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Kinematics and dynamics of 2(3-RPS) manipulators by means of screw theory and the principle of virtual work

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Abstract

In this contribution the kinematic and dynamic analyses, up to the determination of the driving forces, of a specific class of series–parallel manipulators, known as 2(3-RPS) manipulators, are approached via the theory of screws and the principle of virtual work. The generosity of screw theory, and the simplicity of the principle of virtual work, allows to obtain systematically such fundamental analyses. As an initial step, the forward position analysis of the hybrid mechanism is presented in analytical form solution, thus all the feasible solutions, at most 256, can be easily obtained. Afterwards, the velocity and acceleration analyses, as well as the dynamic analysis, are simplified considerably applying the properties of reciprocal screws.

Keywords: Series–parallel; Driving forces; Principle of virtual work; Screw theory; Kinematics and dynamics

1. Introduction

Due to their compact topology, parallel manipulators are more stiff and accurate than their serial counterparts. Owing these merits, over the last decades a growing interest in the development of parallel kinematic machines (PKMs), with heavy payload capacities, have been reported day to day in the literature. Certainly, among flight simulators, which seems to be the first transcendental application [1], parallel manipulators have found interesting applications such as walking machines, pointing devices, machine tools, micro manipulators, and so on. In that way, the Delta robot, invented by Clavel [2,3], proved that parallel robotic manipulators are an excellent option for industrial applications where the accuracy and stiffness are mandatory characteristics. Consider for instance that the Adept Quattro robot, developed by Francois Pierrot in collaboration with

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Fatronik (Int. patent appl. WO/2006/087399), has a 2.0 kg payload capacity and can execute 4 cycles/s. The Adept Quattro robot is considered at this moment the industry's fastest pick-and-place robot. After the success of the Delta robot, specially in packaging operations, PKMs claim to be the next generation of machine tools, a new industrial revolution is coming [4–6]. In the web site http://www.parallemic.org/ an exhaustive list of patents dealing with practical applications, ranging from positioning robotic systems to parallel kinematic machine tools, of parallel mechanisms can be found.

Several PKMs have been constructed using the hexapod as a base mechanism: Octahedral Hexapod HOH-600 (Ingersoll), HEXAPODE CMW 300 (CMW), Cosmo Center PM-600 (Okuma), F-200i (FANUC) and so on. However, one cannot ignore that this kind of PKMs have a more limited and complex-shaped workspace. Furthermore, their rotation and position capabilities are highly coupled and therefore the control and calibration of it are rather complicated. In order to ameliorate such drawbacks, the motion of the moving platform, with respect to the fixed platform, can be decomposed into decoupled motions, this option was investigated in Hunt and Primrose [7] for in-fully parallel manipulators and was applied by Zlatanov et al. [8] in the design of a 6 dof three-legged parallel manipulator. Furthermore, taking into account that the forward position analysis of parallel manipulators with decoupled motions can be easily derived in closed-form, Gallardo-Alvarado et al. [9] proposed a family of non-overconstrained redundantly actuated parallel manipulators. Of course, this option is also applicable to the so-called defective parallel manipulators, see for instance [10–12]; in other words, spatial parallel manipulators with fewer than six degrees of freedom. Alternatively, the manipulability and workspace can be increased by connecting a serial manipulator to the moving platform of a parallel manipulator, a natural an evident possibility. For example, a spherical wrist can be attached at the moving platform of a Tricept yielding a six degrees of freedom spatial mechanism [13], other combinations were reported in [14,15]. Furthermore, if the spherical wrist is an open chain, then, in order to improve the stiffness, the serial manipulator can be remplazed by a parallel manipulator, producing a series–parallel manipulator, see for instance [16–19].

In this work, the kinematic and dynamic analyses of a class of series–parallel manipulators, term proposed in Zoppi et al. [20], known as 2(3-RPS) manipulators (R, P, and S stand for revolute, prismatic and spherical, respectively) are approached by means of the theory of screws and the principle of virtual work. First, the forward position analysis (FPA for brevity) is carried-out using the Sylvester dyalitic elimination method which allows to compute all the feasible solutions of this initial analysis. Later, the kinematics and dynamics of the series–parallel manipulator are solved via the theory of screws and the principle of virtual work. Simple and compact expressions for solving the driving forces of the spatial mechanism are obtained by taking advantage of the Klein form of the Lie algebra $e(3)$. Finally, a numerical example is provided.

2. Description of the series–parallel manipulator considered in this study

The 3-RPS parallel manipulator was introduced by Hunt [21] and consists of a moving platform and a fixed platform connected each other by means of three kinematic subchains type RPS. The limbs are connected at the fixed platform by means of three distinct revolute joints, while the moving platform is connected at the limbs by means of three distinct spherical joints. According to the Kutzbach–Grübler criterion, this spatial mechanism has three degrees of freedom, two rotations and one translation, where usually the prismatic pairs are chosen as the active joints.

In Lu and Leinonen [22] the position analysis of a mechanism composed of two 3-RPS parallel manipulators assembled in tandem, see Fig. 1, called the spatial 2(3-RPS) manipulator is investigated. As it is shown in Fig. 1, this series–parallel manipulator consists of a lower parallel manipulator (LPM), and an upper parallel manipulator (UPM), both spatial mechanisms type 3-RPS. The computation of generalized forces of a similar mechanism, called 2(3-SPR) manipulator, was approached in Lu and Hu [19] applying the principle of virtual work and the so-called CAD variation geometry. In both contributions the middle platform requires of six distinct points for connecting it at the output platform and the fixed platform by means of six extendible rods. It is straightforward to demonstrate that, with such architectures the middle platform can undergo undesirable deflections affecting the accuracy of the manipulator. Furthermore, the presence of bending moments over the kinematic pairs can be a serious problem. In order to overcome, or at least to diminish, such drawbacks, in this

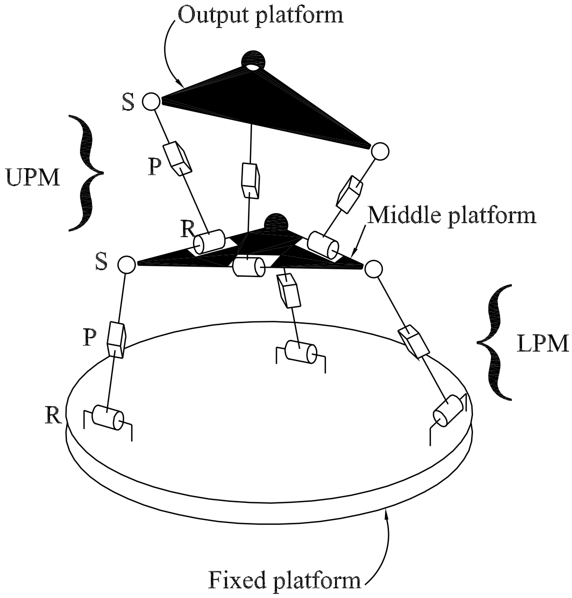

Fig. 1. Conventional 2(3-RPS) manipulator.

contribution a series–parallel manipulator with a more compact topology, see Fig. 2, than the proposed in [19,22] is investigated.

