

Physics 926: Homework #10

Due on April 7, 2020 at 5pm

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Problem 1

Show that:

$$\frac{\Gamma(K_L \rightarrow \pi^- e^+ \nu_e) - \Gamma(K_L \rightarrow \pi^+ e^- \bar{\nu}_e)}{\Gamma(K_L \rightarrow \pi^- e^+ \nu_e) + \Gamma(K_L \rightarrow \pi^+ e^- \bar{\nu}_e)} = 2\text{Re}(\epsilon)$$

to first order in ϵ . This asymmetry is evidence for indirect CP violation, and also allows us to unambiguously define electric charge - positive charge is assigned to the lepton that dominates in the K_L decay.

Solution

$$|K_L\rangle = \frac{1}{\sqrt{1+|\epsilon|}} \left[\frac{1+\epsilon}{\sqrt{2}} |K^0\rangle - \frac{1-\epsilon}{\sqrt{2}} |\bar{K}^0\rangle \right]$$

$\bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu}_e$ and $K^0 \rightarrow \pi^- e^+ \nu_e$ (see Figure 1), therefore to get $K_L \rightarrow \pi^+ e^- \bar{\nu}_e$, take the inner product of \bar{K}^0 with K_L and to get $K_L \rightarrow \pi^- e^+ \nu_e$, take the inner product of K^0 with K_L .

