Foundations1 assignments I 2020
Assignment II: Submit by Wednesday of week 8 (4 November 2020) by 15:30 pm.
worth 15%
Submit only typed material through assessment on vision
No handwritten material
SUBMIT ONE SINGLE FILE IN PDF ONLY FOR THE ASNWERS+ Submit your entire SML program AS ONE SINGLE SML FILE
NO ZIP FILES. NO MULTIPLE FILES. JUST ONE PDF FOR ANSWERS AND ONE SML FOR ALL THE CODE.
BOTH ANSWER FILE AND CODE FILE MUST BE NAMED "FAMILYNAME-NAME.pdf" and "FAMILYNAME-NAME.sml" respectively

WRITE YOUR FAMILYNAME+NAME+STUDENT ID (in this order) HERE

Background

- USE EVERYTHING FROM ASSIGNMENT 1.
  At `http://www.macs.hw.ac.uk/~fairouz/foundations-2020/slides/assign20.pdf` and
  `http://www.macs.hw.ac.uk/~fairouz/foundations-2020/slides/assign20-solutions.pdf` you find your assignment I and its solutions. You can use anything from these two files.

- Remote login to H-W computers and Download the files:
  `http://www.macs.hw.ac.uk/~fairouz/foundations-2020/slides/sml-week1.sml`
  `http://www.macs.hw.ac.uk/~fairouz/foundations-2020/slides/sml-week2.sml`
  `http://www.macs.hw.ac.uk/~fairouz/foundations-2020/slides/sml-week7.sml`
  and put them all in the same directory.

- Recall that you can use these files in your remoted login to H-W computers by typing the following (you need to be in the same directory which contains the files sml-week1.sml, sml-week2.sml and sml-week3.sml):

  ```
  sml
  - use "sml-week1.sml";
  - use "sml-week2.sml";
  - use "sml-week7.sml";
  ```

  Then, you can run all your commands in the above window until you want to quit SML in which case, you type control D.

- Remember the instructions given at:
  `http://www.macs.hw.ac.uk/~fairouz/foundations-2020/slides/running-sml-week1`
  and `http://www.macs.hw.ac.uk/~fairouz/foundations-2020/slides/running-sml-week2`
  Understand and follow these instructions, they will help you in your assignment.

- You are already familiar with sml-week1.sml and sml-week2.sml. The file at `http://www.macs.hw.ac.uk/~fairouz/foundations-2020/slides/sml-week7.sml` contains the SML functions of weeks 1 and 2 as well as the leftmost outermost reduction strategy to reduce a term (and also two other reduction strategies which you don't need to bother about but may want to play with). You can read this file at:
  `http://www.macs.hw.ac.uk/~fairouz/foundations-2020/slides/sml-week7`
  Start practicing with commands like printloreduce t9; or printloreduce (APP(t9,t8));
  The terms t8 and t9 (as well as other terms) are already given in the file sml-week7.sml.

Questions

1. State truthfully whether you have studied very well weeks 1..6 inclusive and have followed all the exercises there. Explain what things you still don't understand in these weeks despite having devoted to each lecture at least 2 hours of study excluding listening to our live sessions. (0.25)

2. State truthfully that you have studied Assignment 1 and its solutions. Explain what things you still don't understand. (0.25)

3. Implement in SML the translation function f from $\mathcal{M}$ to $\mathcal{M}'$ that translates terms in $\mathcal{M}$ to terms in $\mathcal{M}'$. Call this implementation of f, *tran*. (0.75)

4. Test tran by printing running it on t8 and t9. That is, give the output of
   -printIEXP (tran t8);
   -printIEXP (tran t9); (0.25)

5. For each term A given below do the following:
   * Write A as an SML expression (say tA).
   * Run -freeVars tA; and give the SML output.
   * Give A[x:=z].
   * Give A[z:=x].
   * Run printLEXP tA; and give the SML output.
   * Run -subs vx "z" tA; and give the SML output.
   * Run -subs vz "x" tA; and give the SML output.
   * Run printIEXP(tran tA); and give the SML output.

   For example, if A is xz then your answer should be:

```
val vx = (ID "x");
val vz = (ID "z");
val t200 = APP(vx,vz);

- freeVars t200;
val it = ["x","z"] : string list

(xz)[x:=z] = zz
(xz)[z:=x] = xx

- printLEXP t200;
(x z)val it = () : unit

- subs vx "z" t200;
```

```
val it = APP (ID "x",ID "x") : LEXP

- subs vz "x" t200;
val it = APP (ID "z",ID "z") : LEXP

- printIEXP (tran t200);
<z>xval it = () : unit
```

(a) $(\lambda xz.xz)$. (1.5)

(b) $(\lambda xy.xz)$. (1.5)

(c) $xz(\lambda xy.xy)$. (1.5)

6. Run printloreduce on 4 terms of your choice and for each such term, return the SML output followed by the step by step lmo reduction that you carry yourself always underlining the redexes. Each of your 4 chosen terms should have at least the same number of reduction steps as the t9 example I give here. For example,

```
 - printloreduce t9;
((\z.(z ((\x.x) z))) (((\x.x) (\y.x)) z))-->
((((\x.x) (\y.x)) z) ((\x.x) (((\x.x) (\y.x)) z)))-->
(((\y.x) z) ((\x.x) (((\x.x) (\y.x)) z)))-->
(x ((\x.x) (((\x.x) (\y.x)) z)))-->
(x (((\x.x) (\y.x)) z))-->
(x ((\y.x) z))-->
(x x)val it = () : unit
```

$$
\begin{array}{l}
\underline{(\lambda z.z((\lambda x.x)z))((\lambda x.x)(\lambda y.x)z)} \rightarrow_{\beta}^{lmo} \\
\underline{(\lambda x.x)(\lambda y.x)}z((\lambda x.x)((\lambda x.x)(\lambda y.x)z)) \rightarrow_{\beta}^{lmo} \\
\underline{(\lambda y.x)z}((\lambda x.x)((\lambda x.x)(\lambda y.x)z)) \rightarrow_{\beta}^{lmo} \\
x(\underline{(\lambda x.x)((\lambda x.x)(\lambda y.x)z)}) \rightarrow_{\beta}^{lmo} \\
x(\underline{(\lambda x.x)(\lambda y.x)z}) \rightarrow_{\beta}^{lmo} \\
x(\underline{(\lambda y.x)z}) \rightarrow_{\beta}^{lmo} \\
xx
\end{array}
\tag{5}
$$

7. For the term $(\lambda x.x)z(\lambda z.z((\lambda x.x)z))((\lambda x.x)(\lambda y.x)z)$, give the righmost reduction strategy as defined in Lecture 13, always underlining the redexes and showing all the steps. (2)

8. Take the m-strategy defined by:
   $m(AB) = m(A)$ if $m(A)$ is defined
   $m(AB) = m(B)$ if $m(A)$ undefined and $m(B)$ defined
   $m(AB) = AB$ if $m(A)$ undefined and $m(B)$ undefined and AB is a β-redex
   $m(AB) =$ undefined if AB has no β-redexes
   $m(\lambda v.A) = m(A)$
   $m(v) =$ undefined

   Use the m-startegy to reduce t9 showing at every stage all the steps and underlining all redexes. (2)