深蓝学院
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第四章作业分享

主讲人 valuka

纲要

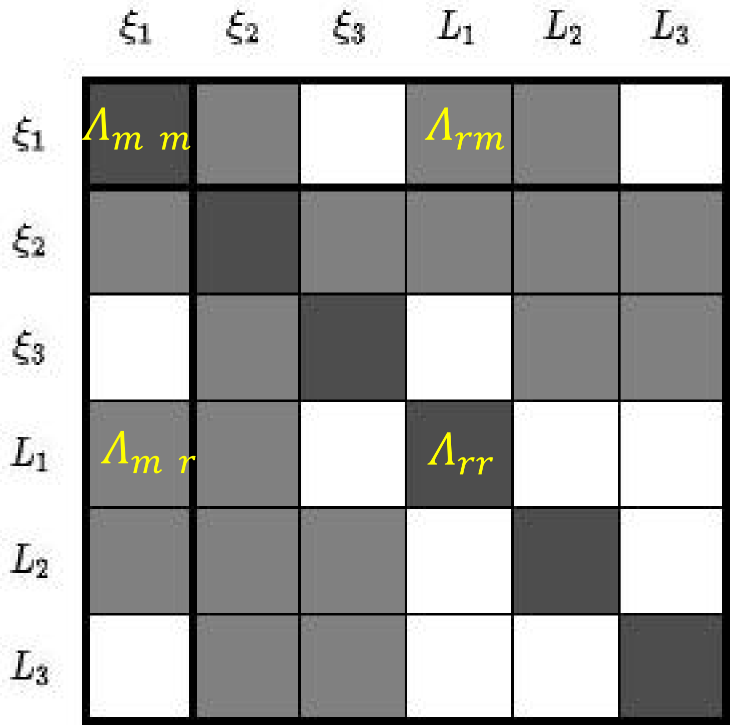
- 任务1：绘制信息矩阵
- 任务2：信息矩阵与协方差矩阵的关系
- 任务3：单目BA问题信息矩阵零空间

绘制信息矩阵

- Marg之前的信息矩阵

  节点i和j之间存在依赖关系：涂灰

  节点i和j之间不存在依赖关系： 留白



- Marginalization

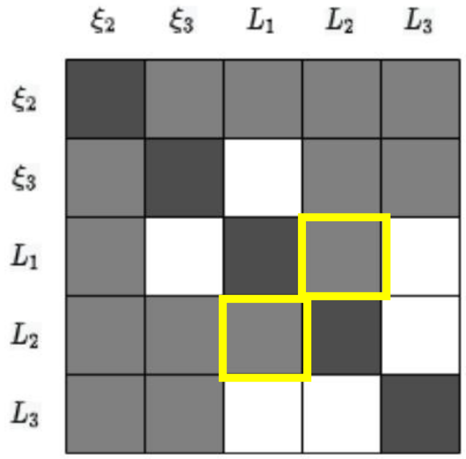
$$\Lambda = \begin{bmatrix} \Lambda_{mm} & \Lambda_{rm} \\ \Lambda_{mr} & \Lambda_{rr} \end{bmatrix},$$