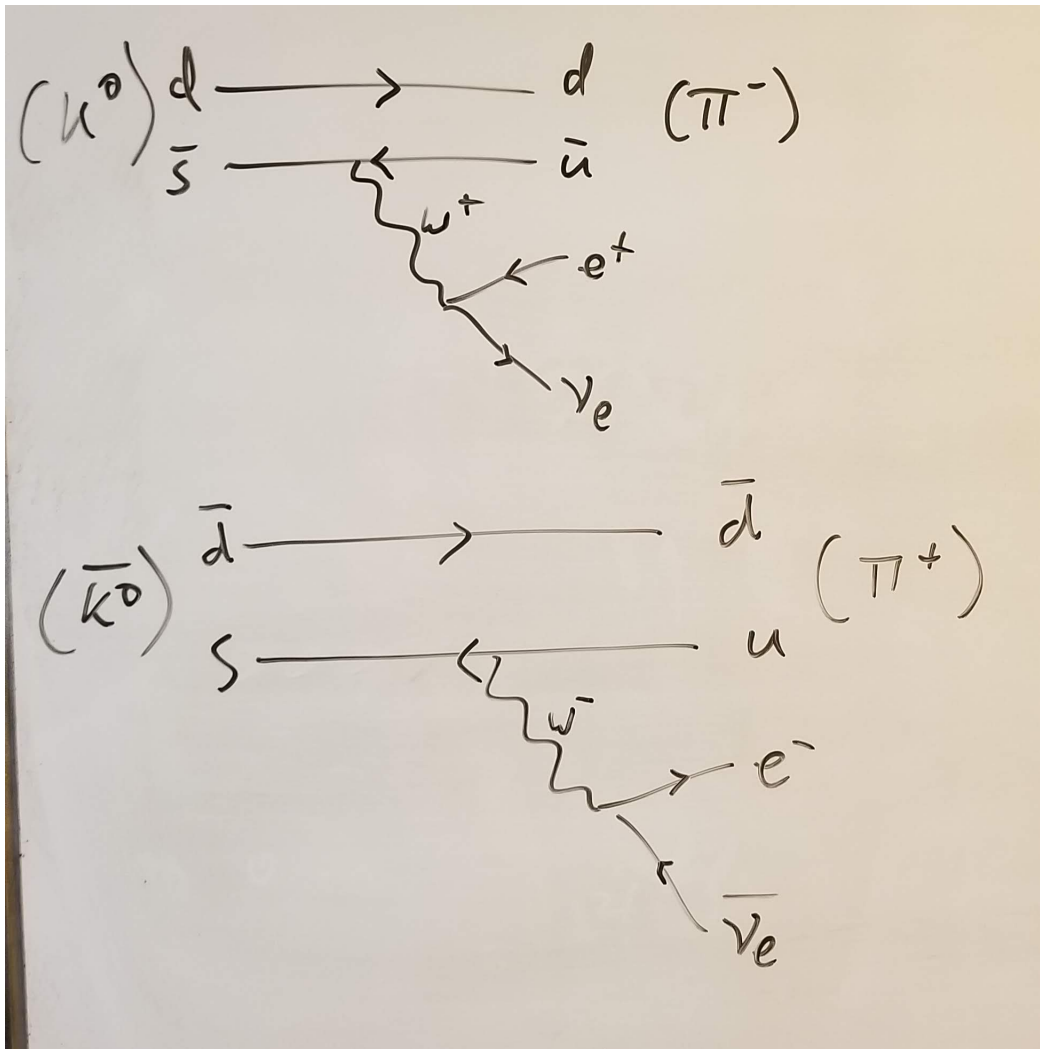


Figure 1: The two neutral kaon decays

$$\Gamma(K_L \rightarrow \pi^+ e^- \bar{\nu}_e) \propto |\langle \bar{K}^0 | K_L \rangle|^2 \propto |1 - \epsilon|^2 = (1 - \epsilon)(1 - \epsilon^*) = 1 - \epsilon^* - \epsilon + |\epsilon|^2 \approx 1 - 2\text{Re}(\epsilon)$$

$$\Gamma(K_L \rightarrow \pi^- e^+ \nu_e) \propto |\langle K^0 | K_L \rangle|^2 \propto |1 + \epsilon|^2 = (1 + \epsilon)(1 + \epsilon^*) = 1 + \epsilon^* + \epsilon + |\epsilon|^2 \approx 1 + 2\text{Re}(\epsilon)$$

Here I dropped the ϵ^2 term since we are only looking to first order in ϵ . I also dropped any constants since they will be the same for each term and will divide out in the end.

$$\begin{aligned} \frac{\Gamma(K_L \rightarrow \pi^- e^+ \nu_e) - \Gamma(K_L \rightarrow \pi^+ e^- \bar{\nu}_e)}{\Gamma(K_L \rightarrow \pi^- e^+ \nu_e) + \Gamma(K_L \rightarrow \pi^+ e^- \bar{\nu}_e)} &= \frac{1 + 2\text{Re}(\epsilon) - (1 - 2\text{Re}(\epsilon))}{1 + 2\text{Re}(\epsilon) + (1 - 2\text{Re}(\epsilon))} \\ &= \frac{4\text{Re}(\epsilon)}{2} = 2\text{Re}(\epsilon) \end{aligned}$$

Problem 2

Defining

$$\eta_{\pm} = |\eta_{\pm}| e^{i\phi_{\pm}} = \frac{\langle \pi^+ \pi^- | K_L \rangle}{\langle \pi^+ \pi^- | K_S \rangle}$$

calculate the probabilities of $\pi^+ \pi^-$ decay as a function of proper time for an initial K^0 or \bar{K}^0 produced at $t = 0$. Express your answer, up to common proportionality constants, in terms of ϵ , $|\eta_{\pm}|$, ϕ_{\pm} , Δm , Γ_S , and Γ_L , where $\Delta m = K_S - K_L$. Keep only leading terms in ϵ . Using the experimental values for these quantities, plot the two probabilities as a function of time in units of the K_S lifetime, going out to 30 K_S lifetimes.

Solution

Here are the definitions of K_L and K_S in terms of K^0 and \bar{K}^0 :

$$\begin{aligned} |K_L\rangle &= \frac{1}{\sqrt{1+|\epsilon|}} \left[\frac{1+\epsilon}{\sqrt{2}} |K^0\rangle - \frac{1-\epsilon}{\sqrt{2}} |\bar{K}^0\rangle \right] \\ |K_S\rangle &= \frac{1}{\sqrt{1+|\epsilon|}} \left[\frac{1+\epsilon}{\sqrt{2}} |K^0\rangle + \frac{1-\epsilon}{\sqrt{2}} |\bar{K}^0\rangle \right] \end{aligned}$$

