Physics 926: Homework #10

Due on April 7, 2020 at 5pm

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Problem 1

Show that:

$$\frac{\Gamma(K_L \to \pi^- e^+ \nu_e) - \Gamma(K_L \to \pi^+ e^- \bar{\nu}_e))}{\Gamma(K_L \to \pi^- e^+ \nu_e) + \Gamma(K_L \to \pi^+ e^- \bar{\nu}_e))} = 2Re(\epsilon)$$

to first order in ϵ. This asymmetry is evidence for indirect CP violation, and also allows us to unambiguously define electric charge - positive charge is assigned to the lepton that dominates in the K_L decay.

Solution

$$|K_L\rangle = \frac{1}{\sqrt{1+|\epsilon|}}\left[\frac{1+\epsilon}{\sqrt{2}}\,|K^0\rangle - \frac{1-\epsilon}{\sqrt{2}}\,|\bar{K}^0\rangle\right]$$

For $K_L \to \pi^- e^+ \nu_e$, the decay has to come from the $|\bar{K}^0\rangle$ component of $|K_L\rangle$ and for $K_L \to \pi^+ e^- \bar{\nu}_e$, the decay must come from the $|K^0\rangle$ component.

$$\Gamma(K_L \to \pi^+ e^- \bar{\nu}_e) \propto \left|\langle\bar{K}^0|K_L\rangle\right|^2 \propto |1-\epsilon|^2 = (1-\epsilon)(1-\epsilon^*) = 1-\epsilon^*-\epsilon+|\epsilon|^2 \approx 1-2Re(\epsilon)$$
$$\Gamma(K_L \to \pi^- e^+ \nu_e) \propto \left|\langle K^0|K_L\rangle\right|^2 \propto |1+\epsilon|^2 = (1+\epsilon)(1+\epsilon^*) = 1+\epsilon^*+\epsilon+|\epsilon|^2 \approx 1+2Re(\epsilon)$$

Here I dropped the ϵ^2 term since we are only looking to first order in ϵ. I also dropped any constants since they will be the same for each term and will divide out in the end.

$$\begin{aligned}\frac{\Gamma(K_L \to \pi^- e^+ \nu_e) - \Gamma(K_L \to \pi^+ e^- \bar{\nu}_e))}{\Gamma(K_L \to \pi^- e^+ \nu_e) + \Gamma(K_L \to \pi^+ e^- \bar{\nu}_e))} &= \frac{1+2Re(\epsilon)-1+2Re(\epsilon)}{1+2Re(\epsilon)+1-2Re(\epsilon)} \\ &= \frac{4Re(\epsilon)}{2} = 2Re(\epsilon)\end{aligned}$$

Problem 2

Problem 3

Problem 4