Date: 2023-06-06
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Report CNC Parametric Curve Interpolation and Trajectory Tracking
Part 1 of 5 Teardrop and Butterfly (x-y) parametric curves

ITEM	DESCRIPTION	TEARDROP CURVE					BUTTERFLY CURVE				
1	Run user feedrate command (mm/s)	**FC10**	**FC20**	**FC25**	**FC30**	**FC40**	**FC10**	**FC20**	**FC25**	**FC30**	**FC40**
2	Total interpolated u-points	10261	7599	7385	7347	7347	35656	18029	14577	12343	9732
3	Parameter completion (reached u-end)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	Pushdown epsilon eps(u) algorithm										
4	Count before pushdown, eps(u) is below (1E-6)	8498	1427	527	0	0	35465	17421	13602	11010	7256
5	Count pushdown points, eps(u) to below (1E-6)	1763	6172	6858	7347	7347	191	608	975	1333	2476
6	Check Total u-points	10261	7599	7385	7347	7347	35656	18029	14577	12343	9732
	Epsilon eps(u) chord error										
7	Count eps(u) above (1E-6)	0	0	0	0	0	0	0	0	0	0
8	Count eps(u) in (1E-7, 1E-6)	10261	7599	7385	7347	7347	2995	12494	13794	12343	9732
9	Count eps(u) in (1E-8, 1E-7)	0	0	0	0	0	32661	5535	783	0	0
10	Count eps(u) in (1E-9, 1E-8)	0	0	0	0	0	0	0	0	0	0
11	Count eps(u) in (1E-10, 1E-9)	0	0	0	0	0	0	0	0	0	0
12	Count eps(u) below (1E-10)	0	0	0	0	0	0	0	0	0	0
13	Check Total eps(u) points	10261	7599	7385	7347	7347	35656	18029	14577	12343	9732
	Count interpolated u-points										
14	Count_rising_S_curve u-points	960	480	389	370	370	1323	693	575	500	418
15	Count_frate is_lower than fratelimit	4734	4342	4260	4202	4049	17751	8968	7255	6129	4772
16	Count_frate is_equal to fratelimit	0	0	0	0	0	0	0	0	0	0
17	Count_frate is_higher than fratelimit	3608	2298	2348	2406	2559	15254	7673	6171	5213	4124
18	Count_falling_S_curve u-points	959	479	388	369	369	1328	695	576	501	418
19	Check Total u-points	10261	7599	7385	7347	7347	35656	18029	14577	12343	9732
	Count u-points histogram (G01 codes)										
20	Count u-points [0.00 <= u < 0.10)	1734	875	768	748	748	3463	1763	1431	1214	952
21	Count u-points [0.10 <= u < 0.20)	1120	791	791	791	791	4332	2167	1733	1444	1112
22	Count u-points [0.20 <= u < 0.30)	809	794	794	794	794	2983	1554	1287	1117	927
24	Count u-points [0.30 <= u < 0.40)	726	710	710	711	711	3220	1611	1293	1098	877
25	Count u-points [0.40 <= u < 0.50)	741	629	629	629	629	3832	1920	1545	1299	998
26	Count u-points [0.50 <= u < 0.60)	742	629	629	628	629	3829	1919	1544	1298	997
27	Count u-points [0.60 <= u < 0.70)	726	710	711	711	711	3222	1612	1294	1098	878
28	Count u-points [0.70 <= u < 0.80)	809	794	793	794	793	2981	1553	1286	1117	926
29	Count u-points [0.80 <= u < 0.90)	1120	791	791	791	792	4323	2162	1730	1441	1110
30	Count u-points [0.90 <= u <= 1.00]	1734	876	769	750	749	3471	1768	1434	1217	955
31	Check Total u-points	10261	7599	7385	7347	7347	35656	18029	14577	12343	9732
32	**Total curve error (sum of epsilon(u))**	5.81E-03	7.14E-03	7.30E-03	7.34E-03	7.34E-03	1.94E-03	3.53E-03	4.23E-03	4.85E-03	5.85E-03
33	**Total dist traversed (sum of chord lengths)**	101.835	101.841	101.834	101.859	101.835	356.075	356.073	356.072	356.072	356.073

	Notes and remarks										
	Pushdown or reducing eps(u) algorithm means reducing chord length, thus reduces u_next and so results in generation of more interpolated u-points. See row Item(7). None of the eps(u) values exceed (1E-6).	See row Item(17) Count_frate is_higher than fratelimit. This count is about adjusting the current feedrate to follow the calculated feedrate_limit, and stay just below this feedrate limit. The calculated net feedrate limit is the minimum of four(4) feedrate limit constraints which comprise:									
		(C1) Absolute constraint not to exceed the user feedrate command, example FC20 (20 mm/s),									
		(C2) Constrain the feedrate to stay within the velocity range (min, max) allowable for the CNC									
		(C3) Constraint the feedrate to have chord error eps(u) absolutely below tolerance (1E-6) mm, as it tracks the curve trajectory, See row Item(7).									
		(C4) Constraint feedrate such that the normal acceleration (not tangential) stay within the acceleration range (min, max) allowable for the CNC machine.									
		Note that, in order to achieve meeting all 4 (C1, C2, C3, and C4) constraints simultaneously, sometimes the current feedrate at point u maybe higher than fratelimit calculated for the point u. Our runs showed that these feedrate overshoots are typically below 0.001 % of the calculated fratelimit									