Physics 926
Problem Set 10
Due Tuesday, April 7

1. Show that
$$\frac{\Gamma(K_L \to \pi^- e^+ \nu_e) - \Gamma(K_L \to \pi^+ e^- \bar{\nu}_e)}{\Gamma(K_L \to \pi^- e^+ \nu_e) + \Gamma(K_L \to \pi^+ e^- \bar{\nu}_e)} = 2\mathrm{Re}(\epsilon) \tag{1}$$
to first order in ϵ. This asymmetry is evidence for indirect CP violation, and also allows us to unambiuously define electric charge – positive charge is assigned to the lepton that dominates in K_L decay.

2. Defining
$$\eta_\pm = |\eta_\pm| e^{i\phi_\pm} = \frac{\langle \pi^+\pi^- | K_L \rangle}{\langle \pi^+\pi^- | K_S \rangle}, \tag{2}$$
calculate the probailities of $\pi^+\pi^-$ decay as a function of (proper) time for an initial K^0 or $\bar{K}^0$ produced at $t = 0$. Express your answer, up to common proportionality constants, in terms of ϵ, $|\eta_\pm|$, $\phi_\pm$, Δm, Γ_S and Γ_L, where Δm is the K_S–K_L mass difference and Γ_S and Γ_L are the widths of the states. Keep only leading terms in ϵ. Using the experimental values for these quantities, plot the two probabilities as a function of time in units of the K_S lifetime, going out to 30 K_S lifetimes. (Hint: K_L and K_S are the eigenstates of the total Hamiltonian, and thus have straightforward time evolution. Write the time evolution of the K^0 and $\bar{K}^0$ in terms of the Hamiltonian eigenstates.)

3. Calculate $P(A, t; B)$, where A and B are either K^0 or $\bar{K}^0$. This is defined to be the probability that a neutral kaon in state B at $t = 0$ has oscillated into a state A after a time t. Express your answer in terms of ϵ, Δm, Γ_S and Γ_L. Keep only leading terms in ϵ. Plot the probabilities as before, going out to 20 K_S lifetimes, and using a factor of ϵ that is a factor of ten larger than the experimental value (so that you can see something interesting).

4. In class, we briefly mentioned the (approximate) $\Delta I = 1/2$ rule, which says that in strange particle decays, transitions with $\Delta I = 1/2$ are enhanced over those with $\Delta I = 3/2$. Use this rule to derive
$$\Gamma(K_L \to 3\pi^0) = \frac{3}{2}\Gamma(K_L \to \pi^+\pi^-\pi^0), \tag{3}$$
and compare this result to experimental data. Make the reasonable assumption that all pairs of pions are in an $L = 0$ state. Don't forget that pions are bosons and you'll need to write totally symmetric wavefunctions.