

# Physics 926: Homework #10

Due on April 7, 2020 at 5pm

*Professor Ken Bloom*

Robert Tabb

## Problem 1

Show that:

$$\frac{\Gamma(K_L \rightarrow \pi^- e^+ \nu_e) - \Gamma(K_L \rightarrow \pi^+ e^- \bar{\nu}_e)}{\Gamma(K_L \rightarrow \pi^- e^+ \nu_e) + \Gamma(K_L \rightarrow \pi^+ e^- \bar{\nu}_e)} = 2\text{Re}(\epsilon)$$

to first order in  $\epsilon$ . This asymmetry is evidence for indirect CP violation, and also allows us to unambiguously define electric charge - positive charge is assigned to the lepton that dominates in the  $K_L$  decay.

**Solution**

$$|K_L\rangle = \frac{1}{\sqrt{1+|\epsilon|}} \left[ \frac{1+\epsilon}{\sqrt{2}} |K^0\rangle - \frac{1-\epsilon}{\sqrt{2}} |\bar{K}^0\rangle \right]$$

$\bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu}_e$  and  $K^0 \rightarrow \pi^- e^+ \nu_e$  (see Figure 1), therefore to get  $K_L \rightarrow \pi^+ e^- \bar{\nu}_e$ , take the inner product of  $\bar{K}^0$  with  $K_L$  and to get  $K_L \rightarrow \pi^- e^+ \nu_e$ , take the inner product of  $K^0$  with  $K_L$ .

