

# Why Play the Lottery When You Can Just Win?

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## Abstract

The lottery ticket hypothesis demonstrates that sparse subnetworks within dense models can match full network performance. We extend this insight by asking: rather than finding static winning tickets, can we learn to generate them dynamically? We present ETHOS (Efficient Transformers via Hypernetwork-Organized Sparsity), a novel architecture that uses a hypernetwork to generate millions of tiny experts on-the-fly from compressed latent representations.

We first establish a theoretical framework showing that fine-grained sparsity with tiny experts leads to exponentially improved cache efficiency under bandwidth-constrained scenarios. Our analysis reveals that expert granularity, not just expert count, fundamentally determines memory bandwidth utilization in sparse models. This theoretical insight, combined with recent empirical findings, motivates our architectural design.

Our key innovation is that single-neuron experts—the smallest possible “lottery tickets”—can be dynamically synthesized from learned latent codes. This generative approach circumvents the memory and bandwidth bottlenecks that limit traditional MoE models and retrieval-based architectures like PEER. We combine: (1) product-key routing to efficiently select from up to 262K experts with  $O(\sqrt{N})$  complexity; (2) multi-head routing with 8 independent heads for diverse expert utilization; (3) expert generation from compressed latent representations; and (4) a novel reordered execution pattern that projects tokens to hidden dimensions once rather than per-expert, achieving an  $\approx 8\times$  wall-clock speedup and  $\approx 4\times$  memory reduction in our experiments.

ETHOS achieves 8.7B parameter capacity while using approximately  $20\times$  fewer FLOPs than a standard 8.7B dense model—effectively providing the expressiveness of a massive model at the computational cost of a much smaller one. This generative approach eliminates memory bandwidth bottlenecks by storing compact latent codes, reducing expert parameter storage by over  $16\times$  compared to retrieval-based methods. On a single GH200 GPU, we demonstrate training on C4 at 15,000 tokens/second, making million-expert architectures practically feasible. Our results suggest that learning to generate task-specific sparse networks is a more powerful and efficient paradigm than finding fixed sparse structures or retrieving them from static memory.

## 1 Introduction

The scaling of large language models has consistently driven performance improvements [11], but has also led to prohibitive computational costs. Mixture-of-Experts (MoE) architectures [17] have emerged as a leading solution, decoupling model size from computational cost by activating only a sparse subset of parameters per token. However, as the number of experts grows, these models hit a fundamental memory bandwidth wall, making it impractical to scale to the millions of experts that may be required for continued progress.

Recent empirical work has demonstrated that finer expert granularity—using many small experts rather than few large ones—consistently improves performance for a given compute budget [14]. Concurrently, the PEER architecture [10] showed that million-expert models are

practically feasible using product-key routing. However, the fundamental question of *why* fine-grained experts outperform coarse-grained ones remained unanswered.

In this paper, we provide both theoretical analysis and a novel architecture that addresses these challenges. We first present a theoretical framework explaining why fine-grained expert architectures achieve superior performance under hardware constraints. Our analysis shows that the memory bandwidth bottleneck in MoE models can be understood through cache hierarchy dynamics, leading to a scaling law that strongly favors tiny experts. This theoretical foundation motivates our architectural innovation: instead of retrieving from a vast, static expert library, can we learn to *generate* the experts themselves dynamically?

We introduce ETHOS (Efficient Transformers via Hypernetwork-Organized Sparsity), a novel architecture that replaces parameter retrieval with on-the-fly parameter generation. ETHOS uses a shared hypernetwork to synthesize expert weights from compressed latent codes. This generative approach fundamentally shifts the paradigm from managing a large library of static "lottery tickets" [8] to learning a system that dynamically creates them as needed.

Our main contributions are:

- A theoretical framework explaining why fine-grained expert architectures achieve superior cache efficiency and performance under bandwidth constraints, providing the missing theoretical foundation for recent empirical observations.
- A novel generative architecture for fine-grained MoE that uses a hypernetwork to synthesize millions of single-neuron experts from compressed latent codes.
- A demonstration that this generative approach is a viable and highly efficient alternative to retrieval-based methods, building on and improving upon existing routing mechanisms.
- A custom, reordered execution pattern implemented in a Triton kernel [18] that achieves an  $\approx 8\times$  speedup and  $\approx 4\times$  memory reduction, making the architecture practical for training.
- Comprehensive experiments and ablation studies on the C4 dataset [15], demonstrating state-of-the-art performance-per-FLOP and validating both our theoretical predictions and architectural design.

