PhD Proposal Writeup

A realtime and parallel look-ahead control and feedrate compensation strategy for CNC reference-pulse interpolation.

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1 Simulation Results

1.1 The Parametric Equations

The ten(10) 2D parametric curves covered in this work are:

1. Teardrop
2. Butterfly
3. Ellipse
4. Skewed-Astroid
5. Circle
6. AstEpi = Astroid + Epicycloid combination
7. Snailshell
8. SnaHyp = Snailshell + Hypotrocoid combination
9. Ribbon-10L
10. Ribbon-100l = 10 times scaleup of Ribbon-10L

The parametric equations describing each of the curves x(u), and y(u) are provided in the next table. The independent parameter u is limited to

$$u \in [0.0, 1.0]$$

The curves were selected based on their different characteristics like closed loop curves, open ended curves, symmetric or non-symmetric about the x-axis and y-axis, and having concave or convex turns. The x and y dimensions (sizes) vary among the different curves.

The main objective of the selection criteria is to ensure that the interpolation algorithm for the parametric curve succeeds and does not break in all cases.

The results for the feedrates in machining the ten(10) curves show continuity, smoothness, with no abrupt jumps as the CNC machine traverse the entire curve from the start (u = 0.0) until the end (u = 1.0).

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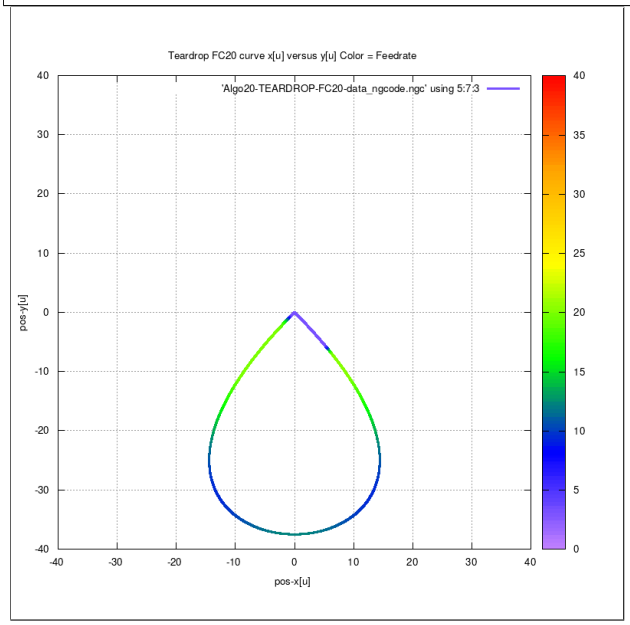
Report CNC Parametric Curve Interpolation and Trajectory Tracking
Part 1 of 5 Teardrop and Butterfly (x-y) parametric curves

ITEM	DESCRIPTION	TEARDROP CURVE					BUTTERFLY CURVE				
		FC10	FC20	FC25	FC30	FC40	FC10	FC20	FC25	FC30	FC40
1	Run user feedrate command (mm/s)										
2	Total interpolated u-points	10261	7599	7385	7347	7347	35656	18029	14577	12343	9732
3	Parameter completion (reached u-end)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	Pushdown epsilon eps(u) algorithm										
4	Count before pushdown, eps(u) is below (1E-6)	8498	1427	527	0	0	35465	17421	13602	11010	7256
5	Count pushdown points, eps(u) to below (1E-6)	1763	6172	6858	7347	7347	191	608	975	1333	2476
	Epsilon eps(u) chord error										
6	Count eps(u) above (1E-6)	0	0	0	0	0	0	0	0	0	0
7	Count eps(u) in (1E-7, 1E-6)	10261	7599	7385	7347	7347	2995	12494	13794	12343	9732
8	Count eps(u) in (1E-8, 1E-7)	0	0	0	0	0	32661	5535	783	0	0
9	Count eps(u) in (1E-9, 1E-8)	0	0	0	0	0	0	0	0	0	0
10	Count eps(u) in (1E-10, 1E-9)	0	0	0	0	0	0	0	0	0	0
11	Count eps(u) below (1E-10)	0	0	0	0	0	0	0	0	0	0
	Count interpolated u-points										
12	Count_rising_S_curve u-points	960	480	389	370	370	1323	693	575	500	418
13	Count_frate is_lower than fratelimit	4734	4342	4260	4202	4049	17751	8968	7255	6129	4772
14	Count_frate is_equal to fratelimit	0	0	0	0	0	0	0	0	0	0
15	Count_frate is_higher than fratelimit	3608	2298	2348	2406	2559	15254	7673	6171	5213	4124
16	Count_falling_S_curve u-points	959	479	388	369	369	1328	695	576	501	418
	Count u-points histogram (G01 codes)										
17	Count u-points [0.00 <= u < 0.10)	1734	875	768	748	748	3463	1763	1431	1214	952
18	Count u-points [0.10 <= u < 0.20)	1120	791	791	791	791	4332	2167	1733	1444	1112
19	Count u-points [0.20 <= u < 0.30)	809	794	794	794	794	2983	1554	1287	1117	927
20	Count u-points [0.30 <= u < 0.40)	726	710	710	711	711	3220	1611	1293	1098	877
21	Count u-points [0.40 <= u < 0.50)	741	629	629	629	629	3832	1920	1545	1299	998
22	Count u-points [0.50 <= u < 0.60)	742	629	629	628	629	3829	1919	1544	1298	997
23	Count u-points [0.60 <= u < 0.70)	726	710	711	711	711	3222	1612	1294	1098	878
24	Count u-points [0.70 <= u < 0.80)	809	794	793	794	793	2981	1553	1286	1117	926
25	Count u-points [0.80 <= u < 0.90)	1120	791	791	791	792	4323	2162	1730	1441	1110
26	Count u-points [0.90 <= u <= 1.00]	1734	876	769	750	749	3471	1768	1434	1217	955
27	Check Total u-points	10261	7599	7385	7347	7347	35656	18029	14577	12343	9732
	Performance										
28	Total curve error (sum of epsilon(u))	0.005809	0.007141	0.007301	0.007337	0.007335	0.001939	0.003534	0.004231	0.004847	0.005851
29	Total dist traversed (sum of chord lengths)	101.8357	101.8419	101.8348	101.8596	101.8356	356.0747	356.0737	356.0723	356.0728	356.0732
30	Percentage (Tot curve error / Tot dist traversed)	0.005704	0.007012	0.00717	0.007203	0.007203	0.000545	0.000993	0.001188	0.001361	0.001643



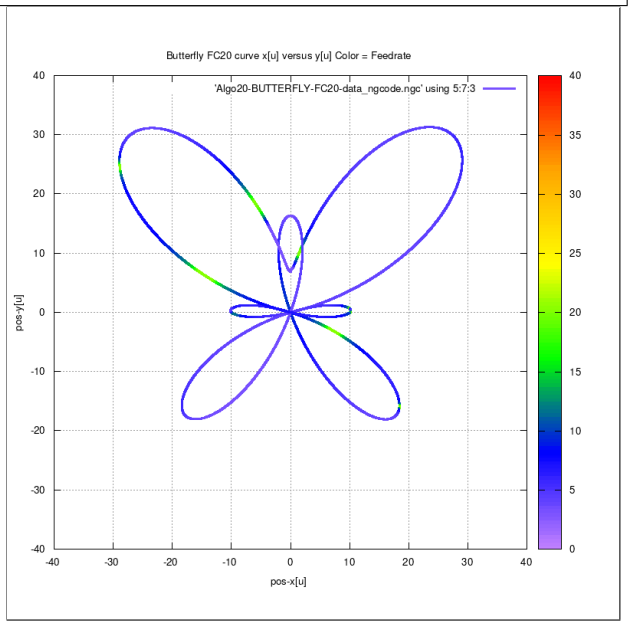


Figure 1.1: Teardrop and Butterfly run data summary

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Report CNC Parametric Curve Interpolation and Trajectory Tracking
Part 2 of 5 Ellipse and Skewed-Astroid (x-y) parametric curves

