

Statistics

The Statistics standard library module contains basic statistics functionality.

`Statistics.std` — Function

```
std(itr; corrected::Bool=true, mean=nothing[, dims])
```

Compute the sample standard deviation of collection `itr`.

The algorithm returns an estimator of the generative distribution's standard deviation under the assumption that each entry of `itr` is an IID drawn from that generative distribution. For arrays, this computation is equivalent to calculating $\sqrt{\text{sum}((\text{itr} .- \text{mean}(\text{itr})) .^2) / (\text{length}(\text{itr}) - 1)}$. If `corrected` is `true`, then the sum is scaled with $n-1$, whereas the sum is scaled with n if `corrected` is `false` with n the number of elements in `itr`.

If `itr` is an `AbstractArray`, `dims` can be provided to compute the standard deviation over dimensions, and `means` may contain means for each dimension of `itr`.

A pre-computed mean may be provided. When `dims` is specified, `mean` must be an array with the same shape as `mean(itr, dims=dims)` (additional trailing singleton dimensions are allowed).

Note

If array contains `NaN` or `missing` values, the result is also `NaN` or `missing` (`missing` takes precedence if array contains both). Use the `skipmissing` function to omit `missing` entries and compute the standard deviation of non-missing values.

`Statistics.stdm` — Function

```
stdm(itr, mean; corrected::Bool=true)
```

Compute the sample standard deviation of collection `itr`, with known mean(s) `mean`.

The algorithm returns an estimator of the generative distribution's standard deviation under the assumption that each entry of `itr` is an IID drawn from that generative distribution. For arrays, this computation is equivalent to calculating `sqrt(sum((itr .- mean(itr)).^2) / (length(itr) - 1))`. If `corrected` is `true`, then the sum is scaled with `n-1`, whereas the sum is scaled with `n` if `corrected` is `false` with `n` the number of elements in `itr`.

If `itr` is an `AbstractArray`, `dims` can be provided to compute the standard deviation over dimensions. In that case, `mean` must be an array with the same shape as `mean(itr, dims=dims)` (additional trailing singleton dimensions are allowed).

Note

If array contains `NaN` or `missing` values, the result is also `NaN` or `missing` (`missing` takes precedence if array contains both). Use the `skipmissing` function to omit `missing` entries and compute the standard deviation of non-missing values.

`Statistics.var` — Function

```
var(itr; corrected::Bool=true, mean=nothing[, dims])
```

Compute the sample variance of collection `itr`.

The algorithm returns an estimator of the generative distribution's variance under the assumption that each entry of `itr` is an IID drawn from that generative distribution. For arrays, this computation is equivalent to calculating `sum((itr .- mean(itr)).^2) / (length(itr) - 1)`. If `corrected` is `true`, then the sum is scaled with $n-1$, whereas the sum is scaled with n if `corrected` is `false` where n is the number of elements in `itr`.

If `itr` is an `AbstractArray`, `dims` can be provided to compute the variance over dimensions.

A pre-computed mean may be provided. When `dims` is specified, `mean` must be an array with the same shape as `mean(itr, dims=dims)` (additional trailing singleton dimensions are allowed).

! Note

If array contains `NaN` or `missing` values, the result is also `NaN` or `missing` (`missing` takes precedence if array contains both). Use the `skipmissing` function to omit `missing` entries and compute the variance of non-missing values.

Statistics.varm — Function

```
varm(itr, mean; dims, corrected::Bool=true)
```

Compute the sample variance of collection `itr`, with known mean(s) `mean`.

The algorithm returns an estimator of the generative distribution's variance under the assumption that each entry of `itr` is an IID drawn from that generative distribution. For arrays, this computation is equivalent to calculating `sum((itr .- mean(itr)).^2) / (length(itr) - 1)`. If `corrected` is `true`, then the sum is scaled with $n-1$, whereas the sum is scaled with n if `corrected` is `false` with n the number of elements in `itr`.

If `itr` is an `AbstractArray`, `dims` can be provided to compute the variance over dimensions. In that case, `mean` must be an array with the same shape as `mean(itr, dims=dims)` (additional trailing singleton dimensions are allowed).

Note

If array contains NaN or `missing` values, the result is also NaN or `missing` (`missing` takes precedence if array contains both). Use the `skipmissing` function to omit `missing` entries and compute the variance of non-missing values.

Statistics.cor — Function

```
cor(x::AbstractVector)
```

Return the number one.

```
cor(X::AbstractMatrix; dims::Int=1)
```

Compute the Pearson correlation matrix of the matrix `X` along the dimension `dims`.

```
cor(x::AbstractVector, y::AbstractVector)
```

Compute the Pearson correlation between the vectors `x` and `y`.

```
cor(X::AbstractVecOrMat, Y::AbstractVecOrMat; dims=1)
```

Compute the Pearson correlation between the vectors or matrices `X` and `Y` along the dimension `dims`.