As it is shown in Fig. 2, the series–parallel manipulator proposed in this work consists of a LPM and an UPM, both with architectures type 3-RPS. Unlike the traditional topology reported in the literature, this manipulator has the advantage that revolute and spherical joints, attached at the middle platform, are conveniently transformed into revolute + spherical joints in such form, that the axes of both kinematic pairs are concurrent at the center of the corresponding spherical joint. Clearly, this arrangement is more compact and does not affect the performance of the series–parallel manipulator. The advantages of this novel mechanism emerge immediately.

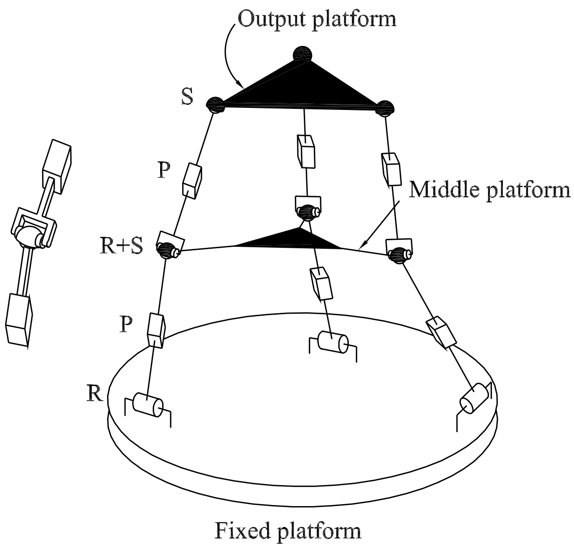

Fig. 2. 2(3-RPS) Manipulator with simplified topology.

3. Finite kinematics of the series–parallel manipulator

In this section the forward position analysis of the proposed manipulator is carried out in analytic form using simple geometric procedures, for a detailed explanation of the FPA of a 3-RPS parallel manipulator, which has a direct connection with this section, the reader is referred to [23–25].

When the six prismatic joints are locked, the series–parallel manipulator, here after SPM for brevity, becomes into the structure shown in Fig. 3.

Under this consideration, the FPA of the SPM is established as follows. Given the limb lengths $q_i i \in \{1,2,3,4,5,6\}$, compute the feasible locations of the output platform, with respect to the fixed platform, through the computation of the coordinates of the centers of the spherical joints attached at the output platform, points $\mathbf{A}_i'' i \in \{1,2,3\}$, and expressed in the reference frame XYZ.

Firstly the FPA of the LPM is formulated. From the geometry of the mechanism, see Fig. 3, the closure equations of the LPM are given by

$$\left.\begin{aligned}
&(\mathbf{A}_i' - \mathbf{A}_i)\cdot \hat{u}_i = 0, \quad i \in \{1,2,3\},\\
&(\mathbf{A}_i' - \mathbf{A}_i)\cdot(\mathbf{A}_i' - \mathbf{A}_i) = q_i^2, \quad i \in \{1,2,3\},\\
&(\mathbf{A}_2' - \mathbf{A}_3')\cdot(\mathbf{A}_2' - \mathbf{A}_3') = b_{23}'^2,\\
&(\mathbf{A}_1' - \mathbf{A}_3')\cdot(\mathbf{A}_1' - \mathbf{A}_3') = b_{13}'^2,\\
&(\mathbf{A}_1' - \mathbf{A}_2')\cdot(\mathbf{A}_1' - \mathbf{A}_2') = b_{12}'^2,
\end{aligned}\right\} \tag{1}$$

where $\hat{u}_i = (u_{Xi}, 0, u_{Zi})$ is the ith unit vector along the screw axis of the ith revolute joint, and $\mathbf{A}_i i \in \{1,2,3\}$ are the nominal coordinates of the three distinct revolute joints attached at the fixed platform. Expression (1) represents a system of nine equations in the unknowns $X_i, Y_i, Z_i i \in \{1,2,3\}$.

After a few computations, a higher non-linear system of three equations in the unknowns Z_1, Z_2 and Z_3, is obtained as follows:

$$\left.\begin{aligned}
&K_1'Z_2^2 + K_2'Z_3^2 + K_3'Z_2^2Z_3 + K_4'Z_2Z_3^2 + K_5'Z_2Z_3 + K_6'Z_2 + K_7'Z_3 + K_8' = 0,\\
&K_1''Z_1^2 + K_2''Z_3^2 + K_3''Z_1^2Z_3 + K_4''Z_1Z_3^2 + K_5''Z_2Z_3 + K_6''Z_1 + K_7''Z_3 + K_8'' = 0,\\
&K_1'''Z_1^2 + K_2'''Z_2^2 + K_3'''Z_1^2Z_2 + K_4'''Z_1Z_2^2 + K_5'''Z_1Z_2 + K_6'''Z_1 + K_7'''Z_2 + K_8''' = 0,
\end{aligned}\right\} \tag{2}$$

where K_*', K_*'', and K_*''' are coefficients that are calculated according to the parameters and generalized coordinates of the LPM. In which follows the non-linear system of equation (2) is solved applying recursively the Sylvester dyalitic elimination method.

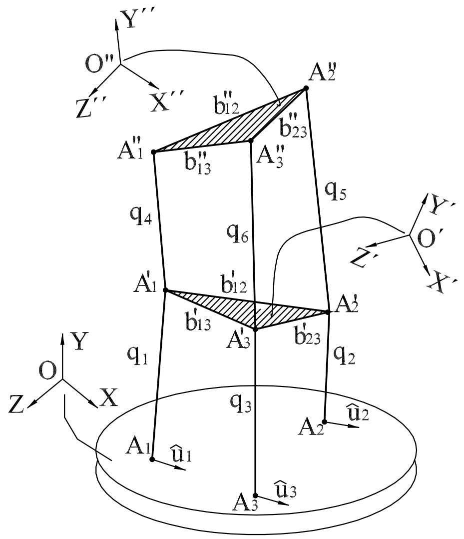

Fig. 3. 2(3RS) Structure.

