Physics 926: Homework #9

Due on March 31, 2020 at 5pm

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Problem 1

Show that in $\pi \rightarrow \mu\nu$ decay, $|\vec{p}_\mu| = |\vec{p}_\nu| = (m_\pi^2 - m_\mu^2)/2m_\pi$

Solution

Start by defining the momentum four-vectors for each particle in the center of momentum frame. Note that σ is the index while μ, ν, and π are the names of the particles.

$$P_\pi^\sigma = (m_\pi, \vec{0})$$
$$P_\mu^\sigma = (E_\mu, \vec{p})$$
$$P_\nu^\sigma = (E_\nu, \vec{-p})$$

Due to conservation of four-momentum, we can write:

$$P_\pi^\sigma = P_\mu^\sigma + P_\nu^\sigma$$
$$P_\pi^\sigma - P_\nu^\sigma = P_\mu^\sigma$$

Now we contract each side with itself which we can do since this operation is Lorentz invariant:

$$(P_\pi - P_\nu)^\sigma (P_\pi - P_\nu)_\sigma = P_\mu^\sigma P_{\mu,\sigma}$$
$$P_\pi^2 + P_\nu^2 - 2P_\nu^\sigma P_{\pi,\sigma} = m_\mu^2$$

Since we are assuming the the neutrino is massless, $P_\nu^2 = 0$ and $E_\nu = |\vec{p}|$

$$m_\pi^2 - 2|\vec{p}|m_\pi = m_\mu^2$$
$$-2|\vec{p}|m_\pi = m_\mu^2 - m_\pi^2$$
$$|\vec{p}| = \frac{m_\pi^2 - m_\mu^2}{2m_\pi}$$

Problem 2

(H&M exercise 12.13) Predict the ratio of the $K^- \to e^- \bar{\nu}_e$ and $K^- \to \mu^- \bar{\nu}_\mu$ decay rates. Given that the lifetime of the K is $\tau = 1.2 \times 10^{-8} s$ and the $K \to \mu\nu$ branching ratio is 64%, estimate the decay constant f_K. Comment on your assumptions and your result.

Solution

Starting with $K^- \to e^- \bar{\nu}_e$, and following the procedure outlined in H&M for π^- decay (p.265):

$$\mathcal{M} = \frac{G}{\sqrt{2}} q^\sigma f_k \bar{u}(p_3)\gamma_\sigma(1-\gamma^5)v(p_4)$$
$$q = p_3 + p_4$$
$$\Rightarrow \mathcal{M} = \frac{G f_k}{\sqrt{2}}(p_3^\sigma + p_4^\sigma)\bar{u}(p_3)\gamma_\sigma(1-\gamma^5)v(p_4)$$

Problem 3

Solution

Problem 4

Solution

Problem 5

Solution

Problem 6

Part a

$$V_{CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

If we let $A = \rho = \eta = 1$ and $\lambda = 0.2$, then we can write this as:

$$V_{CKM} = \begin{pmatrix} 0.98 & 0.2 & 0.008 - i0.008 \\ -0.2 & 0.98 & 0.04 \\ -i0.008 & -0.04 & 1 \end{pmatrix}$$

We can take the dot product between any row and any other row, $row_i row_j^*$ and the dot product between any column and any other column, $col_i col_j^*$ and these dot products must be equal to zero. This is due to the unitarity of the CKM matrix. Each of these six dot products then represents a triangle in the complex plain whose sides are defined by the vector pointing to each of the products in the dot products.

Here I'll write each product of the six dot products out in the form of a column vector defined like this: $v = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ where a_1 = the real component and a_2 = the imaginary component

Triangle 1 ($row_1 row_2^*$):

$$p_1 = \begin{pmatrix} -0.2 \\ 0 \end{pmatrix}, \quad p_2 = \begin{pmatrix} 0.2 \\ 0 \end{pmatrix}, \quad p_3 = \begin{pmatrix} 0.00032 \\ -0.00032 \end{pmatrix}$$

Triangle 2 ($row_1 row_3^*$):

$$p_1 = \begin{pmatrix} 0 \\ 0.008 \end{pmatrix}, \quad p_2 = \begin{pmatrix} -0.008 \\ 0 \end{pmatrix}, \quad p_3 = \begin{pmatrix} 0.008 \\ -0.008 \end{pmatrix}$$

Triangle 3 ($row_2 row_3^*$):

$$p_1 = \begin{pmatrix} 0 \\ -0.0016 \end{pmatrix}, \quad p_2 = \begin{pmatrix} -0.04 \\ 0 \end{pmatrix}, \quad p_3 = \begin{pmatrix} 0.04 \\ 0 \end{pmatrix}$$

Triangle 4 ($col_1 col_2^*$):

$$p_1 = \begin{pmatrix} 0.2 \\ 0 \end{pmatrix}, \quad p_2 = \begin{pmatrix} -0.2 \\ 0 \end{pmatrix}, \quad p_3 = \begin{pmatrix} 0 \\ 0.0032 \end{pmatrix}$$

Triangle 5 ($col_1 col_3^*$):

$$p_1 = \begin{pmatrix} 0.008 \\ 0.008 \end{pmatrix}, \quad p_2 = \begin{pmatrix} -0.008 \\ 0 \end{pmatrix}, \quad p_3 = \begin{pmatrix} 0 \\ -0.008 \end{pmatrix}$$

Triangle 6 ($col_2 col_3^*$):

$$p_1 = \begin{pmatrix} 0.0016 \\ 0.0016 \end{pmatrix}, \quad p_2 = \begin{pmatrix} 0.04 \\ 0 \end{pmatrix}, \quad p_3 = \begin{pmatrix} -0.04 \\ 0 \end{pmatrix}$$

Now these can all be plotted together, which I did using Desmos on the following page

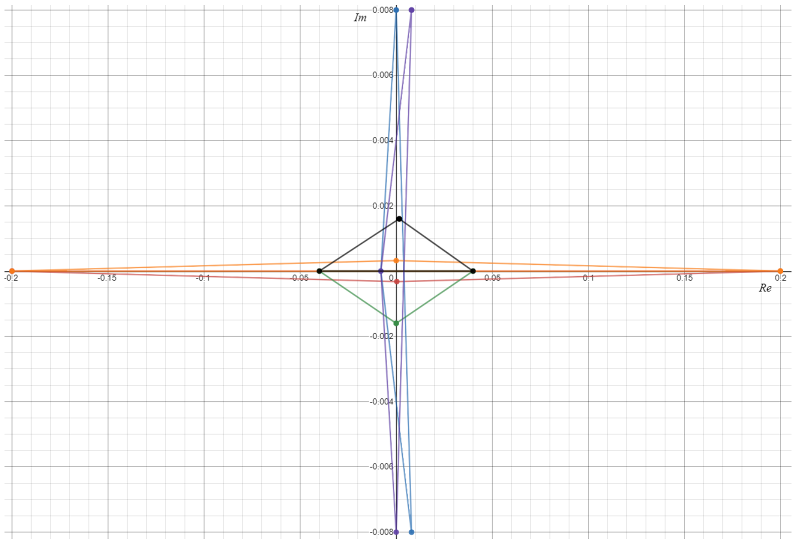

Figure 1: The six triangles plotted for $A = \rho = \eta = 1$ and $\lambda = 0.2$

Part b

Part c