Physics 926: Homework #9

Due on March 31, 2020 at 5pm

Professor Ken Bloom

Robert Tabb

Problem 1

Show that in $\pi \to \mu\nu$ decay, $|\vec{p}_\mu| = |\vec{p}_\nu| = (m_\pi^2 - m_\mu^2)/2m_\pi$

Solution

Start by defining the momentum four-vectors for each particle in the center of momentum frame. Note that σ is the index while μ, ν, and π are the names of the particles.

$$P_\pi^\sigma = (m_\pi, \vec{0})$$
$$P_\mu^\sigma = (E_\mu, \vec{p})$$
$$P_\nu^\sigma = (E_\nu, \vec{-p})$$

Due to conservation of four-momentum, we can write:

$$P_\pi^\sigma = P_\mu^\sigma + P_\nu^\sigma$$
$$P_\pi^\sigma - P_\nu^\sigma = P_\mu^\sigma$$

Now we contract each side with itself which we can do since this operation is Lorentz invariant:

$$(P_\pi - P_\nu)^\sigma (P_\pi - P_\nu)_\sigma = P_\mu^\sigma P_{\mu,\sigma}$$
$$P_\pi^2 + P_\nu^2 - 2P_\nu^\sigma P_{\pi,\sigma} = m_\mu^2$$

Since we are assuming the the neutrino is massless, $P_\nu^2 = 0$ and $E_\nu = |\vec{p}|$

$$m_\pi^2 - 2|\vec{p}|m_\pi = m_\mu^2$$
$$-2|\vec{p}|m_\pi = m_\mu^2 - m_\pi^2$$
$$|\vec{p}| = \frac{m_\pi^2 - m_\mu^2}{2m_\pi}$$

Problem 2

(H&M exercise 12.13) Predict the ratio of the $K^- \to e^- \ \bar{\nu}_e$ and $K^- \to \mu^- \ \bar{\nu}_\mu$ decay rates. Given that the lifetime of the K is $\tau = 1.2 \times 10^{-8} s$ and the $K \to \mu\nu$ branching ratio is 64%, estimate the decay constant f_K. Comment on your assumptions and your result.

Solution

Starting with $K^- \to e^- \ \bar{\nu}_e$, and following the procedure outlined in H&M for π^- decay (p.265):

$$\mathcal{M} = \frac{G}{\sqrt{2}} q^\sigma f_k \bar{u}(p_3)\gamma_\sigma(1-\gamma^5)v(p_4)$$

$$q = p_3 + p_4$$

$$\Rightarrow \mathcal{M} = \frac{G f_k}{\sqrt{2}}(p_3^\sigma + p_4^\sigma)\bar{u}(p_3)\gamma_\sigma(1-\gamma^5)v(p_4)$$

Problem 3

Solution

Problem 4

Solution

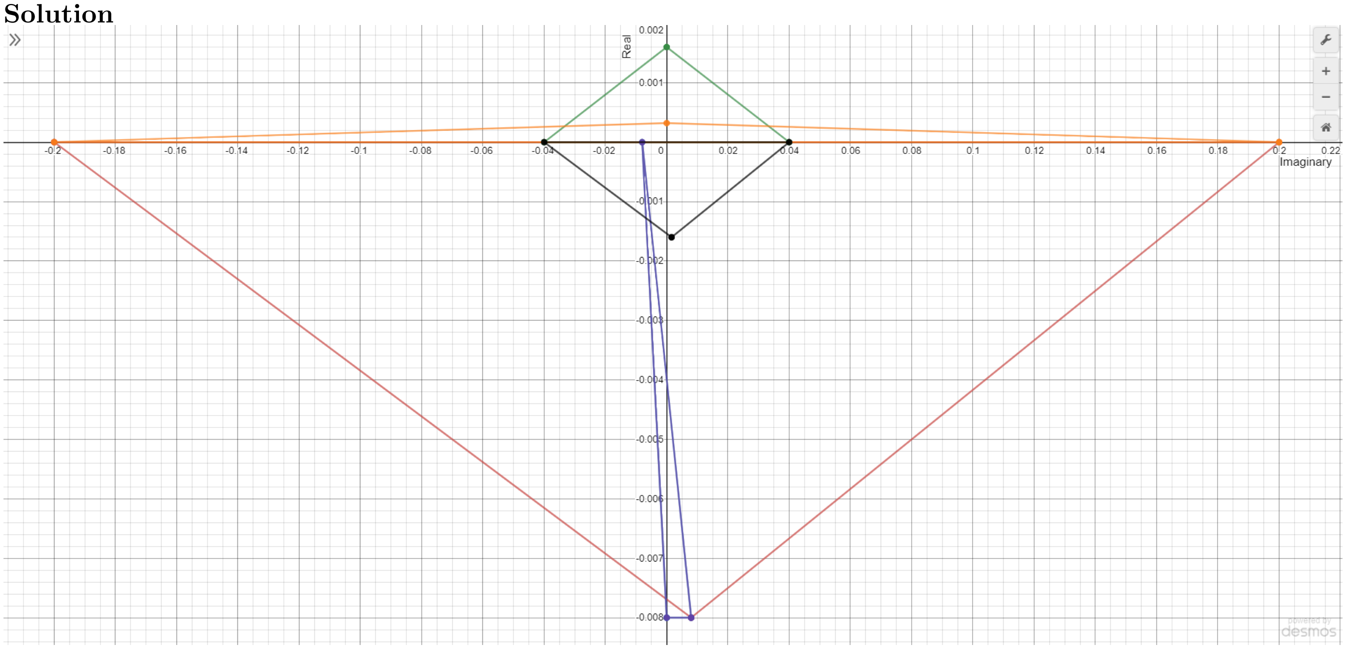
Problem 5

Solution

Problem 6

Part a

Solution



Part b

Solution