Cancelation of Z_3. The corresponding eliminant is obtained from the following expression:

$$M_1 \begin{bmatrix} Z_3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{3}$$

where

$$M_1 = \begin{bmatrix} p_1p_5 - p_2p_4 & p_1p_6 - p_3p_4 \\ p_3p_4 - p_1p_6 & p_3p_5 - p_2p_6 \end{bmatrix}$$

and $p_i \in \{1,2,3\}$ are second-order polynomials in Z_2 while $p_i \in \{4,5,6\}$ are second-order polynomials in Z_1. Thus this eliminant is obtained taking into account that $\det M_1 = 0$, canceling the variable Z_3. Indeed

$$\det M_1 = p_7Z_2^4 + p_8Z_2^3 + p_9Z_2^2 + p_{10}Z_2 + p_{11} = 0, \tag{4}$$

where $p_i i \in \{7,8,9,10,11\}$ are fourth-order polynomials in Z_1.

Cancelation of Z_2. The corresponding eliminant is obtained considering the unknowns Z_2^3, Z_2^2, Z_2, and 1, yielding

$$M_2 \begin{bmatrix} Z_2^3 \\ Z_2^2 \\ Z_2^1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{5}$$

where M_2 is a square matrix given by

$$M_2 = \begin{bmatrix} 0 & p_{12} & p_{13} & p_{14} \\ p_{13}p_7 - p_{12}p_8 & p_{14}p_7 - p_{12}p_9 & -p_{12}p_{10} & -p_{12}p_{11} \\ p_{12} & p_{13} & p_{14} & 0 \\ p_{12}p_9 - p_7p_{14} & p_{12}p_{10} + p_{13}p_9 - p_8p_{14} & p_{12}p_{11} + p_{13}p_{10} & p_{13}p_{11} \end{bmatrix}$$

and $p_i i \in \{12,13,14\}$ are second-order polynomials in Z_1. Finally, a 16th-order polynomial expression in the unknown Z_1 is obtained taking into account that in order to avoid arbitrary solutions necessarily $\det M_2 = 0$.

Once the coordinates of the points $\mathbf{A}'_1, \mathbf{A}'_2$, and $\mathbf{A}'_3$ are calculated,[1] the geometric center of the moving platform expressed in the reference frame XYZ, vector ${}^0\boldsymbol{\rho}^1 = (\rho_X, \rho_Y, \rho_Z)$, results in

$${}^0\boldsymbol{\rho}^1 = (\mathbf{A}'_1 + \mathbf{A}'_2 + \mathbf{A}'_3)/3. \tag{6}$$

Furthermore, since the coordinates of the points $\mathbf{A}'_i i \in \{1,2,3\}$ are easily expressed in the reference frame $X'Y'Z'$. The corresponding 4×4 homogeneous transformation matrix[2] between the reference frames $X'Y'Z'$ and XYZ, ${}^0T^1$, results in

$${}^0T^1 = \begin{bmatrix} {}^0R^1 & {}^0\boldsymbol{\rho}^1 \\ 0_{1\times3} & 1 \end{bmatrix}, \tag{7}$$

where ${}^0R^1$ is the rotation matrix.

Following a similar procedure the transformation matrix between the reference frames $X''Y''Z''$ and $X'Y'Z'$, ${}^1T^2$, is obtained by analyzing the UPM. After, clearly the transformation matrix between the reference frames $X''Y''Z''$ and XYZ, ${}^0T^2$, results in

$${}^0T^2 = {}^0T^{11}T^2. \tag{8}$$

[1] It is straightforward to show that once the feasible values for Z_1 are calculated, the remaining components of the coordinates of the points $\mathbf{A}'_i$ are easily obtained from (1).

[2] It is well known that rigid bodies configurations can be considered as isometric transformations on the three dimensional Euclidean space with respect to a reference frame.

Finally, the coordinates of the centers of the spherical joints attached at the output platform, with respect to the fixed platform, are directly calculated from the expression

$$\begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix} = {}^0T^2 \begin{bmatrix} X''_i \\ Y''_i \\ Z''_i \\ 1 \end{bmatrix} \quad i \in \{1,2,3\}, \tag{9}$$

where (X_i, Y_i, Z_i) and (X''_i, Y''_i, Z''_i) are the coordinates of the centers of the spherical joints according to reference frames attached, respectively, at the fixed platform and the output platform.

4. Infinitesimal kinematics of the series–parallel manipulator

The mathematical resource employed in this contribution for solving the infinitesimal kinematics of the SPM under study is the theory of screws.

Screw theory has been proved to be, since more than two decades ago, an efficient tool for solving the first-order analysis of closed chains. In fact, Mohamed and Duffy [26] demonstrated that the twist, or velocity state, representing the instantaneous motion of the moving platform of a parallel manipulator can be expressed as the sum of the partial screws of the manipulator. Agrawal [27] applied the theory of screws with the purpose to identify the active and passive joints of parallel manipulators undergoing infinitesimal displacements. Zlatanov et al. [28] introduced the concept of constraint singularity which describes the lost of freedoms of the parallel manipulator produced when the screw system loses rank. Joshi and Tsai [29] reported a method, based on the theory of screws, for solving the kinematic analysis of parallel manipulators with fewer than six degrees of freedom. Rico and Dufy [30] demonstrated how easy is to solve the forward kinematics of parallel manipulators when the concept of reciprocal screw is employed; the cancelation of passive joints is the main benefit of that methodology and has been reconsidered in [10,11,18,25] and more recent in Zoppi et al.[20]. Rico and Duffy [31] extended the theory of screws from the velocity analysis to the elusive, in screw form, acceleration analysis of open serial and closed chains. Following the trend introduced in [31], Rico et al. [32] approached the higher-order analyses of parallel manipulators. The jerk analysis of closed chains [33], particularly in Gough–Stewart platforms [34], is a proof that how easy is to solve the higher-order kinematic analyses of parallel manipulators when the theory of screws is employed. Furthermore, as it is shown in [35,36], using screw theory, there are no limits in the required order of analysis.

4.1. Velocity analysis

The Lie algebra $e(3)$ of the Euclidean group $E(3)$ is the mathematical resource required for solving the infinitesimal kinematics of the series–parallel manipulator under study. Taking into account that a RPS-type limb is modeled with only five screws, then in order to satisfy the rank of the Jacobian matrix of each limb, it is necessary to consider an additional, and maybe fictitious, screw. This requirement is satisfied by changing the revolute joint by a cylindrical joint in which the translational displacement is null, see Fig. 4. Clearly, this consideration does not affect the performance of the manipulator and allows to solve systematically the inverse kinematics.