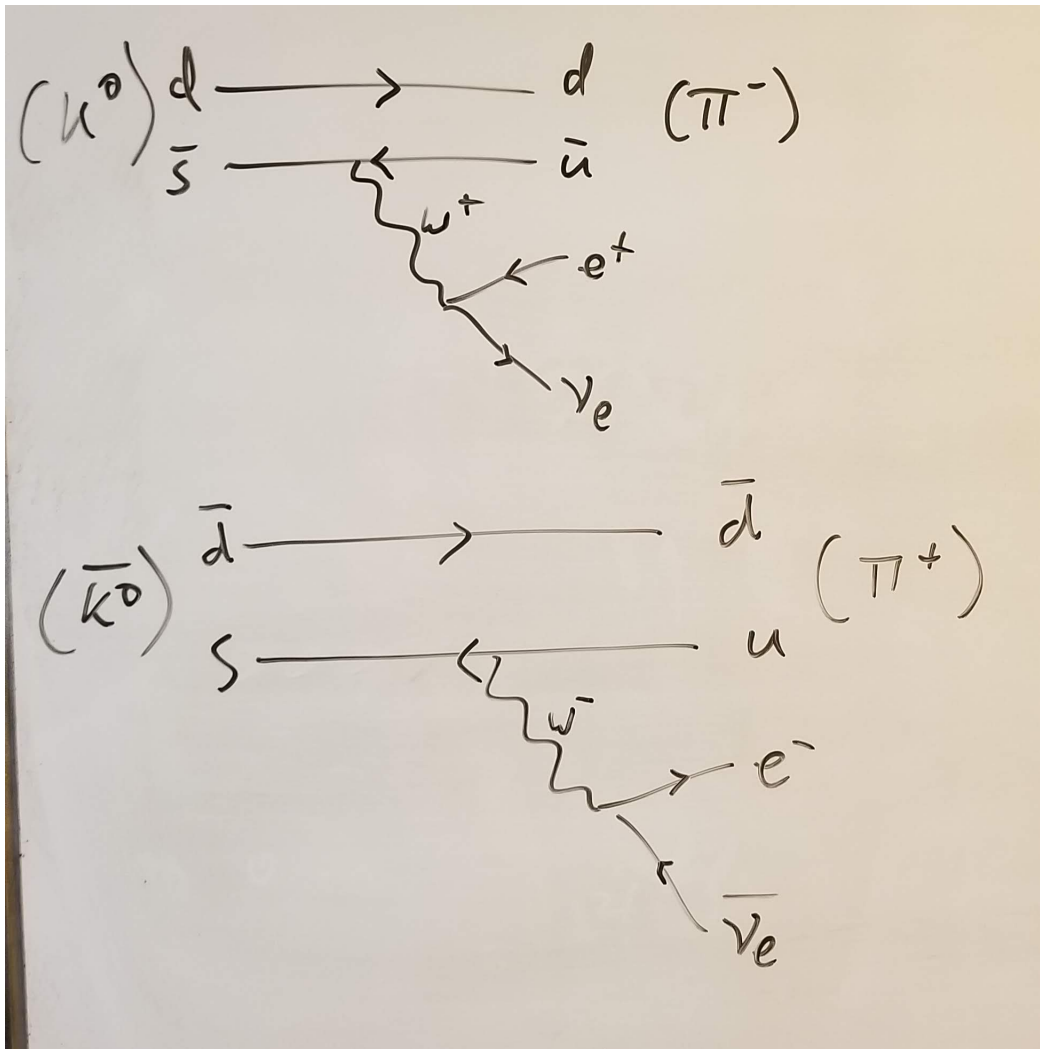


Figure 1: The two neutral kaon decays

$$\Gamma(K_L \rightarrow \pi^+ e^- \bar{\nu}_e) \propto |\langle \bar{K}^0 | K_L \rangle|^2 \propto |1 - \epsilon|^2 = (1 - \epsilon)(1 - \epsilon^*) = 1 - \epsilon^* - \epsilon + |\epsilon|^2 \approx 1 - 2\text{Re}(\epsilon)$$

$$\Gamma(K_L \rightarrow \pi^- e^+ \nu_e) \propto |\langle K^0 | K_L \rangle|^2 \propto |1 + \epsilon|^2 = (1 + \epsilon)(1 + \epsilon^*) = 1 + \epsilon^* + \epsilon + |\epsilon|^2 \approx 1 + 2\text{Re}(\epsilon)$$

Here I dropped the  $\epsilon^2$  term since we are only looking to first order in  $\epsilon$ . I also dropped any constants since they will be the same for each term and will divide out in the end.

$$\begin{aligned} \frac{\Gamma(K_L \rightarrow \pi^- e^+ \nu_e) - \Gamma(K_L \rightarrow \pi^+ e^- \bar{\nu}_e)}{\Gamma(K_L \rightarrow \pi^- e^+ \nu_e) + \Gamma(K_L \rightarrow \pi^+ e^- \bar{\nu}_e)} &= \frac{1 + 2\text{Re}(\epsilon) - (1 - 2\text{Re}(\epsilon))}{1 + 2\text{Re}(\epsilon) + (1 - 2\text{Re}(\epsilon))} \\ &= \frac{4\text{Re}(\epsilon)}{2} = 2\text{Re}(\epsilon) \end{aligned}$$

## Problem 2

Defining

$$\eta_{\pm} = |\eta_{\pm}| e^{i\phi_{\pm}} = \frac{\langle \pi^+ \pi^- | K_L \rangle}{\langle \pi^+ \pi^- | K_S \rangle}$$

calculate the probabilities of  $\pi^+ \pi^-$  decay as a function of proper time for an initial  $K^0$  or  $\bar{K}^0$  produced at  $t = 0$ . Express your answer, up to common proportionality constants, in terms of  $\epsilon, |\eta_{\pm}|, \phi_{\pm}, \Delta m, \Gamma_S$ , and  $\Gamma_L$ , where  $\Delta m = K_S - K_L$ . Keep only leading terms in  $\epsilon$ . Using the experimental values for these quantities, plot the two probabilities as a function of time in units of the  $K_S$  lifetime, going out to 30  $K_S$  lifetimes.

