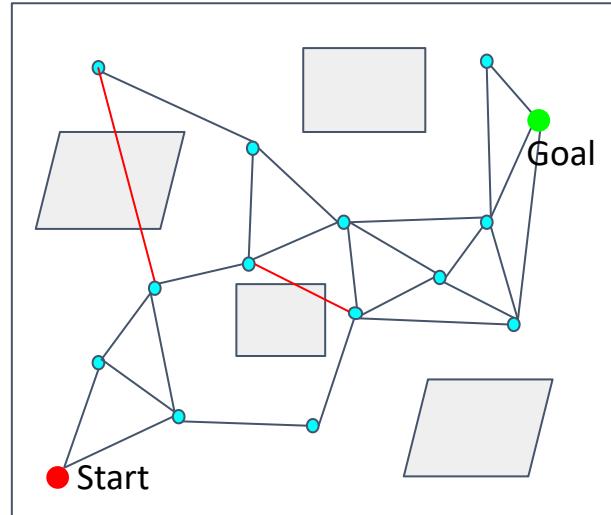


Probabilistic Road Map

Learning phase:

Detect the C-space with random points and construct a graph that represents the connectivity of the environment.

- Connect to nearest points and get collision-free segments.
- Delete segments that are not collision free.



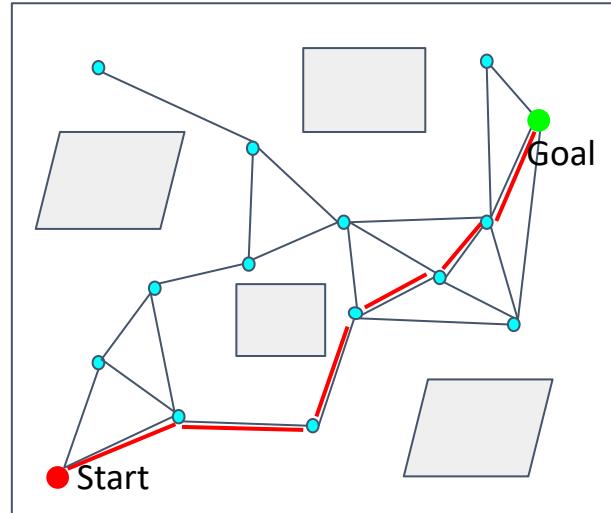


Probabilistic Road Map

Query phase:

Search on the road map to find a path from the start to the goal
(using Dijkstra's algorithm or the A* algorithm).

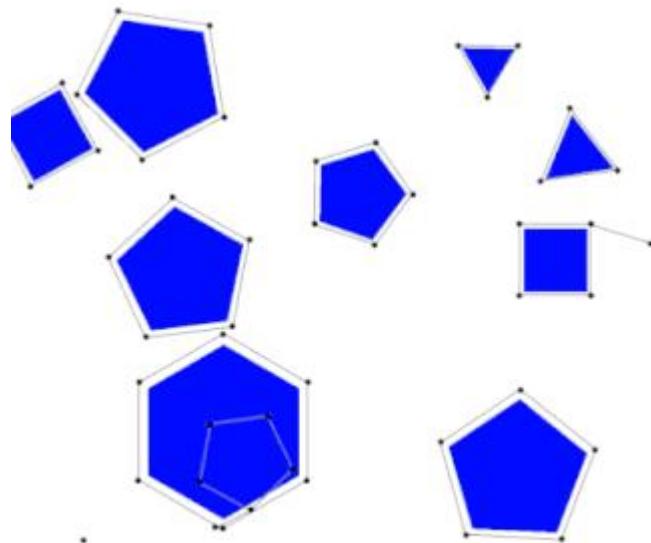
- Road map is now similar with the grid map (or a simplified grid map).



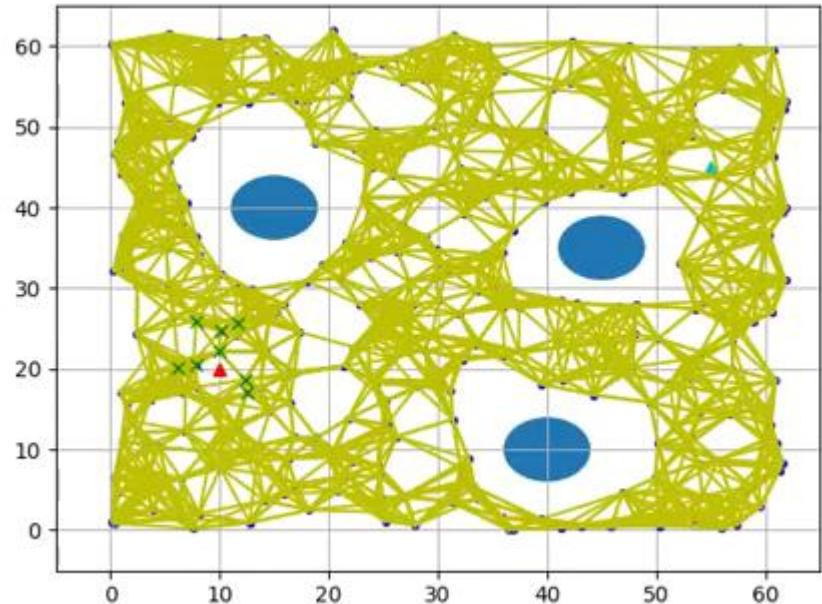


Probabilistic Road Map

Learning phase



Query phase





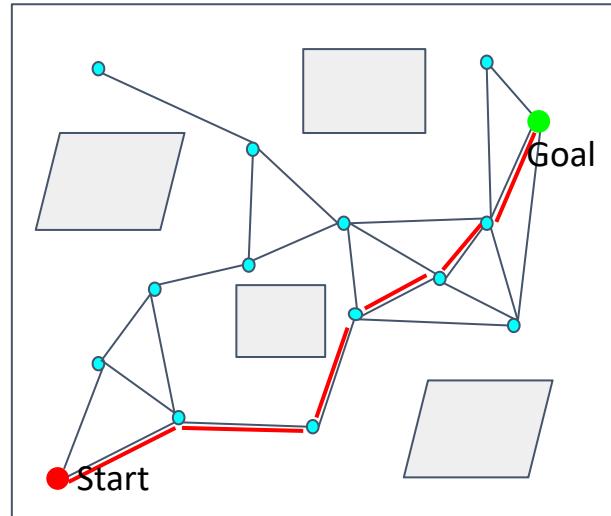
Probabilistic Road Map

Pros

- Can conduct multiple queries (static map).

Cons

- Build graph over state space but no particular focus on generating a path.





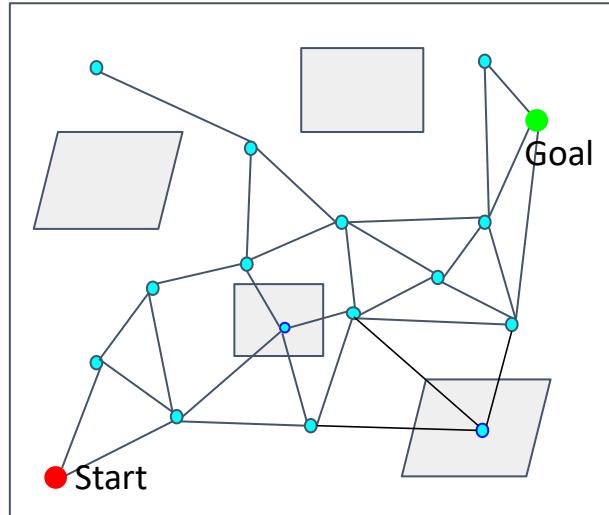
Probabilistic Road Map

Improving efficiency

The collision-checking process is time-consuming, especially in complex or high-dimensional environments.

Lazy collision-checking:

Check collisions only if necessary



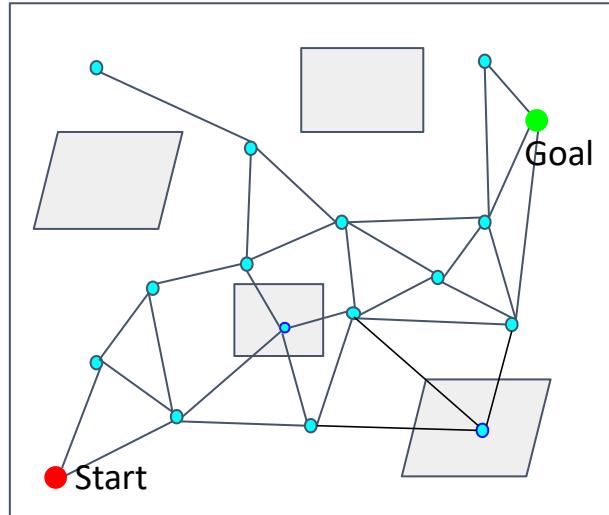


Probabilistic Road Map

Improving efficiency

Lazy collision-checking:

- Sample points and generate segments without considering the collision (Lazy).



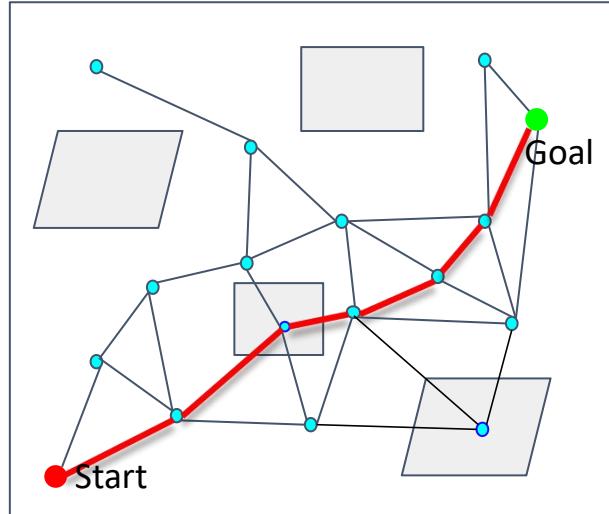


Probabilistic Road Map

Improving efficiency

Lazy collision-checking:

- Sample points and generate segments without considering the collision (**Lazy**).
- Find a path on the road map generated without collision-checking.