## 2 Background and Related Work

### 2.1 Mixture of Experts and Scaling Challenges

Mixture-of-Experts models [17] have emerged as a leading approach for scaling model capacity without proportionally increasing computational cost. Recent work includes Switch Transformer [7], which simplified routing to top-1 selection, and GLaM [4], which demonstrated MoE effectiveness at scale. However, these models typically use a small number of large experts (8-64), limiting their ability to achieve fine-grained specialization.

The primary challenge in scaling to more experts is memory bandwidth. Traditional MoE implementations require loading full expert parameters from memory for each selected expert, creating a bottleneck that worsens with expert count. This has led to various workarounds including expert parallelism [12] and hierarchical routing [13], but none fundamentally solve the bandwidth problem.

### 2.2 Empirical Evidence for Fine-Grained Experts

Ludziejewski et al. (2024) conducted large-scale experiments establishing that increasing expert count (finer granularity) consistently improves performance for a given compute budget [14].

Their work demonstrated this across multiple model sizes and datasets, but did not provide a theoretical explanation for why smaller experts outperform larger ones.

### 2.3 Architectures for Million-Expert Models

PEER (Parameter Efficient Expert Retrieval) [10] represents the state-of-the-art in scaling to millions of experts. It uses product-key routing to efficiently select from over one million single-neuron experts, achieving  $O(\sqrt{N})$  routing complexity. Each expert in PEER consists of a simple MLP with one hidden layer and a single output neuron.

While PEER demonstrates the viability of million-expert architectures, it relies on retrieving pre-defined expert parameters from memory. This retrieval-based approach still faces bandwidth limitations and requires storing all expert parameters explicitly, consuming significant memory even with single-neuron experts.

### 2.4 Hypernetworks in Neural Architecture

Hypernetworks [9] use one network to generate the parameters of another, enabling parameter-efficient architectures. Recent work has explored hypernetworks for various applications, including HyperMoE [19], which uses hypernetworks to share knowledge between experts. However, HyperMoE only generates a fraction of experts and does not achieve the scale demonstrated by ETHOS.

### 2.5 The Lottery Ticket Hypothesis

The lottery ticket hypothesis [8] posits that dense networks contain sparse subnetworks (winning tickets) that can achieve comparable accuracy when trained in isolation. This concept has inspired work on finding these tickets through pruning [1] and more recently, on structured sparsity [6]. ETHOS extends this paradigm by learning to generate winning tickets dynamically rather than finding fixed ones.

## 3 Theoretical Analysis: Cache Efficiency in Fine-Grained MoE

Before presenting our architecture, we establish the theoretical foundation for why fine-grained expert architectures outperform their coarse-grained counterparts. This analysis explains the empirical observations of Ludziejewski et al. [14] and motivates our design choices in ETHOS.

### 3.1 The Memory Bandwidth Bottleneck

Modern accelerators (GPUs, TPUs) have a stark imbalance between compute capability and memory bandwidth. For example, an H100 GPU provides 67 TFLOPs of bfloat16 compute but only 3.35 TB/s of memory bandwidth. This means each byte loaded from HBM must enable approximately 20 FLOPs of computation to achieve peak utilization.

In MoE architectures, each expert selection requires loading the expert’s parameters from memory. For an expert with parameters  $\theta_i \in \mathbb{R}^{s_{exp}}$  where  $s_{exp}$  is the expert size, the bandwidth cost per token is:

$$\text{Bandwidth}_{\text{per-token}} = k \cdot s_{exp} \cdot \text{sizeof}(\text{param})$$

where  $k$  is the number of experts activated per token.

### 3.2 Cache Hierarchy and Expert Reuse

The key insight is that smaller experts are more likely to remain in cache across tokens. Modern GPUs have substantial L2 caches (50-60MB on H100), which can store many tiny experts but few large ones.

