Physics 916: Homework #4

Due on April 13, 2020 at 5pm

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Problem 1

Consider a physical system whose three-dimensional state space is spanned by an orthonormal basis $\{|u_1\rangle, |u_2\rangle, |u_3\rangle\}$. In that state space, consider two operators L_z and S defined by:

$$L_z |u_1\rangle = |u_1\rangle, L_z |u_2\rangle = |0\rangle, L_z |u_3\rangle = -|u_3\rangle$$
$$S |u_1\rangle = |u_3\rangle, S |u_2\rangle = |u_2\rangle, S |u_3\rangle = |u_1\rangle$$

(a) Write the matrices which represent, in the $\{|u_1\rangle, |u_2\rangle, |u_3\rangle\}$ basis, the operators L_z, L_z^2, S, and S^2. Are these operators observables?

Solution

Part a

The matrix representation of these two operators is found by applying to each $|u_i\rangle$ and simply seeing what each row and column must be to bring about the given transformations. They are:

$$L_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, S = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Then we square them to get the other two matrices:

$$L_z^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, S^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Problem 2

Problem 3

Problem 4