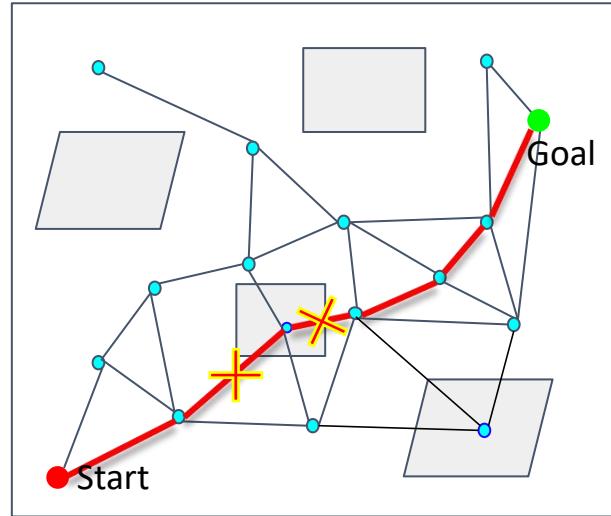


Probabilistic Road Map

Improving efficiency

Lazy collision-checking:

- Sample points and generate segments without considering the collision (**Lazy**).
- Find a path on the road map generated without collision-checking.
- Delete the corresponding edges and nodes if the path is not collision free.



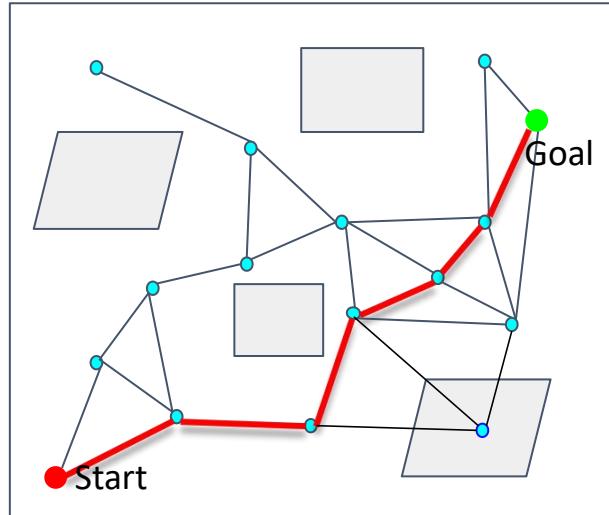


Probabilistic Road Map

Improving efficiency

Lazy collision-checking:

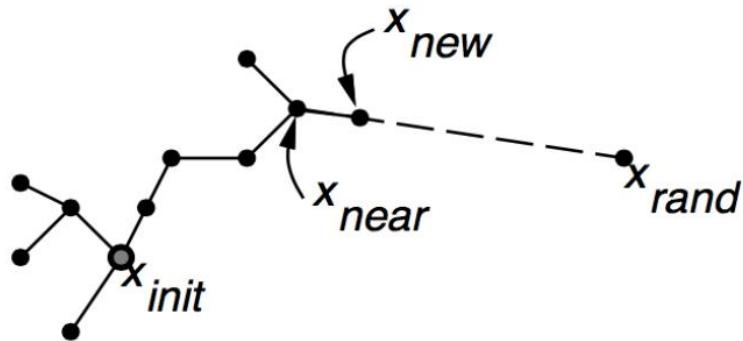
- Sample points and generate segments without considering the collision (**Lazy**).
- Find a path on the road map generated without collision-checking.
- Delete the corresponding edges and nodes if the path is not collision free.
- Restart path finding.





Rapidly-exploring Random Trees

Build up a tree from start to goal through generating “next states” in the tree by executing random controls





Rapidly-exploring Random Trees

Algorithm 1: RRT Algorithm

Input: $\mathcal{M}, x_{init}, x_{goal}$

Result: A path Γ from x_{init} to x_{goal}

$\mathcal{T}.init();$

for $i = 1$ to n **do**

$x_{rand} \leftarrow Sample(\mathcal{M})$;

$x_{near} \leftarrow Near(x_{rand}, \mathcal{T});$

$x_{new} \leftarrow Steer(x_{rand}, x_{near}, StepSize);$

$E_i \leftarrow Edge(x_{new}, x_{near});$

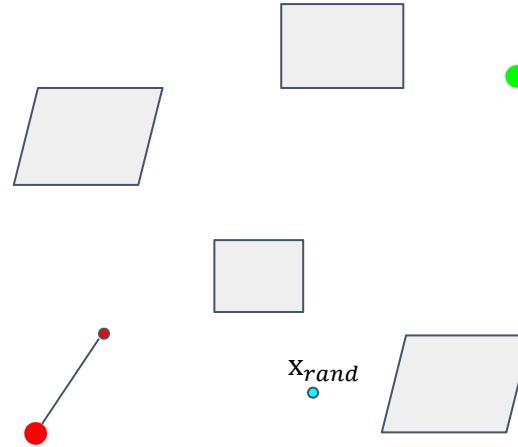
if $CollisionFree(\mathcal{M}, E_i)$ **then**

$\mathcal{T}.addNode(x_{new});$

$\mathcal{T}.addEdge(E_i);$

if $x_{new} = x_{goal}$ **then**

 Success();



Sample a node X_{rand} in the free space



Rapidly-exploring Random Trees

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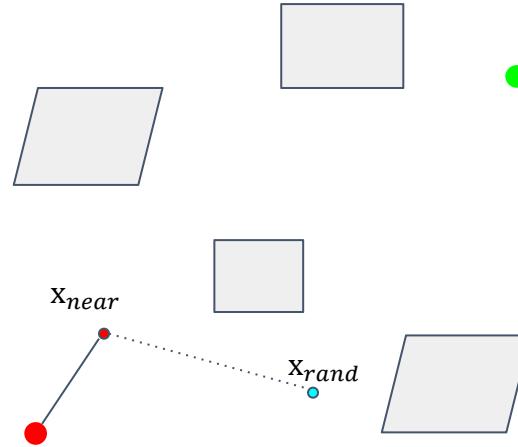
if $CollisionFree(\mathcal{M}, E_i)$ **then**

$\mathcal{T}.addNode(x_{new});$

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if $x_{new} = x_{goal}$ **then**

 Success();



Find the nearest node x_{near} in current tree



Rapidly-exploring Random Trees

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Result: A path Γ from x_{init} to x_{goal}

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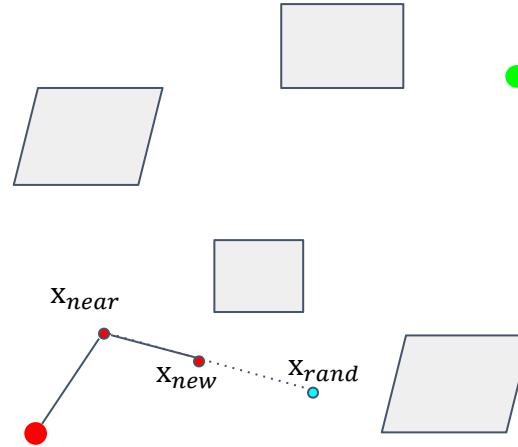
if $CollisionFree(\mathcal{M}, E_i)$ **then**

$\mathcal{T}.addNode(x_{new})$;

$\mathcal{T}.addEdge(E_i)$;

if $x_{new} = x_{goal}$ **then**

 Success();



Grow a new node x_{new} and path E_i from x_{near}



Rapidly-exploring Random Trees

Algorithm 1: RRT Algorithm

Input: $\mathcal{M}, x_{init}, x_{goal}$

Result: A path Γ from x_{init} to x_{goal}

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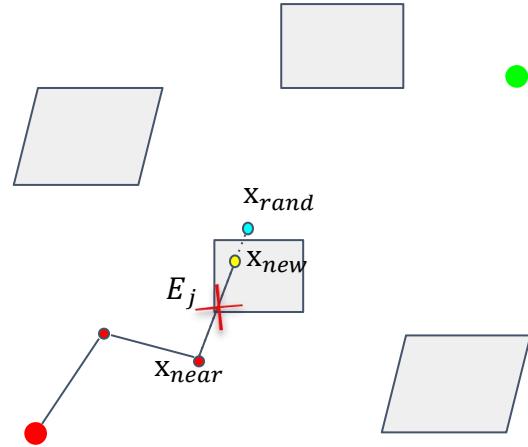
if $CollisionFree(\mathcal{M}, E_i)$ **then**

$\mathcal{T}.addNode(x_{new})$;

$\mathcal{T}.addEdge(E_i)$;

if $x_{new} = x_{goal}$ **then**

 Success();



Do not grow if collision



Rapidly-exploring Random Trees

Algorithm 1: RRT Algorithm

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Result: A path Γ from x_{init} to x_{goal}

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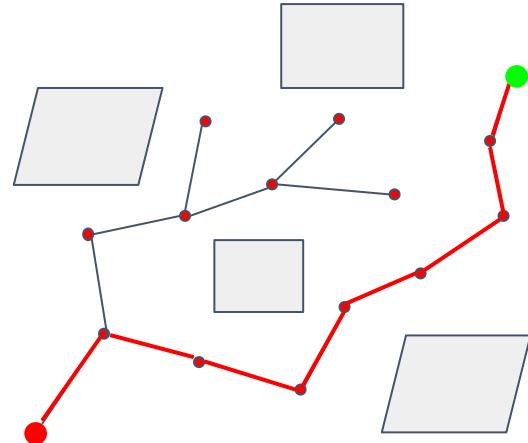
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$\mathcal{T}.addNode(x_{new})$;

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if $x_{new} = x_{goal}$ **then**

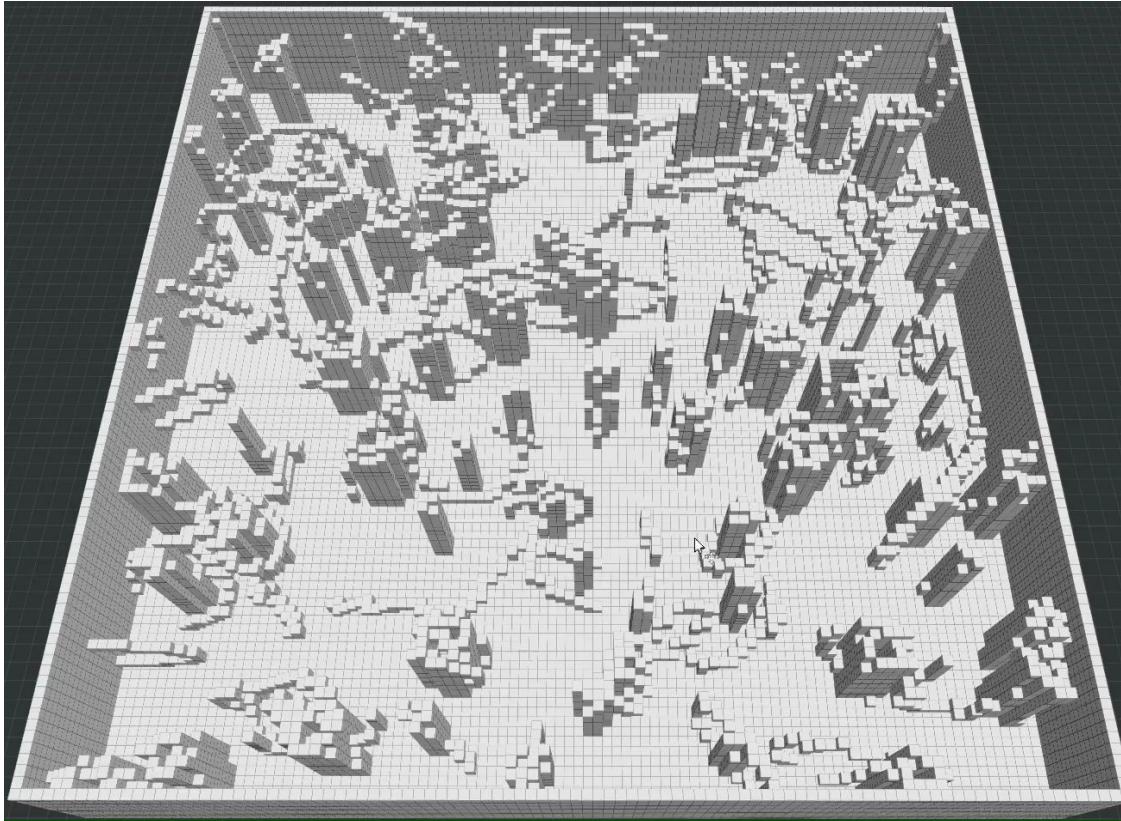
 Success();