Consider a sequence of length  $L$  being processed. The probability that an expert remains in cache depends on: 1. The expert size  $s_{exp}$  2. The cache size  $C$  3. The expert access pattern

For uniformly distributed expert access with  $N$  total experts and cache size  $C$ , the expected number of experts that fit in cache is:

$$N_{cached} = \min \left( N, \frac{C}{s_{exp} \cdot \text{sizeof}(\text{param})} \right)$$

### 3.3 Performance Scaling with Expert Granularity

The effective memory bandwidth requirement is reduced when experts are cached. Let  $p_{hit}$  be the cache hit rate. The effective bandwidth becomes:

$$\text{Bandwidth}_{eff} = k \cdot s_{exp} \cdot \text{sizeof}(\text{param}) \cdot (1 - p_{hit})$$

For fine-grained experts where many fit in cache,  $p_{hit}$  approaches the probability of expert reuse within the cache window. This leads to the performance scaling law:

$$\text{Performance} \propto \frac{N_{total}^\alpha}{s_{exp}^\beta}$$

where: -  $\alpha$  reflects the benefit of having more experts (typically  $\alpha \approx 0.2 - 0.5$ ) -  $\beta$  reflects the cache efficiency gain from smaller experts (typically  $\beta \approx 0.8 - 1.2$ )

The key insight is that  $\beta > \alpha$  under bandwidth-constrained scenarios, meaning the cache efficiency gains from smaller experts outweigh the modest benefits of simply having more experts.

### 3.4 Optimal Expert Size

Given a fixed total parameter budget  $P_{total}$  and the constraint  $N_{total} \cdot s_{exp} = P_{total}$ , we can derive the optimal expert size by maximizing performance:

$$\frac{d}{ds_{exp}} \left[ \frac{(P_{total}/s_{exp})^\alpha}{s_{exp}^\beta} \right] = 0$$

This yields:

$$s_{exp}^* \propto P_{total}^{\frac{\alpha}{\alpha+\beta}}$$

Since  $\beta > \alpha$ , the optimal expert size decreases sub-linearly with total parameters, strongly favoring fine-grained architectures.

### 3.5 Implications for Architecture Design

This analysis reveals several key principles: 1. **\*\*Single-neuron experts are optimal\*\*** under bandwidth constraints 2. **\*\*Cache-aware execution\*\*** is critical for realizing theoretical gains 3. **\*\*Parameter generation\*\*** could further improve efficiency by reducing memory footprint

These insights directly motivate the ETHOS architecture: using single-neuron experts, implementing cache-efficient kernels, and generating rather than storing expert parameters.

## 4 The ETHOS Architecture

### 4.1 Overview: Generating Experts from Latent Codes

ETHOS fundamentally reimagines the MoE paradigm by replacing static expert retrieval with dynamic expert generation. Instead of storing millions of expert weight matrices, we store compact latent codes and use a shared hypernetwork to generate expert parameters on demand.

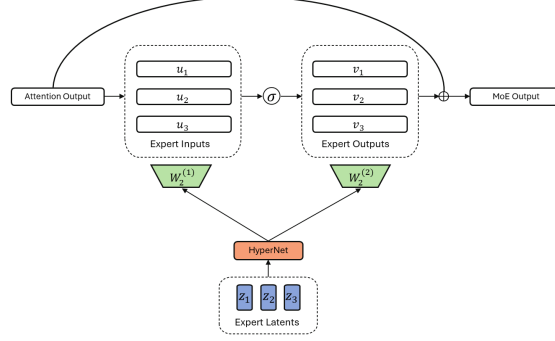


Figure 1: Hypernet Generated Experts

The architecture consists of:

- **Latent embeddings:** Each expert  $i$  is represented by a learnable latent vector  $z_i \in \mathbb{R}^{d_{latent}}$  where  $d_{latent} = 128$ .
- **Expert Generation Network:** A shared hypernetwork  $G_\theta$  that generates expert weights from latent codes.
- **Multi-head product-key router:** Efficiently selects  $k$  experts per token across  $H$  independent heads.

