Physics 926: Homework #12

Due on April 30, 2020 at 5pm

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Problem 1

Suppose that instead of introducing an $SU(2)$ doublet of complex fields to do the symmetry breaking that generates the boson masses, we used an $SU(2)$ triplet instead. In that representation, the generators of the group are

$$T^1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \; T^2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \; T^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

(a) Take

$$\phi_0 = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}$$

assign the hypercharge such that the field is electrically neutral. Calculate M_W/M_Z in this model.

(b) Now take

$$\phi_0 = \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix}$$

and show that only the charged weak bosons acquire mass in this case.

Solution

(a) We want the field be be electrically neutral, this means that $Q\phi_0 = 0$

$$Q = T_3 + \frac{Y}{2}$$
$$T_3\phi_0 = -\phi_0$$
$$Y\phi_0 = 2\phi_0$$

A hypercharge of 2 gives us a field which is electrically neutral.

Problem 2

Solution

Problem 3

Consider the spontaneous symmetry breaking of an $SU(2)$ local gauge symmetry, as we did in class. We made a particular choice of vacuum. Show that for any choice of vacuum, all three gauge bosons still acquire the same mass.

Solution

The kinetic energy term will wind up giving us the mass of the boson(s) as seen in equations (17) AND (18) from the lecture notes.

$$\frac{g^2}{4}\left(\frac{1}{\sqrt{2}}\right)^2\left|\begin{pmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & -W^3_\mu \end{pmatrix}\phi\right|^2$$

We can let ϕ be any generic choice: $\phi = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$$\begin{aligned}
&\frac{g^2}{4}\left(\frac{1}{\sqrt{2}}\right)^2\left|\begin{pmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & -W^3_\mu \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}\right|^2 = \\
=&\frac{g^2}{8}\left[\begin{pmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & -W^3_\mu \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}\right]^\dagger\left[\begin{pmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & -W^3_\mu \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}\right] \\
=&\frac{g^2}{8}\left[\begin{pmatrix} v_1 & v_2 \end{pmatrix}\begin{pmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & -W^3_\mu \end{pmatrix}\right]\left[\begin{pmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & -W^3_\mu \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}\right] \\
=&\frac{g^2}{8}\begin{pmatrix} W^3_\mu v_1 + (W^1_\mu + iW^2_\mu)v_2 & (W^1_\mu - iW^2_\mu)v_1 - W^3_\mu v_2 \end{pmatrix}\begin{pmatrix} W^3_\mu v_1 + (W^1_\mu - iW^2_\mu)v_2 \\ (W^1_\mu + iW^2_\mu)v_1 - W^3_\mu v_2 \end{pmatrix} \\
=&\frac{g^2}{8}(W^3_\mu)^2 v_1^2 + (W^1_\mu + iW^2_\mu)(W^1_\mu - iW^2_\mu)v_2^2 + W^3_\mu(W^1_\mu - iW^2_\mu)v_1v_2 + W^3_\mu(W^1_\mu + iW^2_\mu)v_1v_2 \\
+&\frac{g^2}{8}(W^1_\mu - iW^2_\mu)(W^1_\mu + iW^2_\mu)v_1^2 + (W^3_\mu)^2 v_2^2 - W^3_\mu(W^1_\mu - iW^2_\mu)v_1v_2 - W^3_\mu(W^1_\mu + iW^2_\mu)v_1v_2 \\
=&\frac{g^2}{8}(W^3_\mu)^2 v_1^2 + (W^1_\mu)^2 v_2^2 + (W^2_\mu)^2 v_2^2 + (W^3_\mu)^2 v_2^2 + (W^1_\mu)^2 v_1^2 + (W^2_\mu)^2 v_1^2 \\
=&\frac{g^2}{8}(v_1^2 + v_2^2)(W^1_\mu)^2 + (v_1^2 + v_2^2)(W^2_\mu)^2 + (v_1^2 + v_2^2)(W^3_\mu)^2 \\
=&\frac{g^2(v_1^2 + v_2^2)}{8}\left[(W^1_\mu)^2 + (W^2_\mu)^2 + (W^3_\mu)^2\right]
\end{aligned}$$

And now we have three gauge bosons with the same mass such that

$$m = \frac{g\sqrt{v_1^2 + v_2^2}}{2}$$