And here they are in terms of the CP eigenstates:

$$\begin{aligned} |K_L\rangle &= \frac{1}{\sqrt{1+|\epsilon|}} [|K_2\rangle + \epsilon |K_1\rangle] \\ |K_S\rangle &= \frac{1}{\sqrt{1+|\epsilon|}} [|K_1\rangle + \epsilon |K_2\rangle] \end{aligned}$$

First, define the time-evolution of the wave function in the same way it was done in the lecture notes:

$$\begin{aligned} |K_S(t)\rangle &= |K_S\rangle e^{im_S t - \Gamma_S t/2} \\ |K_L(t)\rangle &= |K_L\rangle e^{im_L t - \Gamma_L t/2} \end{aligned}$$

Here I will write the total wave function as a function of time but in the CP basis since we know that the $\pi^+ \pi^-$ system is a CP eigenstate with an eigenvalue of +1.

$$\begin{aligned} |\psi_{K^0}(t)\rangle &= \frac{1}{\sqrt{2}} [|K_S(t)\rangle + |K_L(t)\rangle] \\ &= \frac{1}{\sqrt{2}} [|K_S\rangle e^{im_S t - \Gamma_S t/2} + |K_L\rangle e^{im_L t - \Gamma_L t/2}] \\ |\psi_{\bar{K}^0}(t)\rangle &= \frac{1}{\sqrt{2}} [|K_S(t)\rangle - |K_L(t)\rangle] \\ &= \frac{1}{\sqrt{2}} [|K_S\rangle e^{im_S t - \Gamma_S t/2} - |K_L\rangle e^{im_L t - \Gamma_L t/2}] \\ |\psi_{K^0}(t)\rangle &= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+|\epsilon|}} \left[(|K_1\rangle + \epsilon |K_2\rangle) e^{im_S t - \Gamma_S t/2} + (|K_2\rangle + \epsilon |K_1\rangle) e^{im_L t - \Gamma_L t/2} \right] \\ &= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+|\epsilon|}} \left[(e^{im_S t - \Gamma_S t/2} + \epsilon e^{im_L t - \Gamma_L t/2}) |K_1\rangle + (e^{im_L t - \Gamma_L t/2} + \epsilon e^{im_S t - \Gamma_S t/2}) |K_2\rangle \right] \\ |\psi_{\bar{K}^0}(t)\rangle &= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+|\epsilon|}} \left[(e^{im_S t - \Gamma_S t/2} - \epsilon e^{im_L t - \Gamma_L t/2}) |K_1\rangle + (-e^{im_L t - \Gamma_L t/2} + \epsilon e^{im_S t - \Gamma_S t/2}) |K_2\rangle \right] \end{aligned}$$

The subscript on the wave function state refers to the initial beam being either purely K^0 or purely \bar{K}^0 .

The probability to find the system in the state $|\pi^+\pi^-\rangle$ can be found like this:

$$\begin{aligned}
 |\langle K_1 | \psi_{K^0}(t) \rangle|^2 &= \frac{1}{2+2|\epsilon|} \left[(e^{im_S t - \Gamma_S t/2} + \epsilon e^{im_L t - \Gamma_L t/2})(e^{im_S t - \Gamma_S t/2} + \epsilon e^{im_L t - \Gamma_L t/2})^* \right] \\
 &= \frac{1}{2+2|\epsilon|} \left[e^{-\Gamma_S t} + |\epsilon|^2 e^{-\Gamma_L t} + \epsilon^* e^{i(m_S - m_L)t} e^{-(\Gamma_S + \Gamma_L)t/2} + \epsilon e^{-i(m_S - m_L)t} e^{-(\Gamma_S + \Gamma_L)t/2} \right] \\
 &\approx \frac{1}{2+2|\epsilon|} \left[e^{-\Gamma_S t} + 2\text{Re}(\epsilon e^{i\Delta m t} e^{-(\Gamma_S + \Gamma_L)t/2}) \right] \\
 |\langle K_1 | \psi_{\bar{K}^0}(t) \rangle|^2 &= \frac{1}{2+2|\epsilon|} \left[e^{-\Gamma_S t} - 2\text{Re}(\epsilon e^{i\Delta m t} e^{-(\Gamma_S + \Gamma_L)t/2}) \right]
 \end{aligned}$$

Problem 3

Problem 4