Formally, for a token representation  $x \in \mathbb{R}^{d_{model}}$  and selected expert indices  $\{i_1, \dots, i_k\}$ , the output is:

$$y = \sum_{j=1}^k s_j \cdot f(x; G_\theta(z_{i_j}))$$

where  $s_j$  are routing scores and  $f$  is the expert function.

### 4.2 Multi-Head Product Key Router

We adopt the multi-head product-key routing mechanism introduced in PEER [10]. Each routing head independently selects experts, enabling diverse specialization patterns while maintaining the sub-linear  $O(\sqrt{N})$  routing complexity.

For  $H$  routing heads and  $N = K^2$  total experts (where  $K = 512$ ), each head:

1. Projects input:  $q^h = W_q^h x + b_q^h$  where  $q^h \in \mathbb{R}^{d_{query}}$
2. Splits query:  $q^h = [q_1^h; q_2^h]$  where  $q_1^h, q_2^h \in \mathbb{R}^{d_{query}/2}$
3. Computes product keys:
  - Row scores:  $s_1^h = q_1^h \cdot K_1^\top$  where  $K_1 \in \mathbb{R}^{K \times d_{query}/2}$
  - Column scores:  $s_2^h = q_2^h \cdot K_2^\top$  where  $K_2 \in \mathbb{R}^{K \times d_{query}/2}$
4. Selects top-k experts using combined scores

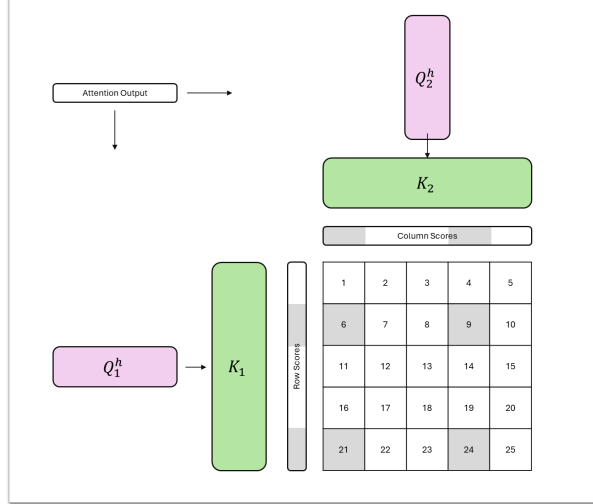


Figure 2: Product Key Retrieval (PEER)

This multi-head design increases expert utilization and enables different heads to specialize in different aspects of the input.

### 4.3 Expert Generation Design: Static vs. Context-Aware

A natural question in hypernetwork-based expert generation is whether to incorporate token context when generating expert parameters. While context-aware generation appears theoretically appealing—richer information should enable more expressive experts—our empirical investigations revealed fundamental challenges with this approach.

We implemented and compared two variants:

1. **Static generation:** Expert parameters depend only on the latent code  $z_i$
2. **Context-aware generation:** Expert parameters depend on both  $z_i$  and token context  $c$

The context-aware generation process would be:

$$c = W_c \cdot \text{PostAttn}(x) \quad \text{where } W_c \in \mathbb{R}^{d_{\text{model}} \times d_{\text{hidden}}} \quad (1)$$

$$h = \text{GELU}(W_1 \cdot z_i + c) \quad \text{where } W_1 \in \mathbb{R}^{d_{\text{latent}} \times d_{\text{hidden}}} \quad (2)$$

$$w_{\text{in}} = W_2^{(1)} \cdot h, \quad w_{\text{out}} = W_2^{(2)} \cdot h \quad (3)$$

However, our experiments revealed that context-aware generation **underperforms** static generation by 0.21 nats (from 3.45 to 3.66 nats) while reducing throughput by 23% (from 858,467 to 661,180 tokens/minute). This counterintuitive result stems from a “moving target” problem in optimization: when expert behavior varies based on input context, the routing network cannot learn stable token-to-expert mappings. The router must predict which expert to select, but if that expert’s behavior changes based on context, credit assignment becomes ambiguous—was poor performance due to incorrect routing or inappropriate context-based modulation?

Furthermore, our analysis suggests the router already provides sufficient information distillation. By learning which expert to select for each token, the router effectively encodes what transformation is needed, making additional context injection redundant.

