HERIOT WATT UNIVERSITY

SCHOOL OF MATHEMATICAL AND COMPUTER SCIENCES

Department of Computer Science

F29FA

FOUNDATIONS I

CLASS TEST — 2020/21

Duration: 0.5 Hour

ANSWER ALL QUESTIONS
Ensure you are familiar with plagiarism rules
Please submit one single PDF file for all your test.
NO ZIP FILES, NO PNG AND NO JPG.

1. $S \equiv \lambda xyz.xz(yz)$.
$I \equiv \lambda x.x$.
$F \equiv \lambda z.zz$.
$O \equiv FF$
$A \equiv \lambda xy.xx$.
$M \equiv x((\lambda x.x)(\lambda xyz.xz(yz))(\lambda xy.xx)(\lambda x.xx)x)$.

(a) Is $(\lambda xyz.xz(yz))(\lambda xy.xx)$ a subterm of M? Justify your answer. (0.5)

(b) Insert the full parenthesis in M. (1.5)

(c) Give the meaning of each of A and F and O. (1.5)

(d) Write M using only the symbols you need from A, S, I, F, x and O (you may not need all these symbols). (0.5)

(e) Is M β-normalising? If yes, β-reduce M until there are no β-redexes left, showing all the β-reduction steps, underlining at each stage the redex you are contracting, and always keeping the term as compact as possible. If the term is not β-normalising, say why it is not. (2.5)

(f) Give the definition of "H has β-normal form G". (0.5)

(g) Recall the lemma which says that if $E \to_\beta G$ then $E[x := H] \to_\beta G[x := H]$. Use this lemma and your result above to show that $M[x := M] \twoheadrightarrow_\beta M(MM)$. (2)

(h) Use the above to deduce whether $M[x := M]$ is β-normalising. If yes, give the β-normal form. If the term is not β-normalising, give a detailed proof why it is not. (2)

(i) carry out the substitution $(\lambda x.xyz)[z := xy]$ showing all the substitution steps and the number of the subtitution rule you use at every step, and always underlining the substitution you are contracting at each step. (3)

(j) Give the η-normal form of $\lambda xy.(\lambda z.Iz)yx$ clearing underlyning the eta redexes you identify. (1)

END OF PAPER