Physics 926: Homework #12

Due on April 21, 2020 at 5pm

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Problem 1

If the vertex factor for the decay of a vector boson X into two spin-1/2 fermions f_1 and f_2 is

$$-igx\gamma^\mu \frac{1}{2}(c_v - c_A\gamma^5)$$

then show that

$$\Gamma(X \to f_1\bar{f}_2) = \frac{g_X^2}{48\pi}(c_v^2 + c_A^2)M_X$$

where M_X is the mass of the boson and where we have neglected the mass of the fermions. Hints: use

$$\sum_\lambda \epsilon_\mu^{(\lambda)*}\epsilon_\nu^\lambda = -g_{\mu\nu} + \frac{p_\mu p_\nu}{M^2}$$

to show that after summing over the fermions and averaging over the boson spins ,

$$\overline{|\mathcal{M}|^2} = \frac{1}{12}g_X^2(c_v^2 + c_A^2)(-g_{\mu\nu})Tr(\gamma^\mu \not{k}\gamma^\nu \not{k}')$$

where k and k' are the four-momenta of the fermions. Work in the boson rest frame, and use

$$\Gamma(X \to f_1\bar{f}_2)\frac{p_f}{32\pi^2 m_X^2}\int \overline{|\mathcal{M}|^2}d\Omega$$

Solution

Using the vertex factor given in the problem, begin by writing the matrix element

$$\begin{aligned}
\mathcal{M} =& \bar{u}(k)\left[-ig_X\gamma^\mu\frac{1}{2}(c_v - c_A\gamma^5)\right]v(k')\epsilon_\mu \\
=& -\frac{ig_X}{2}\left[\bar{u}(k)\gamma^\mu(c_v - c_A\gamma^5)v(k')\epsilon_\mu\right] \\
|\mathcal{M}|^2 =& \frac{g_X^2}{4}\left[\bar{u}(k)\gamma^\mu(c_v - c_A\gamma^5)v(k')\epsilon_\mu\right]\left[\bar{u}(k)\gamma^\nu(c_v - c_A\gamma^5)v(k')\epsilon_\nu\right]^*
\end{aligned}$$

We now need to write the average by summing over the spins and polarizations. In this step, assume the fermion masses can be neglected and use the formula: $\sum_s[\bar{u}(a)\Gamma_1 v(b)][\bar{u}(a)\Gamma_1 v(b)]^* = Tr[\Gamma_1\not{b}\bar{\Gamma}_2\not{a}]$, recalling that $\bar{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0$

$$\begin{aligned}
\overline{|\mathcal{M}|^2} =& \frac{1}{4}\sum_{s,\lambda}|\mathcal{M}|^2 \\
=& \frac{g_X^2}{16}\sum_\lambda(\epsilon_\mu\epsilon_\nu^*)\sum_s\left[\bar{u}(k)\gamma^\mu(c_v - c_A\gamma^5)v(k')\right]\left[\bar{u}(k)\gamma^\nu(c_v - c_A\gamma^5)v(k')\right]^* \\
=& \frac{g_X^2}{16}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_X^2}\right)Tr\left[\gamma^\mu(c_v - c_A\gamma^5)\not{k}'\gamma^0(\gamma^\nu(c_v - c_A\gamma^5))^\dagger\gamma^0\not{k}\right] \\
=& \frac{g_X^2}{16}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_X^2}\right)Tr\left[(c_v\gamma^\mu\not{k}' - c_A\gamma^\mu\gamma^5\not{k}')(c_v\gamma^0\gamma^{\nu\dagger}\gamma^0\not{k} - c_A\gamma^0\gamma^{5\dagger}\gamma^{\nu\dagger}\gamma^0\not{k})\right] \\
=& \frac{g_X^2}{16}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_X^2}\right)Tr\left[(c_v\gamma^\mu\not{k}' - c_A\gamma^\mu\gamma^5\not{k}')(c_v\gamma^\nu\not{k} + c_A\gamma^5\gamma^\nu\not{k})\right]
\end{aligned}$$

Where the plus sign in the last line comes from letting $\gamma^0\gamma^5 \to \gamma^5\gamma^0$ Now let's evaluate that trace

$$\begin{aligned}
Tr\left[(c_v\gamma^\mu \not{k}' - c_A\gamma^\mu\gamma^5\not{k}')(c_v\gamma^\nu\not{k} + c_A\gamma^5\gamma^\nu\not{k})\right] =& Tr\left[c_v^2\gamma^\mu\not{k}'\gamma^\nu\not{k} - c_A^2\gamma^\mu\gamma^5\not{k}'\gamma^5\gamma^\nu\not{k}\right] \\
=& c_v^2 Tr\left[\gamma^\mu\not{k}'\gamma^\nu\not{k}\right] - c_A^2 Tr\left[\gamma^\mu\gamma^5\not{k}'\gamma^5\gamma^\nu\not{k}\right] \\
=& c_v^2 Tr\left[\gamma^\mu\not{k}'\gamma^\nu\not{k}\right] - c_A^2 Tr\left[-\gamma^\mu\not{k}'\gamma^5\gamma^5\gamma^\nu\not{k}\right] \\
=& c_v^2 Tr\left[\gamma^\mu\not{k}'\gamma^\nu\not{k}\right] + c_A^2 Tr\left[\gamma^\mu\not{k}'\gamma^\nu\not{k}\right] \\
=& 2Tr\left[\gamma^\mu\not{k}'\gamma^\nu\not{k}\right](c_v^2 + c_A^2)
\end{aligned}$$

Putting all this together:

$$\overline{|\mathcal{M}|^2} = \frac{g_X^2}{8}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_X^2}\right) Tr\left[\gamma^\mu\not{k}'\gamma^\nu\not{k}\right](c_v^2 + c_A^2)$$

Problem 2

Solution

Problem 3

Solution