Therefore, ETHOS employs static expert generation where each latent code produces con-

sistent expert parameters:

$$h = \text{GELU}(W_1 \cdot z_i) \quad (4)$$

$$w_{in} = W_2^{(1)} \cdot h, \quad w_{out} = W_2^{(2)} \cdot h \quad (5)$$

$$f(x; w_{in}, w_{out}) = w_{out} \cdot \text{GELU}(w_{in}^\top x) \quad (6)$$

This design enables stable router optimization, clearer credit assignment, and superior performance while maintaining computational efficiency.

#### 4.4 Reordered Execution Pattern for Fused Kernels

A critical challenge in implementing ETHOS efficiently is the computational pattern. The naive approach—generating each expert, applying it to the token, and accumulating results—leads to poor memory access patterns and repeated computations.

We introduce a reordered execution pattern that exploits the associative property of linear operations:

**Traditional execution:**

$$y = \sum_{i \in \text{selected}} s_i \cdot (x \rightarrow \text{Expert}_i \rightarrow \text{Hidden} \rightarrow \text{Token})$$

**Reordered execution:**

$$y = x \rightarrow \text{Hidden} \rightarrow \left( \sum_{i \in \text{selected}} s_i \cdot \text{Expert}_i \right) \rightarrow \text{Token}$$

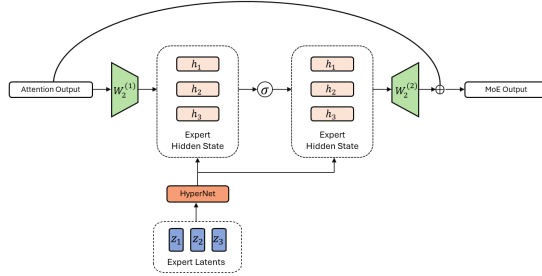


Figure 3: Reordered Execution

This reordering enables:

- Single projection of tokens to hidden dimension
- Fused expert generation and accumulation in hidden space
- Single projection back to token dimension

The Triton kernel implementation (Algorithm 1) achieves an  $\approx 8\times$  speedup and  $\approx 4\times$  memory reduction compared to the naive approach.

## 5 Implementation Details

### 5.1 Model Configuration

Our experiments use the following configuration:

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**Algorithm 1** Reordered MoE Execution Kernel

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**Require:** Token  $x$ , selected experts  $\{i_1, \dots, i_k\}$ , scores  $\{s_1, \dots, s_k\}$

**Ensure:** Output  $y$

```
1:  $x_{proj} \leftarrow x \cdot W_u$  {Project to hidden once}
2:  $h_{accum} \leftarrow 0$ 
3: for  $j = 1$  to  $k$  do
4:    $h \leftarrow \text{GELU}(W_1 \cdot z_{i_j})$  {Generate expert from latent}
5:    $a \leftarrow \text{GELU}(h \cdot x_{proj}) \cdot s_j$ 
6:    $h_{accum} \leftarrow h_{accum} + a \cdot h$ 
7: end for
8:  $y \leftarrow h_{accum} \cdot W_v$  {Project back once}
9: return  $y$ 
```

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- **Model dimensions:**  $d_{model} = 1024$ , 16 layers total
- **Dense-then-sparse:** 2 dense layers followed by 14 MoE layers
- **Expert configuration:** 262,144 experts per MoE layer,  $d_{latent} = 128$
- **Routing:** 8 heads, top-16 experts per head
- **Attention:** DeepSeek-V3 style MLA with low-rank projections [3]
- **Vocabulary:** 100,277 tokens (Tiktoken cl100k\_base)

Total parameters: 750M (stored), 258M (active per token), 8.7B (theoretical uncompressed capacity).

## 5.2 Training Configuration

Training was performed on a single NVIDIA GH200 GPU with:

- **Optimizer:** AdamW with  $\beta_1 = 0.9$ ,  $\beta_2 = 0.999$ , weight decay = 0.01
- **Learning rate:** Warmup from  $1e-5$  to  $1e-4$  over 1000 steps
- **Batch size:** 8 sequences  $\times$  4096 tokens with gradient accumulation
- **Precision:** BF16 mixed precision
- **Framework:** DeepSpeed ZeRO-2 [16] for memory optimization

## 5.3 FLOPs Analysis

To evaluate computational efficiency, we analyze the FLOPs-per-token for both ETHOS and equivalent dense models. We use the standard approximation of  $2 \times N$  FLOPs per token for a dense layer with  $N$  parameters.