ITEM	DESCRIPTION	ELLIPSE CURVE					SKEWED-ASTROID CURVE				
		FC10	FC20	FC25	FC30	FC40	FC10	FC20	FC25	FC30	FC4
1	Run user feedrate command (mm/s)										
2	Total interpolated u-points	21575	7599	9448	8338	7351	116194	58102	46483	38738	2905
3	Parameter completion (reached u-end)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.0
	Pushdown epsilon eps(u) algorithm										
4	Count before pushdown, eps(u) is below (1E-6)	21396	1427	7046	5149	928	116194	58102	46483	38738	2905
5	Count pushdown points, eps(u) to below (1E-6)	179	6172	2402	3189	6423	0	0	0	0	
	Epsilon eps(u) chord error										
6	Count eps(u) above (1E-6)	0	0	0	0	0	0	0	0	0	
7	Count eps(u) in (1E-7, 1E-6)	7058	7599	9448	8338	7351	0	0	0	1455	843
8	Count eps(u) in (1E-8, 1E-7)	14517	0	0	0	0	9483	33004	46483	37283	2062
9	Count eps(u) in (1E-9, 1E-8)	0	0	0	0	0	106711	25098	0	0	
10	Count eps(u) in (1E-10, 1E-9)	0	0	0	0	0	0	0	0	0	
11	Count eps(u) below (1E-10)	0	0	0	0	0	0	0	0	0	
	Count interpolated u-points										
12	Count_rising_S_curve u-points	443	480	370	370	370	3673	1837	1470	1225	91
13	Count_frate is_lower than fratelimit	14764	4342	6120	5270	3308	75069	39802	31837	26531	1993
14	Count_frate is_equal to fratelimit	0	0	0	0	0	0	0	0	0	
15	Count_frate is_higher than fratelimit	5926	2298	2589	2329	3304	33779	14626	11707	9757	727
16	Count_falling_S_curve u-points	442	479	369	369	369	3673	1837	1469	1225	91
	Count u-points histogram (G01 codes)										
17	Count u-points [0.00 <= u < 0.10)	1231	779	748	748	748	7322	3661	2929	2441	183
18	Count u-points [0.10 <= u < 0.20)	2586	1292	1035	878	791	12603	6302	5042	4202	315
19	Count u-points [0.20 <= u < 0.30)	3154	1578	1262	1051	797	18245	9123	7298	6081	456
20	Count u-points [0.30 <= u < 0.40)	2586	1293	1034	863	710	12604	6302	5042	4202	315
21	Count u-points [0.40 <= u < 0.50)	1230	704	644	628	629	7322	3662	2930	2442	183
22	Count u-points [0.50 <= u < 0.60)	1228	704	644	629	629	7323	3662	2930	2442	183
23	Count u-points [0.60 <= u < 0.70)	2586	1293	1034	862	710	12603	6302	5041	4201	315
24	Count u-points [0.70 <= u < 0.80)	3155	1577	1262	1051	796	18244	9122	7298	6082	456
25	Count u-points [0.80 <= u < 0.90)	2586	1293	1035	878	791	12604	6303	5043	4202	315
26	Count u-points [0.90 <= u <= 1.00]	1233	782	750	750	750	7324	3663	2930	2443	183
27	Check Total u-points	21575	11295	9448	8338	7351	116194	58102	46483	38738	2905
	Performance										
28	Total curve error (sum of epsilon(u))	0.002991	0.007141	0.005928	0.006562	0.007332	0.000516	0.001033	0.001291	0.001549	0.0020
29	Total dist traversed (sum of chord lengths)	215.6437	215.6499	215.644	215.6478	215.6439	445.7143	445.7143	445.7143	445.7143	445.714
30	Percent (Tot curve error / Tot dist traversed)	0.001387	0.003311	0.002749	0.003043	0.0034	0.000116	0.000232	0.00029	0.000347	0.00046

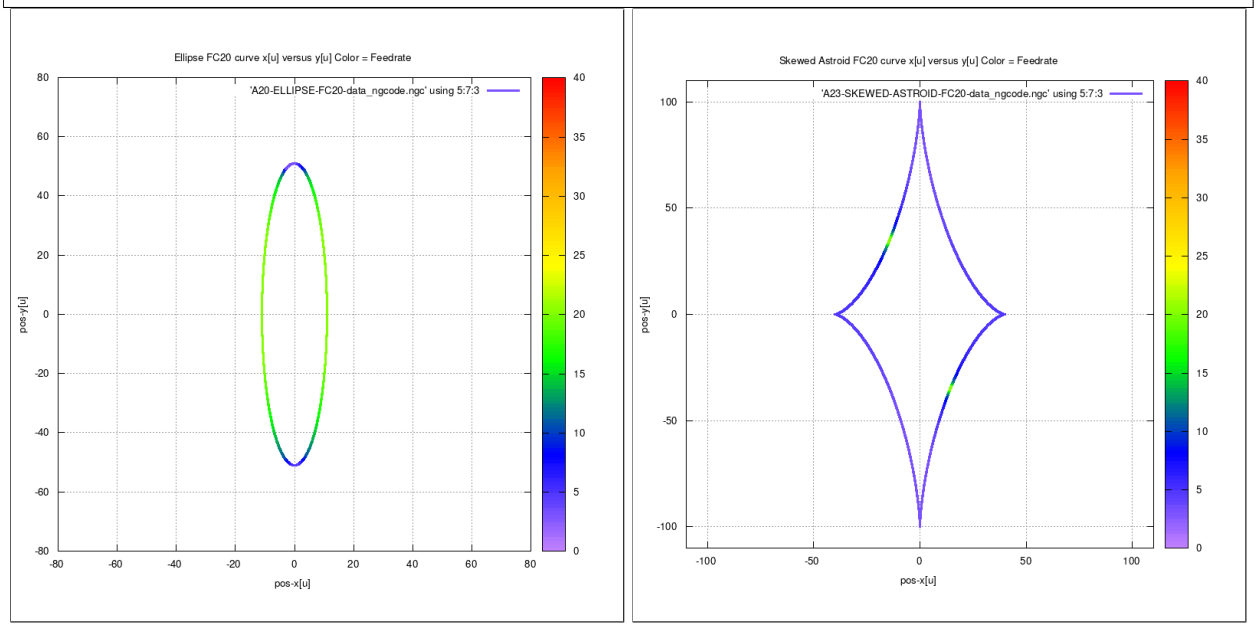

Figure 1.2: Ellipse and Skewed-Astroid run data summary

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Report CNC Parametric Curve Interpolation and Trajectory Tracking
Part 3 of 5 Circle and Astepi (x-y) parametric curves