Repeat sampling for n times until the tree reaches the goal or goal region



Rapidly-exploring Random Trees



<https://github.com/ZJU-FAST-Lab/sampling-based-path-finding>



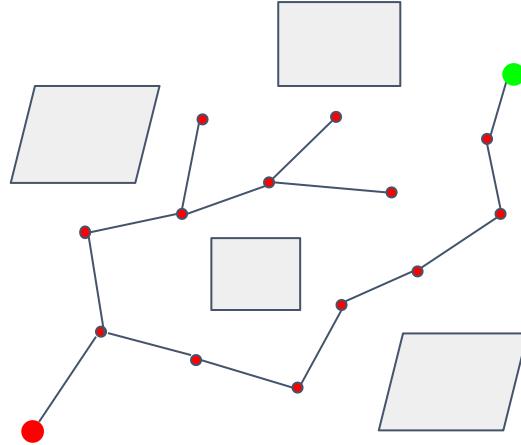
Rapidly-exploring Random Trees

Pros

- Easy to implement
- Aims to find a path from the start to the goal
- More target-oriented than PRM

Cons

- Not optimal solution
- Not efficient (leave room for improvement)
- Sample in the whole space



Optimal Path Planning Methods



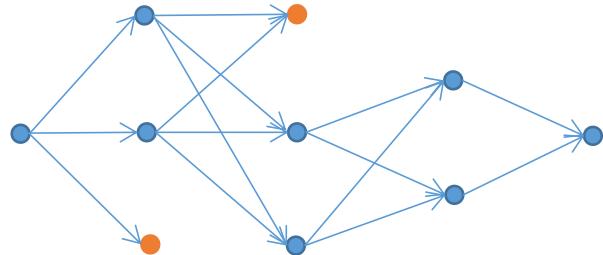
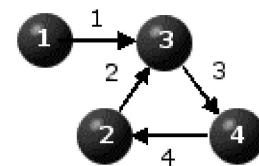
Optimal Criterion

Dynamic Programming (DP) functional equation (discrete form)

$$f(j) = \min_{i \in B(j)} \{f(i) + D(i, j)\}, \quad j \in C \setminus \{1\}$$

$B(j)$ represents for predecessors of j ,

$D(i, j)$ the cost of transition from state i to state j .



$$f(1) = 0$$

$$f(2) = 4 + f(4)$$

$$f(3) = \min\{2 + f(2), 1 + f(1)\}$$

$$f(4) = 3 + f(3)$$

The DP functional equation
does not constitute an algorithm!



Optimal Criterion

Direct DP Methods

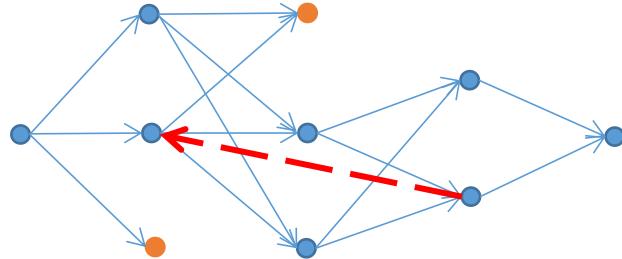
Generic Direct Method

$$D(i, j) = \infty, \forall i, j \in C, i \geq j$$

Initialization: $F(1) = 0$

Iteration: For $j = 2, \dots, n$ Do:

$$F(j) = \min_{i \in B(j)} \{F(i) + D(i, j)\}$$



The cost-to-come value of each state can be determined by processing it **only once**.

- While computing the value of $f(j)$, all the relevant values of $f(i)$ must have **already been computed** somehow.
- The graph must be **acyclic**.



Optimal Criterion

Direct DP Methods

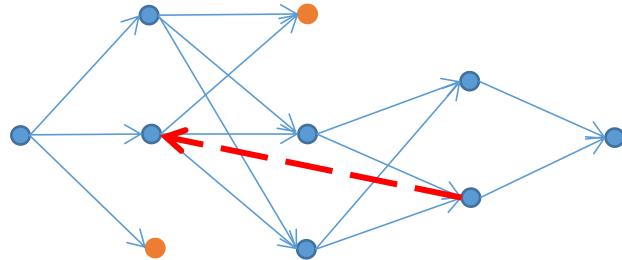
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Iteration: For $j = 2, \dots, n$ Do:

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In motion planning, state graphs (or grids) are almost surely cyclic.

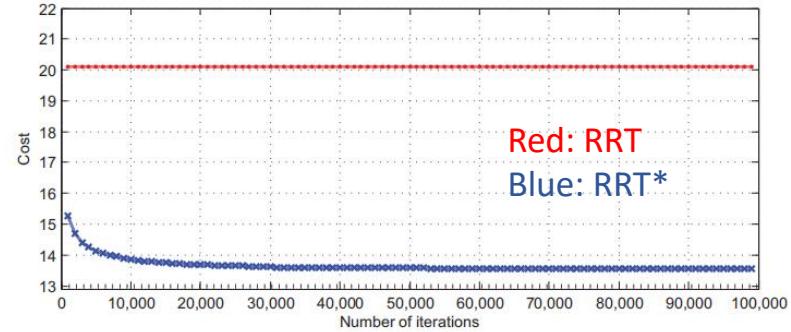
Each state can be in any level (step), and one can only access states by range queries or k-NN queries.

Sampling-based methods introduce some kinds of randomness in generating the **implicit** random geometric graph (**RGG**) **incrementally** or **in batch**. It can still be viewed as graph-searching.

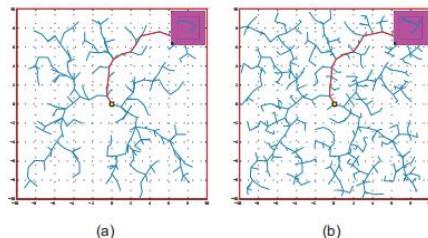


Rapidly-exploring Random Tree* (RRT*)

- An improvement of RRT
- Asymptotically optimal (under conditions)



RRT:



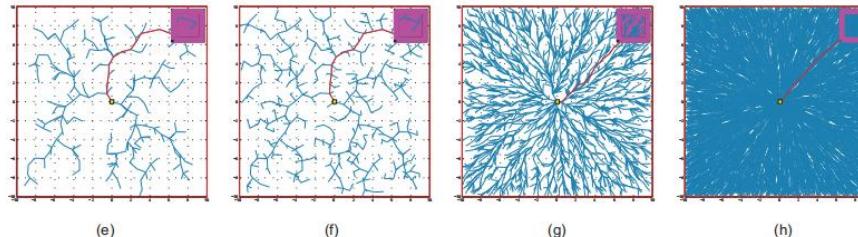
(a)

(b)

(c)

(d)

RRT*:



(e)

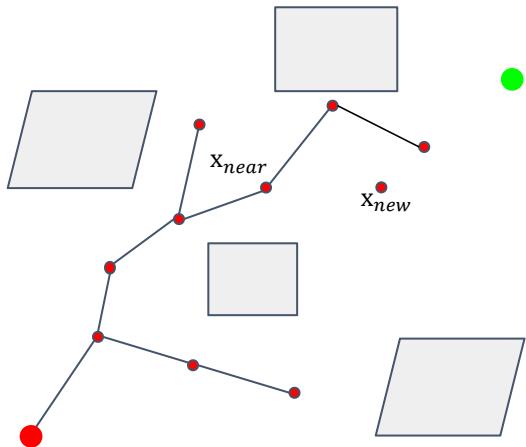
(f)

(g)

(h)



Rapidly-exploring Random Tree* (RRT*)



Algorithm 2: RRT*Algorithm

Input: $\mathcal{M}, x_{init}, x_{goal}$

Result: A path Γ from x_{init} to x_{goal}

$\mathcal{T}.init();$

for $i = 1$ to n **do**

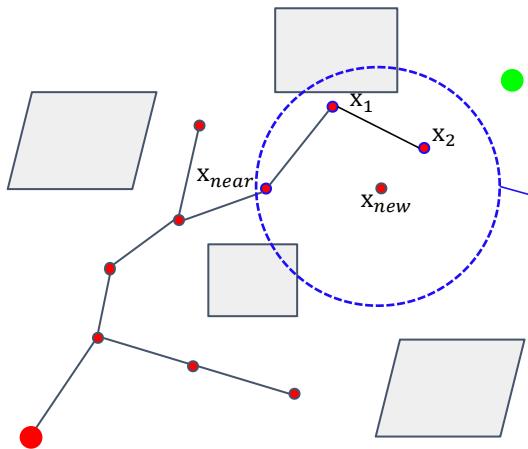
```
 $x_{rand} \leftarrow Sample(\mathcal{M}) ;$ 
 $x_{near} \leftarrow Near(x_{rand}, \mathcal{T}) ;$ 
 $x_{new} \leftarrow Steer(x_{rand}, x_{near}, StepSize) ;$ 
```

if $CollisionFree(x_{new})$ **then**

```
 $X_{near} \leftarrow NearC(\mathcal{T}, x_{new}) ;$ 
 $x_{min} \leftarrow ChooseParent(X_{near}, x_{near}, x_{new}) ;$ 
 $\mathcal{T}.addNodeEdge(x_{min}, x_{new}) ;$ 
 $\mathcal{T}.rewire() ;$ 
```



