

# Physics 926: Homework #9

Due on March 31, 2020 at 5pm

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## Problem 1

Show that in  $\pi \rightarrow \mu \nu$  decay,  $|\vec{p}_\mu| = |\vec{p}_\nu| = (m_\pi^2 - m_\mu^2)/2m_\pi$

### Solution

Start by defining the momentum four-vectors for each particle in the center of momentum frame. Note that  $\sigma$  is the index while  $\mu$ ,  $\nu$ , and  $\pi$  are the names of the particles.

$$\begin{aligned} P_\pi^\sigma &= (m_\pi, \vec{0}) \\ P_\mu^\sigma &= (E_\mu, \vec{p}) \\ P_\nu^\sigma &= (E_\nu, -\vec{p}) \end{aligned}$$

Due to conservation of four-momentum, we can write:

$$\begin{aligned} P_\pi^\sigma &= P_\mu^\sigma + P_\nu^\sigma \\ P_\pi^\sigma - P_\nu^\sigma &= P_\mu^\sigma \end{aligned}$$

Now we contract each side with itself which we can do since this operation is Lorentz invariant:

$$\begin{aligned} (P_\pi - P_\nu)^\sigma (P_\pi - P_\nu)_\sigma &= P_\mu^\sigma P_{\mu,\sigma} \\ P_\pi^2 + P_\nu^2 - 2P_\nu^\sigma P_{\pi,\sigma} &= m_\mu^2 \end{aligned}$$

Since we are assuming the the neutrino is massless,  $P_\nu^2 = 0$  and  $E_\nu = |\vec{p}|$

$$\begin{aligned} m_\pi^2 - 2|\vec{p}|m_\pi &= m_\mu^2 \\ -2|\vec{p}|m_\pi &= m_\mu^2 - m_\pi^2 \\ |\vec{p}| &= \frac{m_\pi^2 - m_\mu^2}{2m_\pi} \end{aligned}$$

## Problem 2

(H&M exercise 12.13) Predict the ratio of the  $K \rightarrow e \bar{\nu}_e$  and  $K \rightarrow \mu \bar{\nu}_\mu$  decay rates. Given that the lifetime of the K is  $\tau = 1.2 \times 10^{-8} s$  and the  $K \rightarrow \mu \nu$  branching ratio is 64%, estimate the decay constant  $f_K$ . Comment on your assumptions and your result.

**Solution** Starting with  $K \rightarrow e \bar{\nu}_e$ :

## Problem 3

## Problem 4

## Problem 5

## Problem 6

Part a

Part b