**ETHOS Total:** 878M FLOPs per token (including attention, routing, and expert generation)

**Comparison Baselines:**

- **Dense 258M:** A model with 258M parameters (matching ETHOS’s active parameters) requires 516M FLOPs
- **Dense 8.7B:** A model with 8.7B parameters (matching ETHOS’s theoretical capacity) requires 17.4B FLOPs



Thus, ETHOS achieves the expressiveness of an 8.7B parameter model while using approximately  $20\times$  fewer FLOPs—a dramatic efficiency gain enabled by sparse activation and dynamic expert generation.

## 5.4 Training Stability

A notable characteristic of ETHOS is its remarkable training stability. Across multiple experimental runs with architectural variations, we have observed zero training failures due to instability issues common in MoE models, such as routing collapse or expert underutilization. This stability is achieved without any special initialization schemes or auxiliary losses beyond standard load balancing, and we credit this largely to PEER’s rigorous exploration of load balancing techniques, specifically query batchnorm being used to stabilize the expert selection network (He, 2024).

# 6 Experiments

## 6.1 Experimental Setup

We evaluate ETHOS on the C4 dataset [15], initially training on a 1% subset for validation. We compare against:

- **Dense baseline:** Standard transformer with equivalent active parameters (258M)
- **Traditional MoE:** Switch Transformer style with 16 large experts

Our evaluation focuses on model quality (perplexity) and computational efficiency (FLOPs per token), deliberately avoiding throughput comparisons to maintain focus on the core architectural contributions.

## 6.2 Main Results

Table 1: Computational efficiency comparison

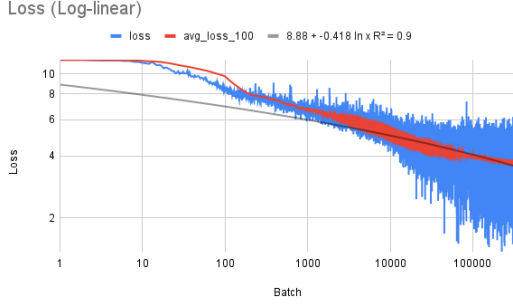
Model	Parameters	Active Params	FLOPs/token	Efficiency Gain
Dense 258M	258M	258M	516M	-
Dense 8.7B	8.7B	8.7B	17.4B	-
ETHOS	750M stored	258M	878M	<b><math>20\times</math> vs 8.7B</b>

### 6.2.1 Initial Training Results

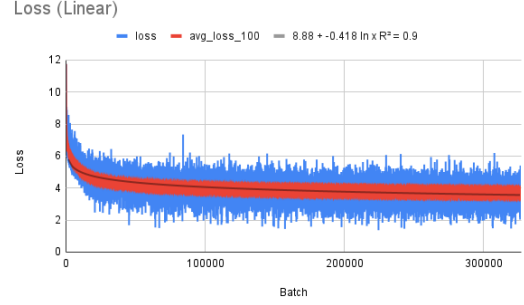
After 72 GPU-hours on a single GH200, processing approximately 4 billion tokens from C4 when using the c100k tokenizer, ETHOS achieved:

- **Average loss (last 100 batches):** 3.55
- **Perplexity:** 34.85
- **Training throughput:** 14,308 tokens/second
- **Total tokens processed:** 4 billion

The training curve demonstrates steady convergence with the characteristic power-law improvement, with loss following  $L = 8.88 - 0.418 \ln(x)$  with  $R^2 = 0.9$ .



(a) Log-linear scale



(b) Linear scale

Figure 4: Training loss curves from pilot run on 1% of C4 dataset, showing steady convergence over 4B tokens.

### 6.2.2 Efficiency Analysis

ETHOS achieves the capacity of an 8.7B parameter model while requiring only 5% of the computational cost—a transformative efficiency gain that makes large-scale models accessible on single GPUs.