Rapidly-exploring Random Tree* (RRT*)



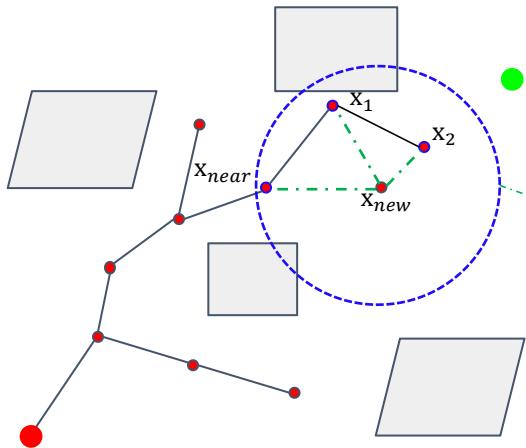
Algorithm 2: RRT*Algorithm

```
Input:  $\mathcal{M}, x_{init}, x_{goal}$ 
Result: A path  $\Gamma$  from  $x_{init}$  to  $x_{goal}$ 
 $\mathcal{T}.init();$ 
for  $i = 1$  to  $n$  do
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```

Consider N nearing nodes



Rapidly-exploring Random Tree* (RRT*)



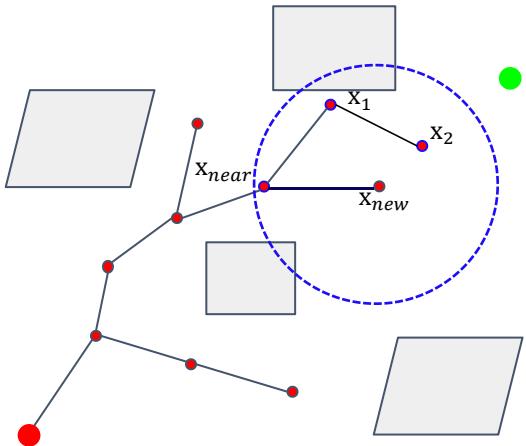
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```

Consider history cost instead of only local information



Rapidly-exploring Random Tree* (RRT*)



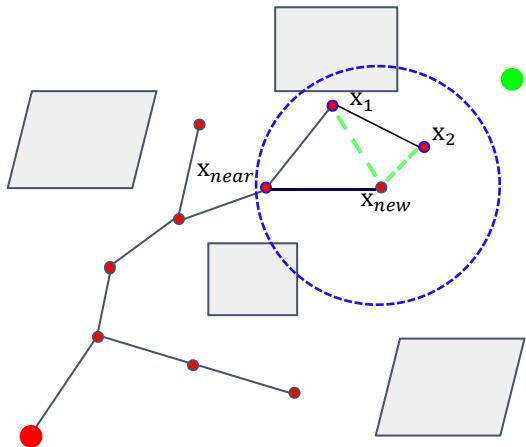
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Consider history cost instead of only local information



Rapidly-exploring Random Tree* (RRT*)



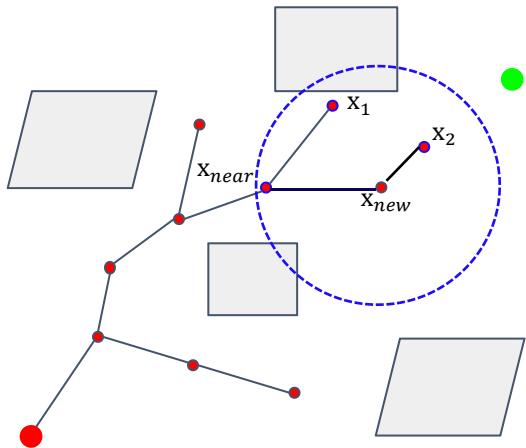
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         $\mathcal{T}.rewire();$ 
```

Rewire to improve local optimality



Rapidly-exploring Random Tree* (RRT*)



Algorithm 2: RRT*Algorithm

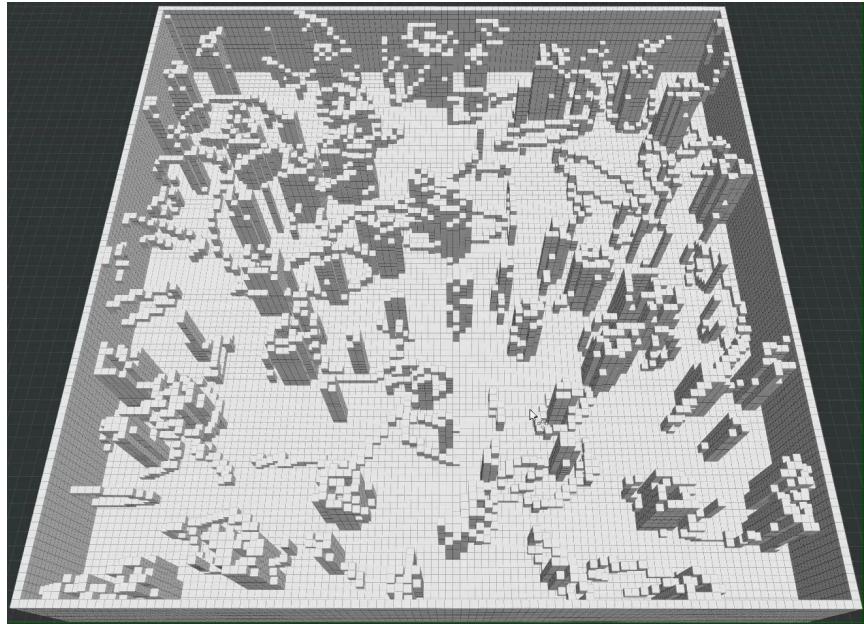
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Rewire to improve local optimality

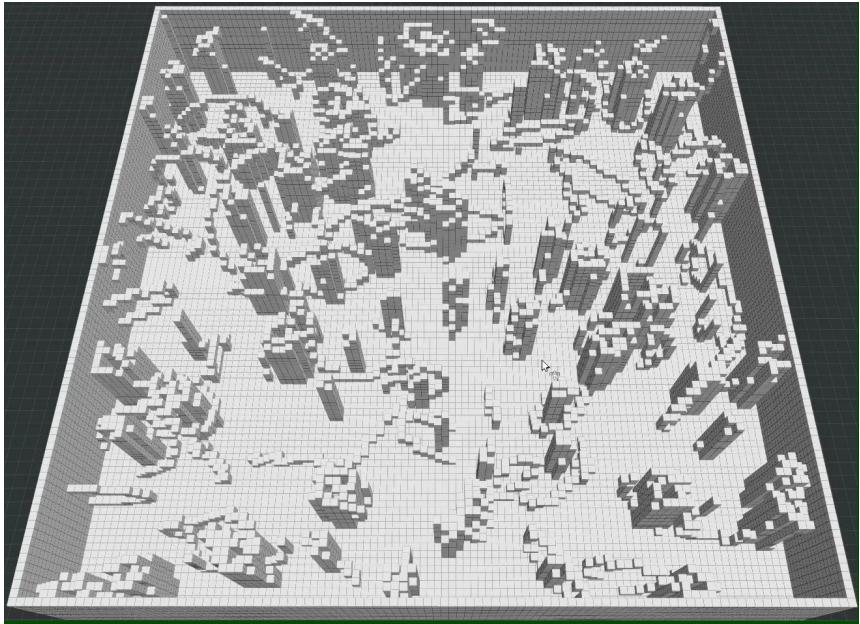


Rapidly-exploring Random Tree* (RRT*)

RRT:



RRT*:





Rapidly-exploring Random Tree* (RRT*)

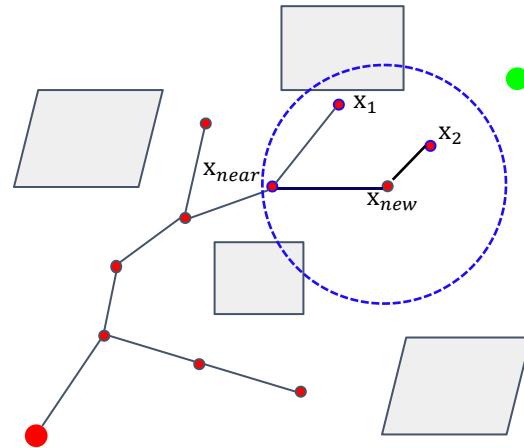
About the query range^[1]

$$range = \min \left\{ \gamma \cdot \left(\frac{\log(card(V))}{card(V)} \right)^{\frac{1}{d}}, \eta \right\}$$

To assure AO:

$$\gamma > \left(2 \left(1 + \frac{1}{d} \right) \right)^{\frac{1}{d}} \left(\frac{\mu(X_{free})}{\xi_d} \right)^{\frac{1}{d}}$$

Empirically, for low dimensional planning, we can set a constant range a bit larger than the steer distance.

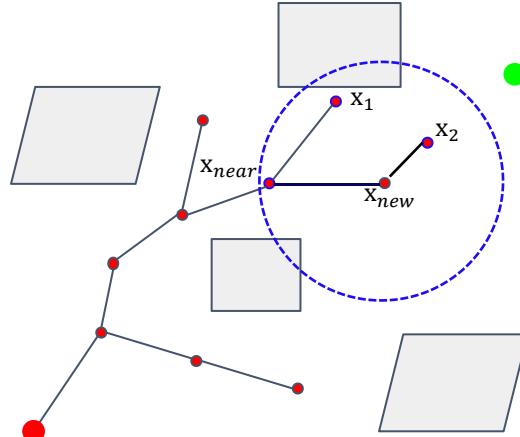


[1] Revised in Kiril Solovey et al., "Revisiting the Asymptotic Optimality of RRT*", ICRA, 2020



Practical implementation

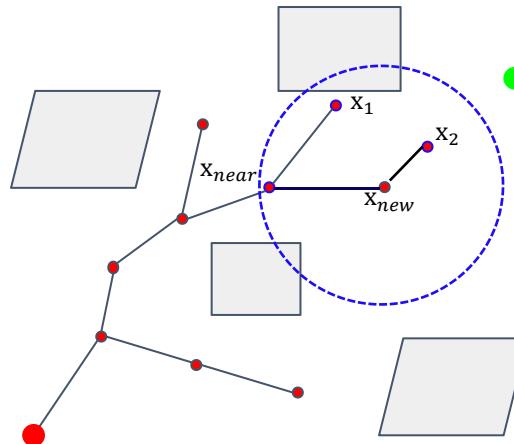
- Bias Sampling
Sample biasing toward the goal
- Sample Rejection
Reject samples with $g + h > c^*$
- Branch-and-bound (Tree Pruning)
Prune the non-promising sub trees to reduce neighbor query cost.
- Graph Sparsify
Reject samples by resolution. Introduce near optimality.