ITEM	DESCRIPTION	CIRCLE CURVE					ASTEROID + EPICYCLOID CURVE				
1	Run user feedrate command (mm/s)	FC10	FC20	FC25	FC30	FC40	FC10	FC20	FC25	FC30	FC40
2	Total interpolated u-points	49641	24822	19859	16549	12413	76275	38169	30563	25499	19184
3	Parameter completion (reached u-end)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	Pushdown epsilon eps(u) algorithm										
4	Count before pushdown, eps(u) is below (1E-6)	49641	24822	19859	16549	12413	76275	38061	30413	25309	18917
5	Count pushdown points, eps(u) to below (1E-6)	0	0	0	0	0	0	108	150	190	267
	Epsilon eps(u) chord error										
6	Count eps(u) above (1E-6)	0	0	0	0	0	0	0	0	0	0
7	Count eps(u) in (1E-7, 1E-6)	0	6959	17603	16549	12413	407	820	4011	6679	13976
8	Count eps(u) in (1E-8, 1E-7)	49641	17863	2256	0	0	23373	37349	26552	18820	5208
9	Count eps(u) in (1E-9, 1E-8)	0	0	0	0	0	52495	0	0	0	0
10	Count eps(u) in (1E-10, 1E-9)	0	0	0	0	0	0	0	0	0	0
11	Count eps(u) below (1E-10)	0	0	0	0	0	0	0	0	0	0
	Count interpolated u-points										
12	Count_rising_S_curve u-points	2483	1242	993	828	621	4437	2219	1775	1479	1110
13	Count_frate is_lower than fratelimit	22363	11171	8936	7449	5586	33349	16686	13352	11144	8085
14	Count_frate is_equal to fratelimit	0	0	0	0	0	0	0	0	0	0
15	Count_frate is_higher than fratelimit	22313	11168	8937	7445	5585	35536	17787	14254	11891	9250
16	Count_falling_S_curve u-points	2482	1241	993	827	621	2953	1477	1182	985	739
	Count u-points histogram (G01 codes)										
17	Count u-points [0.00 <= u < 0.10)	4964	2482	1985	1654	1241	8110	4055	3244	2704	2028
18	Count u-points [0.10 <= u < 0.20)	4964	2482	1986	1655	1241	4901	2478	2009	1702	1334
19	Count u-points [0.20 <= u < 0.30)	4964	2482	1986	1655	1241	7391	3696	2957	2464	1848
20	Count u-points [0.30 <= u < 0.40)	4964	2482	1986	1655	1241	7234	3617	2894	2412	1809
21	Count u-points [0.40 <= u < 0.50)	4964	2482	1986	1655	1242	9182	4592	3673	3061	2297
22	Count u-points [0.50 <= u < 0.60)	4964	2483	1985	1655	1241	7216	3608	2887	2406	1804
23	Count u-points [0.60 <= u < 0.70)	4964	2482	1986	1655	1241	9831	4916	3933	3278	2458
24	Count u-points [0.70 <= u < 0.80)	4964	2482	1986	1655	1241	7525	3763	3010	2508	1882
25	Count u-points [0.80 <= u < 0.90)	4964	2482	1986	1654	1242	8801	4401	3521	2935	2201
26	Count u-points [0.90 <= u <= 1.00]	4965	2483	1987	1656	1242	6084	3043	2435	2029	1523
27	Check Total u-points	49641	24822	19859	16549	12413	76275	38169	30563	25499	19184
	Performance										
28	Total curve error (sum of epsilon(u))	0.001094	0.001094	0.002188	0.002735	0.004375	0.00084	0.001642	0.00202	0.00239	0.003111
29	Total dist traversed (sum of chord lengths)	496.3786	496.3772	496.3964	496.3757	496.3943	426.2622	426.2622	426.2622	426.2622	426.2622
30	Percent (Tot curve error / Tot dist traversed)	0.00022	0.00022	0.000441	0.000551	0.000881	0.000197	0.000385	0.000474	0.000561	0.00073

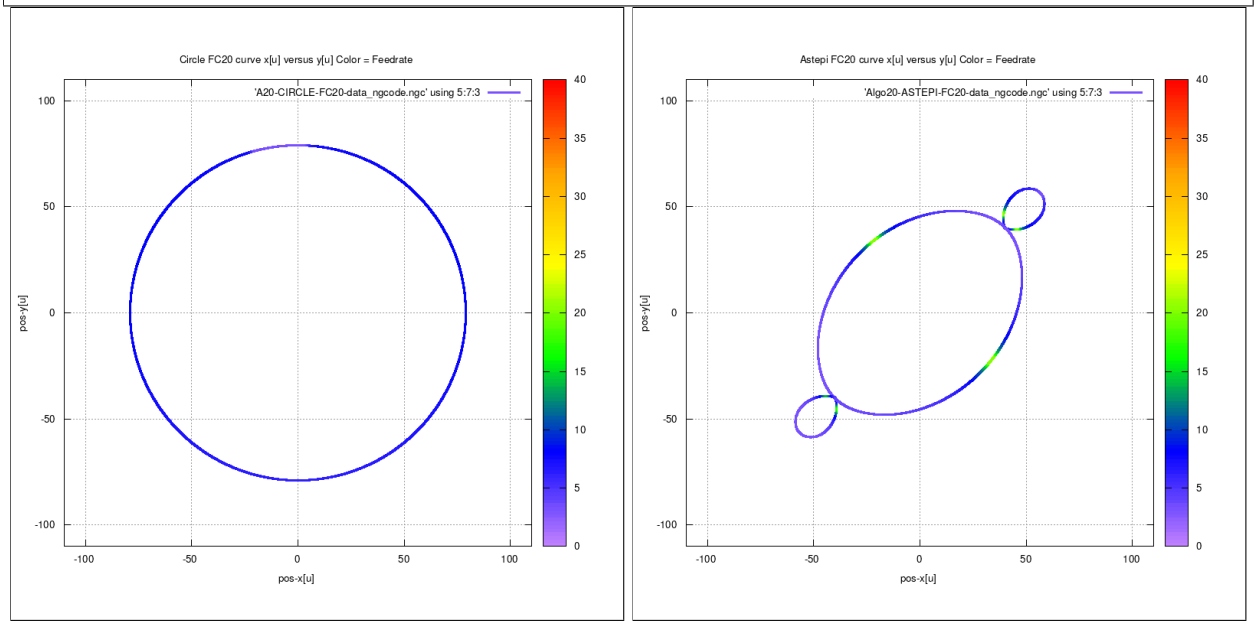

Figure 1.3: Circle and AstEpi run data summary

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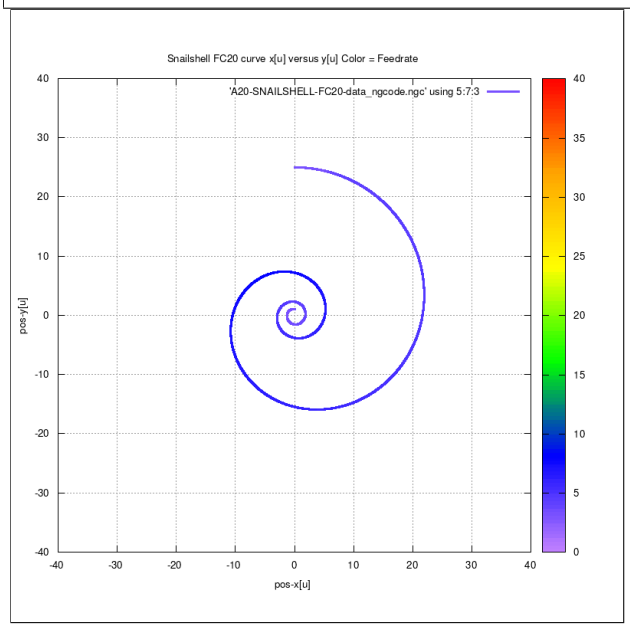
Report CNC Parametric Curve Interpolation and Trajectory Tracking
Part 4 of 5 Snailshell and SnaHyp (x-y) parametric curves

