Physics 926: Homework #10

Due on April 7, 2020 at 5pm

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Problem 1

Show that:

$$\frac{\Gamma(K_L \to \pi^- e^+ \nu_e) - \Gamma(K_L \to \pi^+ e^- \bar{\nu}_e))}{\Gamma(K_L \to \pi^- e^+ \nu_e) + \Gamma(K_L \to \pi^+ e^- \bar{\nu}_e))} = 2Re(\epsilon)$$

to first order in ϵ. This asymmetry is evidence for indirect CP violation, and also allows us to unambiguously define electric charge - positive charge is assigned to the lepton that dominates in the K_L decay.

Solution

$$|K_L\rangle = \frac{1}{\sqrt{1+|\epsilon|}} \left[\frac{1+\epsilon}{\sqrt{2}} |K^0\rangle - \frac{1-\epsilon}{\sqrt{2}} |\bar{K^0}\rangle \right]$$

$\bar{K^0} \to \pi^+ e^- \bar{\nu}_e$ and $K^0 \to \pi^- e^+ \nu_e$ (see Figure 1), therefore to get $K_L \to \pi^+ e^- \bar{\nu}_e$, take the inner product of $\bar{K^0}$ with K_L and to get $K_L \to \pi^- e^+ \nu_e$, take the inner product of K^0 with K_L.

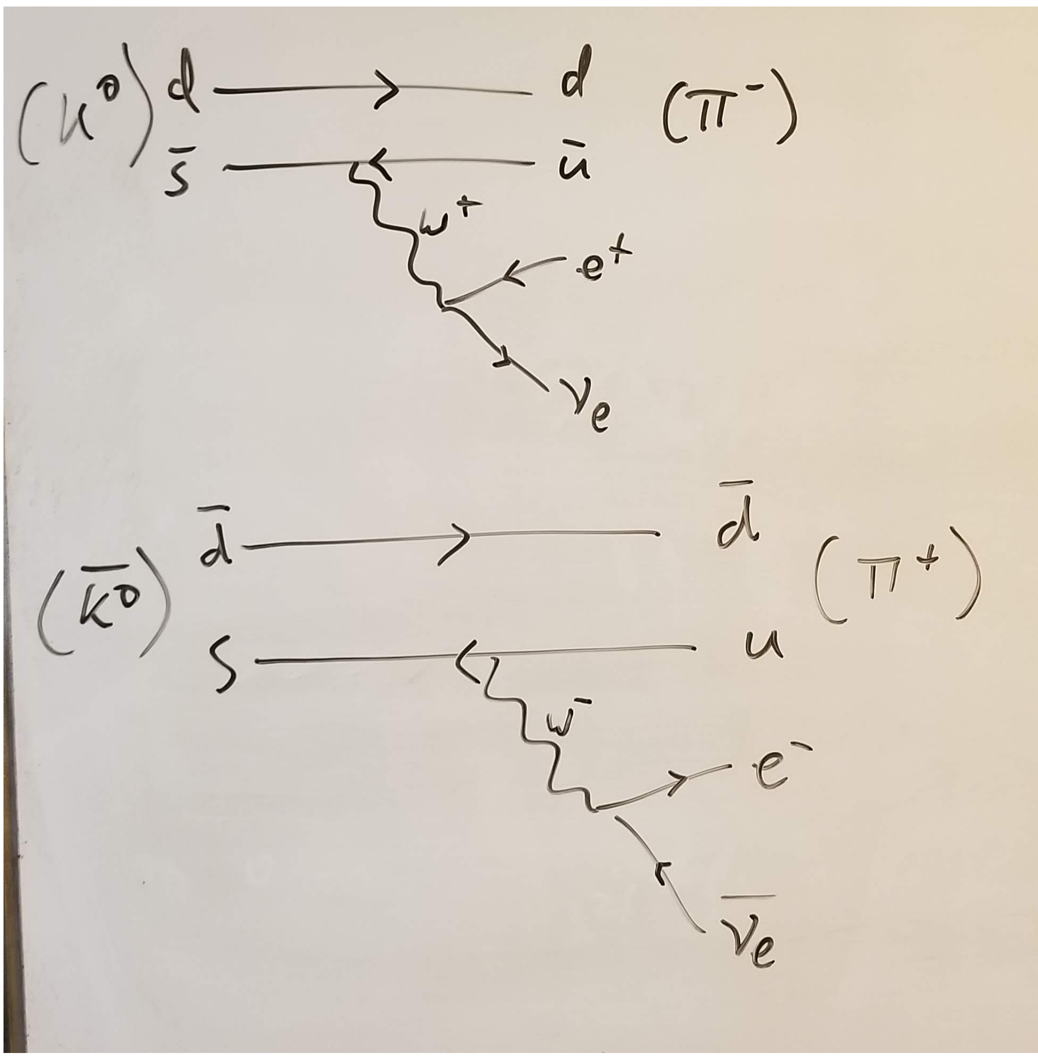

Figure 1: The two neutral kaon decays

$$\Gamma(K_L \to \pi^+ e^- \bar{\nu}_e) \propto \left|\langle \bar{K}^0 | K_L \rangle\right|^2 \propto |1-\epsilon|^2 = (1-\epsilon)(1-\epsilon^*) = 1 - \epsilon^* - \epsilon + |\epsilon|^2 \approx 1 - 2Re(\epsilon)$$
$$\Gamma(K_L \to \pi^- e^+ \nu_e) \propto \left|\langle K^0 | K_L \rangle\right|^2 \propto |1+\epsilon|^2 = (1+\epsilon)(1+\epsilon^*) = 1 + \epsilon^* + \epsilon + |\epsilon|^2 \approx 1 + 2Re(\epsilon)$$