Practical implementation

- Neighbor Query
k-nearest or range near; Approximate nearest neighbor;
Range tree for different dimension.
- Delay Collision Check
Sort the neighbours by potential cost-to-come values.
Check collisions in order and stop once a collision-free edge is found.
- Bi-directional search
- Conditional Rewire
Rewire Until the first solution is found.
- Data Re-use
Store the collision-checking results for ChooseParent and Rewire (only valid for symmetric cost).



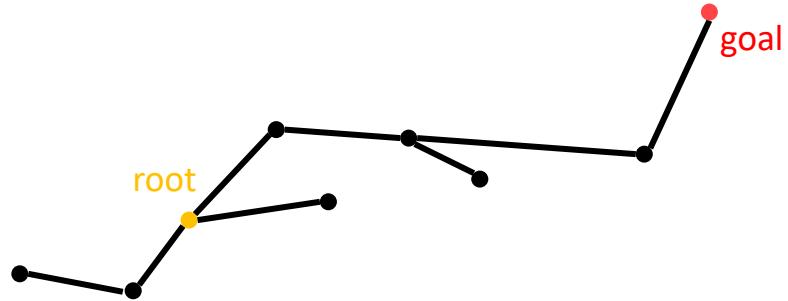
Accelerate Convergence



Flaws in the exploitation of RRT*

Over-exploitation

No need to “Rewire” non-promising vertexes.

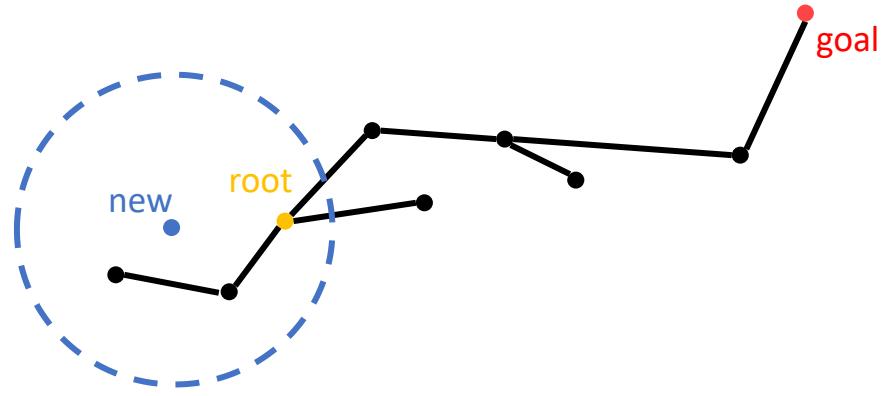




Flaws in the exploitation of RRT*

Over-exploitation

No need to “Rewire” non-promising vertexes.

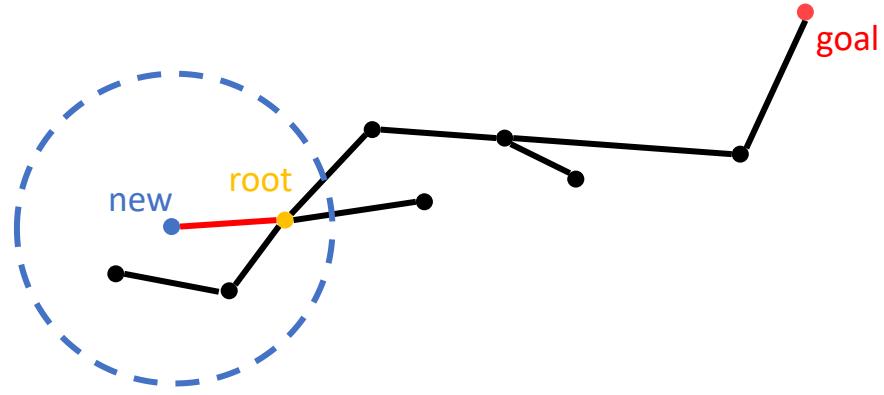




Flaws in the exploitation of RRT*

Over-exploitation

No need to “Rewire” non-promising vertexes.

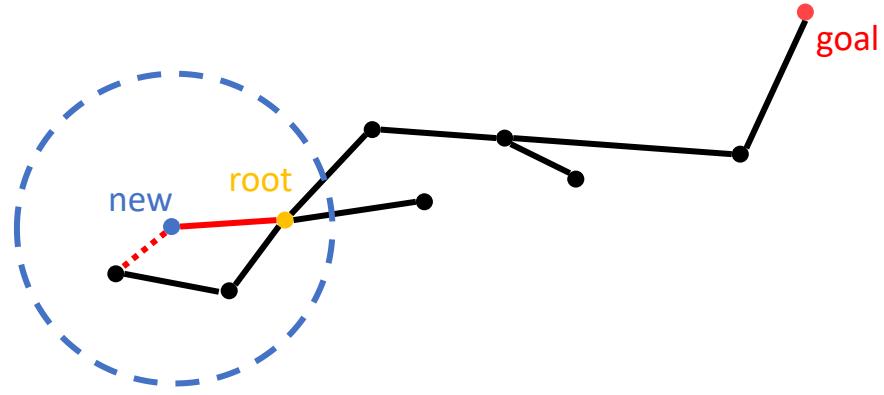




Flaws in the exploitation of RRT*

Over-exploitation

No need to “Rewire” non-promising vertexes.

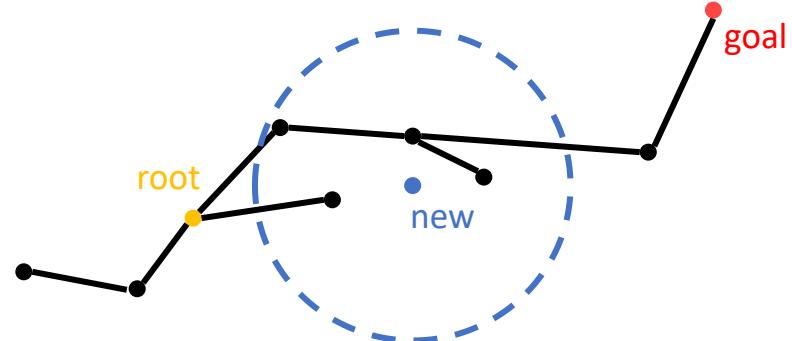




Flaws in the exploitation of RRT*

Over-exploitation

No need to “Rewire” non-promising vertexes.



Under-exploitation

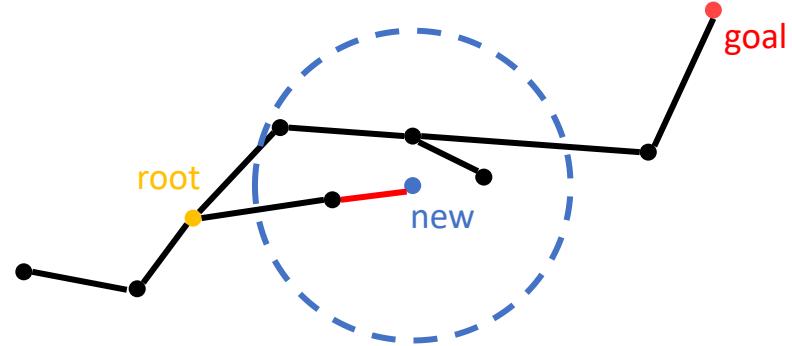
- “Rewire” only happens after a new node is add to the spanning tree and only works on its “Near” nodes.
- It does not offer any guarantees that the interim path at any intermediate iteration is optimal.



Flaws in the exploitation of RRT*

Over-exploitation

No need to “Rewire” non-promising vertexes.



Under-exploitation

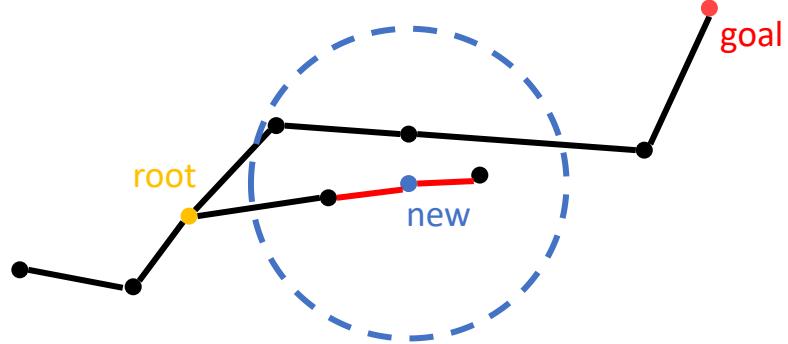
- “Rewire” only happens after a new node is add to the spanning tree and only works on its “Near” nodes.
- It does not offer any guarantees that the interim path at any intermediate iteration is optimal.



Flaws in the exploitation of RRT*

Over-exploitation

No need to “Rewire” non-promising vertexes.



Under-exploitation

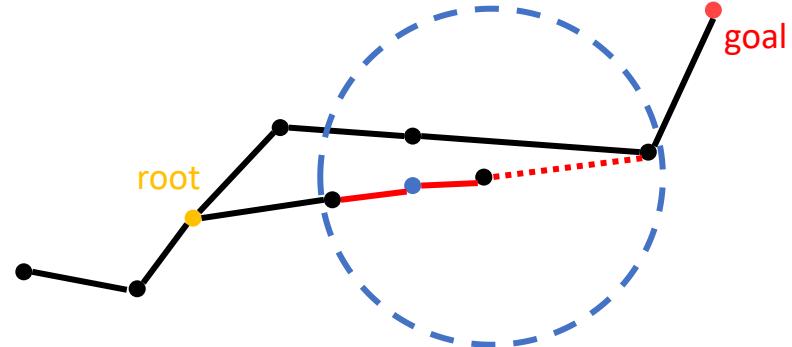
- “Rewire” only happens after a new node is add to the spanning tree and only works on its “Near” nodes.
- It does not offer any guarantees that the interim path at any intermediate iteration is optimal.



Flaws in the exploitation of RRT*

Over-exploitation

No need to “Rewire” non-promising vertexes.



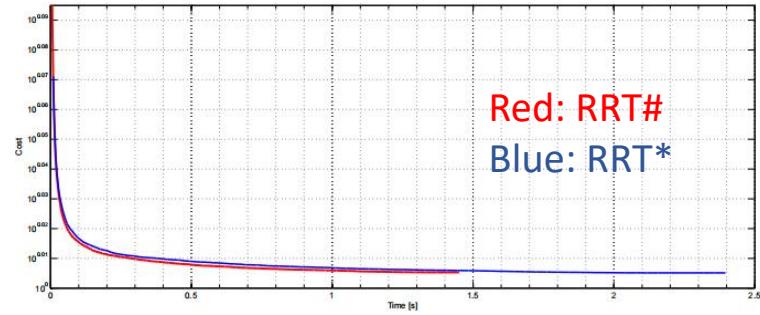
Under-exploitation

- “Rewire” only happens after a new node is add to the spanning tree and only works on its “Near” nodes.
- It does not offer any guarantees that the interim path at any intermediate iteration is optimal.