ITEM	DESCRIPTION	SNAILSHELL CURVE					SNAILSHELL + HYPOTROCOID CURVE				
1	Run user feedrate command (mm/s)	FC10	FC20	FC25	FC30	FC40	FC10	FC20	FC25	FC30	FC40
2	Total interpolated u-points	15621	9883	8935	8370	7766	38672	20223	16618	11497	8889
3	Parameter completion (reached u-end = 1.00)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.706104	0.703536
	Pushdown epsilon eps(u) algorithm									***	***
4	Count before pushdown, eps(u) is below (1E-6)	12245	5010	3569	2671	1592	37227	18167	14266	0	0
5	Count pushdown points, eps(u) to below (1E-6)	3376	4873	5366	5699	6174	1445	2056	2352	0	0
	Epsilon eps(u) chord error										
6	Count eps(u) above (1E-6)	0	0	0	0	0	0	0	0	0	0
7	Count eps(u) in (1E-7, 1E-6)	7893	9323	8935	8370	7766	4106	6971	9900	0	0
8	Count eps(u) in (1E-8, 1E-7)	7728	560	0	0	0	24178	13252	6718	0	0
9	Count eps(u) in (1E-9, 1E-8)	0	0	0	0	0	10388	0	0	0	0
10	Count eps(u) in (1E-10, 1E-9)	0	0	0	0	0	0	0	0	0	0
11	Count eps(u) below (1E-10)	0	0	0	0	0	0	0	0	0	0
	Count interpolated u-points										
12	Count_rising_S_curve u-points	2320	1161	929	774	581	3177	1589	1272	0	0
13	Count_frate is_lower than fratelimit	11722	6605	5826	5386	4860	18305	9773	8142	0	0
14	Count_frate is_equal to fratelimit	0	0	0	0	0	0	0	0	0	0
15	Count_frate is_higher than fratelimit	1210	1747	1811	1859	1956	14859	7695	6271	0	0
16	Count_falling_S_curve u-points	369	370	359	369	369	2331	1166	933	0	0
	Count u-points histogram (G01 codes)									***	***
17	Count u-points [0.00 <= u < 0.10)	4435	2218	1774	1479	1109	4631	2317	1856	1563	1217
18	Count u-points [0.10 <= u < 0.20)	3237	1619	1296	1080	849	8961	4480	3584	2987	2240
19	Count u-points [0.20 <= u < 0.30)	2054	1028	851	796	793	6140	3074	2470	2072	1586
20	Count u-points [0.30 <= u < 0.40)	1312	714	710	711	711	2960	1526	1257	1086	885
21	Count u-points [0.40 <= u < 0.50)	881	629	629	629	629	4860	2431	1945	1620	1216
22	Count u-points [0.50 <= u < 0.60)	657	628	628	629	628	3973	1987	1589	1325	994
23	Count u-points [0.60 <= u < 0.70)	710	711	711	710	711	1324	841	769	732	710
24	Count u-points [0.70 <= u < 0.80)	794	794	794	794	794	794	794	794	112	41
25	Count u-points [0.80 <= u < 0.90)	791	791	792	792	792	1141	828	798	0	0
26	Count u-points [0.90 <= u <= 1.00]	750	751	750	750	750	3888	1945	1556	0	0
27	Check Total u-points	15621	9883	8935	8370	7766	38672	20223	16618	11497	8889
	Performance										
28	Total curve error (sum of epsilon(u))	0.005115	0.00627	0.006558	0.006764	0.007046	0.002847	0.004003	0.004459	0	0
29	Total dist traversed (sum of chord lengths)	138.5595	138.5614	138.5607	138.5602	138.5599	478.9871	478.9987	479.0064	0	0
30	Percent (Tot curve error / Tot dist traversed)	0.003692	0.004525	0.004733	0.004882	0.005085	0.000594	0.000836	0.000931	0	0



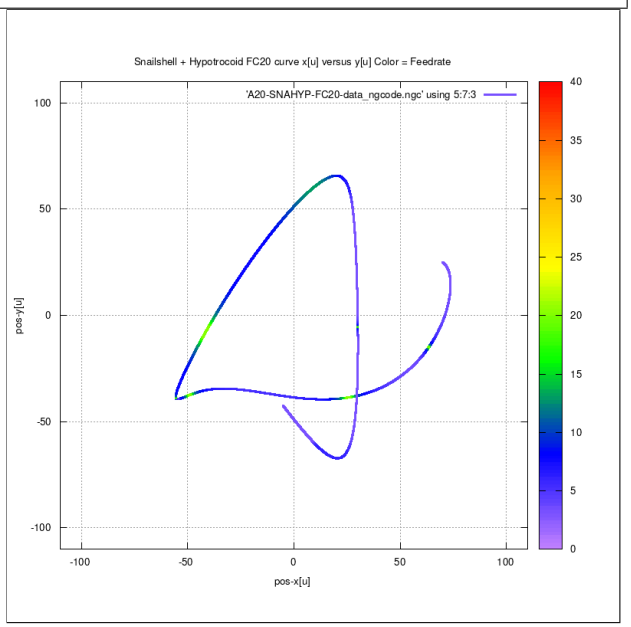


Figure 1.4: Snailshell and SnaHyp run data summary

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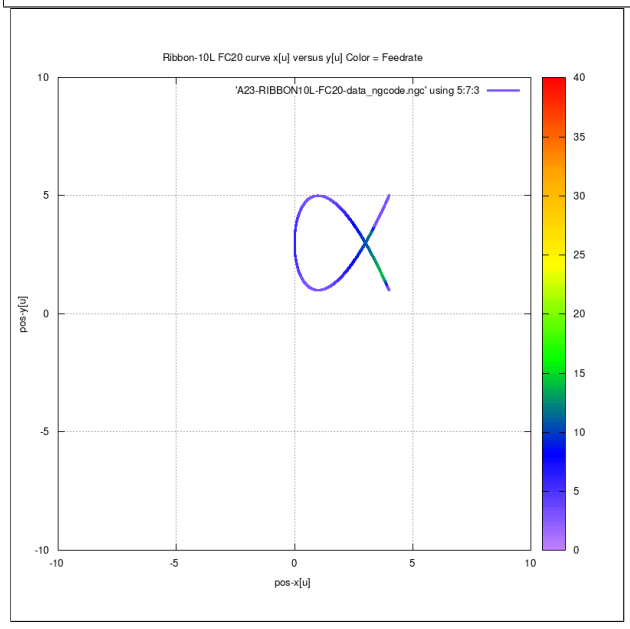
Report CNC Parametric Curve Interpolation and Trajectory Tracking
Part 5 of 5 Ribbon-10L and Ribbon-100L (x-y) parametric curves