Here I dropped the ϵ^2 term since we are only looking to first order in ϵ. I also dropped any constants since they will be the same for each term and will divide out in the end.

$$\begin{aligned}\frac{\Gamma(K_L \to \pi^- e^+ \nu_e) - \Gamma(K_L \to \pi^+ e^- \bar{\nu}_e))}{\Gamma(K_L \to \pi^- e^+ \nu_e) + \Gamma(K_L \to \pi^+ e^- \bar{\nu}_e))} &= \frac{1 + 2Re(\epsilon) - (1 - 2Re(\epsilon))}{1 + 2Re(\epsilon) + (1 - 2Re(\epsilon))} \\ &= \frac{4Re(\epsilon)}{2} = 2Re(\epsilon)\end{aligned}$$

Problem 2

Defining

$$\eta_{\pm} = |\eta_{\pm}| e^{i\phi_{\pm}} = \frac{\langle \pi^+\pi^- | K_L \rangle}{\langle \pi^+\pi^- | K_S \rangle}$$

calculate the probabilities of $\pi^+\pi^-$ decay as a function of proper time for an initial K^0 or $\bar{K^0}$ produced at $t = 0$. Express your answer, up to common proportionality constants, in terms of $\epsilon, |\eta_{\pm}|, \phi_{\pm}, \Delta m, \Gamma_S$, and Γ_L, where $\Delta m = K_S - K_L$. Keep only leading terms in ϵ. Using the experimental values for these quantities, plot the two probabilities as a function of time in units of the K_S lifetime, going out to 30 K_S lifetimes.

Solution

Here are the definitions of K_L and K_S in terms of K^0 and $\bar{K^0}$:

$$|K_L\rangle = \frac{1}{\sqrt{1+|\epsilon|}} \left[\frac{1+\epsilon}{\sqrt{2}} |K^0\rangle - \frac{1-\epsilon}{\sqrt{2}} |\bar{K^0}\rangle \right]$$
$$|K_S\rangle = \frac{1}{\sqrt{1+|\epsilon|}} \left[\frac{1+\epsilon}{\sqrt{2}} |K^0\rangle + \frac{1-\epsilon}{\sqrt{2}} |\bar{K^0}\rangle \right]$$

And here they are in terms of the CP eigenstates:

$$|K_L\rangle = \frac{1}{\sqrt{1+|\epsilon|}} \left[|K_2\rangle + \epsilon |K_1\rangle \right]$$
$$|K_S\rangle = \frac{1}{\sqrt{1+|\epsilon|}} \left[|K_1\rangle + \epsilon |K_2\rangle \right]$$

First, define the time-evolution of the wave function in the same way it was done in the lecture notes:

$$|K_S(t)\rangle = |K_S\rangle e^{im_S t - \Gamma_S t/2}$$
$$|K_L(t)\rangle = |K_L\rangle e^{im_L t - \Gamma_L t/2}$$

Here I will write the total wave function as a function of time but in the CP basis since we know that the $\pi^+\pi^-$ system is a CP eigenstate with an eigenvalue of $+1$. This will be helpful because we know the eigenvalues of $|K_1\rangle$ and $|K_2\rangle$

$$\begin{aligned}
|\psi_{K^0}(t)\rangle =& \frac{1}{\sqrt{2}} \left[|K_S(t)\rangle + |K_L(t)\rangle \right] \\
=& \frac{1}{\sqrt{2}} \left[|K_S\rangle e^{im_S t - \Gamma_S t/2} + |K_L\rangle e^{im_L t - \Gamma_L t/2} \right] \\
|\psi_{\bar{K^0}}(t)\rangle =& \frac{1}{\sqrt{2}} \left[|K_S\rangle e^{im_S t - \Gamma_S t/2} - |K_L\rangle e^{im_L t - \Gamma_L t/2} \right] \\
|\psi_{K^0}(t)\rangle =& \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+|\epsilon|}} \left[(|K_1\rangle + \epsilon |K_2\rangle) e^{im_S t - \Gamma_S t/2} + (|K_2\rangle + \epsilon |K_1\rangle) e^{im_L t - \Gamma_L t/2} \right] \\
=& \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+|\epsilon|}} \left[(e^{im_S t - \Gamma_S t/2} + \epsilon e^{im_L t - \Gamma_L t/2}) |K_1\rangle + (e^{im_L t - \Gamma_L t/2} + \epsilon e^{im_S t - \Gamma_S t/2}) |K_2\rangle \right] \\
|\psi_{\bar{K^0}}(t)\rangle =& \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+|\epsilon|}} \left[(e^{im_S t - \Gamma_S t/2} - \epsilon e^{im_L t - \Gamma_L t/2}) |K_1\rangle + (-e^{im_L t - \Gamma_L t/2} + \epsilon e^{im_S t - \Gamma_S t/2}) |K_2\rangle \right]
\end{aligned}$$

The subscript on the wave function state refers to the initial beam being either purely K^0 or purely $\bar{K^0}$.

