Physics 926: Homework #12

Due on April 21, 2020 at 5pm

Professor Ken Bloom

Robert Tabb

Problem 1

If the vertex factor for the decay of a vector boson X into two spin-1/2 fermions f_1 and f_2 is

$$-igx\gamma^\mu \frac{1}{2}(c_v - c_A\gamma^5)$$

then show that

$$\Gamma(X \to f_1\bar{f}_2) = \frac{g_X^2}{48\pi}(c_v^2 + c_A^2)M_X$$

where M_X is the mass of the boson and where we have neglected the mass of the fermions. Hints: use

$$\sum_\lambda \epsilon_\mu^{(\lambda)*}\epsilon_\nu^\lambda = -g_{\mu\nu} + \frac{p_\mu p_\nu}{M^2}$$

to show that after summing over the fermions and averaging over the boson spins ,

$$\overline{|\mathcal{M}|^2} = \frac{1}{12}g_X^2(c_v^2 + c_A^2)(-g_{\mu\nu})Tr(\gamma^\mu \not{k}\gamma^\nu \not{k}')$$

where k and k' are the four-momenta of the fermions. Work in the boson rest frame, and use

$$\Gamma(X \to f_1\bar{f}_2)\frac{p_f}{32\pi^2 m_X^2}\int \overline{|\mathcal{M}|^2}d\Omega$$

Solution

Using the vertex factor given in the problem, begin by writing the matrix element

$$\begin{aligned}
\mathcal{M} =&\bar{u}(k)\left[-ig_X\gamma^\mu\frac{1}{2}(c_v - c_A\gamma^5)\right]v(k')\epsilon_\mu \\
=&-\frac{ig_X}{2}\left[\bar{u}(k)\gamma^\mu(c_v - c_A\gamma^5)v(k')\epsilon_\mu\right] \\
|\mathcal{M}|^2 =&\frac{g_X^2}{4}\left[\bar{u}(k)\gamma^\mu(c_v - c_A\gamma^5)v(k')\epsilon_\mu\right]\left[\bar{u}(k)\gamma^\nu(c_v - c_A\gamma^5)v(k')\epsilon_\nu\right]^*
\end{aligned}$$

We now need to write the average by summing over the spins and polarizations. In this step, assume the fermion masses can be neglected and use the formula: $\sum_s[\bar{u}(a)\Gamma_1 v(b)][\bar{u}(a)\Gamma_1 v(b)]^* = Tr[\Gamma_1 \not{b}\bar{\Gamma}_2 \not{a}]$, recalling that $\bar{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0$

$$\begin{aligned}
\overline{|\mathcal{M}|^2} =&\frac{1}{3}\sum_{s,\lambda}|\mathcal{M}|^2 \\
=&\frac{g_X^2}{12}\sum_\lambda(\epsilon_\mu\epsilon_\nu^*)\sum_s\left[\bar{u}(k)\gamma^\mu(c_v - c_A\gamma^5)v(k')\right]\left[\bar{u}(k)\gamma^\nu(c_v - c_A\gamma^5)v(k')\right]^* \\
=&\frac{g_X^2}{12}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_X^2}\right)Tr\left[\gamma^\mu(c_v - c_A\gamma^5)\not{k}'\gamma^0(\gamma^\nu(c_v - c_A\gamma^5))^\dagger\gamma^0\not{k}\right] \\
=&\frac{g_X^2}{12}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_X^2}\right)Tr\left[(c_v\gamma^\mu\not{k}' - c_A\gamma^\mu\gamma^5\not{k}')(c_v\gamma^0\gamma^{\nu\dagger}\gamma^0\not{k} - c_A\gamma^0\gamma^{5\dagger}\gamma^{\nu\dagger}\gamma^0\not{k})\right] \\
=&\frac{g_X^2}{12}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_X^2}\right)Tr\left[(c_v\gamma^\mu\not{k}' - c_A\gamma^\mu\gamma^5\not{k}')(c_v\gamma^\nu\not{k} + c_A\gamma^5\gamma^\nu\not{k})\right]
\end{aligned}$$

Problem 1 continued on next page. . .