Statistics.cov — Function

```
cov(x::AbstractVector; corrected::Bool=true)
```

Compute the variance of the vector `x`. If `corrected` is `true` (the default) then the sum is scaled with `n-1`, whereas the sum is scaled with `n` if `corrected` is `false` where `n = length(x)`.

```
cov(X::AbstractMatrix; dims::Int=1, corrected::Bool=true)
```

Compute the covariance matrix of the matrix X along the dimension dims . If `corrected` is `true` (the default) then the sum is scaled with $n-1$, whereas the sum is scaled with n if `corrected` is `false` where $n = \text{size}(X, \text{dims})$.

```
cov(x::AbstractVector, y::AbstractVector; corrected::Bool=true)
```

Compute the covariance between the vectors x and y . If `corrected` is `true` (the default), computes $\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})^*$ where $*$ denotes the complex conjugate and $n = \text{length}(x) = \text{length}(y)$. If `corrected` is `false`, computes $\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})^*$.

```
cov(X::AbstractVecOrMat, Y::AbstractVecOrMat; dims::Int=1, corrected::Bool=true)
```

Compute the covariance between the vectors or matrices X and Y along the dimension dims . If `corrected` is `true` (the default) then the sum is scaled with $n-1$, whereas the sum is scaled with n if `corrected` is `false` where $n = \text{size}(X, \text{dims}) = \text{size}(Y, \text{dims})$.

Statistics.mean! — Function

```
mean!(r, v)
```

Compute the mean of v over the singleton dimensions of r , and write results to r .

Examples

```
julia> using Statistics

julia> v = [1 2; 3 4]
2×2 Array{Int64,2}:
 1  2
 3  4

julia> mean!([1., 1.], v)
2-element Array{Float64,1}:
 1.5
 3.5
```

```
julia> mean!([1. 1.], v)
1×2 Array{Float64,2}:
 2.0  3.0
```

Statistics.mean — Function

```
mean(itr)
```

Compute the mean of all elements in a collection.

! Note

If `itr` contains `NaN` or `missing` values, the result is also `NaN` or `missing` (`missing` takes precedence if array contains both). Use the `skipmissing` function to omit `missing` entries and compute the mean of non-missing values.

Examples

```
julia> using Statistics

julia> mean(1:20)
10.5

julia> mean([1, missing, 3])
missing

julia> mean(skipmissing([1, missing, 3]))
2.0
```

```
mean(f::Function, itr)
```

Apply the function `f` to each element of collection `itr` and take the mean.

```
julia> using Statistics

julia> mean(√, [1, 2, 3])
```

```
1.3820881233139908
```

```
julia> mean([√1, √2, √3])  
1.3820881233139908
```

```
mean(f::Function, A::AbstractArray; dims)
```

Apply the function `f` to each element of array `A` and take the mean over dimensions `dims`.

! Julia 1.3

This method requires at least Julia 1.3.

```
julia> using Statistics
```

```
julia> mean(√, [1, 2, 3])  
1.3820881233139908
```

```
julia> mean([√1, √2, √3])  
1.3820881233139908
```

```
julia> mean(√, [1 2 3; 4 5 6], dims=2)  
2×1 Array{Float64,2}:  
 1.3820881233139908  
 2.2285192400943226
```

```
mean(A::AbstractArray; dims)
```

Compute the mean of an array over the given dimensions.

! Julia 1.1

`mean` for empty arrays requires at least Julia 1.1.

Examples

```
julia> using Statistics
```

```
julia> A = [1 2; 3 4]
2×2 Array{Int64,2}:
 1  2
 3  4

julia> mean(A, dims=1)
1×2 Array{Float64,2}:
 2.0  3.0

julia> mean(A, dims=2)
2×1 Array{Float64,2}:
 1.5
 3.5
```

`Statistics.median!` — Function

```
median!(v)
```

Like `median`, but may overwrite the input vector.

`Statistics.median` — Function

```
median(itr)
```

Compute the median of all elements in a collection. For an even number of elements no exact median element exists, so the result is equivalent to calculating mean of two median elements.

Note

If `itr` contains NaN or `missing` values, the result is also NaN or `missing` (`missing` takes precedence if `itr` contains both). Use the `skipmissing` function to omit `missing` entries and compute the median of non-missing values.

Examples

```
julia> using Statistics
```



```
julia> median([1, 2, 3])
2.0

julia> median([1, 2, 3, 4])
2.5

julia> median([1, 2, missing, 4])
missing

julia> median(skipmissing([1, 2, missing, 4]))
2.0
```

```
median(A::AbstractArray; dims)
```

Compute the median of an array along the given dimensions.