### Solution

Here are the definitions of  $K_L$  and  $K_S$  in terms of  $K^0$  and  $\bar{K}^0$ :

$$\begin{aligned} |K_L\rangle &= \frac{1}{\sqrt{1+|\epsilon|}} \left[ \frac{1+\epsilon}{\sqrt{2}} |K^0\rangle - \frac{1-\epsilon}{\sqrt{2}} |\bar{K}^0\rangle \right] \\ |K_S\rangle &= \frac{1}{\sqrt{1+|\epsilon|}} \left[ \frac{1+\epsilon}{\sqrt{2}} |K^0\rangle + \frac{1-\epsilon}{\sqrt{2}} |\bar{K}^0\rangle \right] \end{aligned}$$

And here they are in terms of the CP eigenstates:

$$\begin{aligned} |K_L\rangle &= \frac{1}{\sqrt{1+|\epsilon|}} [|K_2\rangle + \epsilon |K_1\rangle] \\ |K_S\rangle &= \frac{1}{\sqrt{1+|\epsilon|}} [|K_1\rangle + \epsilon |K_2\rangle] \end{aligned}$$

First, define the time-evolution of the wave function in the same way it was done in the lecture notes:

$$\begin{aligned} |K_S(t)\rangle &= |K_S\rangle e^{im_S t - \Gamma_S t/2} \\ |K_L(t)\rangle &= |K_L\rangle e^{im_L t - \Gamma_L t/2} \end{aligned}$$

We know that the  $\pi^+ \pi^-$  is a CP eigenstate with eigenvalue of +1. So to project  $K_L$  or  $K_S$  onto  $\pi^+ \pi^-$  we project onto  $K_1$ :

$$\begin{aligned} \langle \pi^+ \pi^- | K_L \rangle &= \langle K_1 | K_L \rangle = \frac{\epsilon}{\sqrt{1+|\epsilon|}} e^{im_L t - \Gamma_L t/2} \\ \langle \pi^+ \pi^- | K_S \rangle &= \langle K_1 | K_S \rangle = \frac{1}{\sqrt{1+|\epsilon|}} e^{im_S t - \Gamma_S t/2} \end{aligned}$$

So now we have:

$$\begin{aligned} \frac{\langle \pi^+ \pi^- | K_L \rangle}{\langle \pi^+ \pi^- | K_S \rangle} &= \epsilon e^{i(m_L - m_S)t + \frac{1}{2}(\Gamma_S - \Gamma_L)t} \\ &= \epsilon e^{-i\Delta m t + \frac{1}{2}(\Gamma_S - \Gamma_L)t} \\ &= |\eta_{\pm}| e^{i\phi_{\pm}} \end{aligned}$$

The subscript on the wave function state refers to the initial beam being either purely  $K^0$  or purely  $\bar{K}^0$ . Here I will rewrite the wave functions in terms of the CP eigenstates because the  $\pi^+ \pi^-$  system is a CP

eigenstate with eigenvalue of +1 and we can use this:

$$\begin{aligned}
|\psi(t)\rangle_{K^0} &= \frac{1}{\sqrt{2}} [ |K_S(t)\rangle + |K_L(t)\rangle ] \\
&= \frac{1}{\sqrt{2}} [ |K_S\rangle e^{im_S t - \Gamma_S t/2} + |K_L\rangle e^{im_L t - \Gamma_L t/2} ] \\
|\psi(t)\rangle_{\bar{K}^0} &= \frac{1}{\sqrt{2}} [ |K_S\rangle e^{im_S t - \Gamma_S t/2} - |K_L\rangle e^{im_L t - \Gamma_L t/2} ] \\
|\psi(t)\rangle_{K^0} &= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+|\epsilon|}} [ (|K_1\rangle + \epsilon |K_2\rangle) e^{im_S t - \Gamma_S t/2} + (|K_2\rangle + \epsilon |K_1\rangle) e^{im_L t - \Gamma_L t/2} ] \\
&= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+|\epsilon|}} [ (e^{im_S t - \Gamma_S t/2} + \epsilon e^{im_L t - \Gamma_L t/2}) |K_1\rangle + (e^{im_L t - \Gamma_L t/2} + \epsilon e^{im_S t - \Gamma_S t/2}) |K_2\rangle ] \\
|\psi(t)\rangle_{\bar{K}^0} &= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+|\epsilon|}} [ (e^{im_S t - \Gamma_S t/2} - \epsilon e^{im_L t - \Gamma_L t/2}) |K_1\rangle + (-e^{im_L t - \Gamma_L t/2} + \epsilon e^{im_S t - \Gamma_S t/2}) |K_2\rangle ]
\end{aligned}$$

## Problem 3

## Problem 4