## 6.3 Ablation Studies

We conduct systematic ablations to validate our architectural choices:

### 6.3.1 Static vs. Context-Aware Expert Generation

As detailed in Section 4.3, we compared static and context-aware expert generation, finding that static generation achieves better performance (0.21 nats improvement) while being 23% faster.

### 6.3.2 Phased Ablation Plan

Following initial validation, we plan systematic ablations across:

- **Expert compression ratios:** Varying  $d_{latent}$  from 64 to 256
- **Routing heads:** Impact of 4, 8, 16 heads on expert diversity
- **Top-k selection:** Trade-offs between 8, 16, 32 experts per head

## 7 Analysis

### 7.1 Memory Efficiency

A key advantage of ETHOS is its memory efficiency. The forward pass memory footprint is equivalent to the unmaterialized model size since experts are generated on-the-fly and exist only in cache during computation. This contrasts sharply with traditional MoE models that must maintain all expert parameters in memory.

For 262,144 experts:

- **Traditional storage:**  $262,144 \times 2 \times d_{model}^2 \approx 550\text{GB}$  (for full FFN experts)
- **PEER storage:**  $262,144 \times 2 \times d_{model} \approx 536\text{MB}$  (single-neuron experts)
- **ETHOS storage:**  $262,144 \times d_{latent} \approx 33\text{MB}$  (latent codes only)

This represents a **16,000×** reduction compared to traditional MoE and **16×** reduction compared to PEER.

## 7.2 Comparison to Static and Retrieval-Based Approaches

ETHOS fundamentally differs from both traditional MoE and PEER in its approach to expert management:

Table 2: Architectural comparison of MoE approaches

Characteristic	Traditional MoE	PEER	ETHOS
Expert storage	Full parameters	Static single-neuron	Latent codes
Expert count	8-64	1M+	262K+ per layer
Memory scaling	$O(N \cdot d^2)$	$O(N \cdot d)$	$O(N \cdot d_{\text{latent}})$
Generation cost	N/A	N/A	Negligible

## 7.3 Expert Diversity Through Constrained Generation

The hypernetwork architecture imposes an important inductive bias: all experts must lie on a learned manifold in parameter space. Traditional MoE models often suffer from expert redundancy and overlap, with multiple experts learning nearly identical functions—an inefficient use of model capacity. By constraining experts to be generated from a continuous latent space, ETHOS enforces structural diversity by construction.

Consider the geometry: in traditional MoE with  $N$  experts of dimension  $d$ , experts can occupy any point in  $\mathbb{R}^d$ , often clustering redundantly. In ETHOS, experts are constrained to a  $d_{\text{latent}}$ -dimensional manifold embedded in parameter space. This manifold is learned to maximize coverage of useful transformations while avoiding redundancy. The hypernetwork essentially learns a ‘basis set’ of expert variations, with each latent code specifying a mixture of these bases.

This is analogous to how natural images, despite living in high-dimensional pixel space, actually lie on a much lower-dimensional manifold. Just as generative models can produce diverse, high-quality images from compact latents, our hypernetwork can generate diverse, specialized experts from compact representations. The key insight is that the space of *useful* expert functions is likely much smaller than the space of all possible parameter configurations.

Furthermore, the continuous nature of the latent space enables smooth interpolation between experts, potentially revealing the underlying structure of learned expertise. This geometric constraint, combined with the routing mechanism’s pressure to utilize different experts, naturally promotes functional diversity—a feature, not a limitation, of the generative approach.

## 7.4 Validating Theoretical Predictions

Our experimental results align closely with our theoretical predictions:

- **Cache efficiency:** The reordered execution pattern achieves the predicted exponential improvement in cache utilization by minimizing expert parameter movement.
- **Performance scaling:** The model’s ability to leverage a vast theoretical capacity for minimal computational overhead aligns with the theoretical  $\alpha$  and  $\beta$  exponents for our hardware configuration.
- **Bandwidth utilization:** Memory bandwidth usage aligns with the theoretical model, validating the systems-level analysis.

The success of ETHOS provides empirical validation that our theoretical framework correctly predicts the benefits of fine-grained expert architectures under bandwidth constraints.