With reference to Fig. 4, the velocity state ${}^0\mathbf{V}^1_O = [{}^0\omega^{1\,0}\mathbf{v}^1_O]^{\mathrm{T}}$ of the middle platform, body 1, with respect to the fixed platform, body 0, can be obtained through any of the limbs of the LPM as follows:

$${}_0\omega^i_1\,{}^0\$^1_i + {}_1\omega^i_2\,{}^1\$^2_i + {}_2\omega^i_3\,{}^2\$^3_i + {}_3\omega^i_4\,{}^3\$^4_i + {}_4\omega^i_5\,{}^4\$^5_i + {}_5\omega^i_6\,{}^5\$^6_i = {}^0\mathbf{V}^1_O \quad i \in \{1,2,3\}, \tag{10}$$

where ${}^0\boldsymbol{\omega}^1$ is the angular velocity of the middle platform, while ${}^0\mathbf{v}^1_O$ is the translational velocity of a point O, fixed at the middle platform, which is instantaneously coincident with a point fixed to the reference frame XYZ. Furthermore, ${}_2\omega^i_3 = \dot{q}_i$ is the joint velocity rate of the ith actuated prismatic joint, while ${}_0\omega^i_1 = 0$ is the joint velocity rate of the ith prismatic joint associated to the corresponding cylindrical joint, both in the same limb.

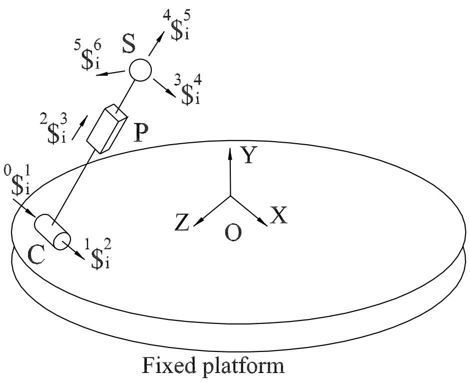

Fig. 4. A limb of the LPM with its infinitesimal screws.

According to expression (10), it follows that the inverse velocity analysis of the LPM is calculated from the expression

$$\Omega_i = J_i^{-1}\,{}^0\mathbf{V}_O^1, \tag{11}$$

wherein

$J_i = [\,{}^0\$_i^1 \quad {}^1\$_i^2 \quad {}^2\$_i^3 \quad {}^3\$_i^4 \quad {}^4\$_i^5 \quad {}^5\$_i^6\,]$ is the ith Jacobian matrix of the ith limb,
$\Omega_i = [\,{}_0\omega_1^i \quad {}_1\omega_2^i \quad {}_2\omega_3^i \quad {}_3\omega_4^i \quad {}_4\omega_5^i \quad {}_5\omega_6^i\,]^{\mathrm{T}}$ is the ith matrix of joint velocity rates of the ith limb.

Similarly, the inverse velocity analysis of the UPM is formulated.

On the other hand, in what follows the forward velocity analysis of the LPM is simplified using the concept of reciprocal screws, via the Klein form.

Given two elements $\$_1 = (\hat{s}_1, \mathbf{s}_{O1})$ and $\$_2 = (\hat{s}_2, \mathbf{s}_{O2})$ of the Lie algebra $e(3)$, the Klein form, $\{*;*\}$, is defined as follows:

$$\{\$_1;\$_2\} = \hat{s}_1 \cdot \mathbf{s}_{O2} + \hat{s}_2 \cdot \mathbf{s}_{O1}.$$

Furthermore, it is said that the screws $\$_1$ and $\$_2$ are reciprocal if $\{\$_1;\$_2\} = 0$.

In order to eliminate the passive joint velocity rates of expression (10), please note that the screw ${}^4\$_i^5$ is reciprocal to all the screws associated to the revolute joints in the same limb. Thus applying the Klein form of this screw with both sides of (10), and taking into account that ${}_0\omega_1^i = 0$, the reduction of terms leads to

$$\{{}^0\mathbf{V}_O^1; {}^4\$_i^5\} = \dot{q}_i \quad i \in \{1,2,3\}. \tag{12}$$

A similar result is obtained when ${}^3\$_i^4$ is chosen as the *cancellator screw*. Indeed,

$$\{{}^0\mathbf{V}_O^1; {}^3\$_i^4\} = 0 \quad i \in \{1,2,3\}. \tag{13}$$

After casting Eqs. (12) and (13) into a matrix–vector form, the velocity state $\mathbf{V}_O = [{}^0\boldsymbol{\omega}^2 \quad {}^0\mathbf{v}_O^2]^{\mathrm{T}}$ can be calculated from

$$J_{\mathrm{LPM}}^{\mathrm{T}} \Delta\, {}^0\mathbf{V}_O^1 = \dot{Q}_{\mathrm{LPM}}, \tag{14}$$

wherein

$J_{\mathrm{LPM}} = [{}^4\$_1^5 \quad {}^4\$_2^5 \quad {}^4\$_3^5 \quad {}^3\$_1^4 \quad {}^3\$_2^4 \quad {}^3\$_3^4]$ is the active Jacobian matrix of the LPM,
$\dot{Q}_{\mathrm{LPM}} = [\dot{q}_1 \quad \dot{q}_2 \quad \dot{q}_3 \quad 0 \quad 0 \quad 0]^{\mathrm{T}}$ is the matrix of generalized joint velocity rates of the LPM,
$\Delta = \begin{bmatrix} 0 & I_3 \\ I_3 & 0 \end{bmatrix}$ is an operator of polarity defined by the identity matrix I_3.

This procedure is repeated for the UPM, and therefore the velocity state of the output platform, body 2, with respect to the middle platform, body 1, can be obtained from the expression

$$J_{\mathrm{UPM}}^{\mathrm{T}} \Delta {}^{1}\mathbf{V}_{O}^{2} = \dot{Q}_{\mathrm{UPM}}, \tag{15}$$

therein

J_{UPM} is the active Jacobian matrix of the UPM,
$\dot{Q}_{\mathrm{UPM}} = [\dot{q}_4 \quad \dot{q}_5 \quad \dot{q}_6 \quad 0 \quad 0 \quad 0]^{\mathrm{T}}$ is the matrix of generalized joint velocity rates of the UPM.