ITEM	DESCRIPTION	RIBBON-10L CURVE					RIBBON-100L CURVE				
1	Run user feedrate command (mm/s)	FC10	FC20	FC25	FC30	FC40	FC10	FC20	FC25	FC30	FC40
2	Total interpolated u-points	7351	7352	7352	7353	7353	7480	7348	7349	7349	7350
3	Parameter completion (reached u-end)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	Pushdown epsilon eps(u) algorithm										
4	Count before pushdown, eps(u) is below (1E-6)	0	0	0	0	0	919	0	0	0	0
5	Count pushdown points, eps(u) to below (1E-6)	7351	7352	7352	7353	7353	6561	7348	7349	7349	7350
	Epsilon eps(u) chord error										
6	Count eps(u) above (1E-6)	0	0	0	0	0	0	0	0	0	0
7	Count eps(u) in (1E-7, 1E-6)	7351	7352	7352	7353	7353	7480	7348	7349	7349	7350
8	Count eps(u) in (1E-8, 1E-7)	0	0	0	0	0	0	0	0	0	0
9	Count eps(u) in (1E-9, 1E-8)	0	0	0	0	0	0	0	0	0	0
10	Count eps(u) in (1E-10, 1E-9)	0	0	0	0	0	0	0	0	0	0
11	Count eps(u) below (1E-10)	0	0	0	0	0	0	0	0	0	0
	Count interpolated u-points										
12	Count_rising_S_curve u-points	370	370	370	370	370	436	370	370	370	370
13	Count_frate is_lower than fratelimit	3491	3310	3310	3308	3309	5562	3939	3733	3545	3307
14	Count_frate is_equal to fratelimit	0	0	0	0	0	0	0	0	0	0
15	Count_frate is_higher than fratelimit	3121	3303	3303	3305	3305	1047	2670	2877	3065	3303
16	Count_falling_S_curve u-points	369	369	369	370	369	435	369	369	369	370
	Count u-points histogram (G01 codes)										
17	Count u-points [0.00 <= u < 0.10)	749	749	749	749	749	815	748	748	748	749
18	Count u-points [0.10 <= u < 0.20)	791	792	792	792	792	791	792	792	792	791
19	Count u-points [0.20 <= u < 0.30)	795	794	794	794	794	794	794	794	794	794
20	Count u-points [0.30 <= u < 0.40)	711	711	711	711	712	711	711	711	711	711
21	Count u-points [0.40 <= u < 0.50)	629	629	629	630	629	629	628	629	629	629
22	Count u-points [0.50 <= u < 0.60)	629	630	630	629	629	628	629	628	629	629
23	Count u-points [0.60 <= u < 0.70)	711	711	711	711	711	711	711	712	711	711
24	Count u-points [0.70 <= u < 0.80)	794	794	794	795	795	794	794	794	794	794
25	Count u-points [0.80 <= u < 0.90)	792	792	792	791	792	791	791	791	791	792
26	Count u-points [0.90 <= u <= 1.00]	750	750	750	751	750	816	750	750	750	750
27	Check Total u-points	7351	7352	7352	7353	7353	7480	7348	7349	7349	7350
	Performance										
28	Total curve error (sum of epsilon(u))	0.007332	0.007331	0.00733	0.007331	0.00733	0.007227	0.007334	0.007335	0.007334	0.007334
29	Total dist traversed (sum of chord lengths)	15.2108	15.21069	15.20945	15.21391	15.21192	152.0974	152.1029	152.1321	152.1103	152.1394
30	Percent (Tot curve error / Tot dist traversed)	0.048201	0.048194	0.048195	0.048185	0.048187	0.004752	0.004822	0.004821	0.004821	0.00482



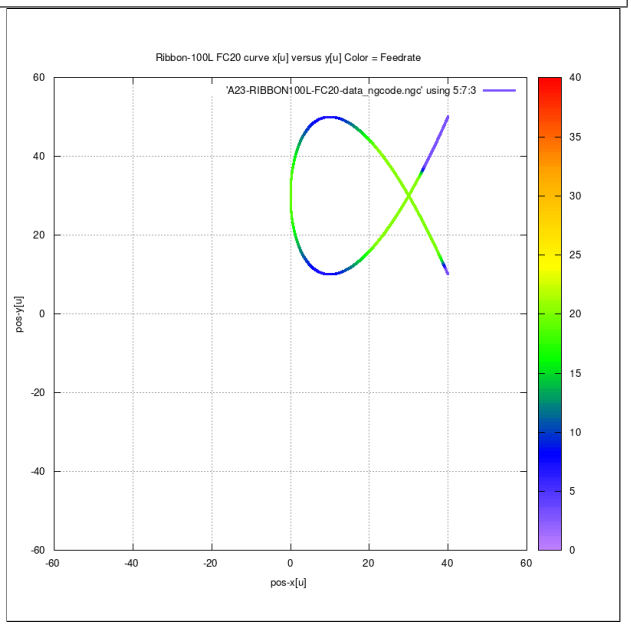


Figure 1.5: Ribbon-10L and Ribbon-100L run data summary

1.1.1 Teardrop parametric equation

No. 1 - Teardrop parametric curve

$$\begin{aligned} x(u) &= -150u + 450u^2 - 300u^3 \\ y(u) &= -150u + 150u^2 \\ u &\in [0.0, 1.0] \end{aligned}$$

Closed loop
Overall Single loop
Reflection x-axis: non-symmetrical
Reflection y-axis: symmetrical

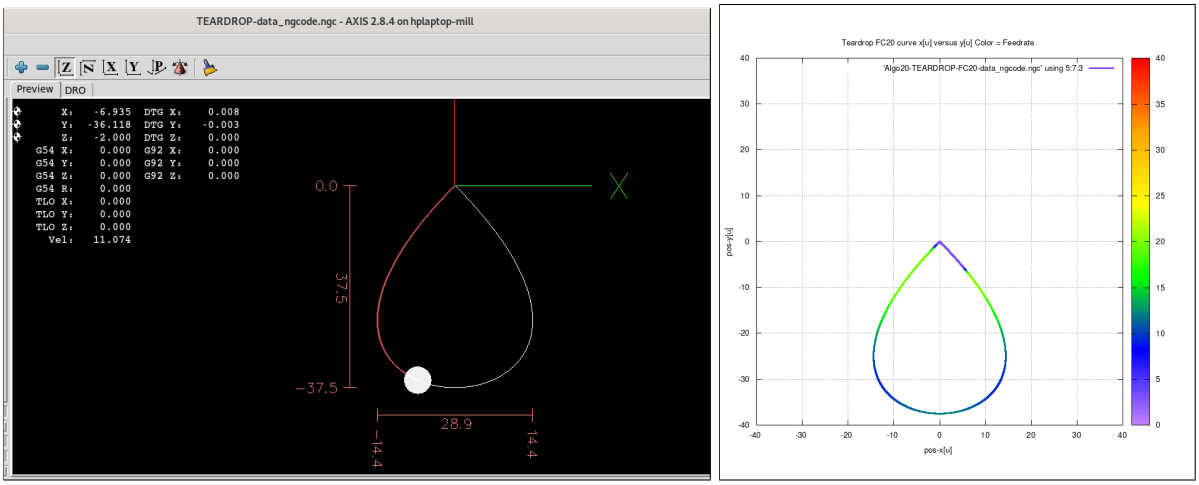


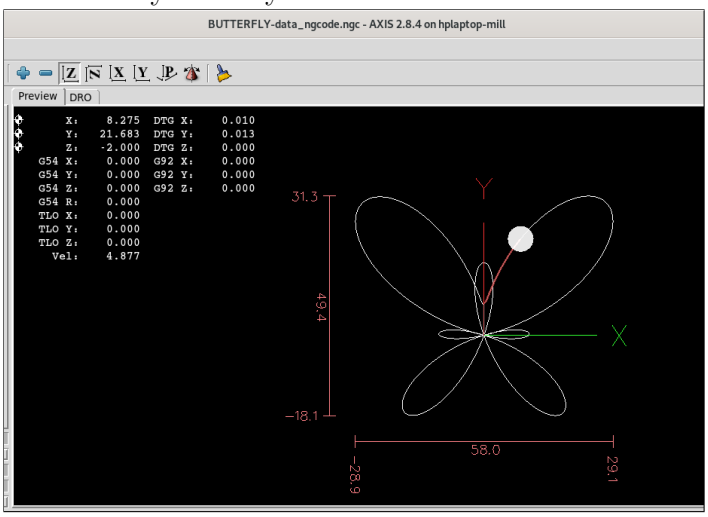
Table 1.1: Teardrop parametric equation and dimensions