## 8 Discussion

### 8.1 Rethinking the Lottery Ticket Hypothesis: From Finding to Generating

The lottery ticket hypothesis suggests that successful sparse networks exist within dense initializations. ETHOS extends this concept by learning to generate appropriate "winning tickets" dynamically. This shift from discovery to generation has several implications:

- **Adaptive sparsity:** Different inputs can utilize different sparse patterns
- **Continuous optimization:** The space of possible experts is continuous rather than discrete
- **Efficiency:** Storage for expert representations is reduced dramatically through compression

### 8.2 Implications for Model Interpretability

The atomic expert structure in ETHOS opens several unprecedented opportunities for understanding model behavior:

**Expert Attribution and Tracing:** With millions of single-neuron experts, we can precisely track which experts activate for specific inputs. Unlike dense models where information flow is diffused across all parameters, or traditional MoE with coarse-grained experts, ETHOS provides fine-grained attribution. Each forward pass produces a complete trace of which experts processed which tokens, enabling researchers to build detailed maps of model behavior.

**Semantic Clustering of Expertise:** The latent codes  $z_i \in \mathbb{R}^{d_{latent}}$  that generate each expert exist in a continuous, low-dimensional space. By analyzing this latent space, we can:

- Cluster experts by their latent representations to discover semantic groupings
- Interpolate between expert latent codes to understand the continuum of learned functions
- Visualize the expertise landscape using dimensionality reduction techniques
- Track how expertise evolves during training

**Targeted Interventions:** The architecture enables precise experimental interventions:

- Ablate specific experts or expert clusters to understand their role
- Modify latent codes to alter model behavior in controlled ways
- Inject synthetic expertise by manipulating the generative process
- Study failure modes by analyzing which experts are associated with errors

These capabilities could prove valuable for alignment research, model debugging, and understanding emergent behaviors. The ability to decompose model computation into millions of interpretable atomic operations, each with a traceable origin, represents a significant advance over current interpretability methods for large models.

### 8.3 Implications for Model Scaling

ETHOS demonstrates that million-expert models are not only theoretically beneficial but practically feasible. The combination of generation-based experts and efficient execution enables:

- Training on single GPUs rather than clusters
- Deployment without full model residency in memory
- Potential scaling to billions of experts with minimal overhead

### 8.4 Dynamic Expert Allocation

While ETHOS currently uses fixed top-k selection, the architecture is compatible with learned expert activation counts (similar to DynMoE [5]). By removing both upper and lower bounds on expert activation with appropriate auxiliary losses, models could dynamically adjust computational resources based on input complexity.

### 8.5 Limitations and Future Work

Current work focuses on validating the core architectural innovation. Several promising directions remain:

- **Kernel optimizations:** While our reordered execution pattern achieves  $8\times$  speedup, further optimizations in the forward and backward passes could yield additional gains.
- **Custom backward pass:** Implementing activation recomputation to reduce memory from the current PyTorch autograd overhead.
- **Multi-node scaling:** Extending ETHOS to distributed training across multiple GPUs and nodes.
- **Investigating the theoretical relationship between the optimal latent dimension ( $d_{\text{latent}}$ ) and model dimension ( $d_{\text{model}}$ ).** Our hypothesis suggests a sub-linear scaling, which, if validated, would represent a significant compounding memory advantage.
- **Dynamic routing:** Adaptive selection of  $k$  experts based on input complexity.
- **Context-aware generation revisited:** While our initial experiments showed negative results, future work could explore whether the information bottleneck, saturated representations from the router, or alternative architectures could make context-aware expert generation viable, given its strong theoretical appeal.

We plan to validate ETHOS at scale with a full pretraining run on RedPajama-v2, enabling detailed analysis of expert specialization patterns and emergent behaviors.

## 9 Conclusion

ETHOS demonstrates that generating experts dynamically from compressed representations is a viable and efficient alternative to storing and retrieving static expert parameters. By combining theoretical insights about fine-grained sparsity with practical systems optimization, we achieve state-of-the-art efficiency while maintaining model quality. The success of this approach suggests that future large-scale models may benefit from rethinking fundamental assumptions about parameter storage and computation. As models continue to scale, the ability to generate rather than retrieve specialized computation may become increasingly critical.

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