Finally, the velocity state of the output platform with respect to the fixed platform, six-dimensional vector ${}^{0}\mathbf{V}_{O}^{2}$, is calculated as follows:

$${}^{0}\mathbf{V}_{O}^{2} = {}^{0}\mathbf{V}_{O}^{1} + {}^{1}\mathbf{V}_{O}^{2}. \tag{16}$$

Furthermore, the angular velocity of the output platform, with respect to the fixed platform, is obtained as the primal part of the velocity state, ${}^{0}\boldsymbol{\omega}^{2} = P({}^{0}\mathbf{V}_{O}^{2})$, while the translational velocity of the center of the output platform with respect to the fixed platform, vector $\mathbf{v}_{C2}$, is calculated according to the condition of helicoidal fields [35,36]; using the dual part of the six-dimensional vector ${}^{0}\mathbf{V}_{O}^{2}, D({}^{0}\mathbf{V}_{O}^{2})$, as follows:

$$\mathbf{v}_{C2} = D({}^{0}\mathbf{V}_{O}^{2}) + P({}^{0}\mathbf{V}_{O}^{2}) \times {}^{0}\boldsymbol{\rho}^{2}, \tag{17}$$

where ${}^{0}\boldsymbol{\rho}^{2}$ is a vector from the origin of the reference frame XYZ to the center of the output platform.

4.2. Acceleration analysis

Let ${}^{0}\mathbf{A}_{O}^{1} = [{}^{0}\dot{\boldsymbol{\omega}}^{1} \quad {}^{0}\mathbf{a}_{O}^{1} - {}^{0}\boldsymbol{\omega}^{1} \times {}^{0}\mathbf{v}_{O}^{1}]^{\mathrm{T}}$ be the reduced acceleration state, also known as accelerator, of the middle platform with respect to the fixed platform, in which ${}^{0}\dot{\boldsymbol{\omega}}^{1}$ is the angular acceleration of the middle platform with respect to the fixed platform, whereas ${}^{0}\mathbf{a}_{O}^{1}$ is the translational acceleration of a point fixed at the middle platform that is instantaneously coincident with a point O fixed at the reference frame XYZ. Then, the accelerator ${}^{0}\mathbf{A}_{O}^{1}$ can be written in screw form, see Rico and Duffy [31], through any of the limbs of the LPM as follows:

$${}_{0}\dot{\omega}_{1}^{i}\,{}^{0}\$_{i}^{1} + {}_{1}\dot{\omega}_{2}^{i}\,{}^{1}\$_{i}^{2} + {}_{2}\dot{\omega}_{3}^{i}\,{}^{2}\$_{i}^{3} + {}_{3}\dot{\omega}_{4}^{i}\,{}^{3}\$_{i}^{4} + {}_{4}\dot{\omega}_{5}^{i}\,{}^{4}\$_{i}^{5} + {}_{5}\dot{\omega}_{6}^{i}\,{}^{5}\$_{i}^{6} + \$_{\mathrm{Lie}_i} = {}^{0}\mathbf{A}_{O}^{1} \quad i \in \{1,2,3\}, \tag{18}$$

where $\$_{\mathrm{Lie}_i}$ is the ith *Lie screw* given by

$$\$_{\mathrm{Lie}_i} = [{}_{0}\omega_{1}^{i}\,{}^{0}\$_{i}^{1} \quad {}_{1}\omega_{2}^{i}\,{}^{1}\$_{i}^{2} + \cdots + {}_{5}\omega_{6}^{i}\,{}^{5}\$_{i}^{6}] + [{}_{1}\omega_{2}^{i}\,{}^{1}\$_{i}^{2} \quad {}_{2}\omega_{3}^{i}\,{}^{2}\$_{i}^{3} + \cdots + {}_{5}\omega_{6}^{i}\,{}^{5}\$_{i}^{6}] + \cdots + [{}_{4}\omega_{5}^{i}\,{}^{4}\$_{i}^{5} \quad {}_{5}\omega_{6}^{i}\,{}^{5}\$_{i}^{6}],$$

in which the brackets $[* \quad *]$ denote the Lie product.

Given the accelerator ${}^{0}\mathbf{A}_{O}^{1}$, the inverse acceleration analysis of the LPM, or in other words the computation of the joint acceleration rates of the LPM, can be calculated from Eq. (18) as follows:

$$\dot{\Omega}_i = J_i^{-1}({}^{0}\mathbf{A}_{O}^{1} - \$_{\mathrm{Lie}_i}), \tag{19}$$

where

$$\dot{\Omega}_i = [{}_{0}\dot{\omega}_{1}^{i} \quad {}_{1}\dot{\omega}_{2}^{i} \quad {}_{2}\dot{\omega}_{3}^{i} \quad {}_{3}\dot{\omega}_{4}^{i} \quad {}_{4}\dot{\omega}_{5}^{i} \quad {}_{5}\dot{\omega}_{6}^{i}]^{\mathrm{T}}.$$

It is straightforward to show that a similar procedure can be used in order to solve the inverse acceleration analysis of the UPM, of course taking into proper account the different reference frames.

On the other hand, the forward acceleration analysis of the LPM, or in other words the computation of the reduced acceleration state or accelerator of the middle platform with respect to the fixed platform, for the given the joint accelerations $\{\ddot{q}_1, \ddot{q}_2, \ddot{q}_3\}$ of the actuated prismatic joints, are calculated following the trend indicated in Section 4.1 from

$$J_{\mathrm{LPM}}^{\mathrm{T}} \Delta {}^{0}\mathbf{A}_{O}^{1} = \ddot{Q}_{\mathrm{LPM}}, \tag{20}$$

where

$$\ddot{Q}_{\mathrm{LPM}} = \begin{bmatrix} \ddot{q}_1 + \{{}^4\$_1^5; \$_{\mathrm{Lie}_1}\} \\ \ddot{q}_2 + \{{}^4\$_2^5; \$_{\mathrm{Lie}_2}\} \\ \ddot{q}_3 + \{{}^4\$_3^5; \$_{\mathrm{Lie}_3}\} \\ \{{}^3\$_1^4; \$_{\mathrm{Lie}_1}\} \\ \{{}^3\$_2^4; \$_{\mathrm{Lie}_2}\} \\ \{{}^3\$_3^4; \$_{\mathrm{Lie}_3}\} \end{bmatrix}.$$

It is worth mentioning that Eq. (20) does not require the values of the passive joint acceleration rates of the LPM. This follows that the acceleration analysis used in this study is computationally efficient. Furthermore, it is straightforward to show that a similar procedure allows to compute the forward acceleration analysis of the UPM.