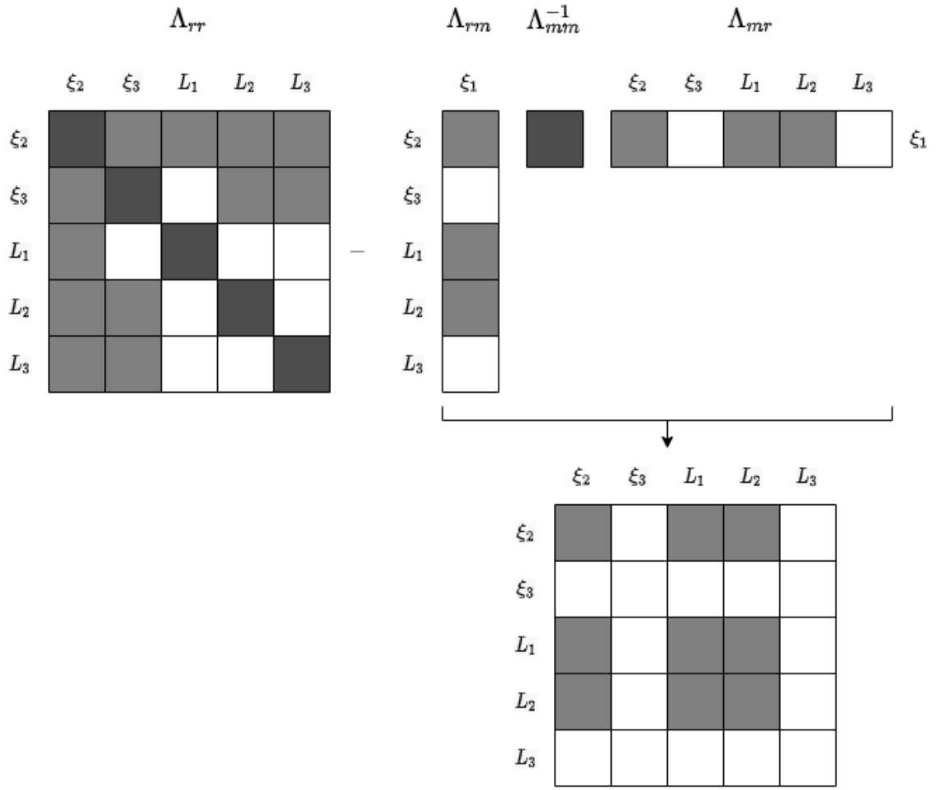
舒尔补

- 舒尔补

$$\Lambda_p = \Lambda_{rr} - \Lambda_{rm}\Lambda_{mm}^{-1}\Lambda_{mr}$$

- Marg之后的信息矩阵





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信息矩阵

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- 信息矩阵的定义

Fisher information

From Wikipedia, the free encyclopedia

In mathematical statistics, the **Fisher information** (sometimes simply called **information**[1]) is a way of measuring the amount of information that an observable random variable *X* carries about an unknown parameter *θ* of a distribution that models *X*. Formally, it is the variance of the score, or the

概率分布相对某个参数变化越显著，则包含了关于这个参数更多的信息。

- 矩阵形式

$$[\mathcal{I}(\theta)]_{i,j} = -\operatorname{E}\left[\frac{\partial^2}{\partial\theta_i\,\partial\theta_j}\log f(X;\theta)\middle|\theta\right].$$

信息矩阵与协方差矩阵

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- 高斯分布

$$p(\boldsymbol{\theta}) = (2\pi)^{-\frac{N_\theta}{2}} |\boldsymbol{\Sigma}_\theta|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}(\boldsymbol{\theta}-\boldsymbol{\theta}^\star)^T \boldsymbol{\Sigma}_\theta^{-1}(\boldsymbol{\theta}-\boldsymbol{\theta}^\star)\right]$$

$$J(\boldsymbol{\theta}) \equiv -\ln p(\boldsymbol{\theta}) = \frac{N_\theta}{2}\ln 2\pi + \frac{1}{2}\ln|\boldsymbol{\Sigma}_\theta| + \frac{1}{2}(\boldsymbol{\theta}-\boldsymbol{\theta}^\star)^T \boldsymbol{\Sigma}_\theta^{-1}(\boldsymbol{\theta}-\boldsymbol{\theta}^\star)$$

- Hessian

$$\begin{aligned}
-\frac{\partial^2 \log p(\theta)}{\partial\theta_i\,\partial\theta_j} &= \frac{1}{2}\frac{\partial^2}{\partial\theta_i\,\partial\theta_j}(\theta-\theta^\star)^\top \Sigma_\theta^{-1}(\theta-\theta^\star), \\
&= \frac{1}{2}\frac{\partial^2}{\partial\theta_i\,\partial\theta_j}\sum_{k,l}(\theta_k-\theta_k^\star)(\Sigma_\theta^{-1})_{kl}(\theta_l-\theta_l^\star), \\
&= \frac{1}{2}\sum_{k,l}(\Sigma_\theta^{-1})_{kl}(\delta_{k,i}\delta_{l,j}+\delta_{k,j}\delta_{l,i}), \\
&= \frac{1}{2}[(\Sigma_\theta^{-1})_{ij}+(\Sigma_\theta^{-1})_{ji}] = (\Sigma_\theta^{-1})_{ij},
\end{aligned}$$

