Physics 926
Homework 13
Due Tuesday, April 28

1. Suppose that instead of introducing an $SU(2)$ doublet of complex fields to do the symmetry breaking that generates the boson masses, we used an $SU(2)$ triplet instead. In that representation, the generators of the group are

$$T^1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \; T^2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \; T^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \tag{1}$$

which should look familiar from the formalism for spin-1 systems.

   (a) Take

$$\phi_0 = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}. \tag{2}$$

   Assign the hypercharge such that the field is electrically neutral. Calculate M_W/M_Z in this model.

   (b) Now take

$$\phi_0 = \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix}. \tag{3}$$

   Show that only the charged weak bosons acquire mass in this case.

   Clearly neither of these cases correspond to what we observe in nature.

2. Consider using *polarized* electrons in the process $e^+e^- \to Z^0 \to f\bar{f}$, where f is a fermion that is assumed to be massless. You can learn a lot about this just from the nature of the $Z^0 ff$ coupling and helicity arguments; no Feynman diagram computations are necessary to complete the problem.

   (a) First, write the $Z^0 ff$ coupling, which is proportional to $\gamma^\mu(c_V^f - c_A^f\gamma^5)$, in terms of left- and right-handed couplings, *i.e.* determine the g^f factors in $g_L^f\gamma^\mu(1-\gamma^5) + g_R^f\gamma^\mu(1+\gamma^5)$. Determine the g^f factors in terms of $x_W = \sin^2\theta_W$ when f is an electron.

   (b) Draw the allowed helicity configurations for this process; remember arguments about helicity conservation at the vertex that we made in class which are also in H&M Section 6.6. For each configuration, give the dependence of the amplitude on the fermion scattering angle θ with respect to the initial momentum of the electron. (For this, see especially the rotation matrices on page 128.) Include the relevant *relative* weak coupling parameters. (Overall normalization doesn't matter for this problem, as we will see.)

(c) Compute the relative total cross sections for each initial e^+e^- helicity configuration, assuming that the polarization of the final-state fermions is not measured.

(d) Show that the left-right asymmetry

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = \frac{1 - 4x_W}{1 - 4x_W + 8x_W^2}, \tag{4}$$

where σ_L and σ_R are the total cross sections for left- and right-handed electrons respectively. (Note that this is independent of the final-state fermions.)

(e) A_{LR} has been measured by the SLD experiment at SLAC as 0.1513 ± 0.0021. From this measurement, extract $\sin^2\theta_W$ and its (tiny!) uncertainty. This is the world's most precise single measurement of this quantity.

3. Consider the spontaneous symmetry breaking of an $SU(2)$ local gauge symmetry, as we did in class (or Section 14.9 of H&M). We made a particular choice of vacuum. Show that for any choice of vacuum, all three gauge bosons still acquire the same mass.

4. (Some of H&M Exercise 15.5) The Lagrangian for the scalar field

$$\mathcal{L} = \left| \left(i\partial_\mu - \frac{g}{2}\vec{\tau} \cdot \vec{W} - \frac{g'}{2} Y B_\mu \right) \phi \right| - V(\phi) \tag{5}$$

contains trilinear hW^+W^- and quadrilinear hhW^+W^- Higgs boson couplings. Use

$$\phi = \sqrt{\frac{1}{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \tag{6}$$

to show that in the standard model the vertex factors are igM_W and $ig^2/4$, respectively. (Forget the bit of the problem about the Z couplings; this will keep you busy enough.)

5. Show that

$$\Gamma(H \to W^+W^-) = \frac{G_F M_H^3}{8\sqrt{2}\pi} \sqrt{1 - 4\lambda_W} (12\lambda_W^2 - 4\lambda_W + 1) \tag{7}$$

where $\lambda_W = (M_W/M_H)^2$. Note that $\Gamma(H \to ZZ)$ is given by an identical expression with M_Z replacing M_W and an additional factor of $1/2$ due to the different HZZ coupling and the identical bosons in the final state.

6. Show that the partial width for the decay $H \to f\bar{f}$ is given by

$$\Gamma(H \to f\bar{f}) = \frac{\sqrt{2} N_c G_F M_H m^2}{8\pi} \left(1 - \frac{4m^2}{M_H^2} \right)^{3/2}, \tag{8}$$

where m is the mass of the fermion and N_c is the number of colors. Had the Higgs been heavier, $H \to t\bar{t}$ would have been a possible decay mode. Compare with results of the previous problem and demonstrate that the branching fraction to $t\bar{t}$ is small compared to that for WW or ZZ.