Afterwards, the accelerator of the output platform with respect to the fixed platform, vector ${}^0\mathbf{A}_O^2$, is calculated as follows:

$${}^0\mathbf{A}_O^2 = {}^0\mathbf{A}_O^1 + {}^1\mathbf{A}_O^2 + [{}^0\mathbf{V}_O^1 \quad {}^1\mathbf{V}_O^2]. \tag{21}$$

Finally, the angular acceleration of the output platform, with respect to the fixed platform, is obtained as the primal part of the accelerator ${}^0\mathbf{A}_O^2$, indeed ${}^0\dot{\boldsymbol{\omega}}^2 = P({}^0\mathbf{A}_O^2)$, while the translational acceleration of the center of the output platform, vector $\mathbf{a}_{C2}$, expressed in the reference frame XYZ using classical kinematics results in

$$\mathbf{a}_{C2} = {}^0\mathbf{a}_O^2 + {}^0\dot{\boldsymbol{\omega}}^2 \times {}^0\boldsymbol{\rho}^2 + {}^0\boldsymbol{\omega}^2 \times ({}^0\boldsymbol{\omega}^2 \times {}^0\boldsymbol{\rho}^2), \tag{22}$$

where the translational acceleration ${}^0\mathbf{a}_O^2$ is calculated from the dual part of the accelerator, $D({}^0\mathbf{A}_O^2)$, as follows:

$${}^0\mathbf{a}_O^2 = D({}^0\mathbf{A}_O^2) + {}^0\boldsymbol{\omega}^2 \times {}^0\mathbf{v}_O^2.$$

5. Driving forces

In this section the principle of virtual work and the theory of screws are used for computing the generalized forces of the series–parallel manipulator, for a detailed explanation of this systematic method, the reader is referred to [37].

Fig. 5 shows a closed chain in which an arbitrary body n of mass m_n and centroidal inertia matrix I_n, is under the action of gravitational and inertial forces. In addition, the body n is supporting an external force

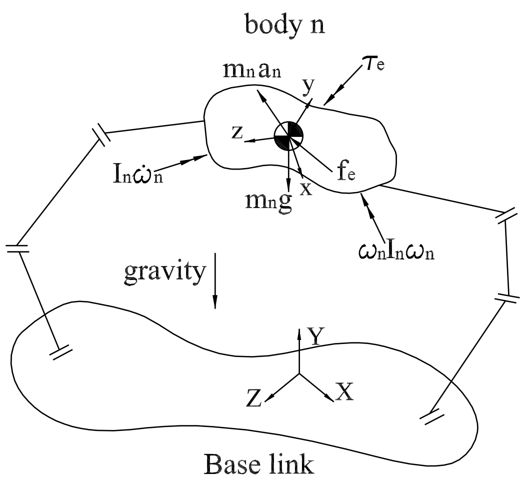

Fig. 5. Closed chain under the action of inertial, gravitational and external forces.

f_e and an external torque $\boldsymbol{\tau}_e$. Naturally, the body n could be the moving platform of a parallel manipulator or a simple link.

Assuming that the velocity state $\mathbf{V}_n = [\boldsymbol{\omega}_n \quad \mathbf{v}_n]^{\mathrm{T}}$ and the reduced acceleration state $\mathbf{A}_n = [\dot{\boldsymbol{\omega}}_n \quad \mathbf{a}_n - \boldsymbol{\omega}_n \times \mathbf{v}_n]^{\mathrm{T}}$ describe the motion of rigid body n taking its mass center as representation point, in other words, $\mathbf{v}_n$ and $\mathbf{a}_n$ are, respectively, the translational velocity and acceleration of the mass center. Then, the overall wrench acting on body $n, \mathbf{F}_n = [\mathbf{f}_n \quad \boldsymbol{\tau}_n]^{\mathrm{T}}$, is given by

$$\mathbf{F}_n = \mathbf{F}_{\text{inertial}} + \mathbf{F}_{\text{grav}} + \mathbf{F}_{\text{external}}, \tag{23}$$

where $\mathbf{F}_{\text{inertial}} = \begin{bmatrix} -m_n\mathbf{a}_n \\ -I_n\dot{\boldsymbol{\omega}}_n - \boldsymbol{\omega}_n \times I_n\boldsymbol{\omega}_n \end{bmatrix}$ is the inertial wrench according with D'Alembert's principle. In this wrench, the centroidal inertia matrix I_n can be calculated as follows:

$$I_n = RI_{xyz}(R)^{\mathrm{T}},$$

where I_{xyz} is the centroidal inertia matrix of body n expressed in the reference frame xyz and R is the rotation matrix between the frames xyz and XYZ, $\mathbf{F}_{\text{grav}} = \begin{bmatrix} m_n\mathbf{g} \\ \mathbf{0} \end{bmatrix}$ is the gravity wrench, $\mathbf{F}_{\text{external}} = \begin{bmatrix} \mathbf{f}_e \\ \boldsymbol{\tau}_e \end{bmatrix}$ is the external wrench.

Under this considerations, the analysis is formulated as follows: Given the inertial, gravitational and external wrenches; compute the required driving forces for executing the chosen velocity and reduced acceleration states for the arbitrary body n. Furthermore, taking into account that the kinematic properties of the rigid body depend on the generalized coordinates, and their time derivatives, then the kinematic terms associated to the actuation wrenches (generalized forces) are present, in implicit form, in the velocity and reduced acceleration states.

If the closed chain undergoes virtual velocities $\delta\dot{q}$, then the virtual power δw_n produced by the resulting wrench $\mathbf{F}_n$, with $\mathbf{f}_n = f_{nx}\hat{i} + f_{ny}\hat{j} + f_{nz}\hat{k}$, acting on body n on a generic motion $\mathbf{V}_n = [\boldsymbol{\omega}_n \quad \mathbf{v}_n]^{\mathrm{T}}$, can be calculated upon the Klein form as follows:

$$\delta w_n = \{\mathbf{F}_n; \mathbf{V}_n\}. \tag{24}$$

The principle of virtual work states that if a closed chain is in equilibrium under the effect of external actions, then the global work produced by the external forces with any virtual velocity must be null. Furthermore, taking into account that the vectors $\boldsymbol{\omega}_n$ and $\mathbf{v}_n$ can be expressed in terms of first-order influence coefficients, see Gallardo and Rico [33], as follows:

