

SCHOOL OF MATHEMATICAL AND COMPUTER SCIENCES

Department of Computer Science

F29FA

FOUNDATIONS I

CLASS TEST 2 — 2020/21

Duration: 1 Hour

ANSWER ALL QUESTIONS

Ensure you are familiar with plagiarism rules
Please submit one single PDF file for all your test.
NO ZIP FILES, NO PNG AND NO JPG.

1. $S \equiv \lambda xyz.xz(yz).$
 $K \equiv \lambda xy.x.$
 $I \equiv \lambda x.x.$
 $\Omega \equiv (\lambda z.zz)(\lambda z.zz).$
 $O \equiv \lambda xyu.xuy.$
 $A \equiv \lambda x.xSK.$
 $B = \lambda xy.y(xx).$
 $C \equiv Oz\Omega I.$
 $D \equiv \lambda x.y.$

(a) Recall the terms C and K given at the top of this page. For each of the terms below, answer whether the term is β -normalising? If yes, β -reduce the term using the leftmost outermost reduction strategy until there are no β -redexes left, showing all the β -reduction steps, underlining at each stage the redex you are contracting, and always keeping the term as compact as possible, and finally give its β -normal form. If the term is not β -normalising, justify why not.

1. $C.$ (1.5)
2. $C[z := K].$ (1.5)

(b) Recall the term O given at the top of this page.

1. Find a term E such that $E =_{\beta} OE$; justify your answer. (1)
2. Show that this term E you found in (b).1. satisfies $ERT =_{\beta} ETR$ for any terms R and T . (1)

(c) Recall the terms S and I given at the top of this page.

1. Find a term M such that $Mx =_{\beta} MIdx$; justify your answer. (1)
2. Show that this term M you found in (c).1. satisfies $MS =_{\beta} MISS$. (1)

(d) Recall the term A given at the top of this page. β -reduce the term AAx to a β -normal form using the rightmost reduction strategy showing all the β -reduction steps, underlining at each stage the redex you are contracting, and always keeping the term as compact as possible, and finally give its β -normal form. (3.5)

(e) Recall the terms D and B given at the top of this page.

1. Is $D(BB)$ β -normalising? Fully justify your answer. (1)
2. Is $D(BB)$ strongly β -normalising? Fully justify your answer. (1.5)

(f) Recall the terms S , K and I given at the top of this page.

- We say that a term P is *sick* if the term $SIKP$ is not strongly β -normalising.
- We say that a term P is *sinful*, if for every term N , the term $SINP$ is not strongly β -normalising.

1. Show that if P is sinful then P is sick. (1)
2. Give a term which is sinful and justify your answer. (1)

END OF PAPER