Problem 2 continued on next page. . .

The probability to find the system in the state $|\pi^+\pi^-\rangle$ can be found like this:

$$\begin{aligned}
|\langle K_1|\psi_{K^0}(t)\rangle|^2 =& \frac{1}{2+2|\epsilon|}\left[(e^{im_S t-\Gamma_S t/2}+\epsilon e^{im_L t-\Gamma_L t/2})(e^{im_S t-\Gamma_S t/2}+\epsilon e^{im_L t-\Gamma_L t/2})^*\right] \\
=& \frac{1}{2+2|\epsilon|}\left[e^{-\Gamma_S t}+|\epsilon|^2 e^{-\Gamma_L t}+\epsilon^* e^{i(m_S-m_L)t}e^{-(\Gamma_S+\Gamma_L)t/2}+\epsilon e^{-i(m_S-m_L)t}e^{-(\Gamma_S+\Gamma_L)t/2}\right] \\
\approx& \frac{1}{2+2|\epsilon|}\left[e^{-\Gamma_S t}+2Re(\epsilon e^{i\Delta m t}e^{-(\Gamma_S+\Gamma_L)t/2})\right] \\
|\langle K_1|\psi_{\bar{K^0}}(t)\rangle|^2 \approx& \frac{1}{2+2|\epsilon|}\left[e^{-\Gamma_S t}-2Re(\epsilon e^{i\Delta m t}e^{-(\Gamma_S+\Gamma_L)t/2})\right]
\end{aligned}$$

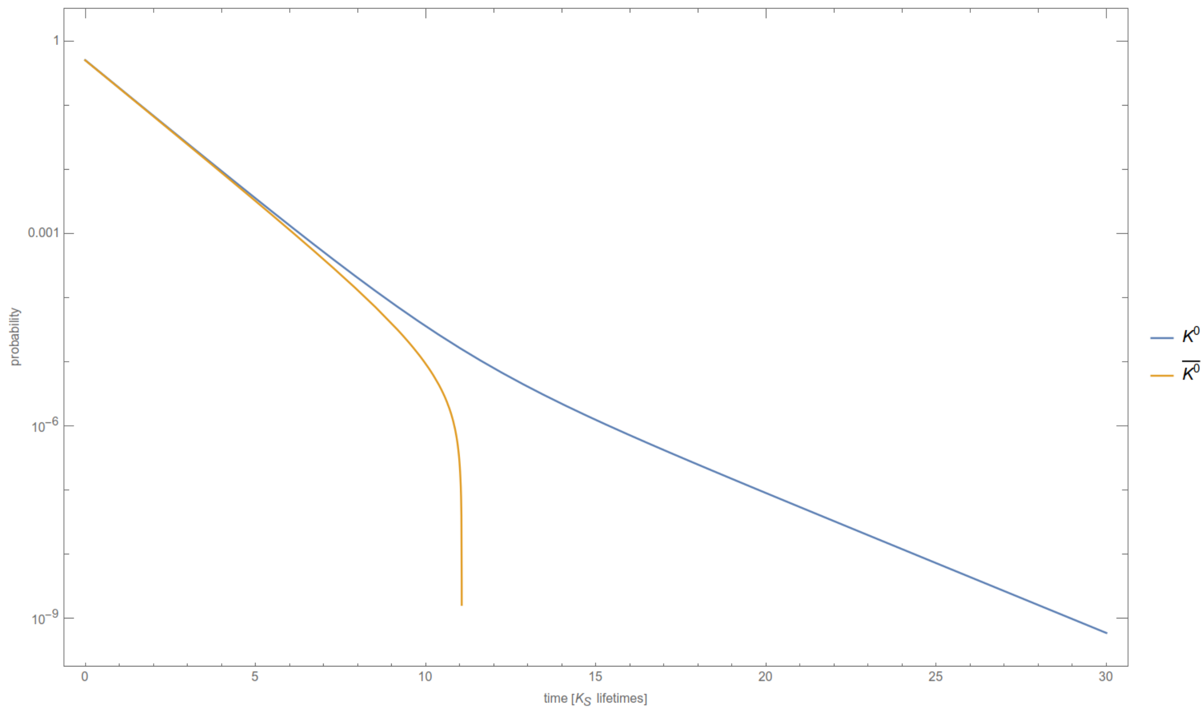

Figure 2: Probability for K^0 and $\bar{K^0}$ to decay to $\pi^+\pi^-$ as a function of K_S lifetime.

Problem 3

Problem 4