1.1.2 Butterfly parametric equation

No. 2 - Butterfly parametric curve

$$
\begin{aligned}
x(u) &= \sin(2\pi u)\left[e^{\cos(2\pi u)} - 2\cos(8\pi u) - (\sin(2\pi u/12))^5\right] \\
y(u) &= \cos(2\pi u)\left[e^{\cos(2\pi u)} - 2\cos(8\pi u) - (\sin(2\pi u/12))^5\right] \\
u &\in [0.0, 1.0]
\end{aligned}
$$

Closed loop
Overall Multiple loops
Reflection x-axis: non-symmetrical
Reflection y-axis: symmetrical



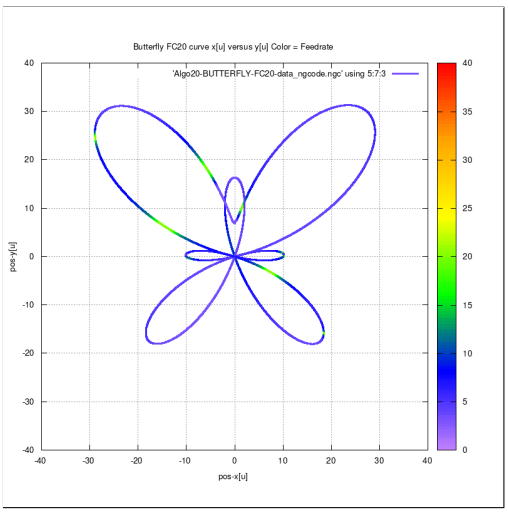


Table 1.2: Butterfly parametric equation and dimensions

1.1.3 Ellipse parametric equation

No. 3 - Ellipse parametric curve

$$\begin{aligned} x(u) &= 11\sin(2\pi u) \\ y(u) &= 51\cos(2\pi u) \\ u &\in [0.0, 1.0] \end{aligned}$$

Closed loop
Overall Single loop, smooth convex curves
Reflection x-axis: symmetrical
Reflection y-axis: symmetrical

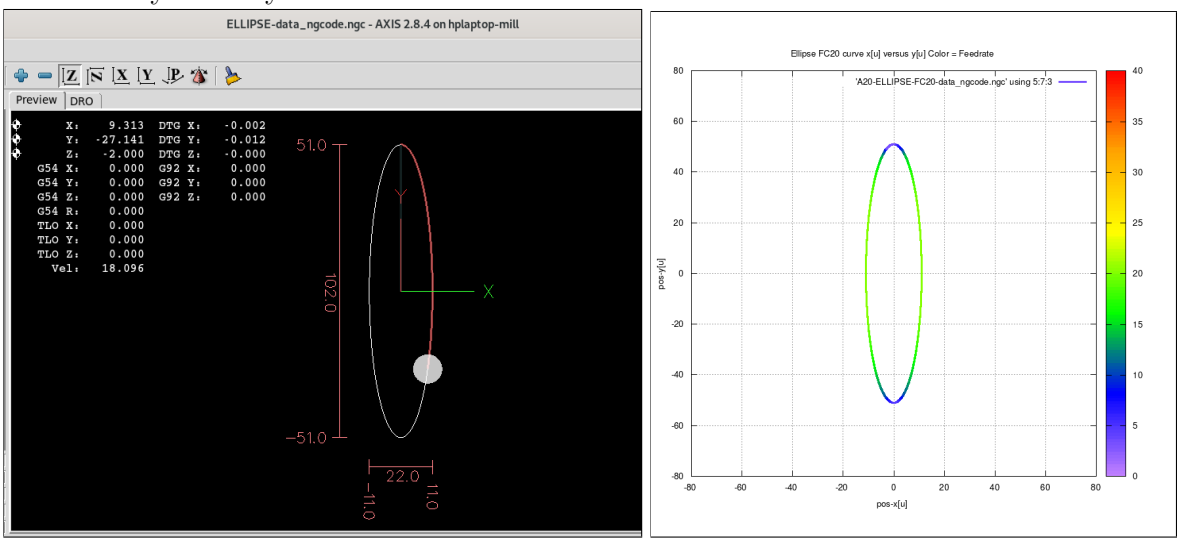


Table 1.3: Ellipse equation and dimensions

1.1.4 Skewed-Astroid parametric equation

No. 4 - Skewed-Astroid parametric curve

$$\begin{aligned} x(u) &= 40[\sin(2\pi u)]^3 \\ y(u) &= 100[\cos(2\pi u)]^3 \\ u &\in [0.0, 1.0] \end{aligned}$$

Closed loop
Overall Single loop, 4 cusps and 4 concave curves
Reflection x-axis: symmetrical
Reflection y-axis: symmetrical

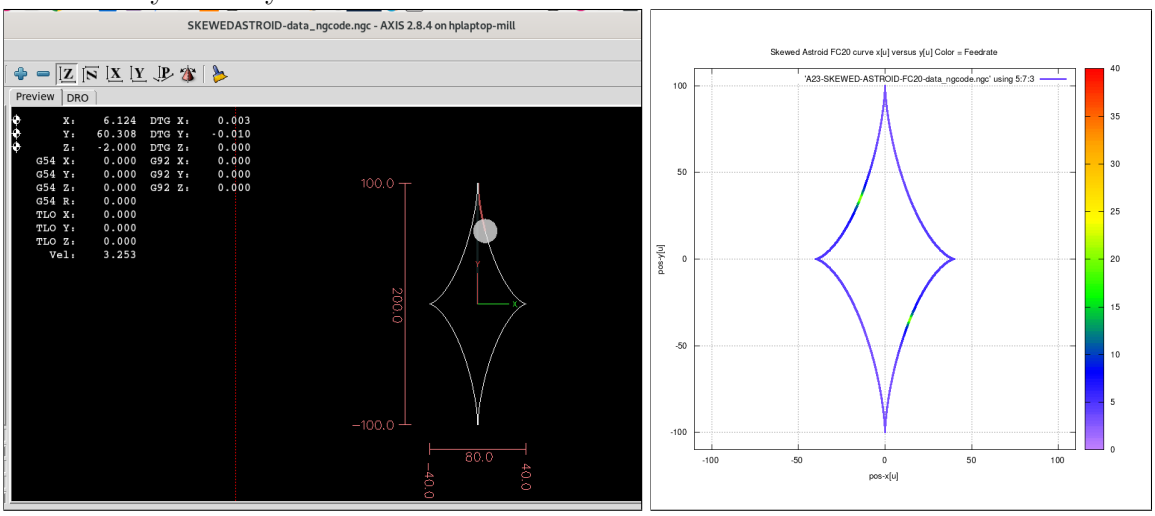


Table 1.4: Skewed-Astroid and dimensions

1.1.5 Circle parametric equation

No. 5 - Circle parametric curve

$$\begin{aligned} x(u) &= 79\sin(2\pi u) \\ y(u) &= 79\cos(2\pi u) \\ u &\in [0.0, 1.0] \end{aligned}$$

Closed loop
Overall Single loop, smooth convex curves
Reflection x-axis: symmetrical
Reflection y-axis: symmetrical

