Physics 926: Homework #11

Due on April 14, 2020 at 5pm

Professor Ken Bloom

Robert Tabb

*In addition to the lecture notes, the following resources were used to better understand the material: `https://arxiv.org/ftp/arxiv/papers/1511/1511.06752.pdf`

Problem 1

Show that

$$P(\nu_1 \to \nu_2) = \sin^2 2\theta \sin^2 \frac{\Delta m_{12}^2 L}{4E}$$

for a two neutrino system in which the mixing matrix is

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

and $\Delta m_{12}^2 = m_1^2 - m_2^2$

Solution

Equation (7) from the lecture gives us a good starting point. This equation is an approximation which is valid when mass is small, which in this case it is

$$P(\nu_1 \to \nu_2) = \left| \sum_i U_{1i}^* U_{2i} e^{-im_i^2 L/2E} \right|^2$$

Using this equation as a starting point, we can plug in the given values of U and explicitly do the sum.

$$\begin{aligned}
\left| \sum_i U_{1i}^* U_{2i} e^{-im_i^2 L/2E} \right|^2 &= \left[\sum_i U_{1i}^* U_{2i} e^{-im_i^2 L/2E} \right] \left[\sum_j U_{1j}^* U_{2j} e^{-im_j^2 L/2E} \right]^* \\
&= \left[\sum_i U_{1i}^* U_{2i} e^{-im_i^2 L/2E} \right] \left[\sum_j U_{2j}^* U_{1j} e^{im_j^2 L/2E} \right] \\
\sum_i U_{1i}^* U_{2i} e^{-im_i^2 L/2E} &= \cos\theta(-\sin\theta) e^{-im_1^2 L/2E} + \sin\theta\cos\theta e^{-im_2^2 L/2E} \\
&= -\cos\theta\sin\theta e^{-im_1^2 L/2E} + \cos\theta\sin\theta e^{-im_2^2 L/2E} \\
&= \cos\theta\sin\theta \left(e^{-im_2^2 L/2E} - e^{-im_1^2 L/2E} \right) \\
\sum_j U_{2j}^* U_{1j} e^{im_j^2 L/2E} &= -\sin\theta\cos\theta e^{im_1^2 L/2E} + \cos\theta\sin\theta e^{im_2^2 L/2E} \\
&= \cos\theta\sin\theta \left(e^{im_2^2 L/2E} - e^{im_1^2 L/2E} \right)
\end{aligned}$$

$$\begin{aligned}
\left[\sum_i U_{1i}^* U_{2i} e^{-im_i^2 L/2E} \right] \left[\sum_j U_{2j}^* U_{1j} e^{im_j^2 L/2E} \right] &= \cos^2\theta\sin^2\theta \left(e^{-im_2^2 L/2E} - e^{-im_1^2 L/2E} \right) \left(e^{im_2^2 L/2E} - e^{im_1^2 L/2E} \right) \\
&= \cos^2\theta\sin^2\theta \left(2 - e^{i(m_1^2 - m_2^2)L/2E} - e^{-i(m_1^2 - m_2^2)L/2E} \right) \\
&= \cos^2\theta\sin^2\theta \left(2 - e^{2i(m_1^2 - m_2^2)L/4E} - e^{-2i(m_1^2 - m_2^2)L/4E} \right) \\
&= \cos^2\theta\sin^2\theta \left(2 - 2Re\left[e^{i(m_1^2 - m_2^2)L/4E} \right]^2 \right) \\
&=2\cos^2\theta\sin^2\theta \left(1 - \cos^2 \frac{\Delta m_{12} L}{4E} \right) \\
&= \sin^2 2\theta sin^2 \frac{\Delta m_{12} L}{4E}
\end{aligned}$$

Problem 2

Problem 3