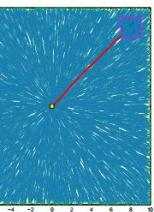
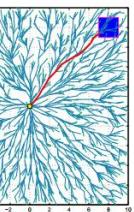
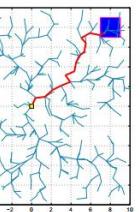
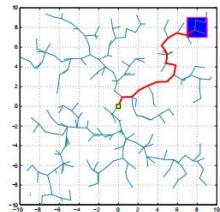


RRT#

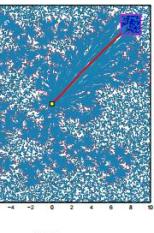
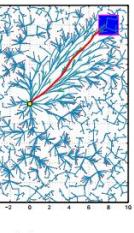
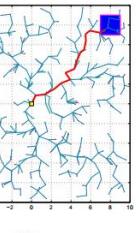
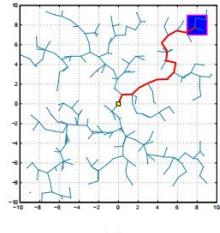
- An improvement of RRT*
- Asymptotically optimal (under conditions)



RRT*



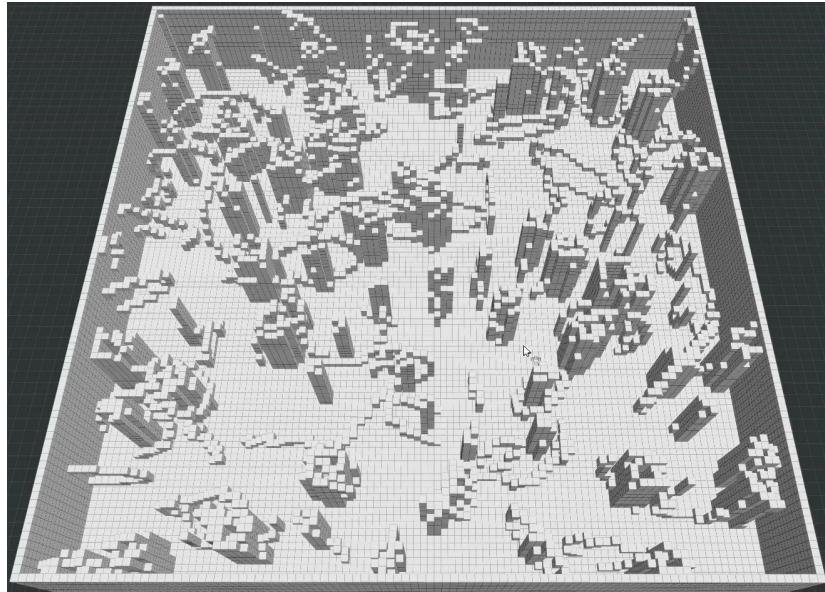
RRT#



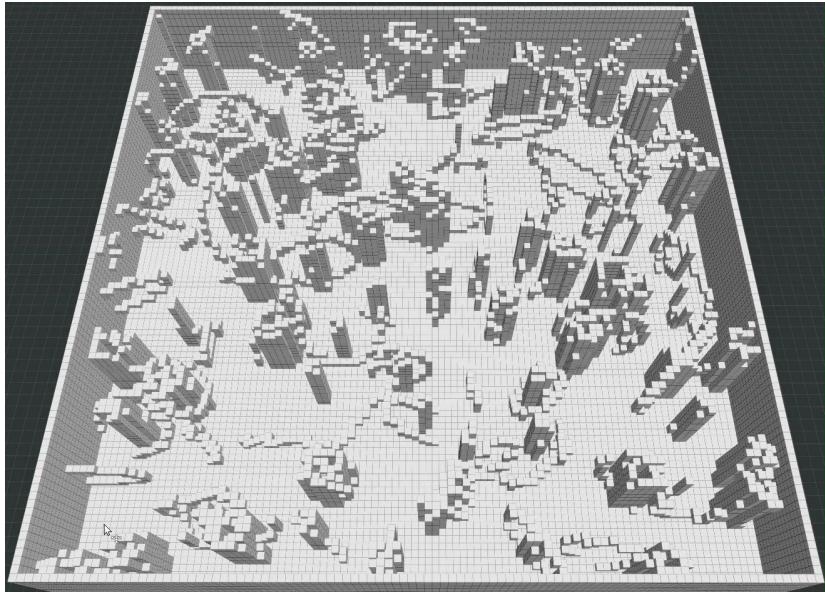


RRT#

RRT*



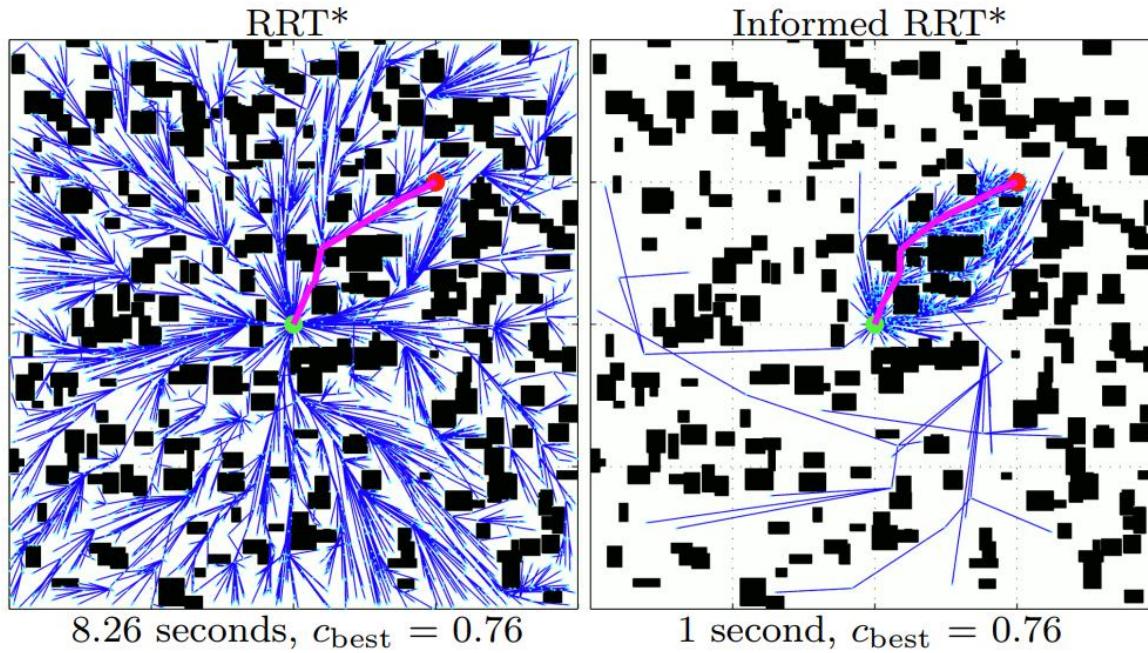
RRT#





Informed Sampling

Improve sampling efficiency



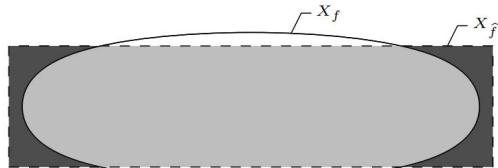


Informed Sampling

Informed sets:

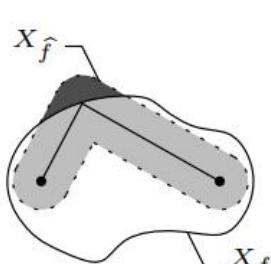
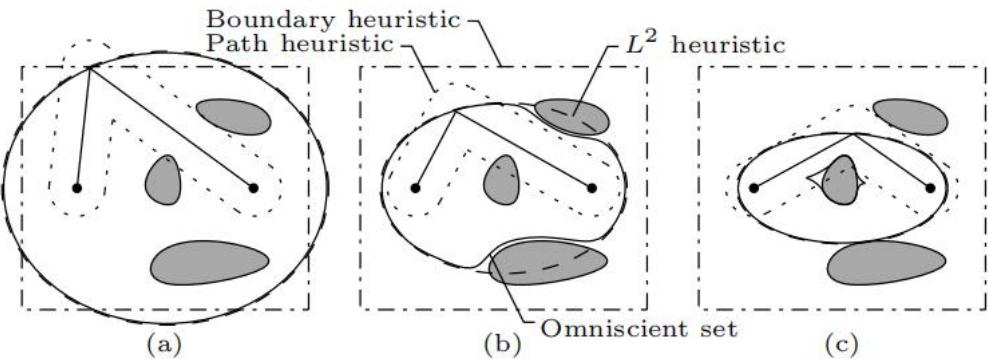
estimates of the omniscient set

- Bounding boxes
- Path heuristics
- L^2 heuristic

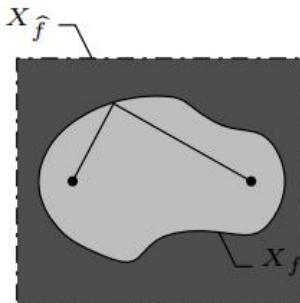


$$\text{Precision} (X_{\hat{f}}) := \frac{\lambda(X_{\hat{f}} \cap X_f)}{\lambda(X_{\hat{f}})} \equiv \frac{\square}{\square + \blacksquare}$$

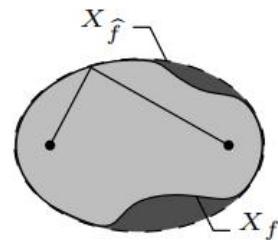
$$\text{Recall} (X_f) := \frac{\lambda(X_{\hat{f}} \cap X_f)}{\lambda(X_f)} \equiv \frac{\blacksquare}{\square + \blacksquare}$$