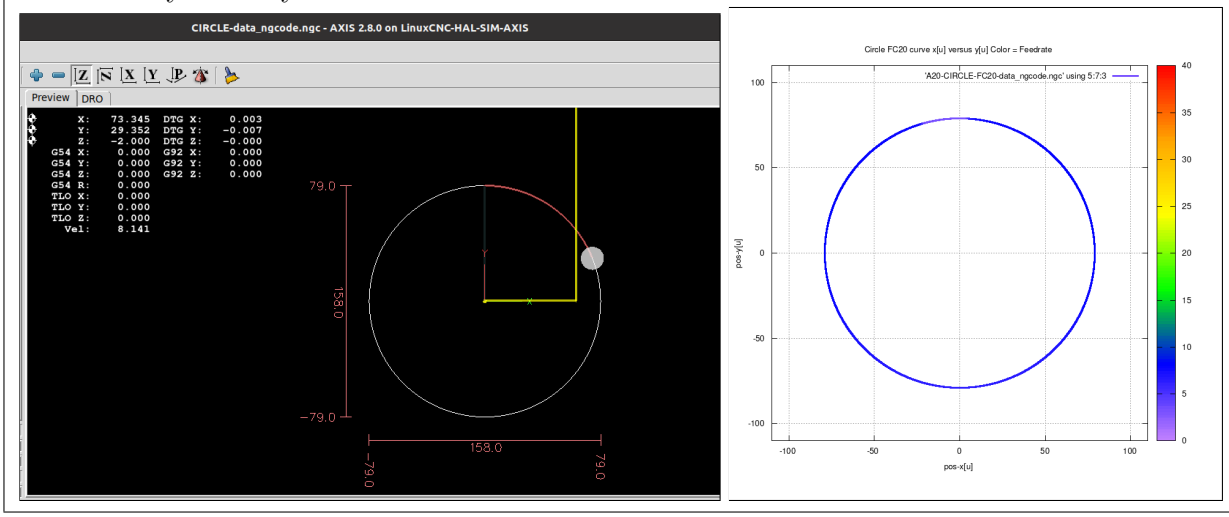


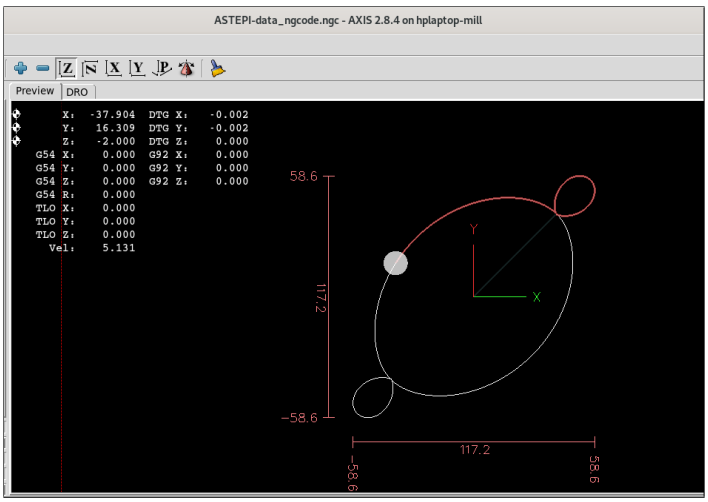
Table 1.5: Circle equation and dimensions

1.1.6 AstEpi parametric equation

No. 6 - AstEpi = Sum of (Astroid + Epicycloid) parametric curves

$$
\begin{aligned}
tiny &= 1.0\times10^{-10} \\
x(u) &= 40[\sin(2\pi u)]^3 + 50\cos(2\pi u + tiny) - 10\cos(10\pi u - tiny) \\
y(u) &= 40[\cos(2\pi u)]^3 + 50\sin(2\pi u + tiny) - 10\sin(10\pi u - tiny) \\
u &\in [0.0, 1.0]
\end{aligned}
$$

Closed loop
Overall Three loops, all convex curves
Reflection x-axis: non-symmetrical
Reflection y-axis: non-symmetrical



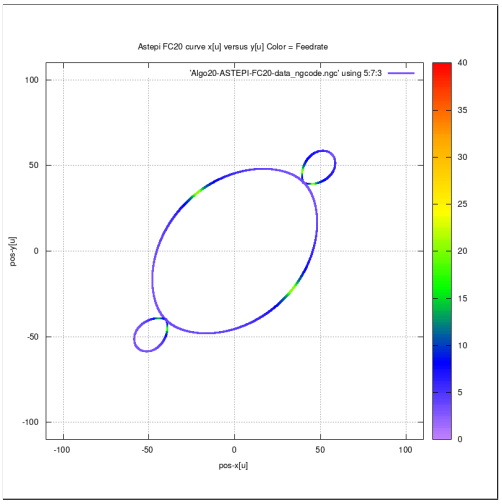


Table 1.6: Astepi equation and dimensions

1.1.7 Snailshell parametric equation

No. 7 - Snailshell parametric curve

$$\begin{aligned} x(u) &= 100\sin(6\pi u)/[9(\pi u)^2+4] \\ y(u) &= 100\cos(6\pi u)/[9(\pi u)^2+4] \\ u &\in [0.0, 1.0] \end{aligned}$$

Open ended curve
Overall No loop, smooth and continuous convex curves
Reflection x-axis: non-symmetrical
Reflection y-axis: non-symmetrical

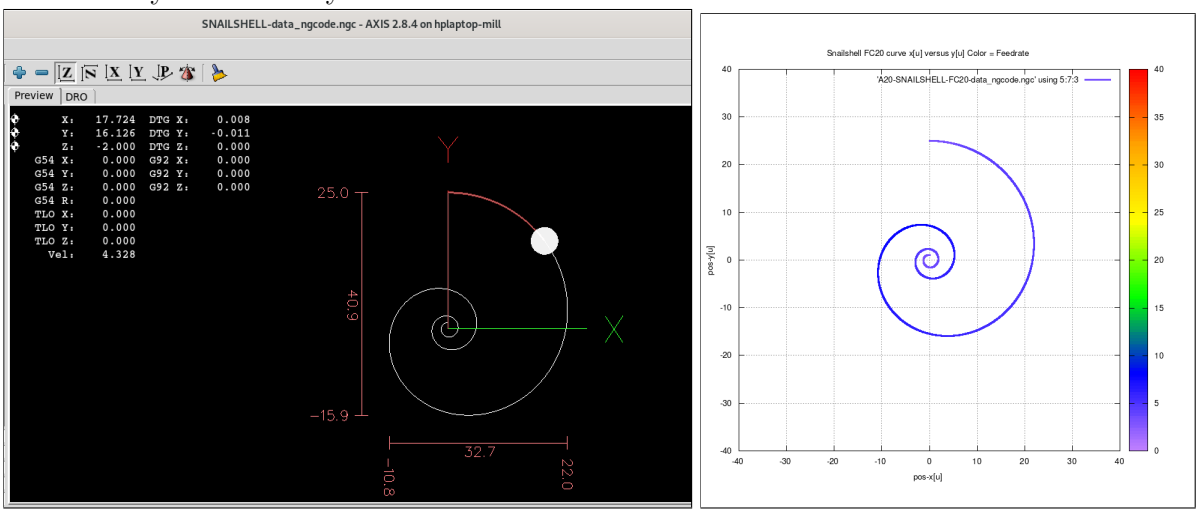


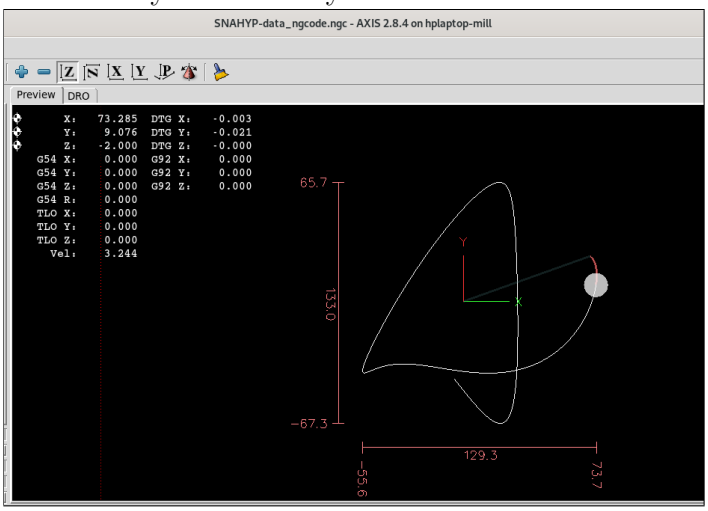
Table 1.7: Snailshell equation and dimensions

