Physics 926: Homework #11

Due on April 14, 2020 at 5pm

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*In addition to the lecture notes, the following resources were used to better understand the material:
`https://arxiv.org/ftp/arxiv/papers/1511/1511.06752.pdf`

Problem 1

Show that

$$P(\nu_1 \to \nu_2) = \sin^2 2\theta \frac{\Delta m_{12}^2 L}{4E}$$

for a two neutrino system in which the mixing matrix is

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \quad U^* = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

and $\Delta m_{12}^2 = m_1^2 - m_2^2$

Solution

Equation (7) from the lecture gives us a good starting point. This equation is an approximation which is valid when mass is small, which in this case it is

$$P(\nu_1 \to \nu_2) = \left| \sum_i U_{1i}^* U_{2i} e^{-im_i^2 L/2E} \right|^2$$

Using this equation as a starting point, we can plug in the given values of U and explicitly do the sum.

$$\begin{aligned}
\left| \sum_i U_{1i}^* U_{2i} e^{-im_i^2 L/2E} \right|^2 &= \left[\sum_i U_{1i}^* U_{2i} e^{-im_i^2 L/2E} \right] \left[\sum_j U_{1j}^* U_{2j} e^{-im_j^2 L/2E} \right]^* \\
&= \left[\sum_i U_{1i}^* U_{2i} e^{-im_i^2 L/2E} \right] \left[\sum_j U_{2j}^* U_{1j} e^{im_j^2 L/2E} \right] \\
\sum_i U_{1i}^* U_{2i} e^{-im_i^2 L/2E} &= \cos\theta(-\sin\theta) e^{-im_1^2 L/2E} + (-\sin\theta)\cos\theta e^{-im_2^2 L/2E} \\
&= -\cos\theta\sin\theta e^{-im_1^2 L/2E} - \cos\theta\sin\theta e^{-im_2^2 L/2E} \\
&= -\cos\theta\sin\theta \left(e^{-im_1^2 L/2E} + e^{-im_2^2 L/2E} \right) \\
\sum_j U_{2j}^* U_{1j} e^{im_j^2 L/2E} &= \sin\theta\cos\theta e^{im_1^2 L/2E} + \cos\theta\sin\theta e^{im_2^2 L/2E} \\
&= \cos\theta\sin\theta \left(e^{im_1^2 L/2E} + e^{im_2^2 L/2E} \right)
\end{aligned}$$

Problem 2

Problem 3