(a) Path biasing



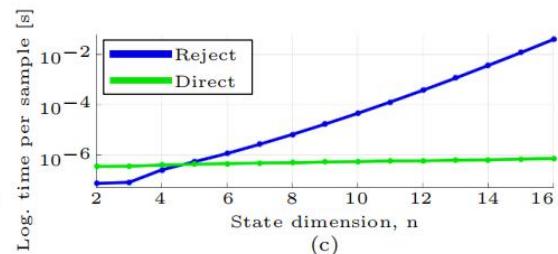
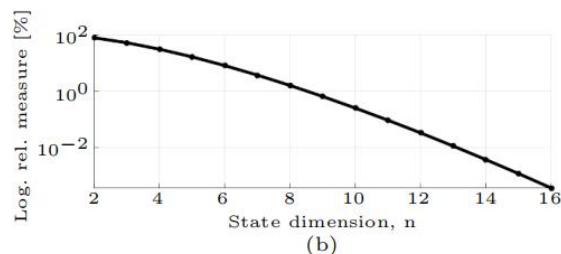
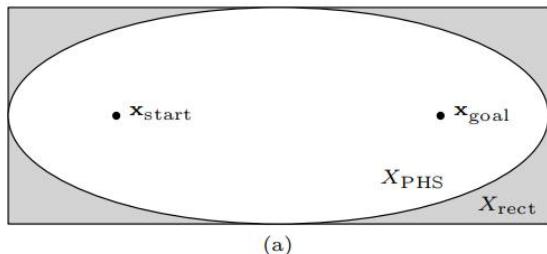
(b) Bounding box



(c) L^2 informed set



Informed Sampling



Direct Sampling in the L2 Informed set

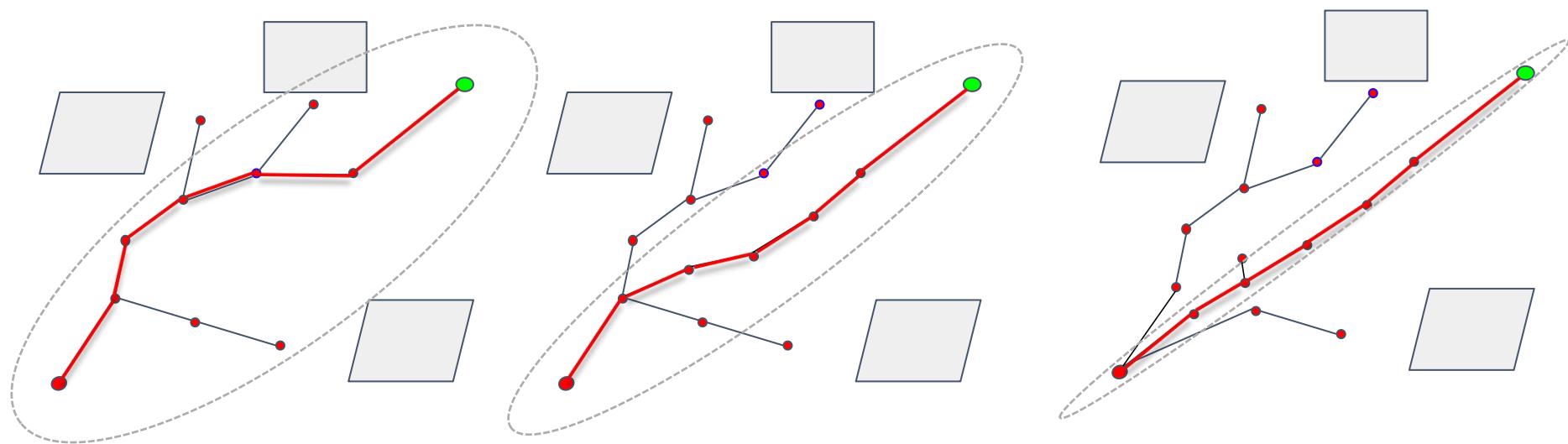
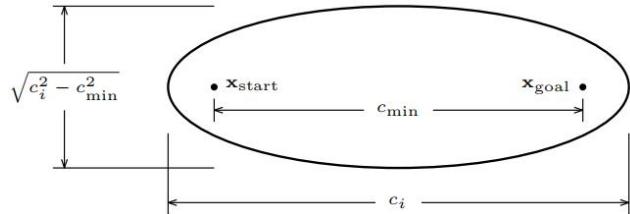
VS

Reject sampling in the bounding rectangular



Informed Sampling

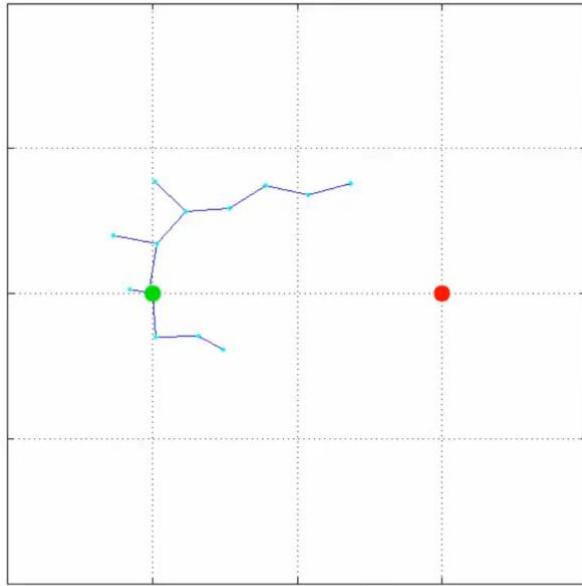
L2 Informed set





Informed Sampling

000013



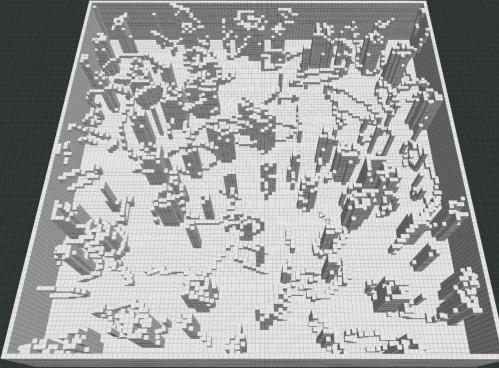
Cons: Can hardly apply to other cost except L2 norm cost (Euclidean distance).



Informed Sampling

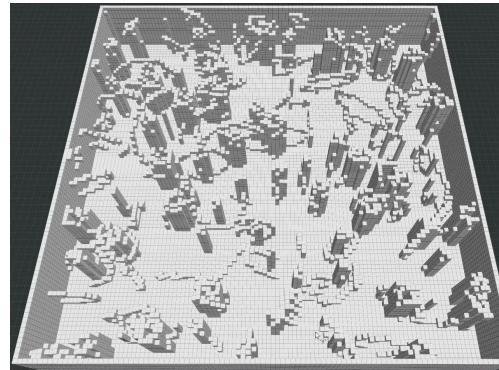
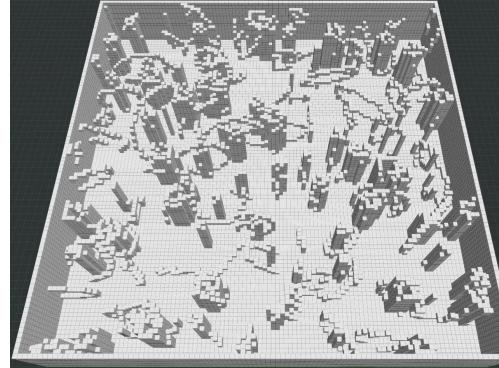
RRT*

random uniform sampling



RRT#

L2-informed sampling



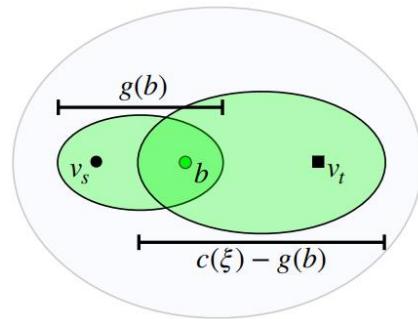


GULLD *Guided Incremental Local Densification*

Although the iteration may not have found a shorter path to the goal, new shorter paths to other vertices in the search tree can **immediately** improve the sampling strategy.



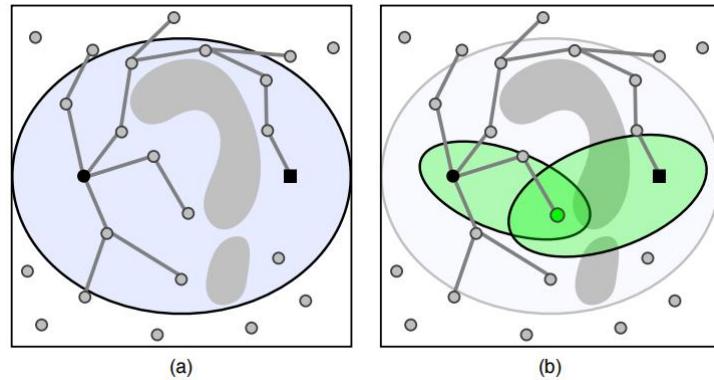
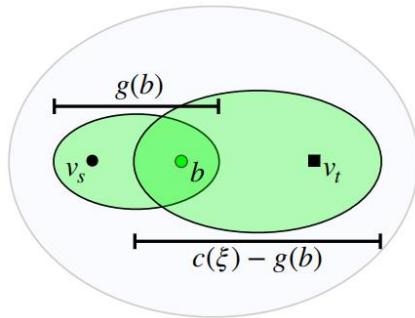
III informed set cases



Local Subsets (green):

Defined by

- A beacon node b ;
- Its cost-to-come on the search tree $g(b)$;
- The current best solution cost $c(\xi)$.



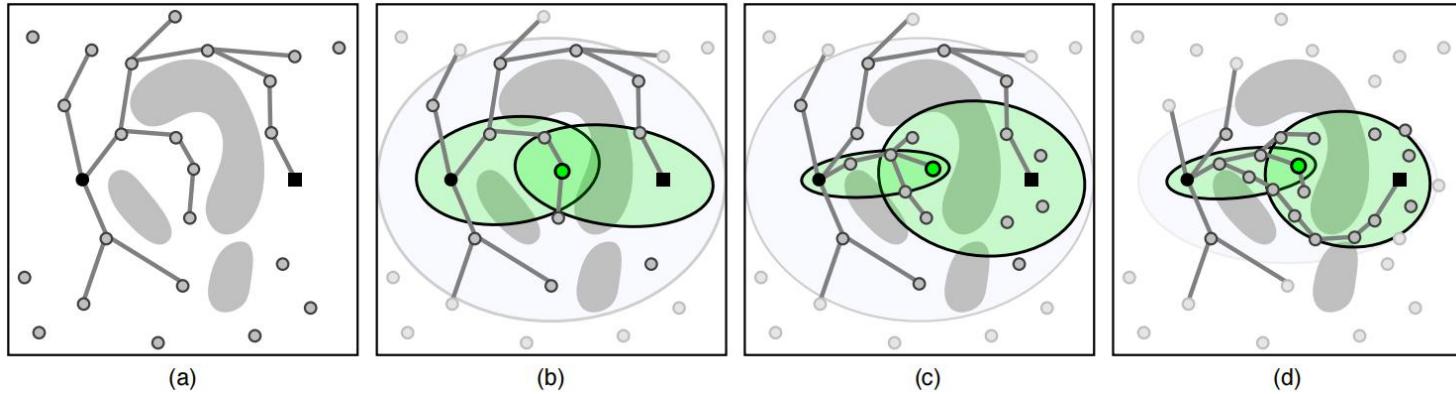
Properties of Local Subsets

- LSs are subsets of the IS;
- The measure of LSs is upper-bounded by that of the IS.