$$\left.\begin{aligned} \boldsymbol{\omega}_n &= (G\omega x_1\delta\dot{q}_1 + G\omega x_2\delta\dot{q}_2 + \cdots + G\omega x_{\text{dof}}\delta\dot{q}_{\text{dof}})\hat{i} \\ &\quad + (G\omega y_1\delta\dot{q}_1 + G\omega y_2\delta\dot{q}_2 + \cdots + G\omega y_{\text{dof}}\delta\dot{q}_{\text{dof}})\hat{j} \\ &\quad + (G\omega z_1\delta\dot{q}_1 + G\omega z_2\delta\dot{q}_2 + \cdots + G\omega z_{\text{dof}}\delta\dot{q}_{\text{dof}})\hat{k}, \\ \mathbf{v}_n &= (Gvx_1\delta\dot{q}_1 + Gvx_2\delta\dot{q}_2 + \cdots + Gvx_{\text{dof}}\delta\dot{q}_{\text{dof}})\hat{i} \\ &\quad + (Gvy_1\delta\dot{q}_1 + Gvy_2\delta\dot{q}_2 + \cdots + Gvy_{\text{dof}}\delta\dot{q}_{\text{dof}})\hat{j} \\ &\quad + (Gvz_1\delta\dot{q}_1 + Gvz_2\delta\dot{q}_2 + \cdots + Gvz_{\text{dof}}\delta\dot{q}_{\text{dof}})\hat{k}, \end{aligned}\right\}$$

where dof is the number of freedoms of the closed chain. Then Eq. (24) leads to

$$\begin{aligned} &(Gvx_1f_{nx} + Gvy_1f_{ny} + Gvz_1f_{nz} + G\omega x_1\tau_{nx} + G\omega y_1\tau_{ny} + G\omega z_1\tau_{nz} + \xi_1)\delta\dot{q}_1 + (Gvx_2f_{nx} + Gvy_2f_{ny} \\ &\quad + Gvz_2f_{nz} + G\omega x_2\tau_{nx} + G\omega y_2\tau_{ny} + G\omega z_2\tau_{nz} + \xi_2)\delta\dot{q}_2 + \cdots + (Gvx_{\text{dof}}f_{nx} + Gvy_{\text{dof}}f_{ny} + Gvz_{\text{dof}}f_{nz} \\ &\quad + G\omega x_{\text{dof}}\tau_{nx} + G\omega y_{\text{dof}}\tau_{ny} + G\omega z_{\text{dof}}\tau_{nz} + \xi_{\text{dof}})\delta\dot{q}_{\text{dof}} \\ &\quad = 0, \end{aligned} \tag{25}$$

therein $\xi_i i \in \{1, 2, \ldots, \text{dof}\}$ is the ith generalized force also known as the driver force.

Table 1
Parameters of the numerical example

$\mathbf{A}_1 = (.353, 0, .353),\ \mathbf{A}_2 = (.129, 0, -.482),\ \mathbf{A}_3 = (-.482, 0, .129)$ frame XYZ
$\mathbf{A}'_1 = (.353, 0, .353),\ \mathbf{A}'_2 = (.129, 0, -.482),\ \mathbf{A}'_3 = (-.482, 0, .129)$ frame $X'Y'Z'$
$\mathbf{A}''_1 = (.353, 0, .353),\ \mathbf{A}''_2 = (.129, 0, -.482),\ \mathbf{A}''_3 = (-.482, 0, .129)$ frame $X''Y''Z''$
$\hat{u}_1 = (.7071, 0, -.7071),\ \hat{u}_2 = (-.965, 0, -.258),\ \hat{u}_3 = (.258, 0, .965)$ frame XYZ
$\hat{u}'_1 = (.7071, 0, -.7071),\ \hat{u}'_2 = (-.965, 0, -.258),\ \hat{u}'_3 = (.258, 0, .965)$ frame $X'Y'Z'$
$b_{12} = b_{13} = b_{23} = b'_{12} = b'_{13} = b'_{23} = b''_{12} = b''_{13} = b''_{23} = \sqrt{3}/2$
$q_1 = 1.0 + 0.25\sin(t),\ q_2 = 1.0 + 0.225\sin(t),\ q_3 = 1.0 + 0.275\sin(t)$
$q_4 = 1.0 + 0.25\sin(t)\cos(t),\ q_5 = 1.0 + 0.3\sin(t)\cos(t),\ q_6 = 1.0 + 0.275\sin(t)\cos(t)$
$0 \leqslant t \leqslant 2\pi$
$g = -9.80665\hat{j}$
$m_{\text{middle}} = m_{\text{output}} = 18.914$
$I_{\text{middle}} = I_{\text{output}} = \begin{bmatrix} 0.809 & 0 & 0 \\ 0 & 1.615 & 0 \\ 0 & 0 & 0.809 \end{bmatrix}$ expressed in their corresponding local frames
$\mathbf{F}_{\text{external}} = \begin{bmatrix} 0 \\ -250 \\ 0 \\ 0 \\ 10 \\ 0 \end{bmatrix}$ expressed in the reference frame $X''Y''Z''$

Table 2
Initial configuration, or home, of the SPM

$\mathbf{A}'_1 = (.353, 1.0, .353),\ \mathbf{A}'_2 = (.129, 1.0, -.482),\ \mathbf{A}'_3 = (-.482, 1.0, .129)$
$\mathbf{A}''_1 = (.353, 2.0, .353),\ \mathbf{A}''_2 = (.129, 2.0, -.482),\ \mathbf{A}''_3 = (-.482, 2.0, .129)$

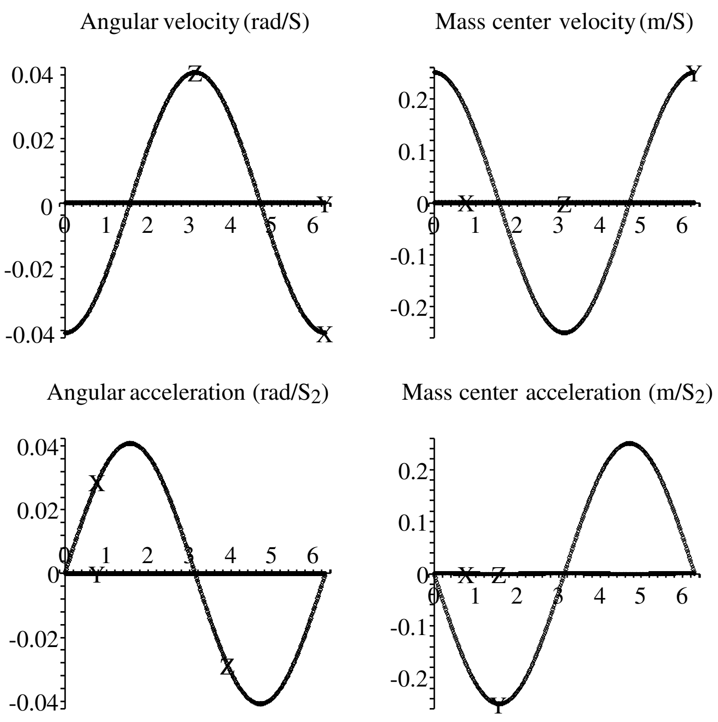

Fig. 6. Time history of the forward kinematics of the middle platform.

