Physics 926
Homework 12
Due Tuesday, April 21

1. (Halzen & Martin, Exercise 13.2) If the vertex factor for the decay of a vector boson X into two spin-1/2 fermions f_1 and f_2 is

$$-ig_X\gamma^\mu\frac{1}{2}(c_v - c_A\gamma^5), \tag{1}$$

then show that

$$\Gamma(X \to f_1\bar{f}_2) = \frac{g_X^2}{48\pi}(c_v^2 + c_A^2)M_X, \tag{2}$$

where M_x is the mass of the boson and where we have neglected the mass of the fermions. Hints: Use

$$\sum_\lambda \epsilon_\mu^{(\lambda)*}\epsilon_\nu^\lambda = -g_{\mu\nu} + \frac{p_\mu p_\mu}{M^2} \tag{3}$$

to show that after summing over the fermion and averaging over the boson spins,

$$\overline{|\mathcal{M}|^2} = \frac{1}{12}g_X^2(c_V^2 + c_A^2)(-g_{\mu\nu})\mathrm{Tr}(\gamma^\mu \not{k}\gamma^\nu \not{k}') \tag{4}$$

where k, k' are the four-momenta of the fermions. Work in the boson rest frame, and use

$$\Gamma(X \to f_1\bar{f}_2) = \frac{p_f}{32\pi^2 m_X^2}\int \overline{|\mathcal{M}|^2}d\Omega. \tag{5}$$

2. Using the result of the previous problem, compute the total widths and branching ratios for Z and W decays into all possible final-state fermions. Use $\sin^2\theta_W = 0.23$, $M_Z = 91$ GeV and $G_F = 1.17 \times 10^{-5}$ GeV^{-2}. Be careful about the definition of "possible" here.

3. (H&M Exercise 14.12) The Lagrangian for three interacting real fields ϕ_1, ϕ_2, ϕ_3 is

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi_i)^2 - \frac{1}{2}\mu^2\phi_i^2 - \frac{1}{4}\lambda(\phi_i^2)^2 \tag{6}$$

with $\mu^2 < 0$ and $\lambda > 0$, and where a summation of ϕ_i^2 over i is implied. Show that it describes a massive field of mass $\sqrt{-2\mu^2}$ and two massless Goldstone bosons.