Instructions: For question 1, you may:

- do the work by hand and upload a ***neatly*** scanned PDF file,
- type up your work in Word or LaTeX and save as a PDF, OR
- type up your work in RMarkdown and include it with question 2 in your HTML file.

Questions:

1. We have learned that $\hat{\beta}_0$ and $\hat{\beta}_1$ are linear estimators or linear functions of $y_1, \ldots, y_n$, i.e., there exist $k_1, \ldots, k_n$ and $a_1, \ldots, a_n$ such that

$$\hat{\beta}_0 = \sum a_i y_i \text{ and } \hat{\beta}_1 = \sum k_i y_i$$

Rewrite the equations for $\hat{\beta}_1$ and $\hat{\beta}_0$ to show this is true, i.e., determine the values for k_i and a_i for $i = 1, \ldots, n$.

2. Assume the simple linear regression model

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i, \quad i = 1, \ldots, n$$

where $\epsilon_1, \ldots, \epsilon_n \overset{iid}{\sim} N(0, \sigma^2)$. Let $x_i = i$ (e.g. $x_1 = 1$, $x_2 = 2$, etc.). Set $\beta_0 = 10, \beta_1 = -2.5, \sigma^2 = 9, n = 35$.

   a) Randomly generate and display the n error terms. Before doing this, set a random seed using the last four digits of your student id number.
   b) Obtain your data set of pairs $(x_1, y_1), \ldots, (x_n, y_n)$. Create a scatterplot of y against x. Comment about the main characteristics.
   c) Estimate the regression coefficients $\hat{\beta}_0$ and $\hat{\beta}_1$. Use the equations we learned in class **and** the `lm` function in R and show those results are equivalent.
   d) Compare the estimates $\hat{\beta}_0$ and $\hat{\beta}_1$ with the true parameters β_0 and β_1, respectively. Are $\hat{\beta}_0$ and $\hat{\beta}_1$ good estimators of β_0 and β_1? Explain why or why not.
   e) Compute the residuals and the estimated variance. Use the equations learned in class **and** verify your numbers using the `lm` function in R.