Where the plus sign in the last line comes from letting $\gamma^0\gamma^5 \to \gamma^5\gamma^0$ Now let's evaluate that trace

$$\begin{aligned}
Tr\left[(c_v\gamma^\mu \not{k}' - c_A\gamma^\mu\gamma^5\not{k}')(c_v\gamma^\nu\not{k} + c_A\gamma^5\gamma^\nu\not{k})\right] =& Tr\left[c_v^2\gamma^\mu\not{k}'\gamma^\nu\not{k} - c_A^2\gamma^\mu\gamma^5\not{k}'\gamma^5\gamma^\nu\not{k}\right]\\
=& c_v^2 Tr\left[\gamma^\mu\not{k}'\gamma^\nu\not{k}\right] - c_A^2 Tr\left[\gamma^\mu\gamma^5\not{k}'\gamma^5\gamma^\nu\not{k}\right]\\
=& c_v^2 Tr\left[\gamma^\mu\not{k}'\gamma^\nu\not{k}\right] - c_A^2 Tr\left[-\gamma^\mu\not{k}'\gamma^5\gamma^5\gamma^\nu\not{k}\right]\\
=& c_v^2 Tr\left[\gamma^\mu\not{k}'\gamma^\nu\not{k}\right] + c_A^2 Tr\left[\gamma^\mu\not{k}'\gamma^\nu\not{k}\right]\\
=& Tr\left[\gamma^\mu\not{k}'\gamma^\nu\not{k}\right](c_v^2 + c_A^2)
\end{aligned}$$

Putting all this together:

$$\overline{|\mathcal{M}|^2} = \frac{g_X^2}{12}(-g_{\mu\nu})\,Tr\left[\gamma^\mu\not{k}'\gamma^\nu\not{k}\right](c_v^2 + c_A^2)$$

Now we calculate the decay rate, Γ, but first let's go ahead and evaluate that trace

$$\Gamma(X \to f_1\bar{f}_2) = \frac{p_f}{32\pi^2 M_X^2}\int \overline{|\mathcal{M}|^2}\, d\Omega$$

Problem 2

Using the result of the previous problem, compute the total widths and branching ratios for the Z and W decays into all possible final-state fermions. Use $\sin^2\theta_W = 0.23$, $M_Z = 91\ GeV$, and $G_F = 1.17 \times 10^{-5}\ GeV^{-2}$.

Solution

First, the Z decays. The Z boson does not change flavor, so all decays must be of the same flavor, have zero net charge, and conserve lepton number, meaning for example that you can't get a final product containing two neutrinos that are not antiparticle versions of each other. Note that the top quark is too heavy to be a decay product from the Z boson.

quarks	$Z \to u\bar{u}$	$Z \to d\bar{d}$	$Z \to c\bar{c}$	$Z \to s\bar{s}$	$Z \to b\bar{b}$
leptons	$Z \to e^+e^-$	$Z \to \mu^+\mu^-$	$Z \to \tau^+\tau^-$	X	X
neutrinos	$Z \to \nu_e\bar{\nu}_e$	$Z \to \nu_\mu\bar{\nu}_\mu$	$Z \to \nu_\tau\bar{\nu}_\tau$	X	X

The decay widths: (all of these have the form $Z \to ff$, so in the equations I just put which type of fermion it decays to)

$$\begin{aligned}
\Gamma(q^+) =& \frac{G^2 M_Z (c_{v,q^+}^2 + c_{A,q^+}^2)}{48\pi} \\
=& 2.37 \times 10^{-11} \\
\Gamma(q^-) =& \frac{G^2 M_Z (c_{v,q^-}^2 + c_{A,q^-}^2)}{48\pi} \\
=& 3.08 \times 10^{-11} \\
\Gamma(l) =& \frac{G^2 M_Z (c_{v,l}^2 + c_{A,l}^2)}{48\pi} \\
=& 2.08 \times 10^{-11} \\
\Gamma(\nu) =& \frac{G^2 M_Z (c_{v,\nu}^2 + c_{A,\nu}^2)}{48\pi} \\
=& 4.13 \times 10^{-11} \\
\Gamma_Z =& 2\Gamma(q^+) + 3(\Gamma(q^-) + \Gamma(l) + \Gamma(\nu)) \\
=& 3.24 \times 10^{-10}
\end{aligned}$$

Then the branching ratios are:

$$\begin{aligned}
\Gamma(q^+)/\Gamma_Z =& 0.073 \\
\Gamma(q^-)/\Gamma_Z =& 0.094 \\
\Gamma(l)/\Gamma_Z =& 0.064 \\
\Gamma(\nu)/\Gamma_Z =& 0.13
\end{aligned}$$

Problem 3

Solution