Finally, in order to avoid arbitrary and trivial solutions, Eq. (25) requires that

$$\left.\begin{aligned}
&Gvx_1 f_{nx} + Gvy_1 f_{ny} + Gvz_1 f_{nz} + G\omega x_1 \tau_{nx} + G\omega y_1 \tau_{ny} + G\omega z_1 \tau_{nz} + \xi_1 = 0,\\
&Gvx_2 f_{nx} + Gvy_2 f_{ny} + Gvz_2 f_{nz} + G\omega x_2 \tau_{nx} + G\omega y_2 \tau_{ny} + G\omega z_2 \tau_{nz} + \xi_2 = 0,\\
&\vdots\\
&Gvx_{\mathrm{dof}} f_{nx} + Gvy_{\mathrm{dof}} f_{ny} + Gvz_{\mathrm{dof}} f_{nz} + G\omega x_{\mathrm{dof}} \tau_{nx} + G\omega y_{\mathrm{dof}} \tau_{ny} + G\omega z_{\mathrm{dof}} \tau_{nz} + \xi_{\mathrm{dof}} = 0,
\end{aligned}\right\} \tag{26}$$

which allows to compute directly the driving forces required for controlling the motion of body n, and, of course, are applicable to any body of the closed chain.

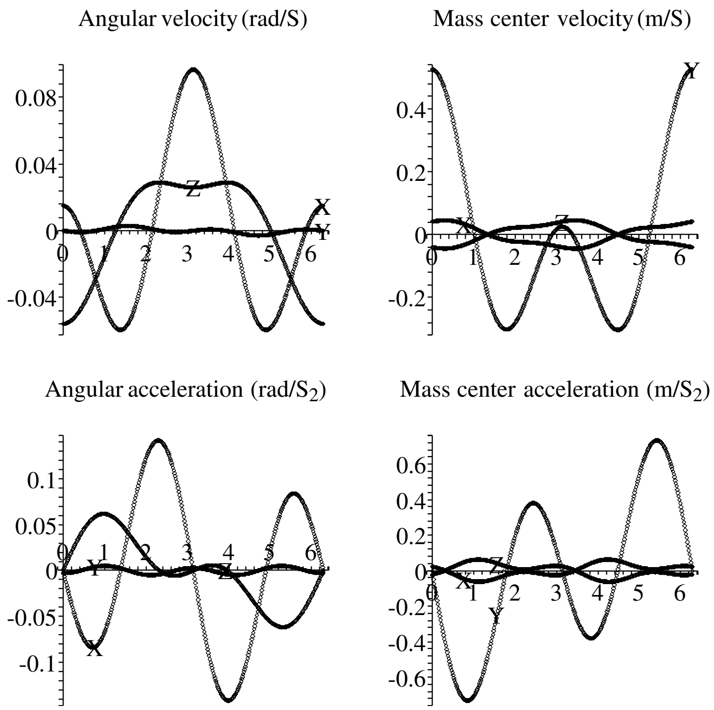

Fig. 7. Time history of the forward kinematics of the output platform.

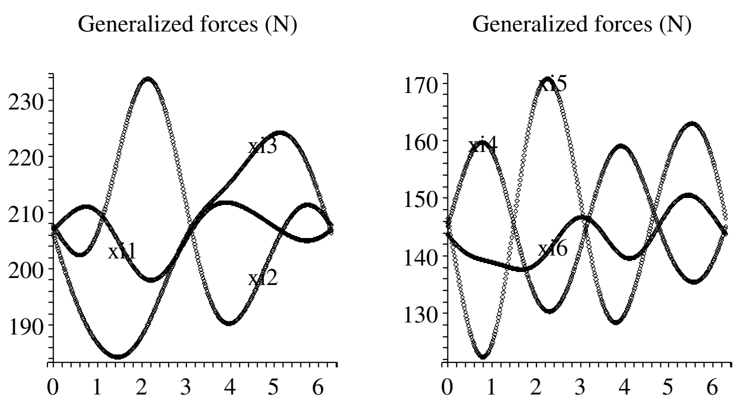

Fig. 8. Time history of the driving forces of the numerical example.

6. Case study

In order to exemplify the proposed methodology, in this section a numerical example is provided. The parameters of the SPM, using SI units, are listed in Table 1.[3]

Furthermore, the home position of the SPM is provided in Table 2.

With these data, the time history of the forward kinematics of the numerical example is summarized in Figs. 6 and 7.

Finally, the resulting driving forces of the lower parallel manipulator, left graphic, and the upper parallel manipulator, right graphic, are provided in Fig. 8.

7. Conclusions

The computation of driving forces in parallel manipulators becomes a hazardous task when traditional strategies such as the Newton–Euler method or the Lagrangian formulation are used. In fact, the Newton–Euler method usually requires large computation time, since it needs the calculation of all the internal reactions of constraint of the system, even if they are not employed in the control law of the manipulator. On the other hand, the Lagrangian formulation is based on the computation of the energy of the whole system with the adoption of a generalized coordinate framework. These time consuming computations are unnecessary when the theory of screws and the principle of virtual work are used systematically.

In this work, the kinematic and dynamic analyses of a class of series–parallel manipulator known as 2(3-RPS) manipulator are approached by means of the theory of screws and the principle of virtual work.

The class of series–parallel manipulator considered here has a more compact topology than the proposed in the literature, due to the introduction of compound joints, spherical + revolute joint assembly, leading to a compact structure. It is straightforward to show that this arrangement will cause less interference among the manipulator links and joints, and will generate a better force transmission when compared to the same manipulator topology with non-concentric spherical and revolute joint pairs attached to the middle platform.

First, the forward position analysis is carried out using the Sylvester dyalitic elimination method, which leads to a multiple solution. Thus, with this procedure the 256 feasible, real or imaginary, solutions of the forward position analysis are calculated. Later, simple and compact expressions are derived using the theory of screws for solving the velocity and acceleration analyses of the spatial mechanisms. It is worth mentioning that the Klein form, $\{*;*\}$, of the Lie algebra, $e(3)$, allows to solve the forward acceleration analysis without computing the passive joint acceleration rates of the series–parallel manipulator. Finally, the driving forces for controlling any body of the series–parallel manipulator are calculated by means of an harmonious combination of the theory of screws and the principle of virtual work, the proposed method is simple and does not require the instantaneous values of the internal reactions of constraint nor the computation of the energy of the whole system.

In order to illustrate the efficacy of the chosen method of analysis, a case study is included.

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