Examples

```
julia> using Statistics

julia> median([1 2; 3 4], dims=1)
1×2 Array{Float64,2}:
 2.0  3.0
```

Statistics.middle — Function

```
middle(x)
```

Compute the middle of a scalar value, which is equivalent to x itself, but of the type of `middle(x, x)` for consistency.

```
middle(x, y)
```

Compute the middle of two reals x and y , which is equivalent in both value and type to computing their mean $((x + y) / 2)$.

```
middle(range)
```

Compute the middle of a range, which consists of computing the mean of its extrema. Since a range is sorted, the mean is performed with the first and last element.

```
julia> using Statistics

julia> middle(1:10)
5.5
```

```
middle(a)
```

Compute the middle of an array `a`, which consists of finding its extrema and then computing their mean.

```
julia> using Statistics

julia> a = [1,2,3.6,10.9]
4-element Array{Float64,1}:
 1.0
 2.0
 3.6
10.9

julia> middle(a)
5.95
```

`Statistics.quantile!` — Function

```
quantile!([q::AbstractArray, ] v::AbstractVector, p; sorted=false, alpha::Real=
```

Compute the quantile(s) of a vector `v` at a specified probability or vector or tuple of probabilities `p` on the interval `[0,1]`. If `p` is a vector, an optional output array `q` may also be specified. (If not provided, a new output array is created.) The keyword argument `sorted` indicates whether `v` can be assumed to be sorted; if `false` (the default), then the elements of `v` will be partially sorted in-place.

By default ($\alpha = \beta = 1$), quantiles are computed via linear interpolation between the points $((k-1)/(n-1), v[k])$, for $k = 1:n$ where $n = \text{length}(v)$. This corresponds to Definition 7 of Hyndman and Fan (1996), and is the same as the R and NumPy default.

The keyword arguments `alpha` and `beta` correspond to the same parameters in Hyndman and Fan, setting them to different values allows to calculate quantiles with any of the methods 4-9 defined in this paper:

- Def. 4: $\alpha=0, \beta=1$
- Def. 5: $\alpha=0.5, \beta=0.5$
- Def. 6: $\alpha=0, \beta=0$ (Excel `PERCENTILE.EXC`, Python default, Stata `altdf`)
- Def. 7: $\alpha=1, \beta=1$ (Julia, R and NumPy default, Excel `PERCENTILE` and `PERCENTILE.INC`, Python `'inclusive'`)
- Def. 8: $\alpha=1/3, \beta=1/3$
- Def. 9: $\alpha=3/8, \beta=3/8$

Note

An `ArgumentError` is thrown if `v` contains `NaN` or `missing` values.

References

- Hyndman, R.J and Fan, Y. (1996) "Sample Quantiles in Statistical Packages", *The American Statistician*, Vol. 50, No. 4, pp. 361-365
- [Quantile on Wikipedia](#) details the different quantile definitions

Examples

```
julia> using Statistics

julia> x = [3, 2, 1];

julia> quantile!(x, 0.5)
2.0

julia> x
3-element Array{Int64,1}:
 1
 2
 3
```

```
julia> y = zeros(3);

julia> quantile!(y, x, [0.1, 0.5, 0.9]) === y
true

julia> y
3-element Array{Float64,1}:
 1.2000000000000002
 2.0
 2.8000000000000003
```

Statistics.quantile — Function

```
quantile(itr, p; sorted=false, alpha::Real=1.0, beta::Real=alpha)
```

Compute the quantile(s) of a collection `itr` at a specified probability or vector or tuple of probabilities `p` on the interval `[0,1]`. The keyword argument `sorted` indicates whether `itr` can be assumed to be sorted.

Samples quantile are defined by $Q(p) = (1-\gamma)x[j] + \gamma x[j+1]$, where $x[j]$ is the j -th order statistic, and γ is a function of $j = \text{floor}(n*p + m)$, $m = \alpha + p*(1 - \alpha - \beta)$ and $g = n*p + m - j$.

By default ($\alpha = \beta = 1$), quantiles are computed via linear interpolation between the points $((k-1)/(n-1), v[k])$, for $k = 1:n$ where $n = \text{length}(itr)$. This corresponds to Definition 7 of Hyndman and Fan (1996), and is the same as the R and NumPy default.

The keyword arguments `alpha` and `beta` correspond to the same parameters in Hyndman and Fan, setting them to different values allows to calculate quantiles with any of the methods 4-9 defined in this paper:

- Def. 4: $\alpha=0, \beta=1$
- Def. 5: $\alpha=0.5, \beta=0.5$
- Def. 6: $\alpha=0, \beta=0$ (Excel `PERCENTILE.EXC`, Python default, Stata `altdf`)
- Def. 7: $\alpha=1, \beta=1$ (Julia, R and NumPy default, Excel `PERCENTILE` and `PERCENTILE.INC`, Python `'inclusive'`)
- Def. 8: $\alpha=1/3, \beta=1/3$
- Def. 9: $\alpha=3/8, \beta=3/8$

Note

An `ArgumentError` is thrown if `v` contains `NaN` or `missing` values. Use the `skipmissing` function to omit missing entries and compute the quantiles of non-missing values.

References

- Hyndman, R.J and Fan, Y. (1996) "Sample Quantiles in Statistical Packages", *The American Statistician*, Vol. 50, No. 4, pp. 361-365
- [Quantile on Wikipedia](#) details the different quantile definitions

Examples

```
julia> using Statistics

julia> quantile(0:20, 0.5)
10.0

julia> quantile(0:20, [0.1, 0.5, 0.9])
3-element Array{Float64,1}:
 2.0
10.0
18.000000000000004

julia> quantile(skipmissing([1, 10, missing]), 0.5)
5.5
```