1.1.8 SnaHyp parametric equation

No. 8 - SnaHyp = Sum of (Snailshell + Hypotrocoid) parametric curves

$$
\begin{aligned}
xsna(u) &= [4\sin(8\pi u)]/[16(\pi u)^2 + 4] \\
xhyp(u) &= [2\cos(4\pi u) + 5\cos(8\pi u/3)] \\
x(u) &= 10[xsna(u) + xhyp(u)] \\
ysna(u) &= [10\cos(8\pi u)]/[16(\pi u)^2 + 4] \\
yhyp(u) &= [2\sin(8\pi u) - 5\sin(8\pi u/3)] \\
y(u) &= 10[ysna(u) + yhyp(u)] \\
u &\in [0.0, 1.0]
\end{aligned}
$$

Open ended curve
Overall 1 loop, except for 1 concave curve, the rest are convex curves
Reflection x-axis: non-symmetrical
Reflection y-axis: non-symmetrical



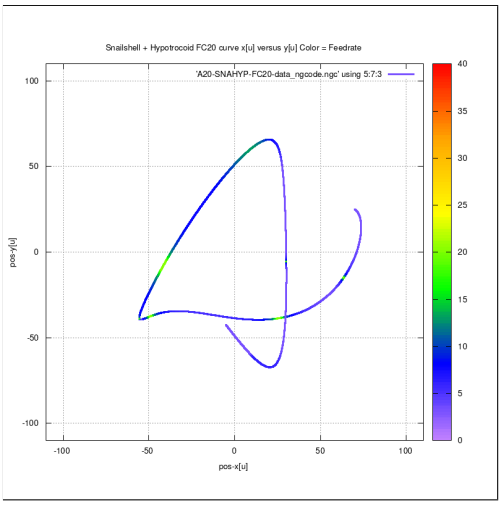


Table 1.8: SnaHyp equation and dimensions

1.1.9 Ribbon-10L parametric equation

No. 9 - Ribbon-10L parametric curve

$$
\begin{aligned}
t(u) &= 4(u - 0.50) \\
x(u) &= t^2 \\
y(u) &= t^3 - 3t + 3 \\
u &\in [0.0, 1.0]
\end{aligned}
$$

Open ended curve
Overall Single loop, smooth convex curves
Reflection x-axis: non-symmetrical
Reflection y-axis: non-symmetrical

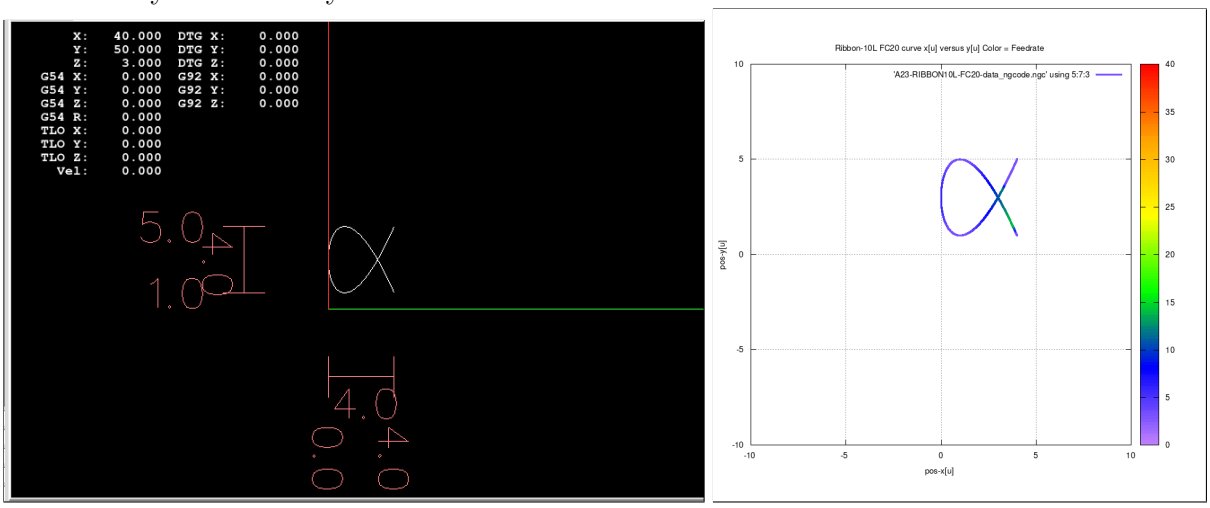


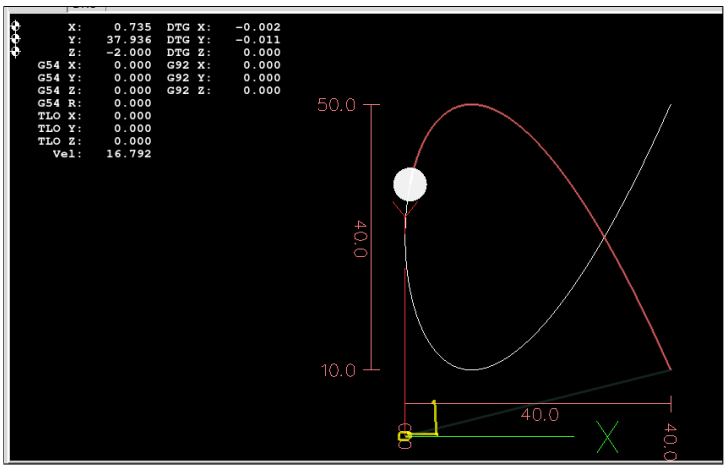
Table 1.9: Ribbon-10L equations and dimensions

1.1.10 Ribbon-100L parametric equation

No. 10 - Ribbon-100L parametric curve

$$
\begin{aligned}
t(u) &= 4(u - 0.50) \\
x(u) &= 10t^2 \\
y(u) &= 10t^3 - 30t + 30 \\
u &\in [0.0, 1.0]
\end{aligned}
$$

Open ended curve (10 times larger than RIBBON-10L)
Overall Single loop, smooth convex curves
Reflection x-axis: non-symmetrical
Reflection y-axis: non-symmetrical



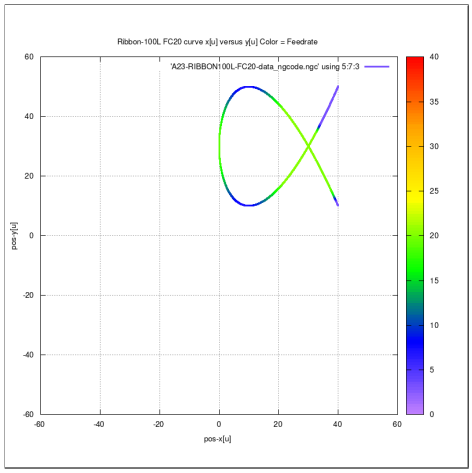


Table 1.10: Ribbon-100L equation and dimensions