Physics 926: Homework #10

Due on April 7, 2020 at 5pm

Professor Ken Bloom

Robert Tabb

Problem 1

Show that:

$$\frac{\Gamma(K_L \to \pi^- e^+ \nu_e) - \Gamma(K_L \to \pi^+ e^- \bar{\nu}_e))}{\Gamma(K_L \to \pi^- e^+ \nu_e) + \Gamma(K_L \to \pi^+ e^- \bar{\nu}_e))} = 2Re(\epsilon)$$

to first order in ϵ. This asymmetry is evidence for indirect CP violation, and also allows us to unambiguously define electric charge - positive charge is assigned to the lepton that dominates in the K_L decay.

Solution

$$|K_L\rangle = \frac{1}{\sqrt{1+|\epsilon|}}\left[\frac{1+\epsilon}{\sqrt{2}}|K^0\rangle - \frac{1-\epsilon}{\sqrt{2}}|\bar{K^0}\rangle\right]$$

$\bar{K^0} \to \pi^+ e^- \bar{\nu}_e$ and $K^0 \to \pi^- e^+ \nu_e$ (see Figure 1), therefore to get $K_L \to \pi^+ e^- \bar{\nu}_e$, take the inner product of $\bar{K^0}$ with K_L and to get $K_L \to \pi^- e^+ \nu_e$, take the inner product of K^0 with K_L.

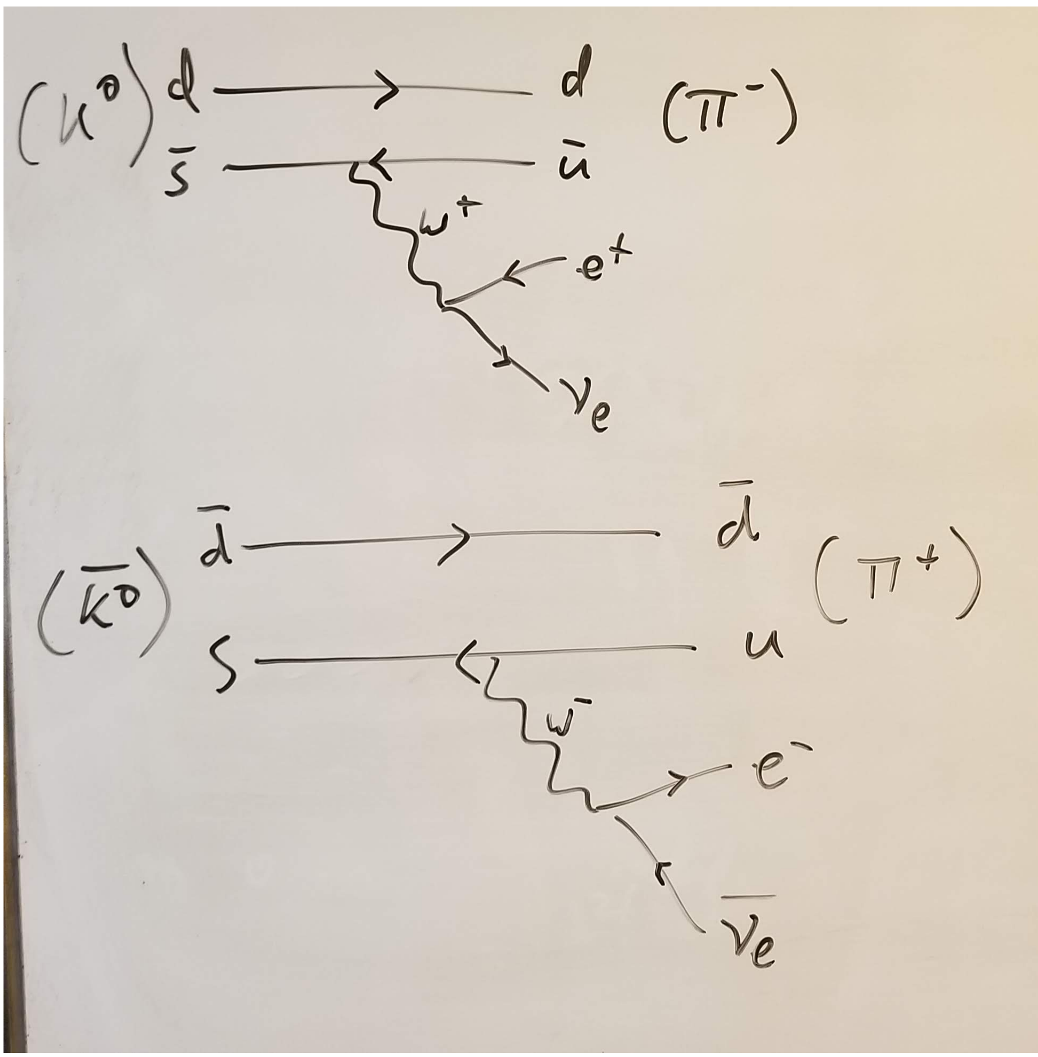

Figure 1: The two neutral kaon decays

$$\Gamma(K_L \to \pi^+ e^- \bar{\nu}_e) \propto \left|\langle \bar{K}^0 | K_L \rangle\right|^2 \propto |1-\epsilon|^2 = (1-\epsilon)(1-\epsilon^*) = 1 - \epsilon^* - \epsilon + |\epsilon|^2 \approx 1 - 2Re(\epsilon)$$
$$\Gamma(K_L \to \pi^- e^+ \nu_e) \propto \left|\langle K^0 | K_L \rangle\right|^2 \propto |1+\epsilon|^2 = (1+\epsilon)(1+\epsilon^*) = 1 + \epsilon^* + \epsilon + |\epsilon|^2 \approx 1 + 2Re(\epsilon)$$

Here I dropped the ϵ^2 term since we are only looking to first order in ϵ. I also dropped any constants since they will be the same for each term and will divide out in the end.

$$\begin{aligned}\frac{\Gamma(K_L \to \pi^- e^+ \nu_e) - \Gamma(K_L \to \pi^+ e^- \bar{\nu}_e))}{\Gamma(K_L \to \pi^- e^+ \nu_e) + \Gamma(K_L \to \pi^+ e^- \bar{\nu}_e))} &= \frac{1 + 2Re(\epsilon) - 1 + 2Re(\epsilon)}{1 + 2Re(\epsilon) + 1 - 2Re(\epsilon)} \\ &= \frac{4Re(\epsilon)}{2} = 2Re(\epsilon)\end{aligned}$$

Problem 2

Problem 3

Problem 4