- 信息矩阵 $\mathbf{I}(\theta)_{ij} = \mathbb{E}[(\Sigma_\theta^{-1})_{ij}] = (\Sigma_\theta^{-1})_{ij},$ 矩阵形式 $\mathbf{I}(\theta) = \Sigma_\theta^{-1}.$

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单目BA问题

- 参考十四讲7.7, 9.2
- 状态向量 $$\mathbf{x} = [\xi_1, \cdots . \xi_n, L_1, \cdots, L_m] \in \mathbb{R}^{6n+3m}$$

  位姿$\xi_i \in R^6$，$i \in [n]$，路标$L_j \in R^3$，$j \in [m]$

- 残差 $$r_{ij} = z_{ij} - h(\xi_i, L_j) \in \mathbb{R}^2,$$

  z_{ij}是位姿i处观察到路标j的像素坐标

- 代价函数

$$R(\mathbf{x}) = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{m} ||r_{ij}||^2 = \frac{1}{2} \mathbf{r}^\top \mathbf{r},$$

单目BA问题

- 雅可比

$$\mathbf{J} = \frac{\partial \mathbf{r}}{\partial \mathbf{x}} \in \mathbb{R}^{(2nm)\times(6n+3m)}$$

- 每2行对应一个残差

$$J_{ij} = \frac{\partial r_{ij}}{\partial \mathbf{x}} = \left[\mathbf{0}_{2\times 6}, \cdots, \mathbf{0}_{2\times 6}, \frac{\partial r_{ij}}{\partial \xi_i}, \mathbf{0}_{2\times 6}, \cdots, \mathbf{0}_{2\times 3}, \cdots, \mathbf{0}_{2\times 3}, \frac{\partial r_{ij}}{\partial L_j}, \mathbf{0}_{2\times 3}, \cdots, \mathbf{0}_{2\times 3}\right].$$

非常稀疏，其中只有2个block不为0。

单目BA问题

- s Hessian

$$\mathbf{H} = \mathbf{J}^\top \mathbf{J} = \sum_{i=1}^{n}\sum_{j=1}^{m} J_{ij}^\top J_{ij} = \sum_{i=1}^{n}\sum_{j=1}^{m} H_{ij},$$

- H_{ij}的稀疏性

$$H_{ij} = \begin{bmatrix}
\mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\
\vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\
\mathbf{0} & \cdots & \mathbf{0} & \frac{\partial r_{ij}}{\partial \xi_i}^\top \frac{\partial r_{ij}}{\partial \xi_i} & \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{0} & \frac{\partial r_{ij}}{\partial \xi_i}^\top \frac{\partial r_{ij}}{\partial L_j} & \mathbf{0} & \cdots & \mathbf{0} \\
\mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\
\vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\
\mathbf{0} & \cdots & \mathbf{0} & \frac{\partial r_{ij}}{\partial L_j}^\top \frac{\partial r_{ij}}{\partial \xi_i} & \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{0} & \frac{\partial r_{ij}}{\partial L_j}^\top \frac{\partial r_{ij}}{\partial L_j} & \mathbf{0} & \cdots & \mathbf{0} \\
\vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0}
\end{bmatrix}.$$

代码和结果

```
H.block(i * 6, i * 6, 6, 6) += jacobian_Ti.transpose() * jacobian_Ti;
/// 请补充完整作业信息矩阵块的计算
H.block(poseNums * 6 + j * 3, poseNums * 6 + j * 3, 3, 3) += jacobian_Pj.transpose() *
jacobian_Pj;
H.block(i * 6, poseNums * 6 + j * 3, 6, 3) += jacobian_Ti.transpose() * jacobian_Pj;
H.block(poseNums * 6 + j * 3, i * 6, 3, 6) += jacobian_Pj.transpose() * jacobian_Ti;
```

- H的特征值

  H是半正定对称矩阵，奇异值=特征值

- 零空间维度为7

  3维平移、3维转动、1维尺度

```
...
3.21708e-17
2.06732e-17
1.43188e-17
7.66992e-18
6.08423e-18
6.05715e-18
3.94363e-18
```

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感谢各位聆听！

Thanks for Listening