Sampling in the L2 Informed Set
vs
Sampling in the Local Subsets



GuILD *Guided Incremental Local Densification*



The Informed Set is unchanged. However, GuILD leverages the improved cost-to-come in the search tree to update the Local Subsets.

The start-beacon set shrinks to further focus sampling, and the remaining slack between the beacon's and goal's cost-to-comes is used to expand the beacon-target set.

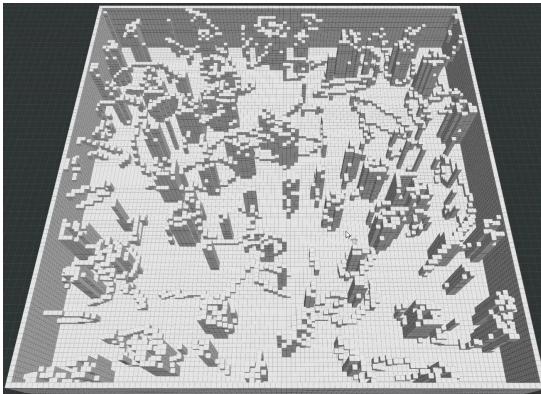
There is a larger path length budget that can be expended between the beacon and target that could **still** yield a shorter path overall.



GullD *Guided Incremental Local Densification*

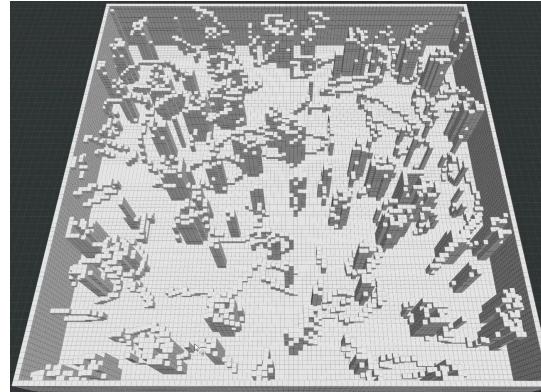
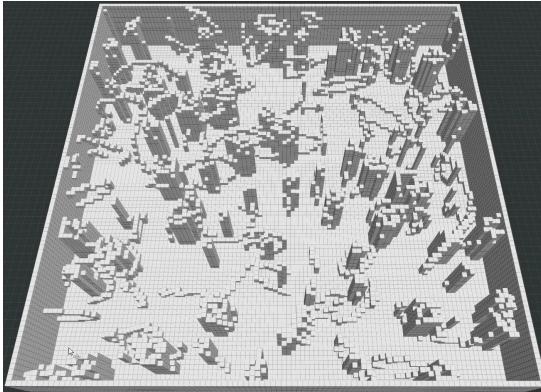
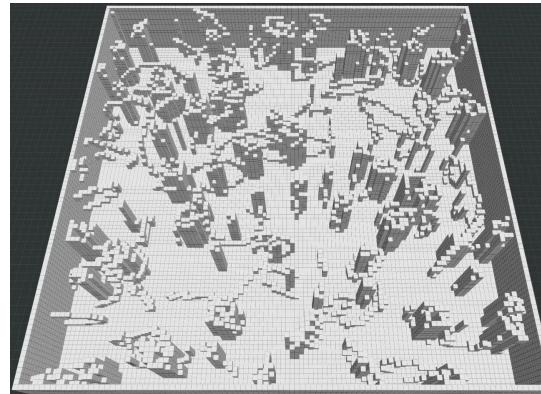
RRT*

random uniform sampling



RRT#

GullD sampling





Development Trend

Sampling Strategy

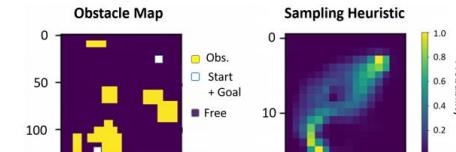
- Deterministic/Random;
- Batch/Incremental;
- Informed sampling;
- *Learned sampling*;

Hybrid Exploration-Exploitation Scheme

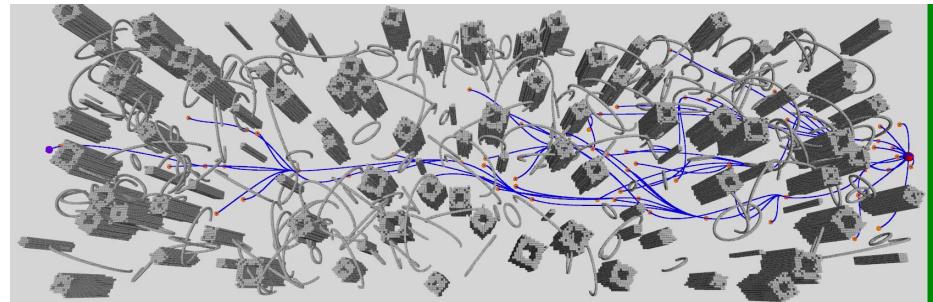
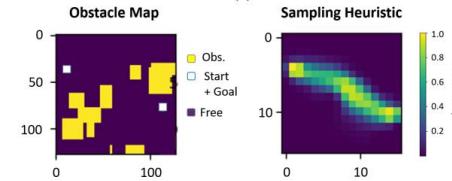
- Introduce local optimization.

Apply to Kinodynamic Systems

- The original purpose



(a)





Libraries off-the-shelf

- Open Motion Planning Library (OMPL) [1]
- Moveit with ROS [2]
- Fast-lab [3]
- ...

[1] <https://ompl.kavrakilab.org/>

[2] <https://moveit.ros.org/>

[3] <https://github.com/ZJU-FAST-Lab/sampling-based-path-finding>



Assignments

1) Implement RRT in Matlab;

2) Implement RRT* in C++;

The framework is ready, only coding for ChooseParent and Rewire are required.

```
275 // TODO Choose a parent according to potential cost-from-start values
276 // Hints:
277 // 1. Use map ptr->isSegmentValid(p1, p2) to check line edge validity;
278 // 2. Default parent is [nearest node];
279 // 3. Set curr_node->parent to the according cost-from-parent and cost-from-start
280 //    in [min node], [cost from p], and [min dist from start], respectively;
281 // 4. [Optional] You can sort the potential parents first in increasing order by cost-from-start value;
282 // 5. [Optional] You can store the collision-checking results for later usage in the Rewire procedure.
283 // Implement your own code inside the following loop
284 for (auto &curr_node : neighbour_nodes)
285 {
286 }
287 // Implement your own code inside the above loop
```

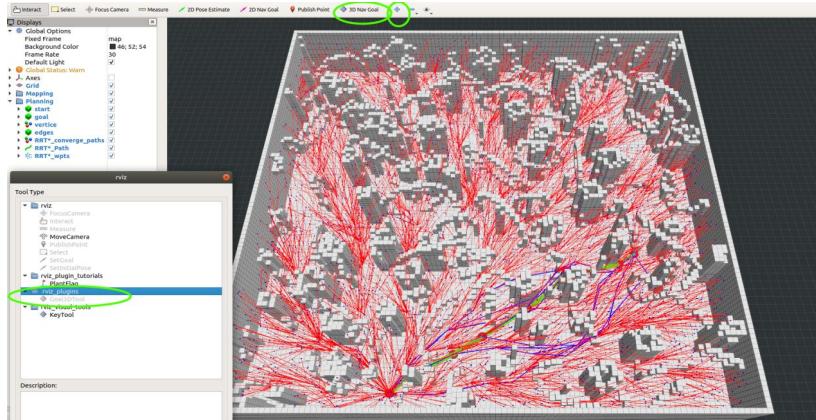
```
320 // TODO Rewire according to potential cost-from-start values
321 // Hints:
322 // 1. Use map_ptr->isSegmentValid(p1, p2) to check line edge validity;
323 // 2. Set curr_node->parent to the new parent (curr_node->parent) to change a node's parent;
324 // 3. The variable [new_node] is the pointer of x new;
325 // 4. [Optional] You can test whether the node is promising before checking edge collision.
326 // Implement your own code after the dash lines (-----) in the following loop
327 for (auto &curr_node : neighbour_nodes)
328 {
329     double best_cost_before_rewire = goal_node->cost_from_start;
330     // -----
331     // -----
332     if (best_cost_before_rewire > goal_node->cost_from_start)
333     {
334         vector<Eigen::Vector3d> curr_best_path;
335         fillPath(goal_node, curr_best_path);
336         path_list.emplace_back(curr_best_path);
337         solution_cost_time_pair_list.emplace_back(goal_node->cost_from_start, (ros::Time::now() - rrt_start_time).toSec());
338     }
339 }
340 }
```

Grading

Qualified: Finish 1)

Good: Finish 1) and 2)

Excellent: Implement Informed-RRT* based on 2)



Expected result



Reference

- L. E. Kavraki, P. Svestka, J. -C. Latombe and M. H. Overmars, "Probabilistic roadmaps for path planning in high-dimensional configuration spaces," in *IEEE Transactions on Robotics and Automation*, 1996
- S. M. LaValle and J. J. Kuffner, Jr., "Randomized Kinodynamic Planning," *The International Journal of Robotics Research*, 2001
- S. Karaman and E. Frazzoli, "Sampling-Based Algorithms for Optimal Motion Planning," *The International Journal of Robotics Research*, 2011
- Sniedovich, Moshe. "Dijkstra's algorithm revisited: the dynamic programming connexion." *Control and Cybernetics*, 2006
- O. Arslan and P. Tsotras, "Use of relaxation methods in sampling-based algorithms for optimal motion planning," *IEEE International Conference on Robotics and Automation*, 2013
- J. D. Gammell, T. D. Barfoot and S. S. Srinivasa, "Informed Sampling for Asymptotically Optimal Path Planning," in *IEEE Transactions on Robotics*, 2018
- Aditya Mandalika, Rosario Scalise, Brian Hou, Sanjiban Choudhury, Siddhartha S. Srinivasa, "Guided Incremental Local Densification for Accelerated Sampling-based Motion Planning", in *arXiv:2104.05037*, 2021

Thanks for Listening!