

Physics 926: Homework #11

Due on April 14, 2020 at 5pm

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*In addition to the lecture notes, the following resources were used to better understand the material:
<https://arxiv.org/ftp/arxiv/papers/1511/1511.06752.pdf>

Problem 1

Show that

$$P(\nu_1 \rightarrow \nu_2) = \sin^2 2\theta \sin^2 \frac{\Delta m_{12}^2 L}{4E}$$

for a two neutrino system in which the mixing matrix is

$$U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}, \quad U^* = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

and $\Delta m_{12}^2 = m_1^2 - m_2^2$

Solution

Equation (7) from the lecture gives us a good starting point. This equation is an approximation which is valid when mass is small, which in this case it is

$$P(\nu_1 \rightarrow \nu_2) = \left| \sum_i U_{1i}^* U_{2i} e^{-im_i^2 L/2E} \right|^2$$

Using this equation as a starting point, we can plug in the given values of U and explicitly do the sum.

$$\begin{aligned} \left| \sum_i U_{1i}^* U_{2i} e^{-im_i^2 L/2E} \right|^2 &= \left[\sum_i U_{1i}^* U_{2i} e^{-im_i^2 L/2E} \right] \left[\sum_j U_{1j}^* U_{2j} e^{-im_j^2 L/2E} \right]^* \\ &= \left[\sum_i U_{1i}^* U_{2i} e^{-im_i^2 L/2E} \right] \left[\sum_j U_{2j}^* U_{1j} e^{im_j^2 L/2E} \right] \\ \sum_i U_{1i}^* U_{2i} e^{-im_i^2 L/2E} &= \cos \theta (-\sin \theta) e^{-im_1^2 L/2E} + (-\sin \theta) \cos \theta e^{-im_2^2 L/2E} \\ &= -\cos \theta \sin \theta e^{-im_1^2 L/2E} - \cos \theta \sin \theta e^{-im_2^2 L/2E} \\ &= -\cos \theta \sin \theta \left(e^{-im_1^2 L/2E} + e^{-im_2^2 L/2E} \right) \\ \sum_j U_{2j}^* U_{1j} e^{im_j^2 L/2E} &= \sin \theta \cos \theta e^{im_1^2 L/2E} + \cos \theta \sin \theta e^{im_2^2 L/2E} \\ &= \cos \theta \sin \theta \left(e^{im_1^2 L/2E} + e^{im_2^2 L/2E} \right) \\ \left[\sum_i U_{1i}^* U_{2i} e^{-im_i^2 L/2E} \right] \left[\sum_j U_{2j}^* U_{1j} e^{im_j^2 L/2E} \right] &= -\cos^2 \theta \sin^2 \theta \left(e^{-im_1^2 L/2E} + e^{-im_2^2 L/2E} \right) \left(e^{im_1^2 L/2E} + e^{im_2^2 L/2E} \right) \\ &= -\cos^2 \theta \sin^2 \theta \left(2 + e^{-i(m_1^2 - m_2^2)L/2E} + e^{i(m_1^2 - m_2^2)L/2E} \right) \\ &= -\cos^2 \theta \sin^2 \theta \left(2 + 2\operatorname{Re} \left[e^{-i2(m_1^2 - m_2^2)L/4E} \right] \right) \\ &= -\frac{1}{2} \sin^2 2\theta \left(1 + \cos^2 \frac{\Delta m_{12}^2 L}{4E} \right) \end{aligned}$$

Problem 2

Problem 3