





Sparse Arrays

Julia has support for sparse vectors and sparse matrices in the SparseArrays stdlib module. Sparse arrays are arrays that contain enough zeros that storing them in a special data structure leads to savings in space and execution time, compared to dense arrays.

Compressed Sparse Column (CSC) Sparse Matrix **Storage**

In Julia, sparse matrices are stored in the Compressed Sparse Column (CSC) format. Julia sparse matrices have the type SparseMatrixCSC (Tv, Ti), where Tv is the type of the stored values, and Ti is the integer type for storing column pointers and row indices. The internal representation of SparseMatrixCSC is as follows:

```
struct SparseMatrixCSC{Tv,Ti<:Integer} <: AbstractSparseMatrix{Tv,Ti}</pre>
    m::Int
                            # Number of rows
    n::Int
                             # Number of columns
                            # Column j is in colptr[j]:(colptr[j+1]-1)
    colptr::Vector{Ti}
    rowval::Vector{Ti}
                            # Row indices of stored values
    nzval::Vector{Tv}
                            # Stored values, typically nonzeros
end
```

The compressed sparse column storage makes it easy and quick to access the elements in the column of a sparse matrix, whereas accessing the sparse matrix by rows is considerably slower. Operations such as insertion of previously unstored entries one at a time in the CSC structure tend to be slow. This is because all elements of the sparse matrix that are beyond the point of insertion have to be moved one place over.

All operations on sparse matrices are carefully implemented to exploit the CSC data structure for performance, and to avoid expensive operations.

If you have data in CSC format from a different application or library, and wish to import it in Julia, make sure that you use 1-based indexing. The row indices in every column need to be sorted. If your SparseMatrixCSC object contains unsorted row indices, one quick way to sort them is by doing a double transpose.

In some applications, it is convenient to store explicit zero values in a SparseMatrixCSC. These are

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accepted by functions in Base (but there is no guarantee that they will be preserved in mutating operations). Such explicitly stored zeros are treated as structural nonzeros by many routines. The nnz function returns the number of elements explicitly stored in the sparse data structure, including structural nonzeros. In order to count the exact number of numerical nonzeros, use count(!iszero, x), which inspects every stored element of a sparse matrix. dropzeros, and the in-place dropzeros!, can be used to remove stored zeros from the sparse matrix.

```
julia> A = sparse([1, 1, 2, 3], [1, 3, 2, 3], [0, 1, 2, 0])
3×3 SparseMatrixCSC{Int64,Int64} with 4 stored entries:
    [1, 1] = 0
    [2, 2] = 2
    [1, 3] = 1
    [3, 3] = 0

julia> dropzeros(A)
3×3 SparseMatrixCSC{Int64,Int64} with 2 stored entries:
    [2, 2] = 2
    [1, 3] = 1
```

Sparse Vector Storage

Sparse vectors are stored in a close analog to compressed sparse column format for sparse matrices. In Julia, sparse vectors have the type SparseVector{Tv, Ti} where Tv is the type of the stored values and Ti the integer type for the indices. The internal representation is as follows:

As for SparseMatrixCSC, the SparseVector type can also contain explicitly stored zeros. (See Sparse Matrix Storage.).

Sparse Vector and Matrix Constructors

The simplest way to create a sparse array is to use a function equivalent to the zeros function that Julia provides for working with dense arrays. To produce a sparse array instead, you can use the same name with an sp prefix:

```
julia> spzeros(3)
3-element SparseVector{Float64, Int64} with 0 stored entries
```

The sparse function is often a handy way to construct sparse arrays. For example, to construct a sparse matrix we can input a vector I of row indices, a vector J of column indices, and a vector V of stored values (this is also known as the COO (coordinate) format). sparse(I, J, V) then constructs a sparse matrix such that S[I[k], J[k]] = V[k]. The equivalent sparse vector constructor is sparsevec, which takes the (row) index vector I and the vector V with the stored values and constructs a sparse vector R such that R[I[k]] = V[k].

```
julia> I = [1, 4, 3, 5]; J = [4, 7, 18, 9]; V = [1, 2, -5, 3];
julia > S = sparse(I, J, V)
5×18 SparseMatrixCSC{Int64, Int64} with 4 stored entries:
  [1,
      4] = 1
  [4,
     7] = 2
  [5,
      9] = 3
  [3, 18] = -5
julia> R = sparsevec(I,V)
5-element SparseVector{Int64, Int64} with 4 stored entries:
  [1] = 1
  [3] = -5
  [4] = 2
  [5] = 3
```

The inverse of the sparse and sparsevec functions is findnz, which retrieves the inputs used to create the sparse array. findall(!iszero, x) returns the cartesian indices of non-zero entries in x (including stored entries equal to zero).

```
julia> findnz(S)
([1, 4, 5, 3], [4, 7, 9, 18], [1, 2, 3, -5])

julia> findall(!iszero, S)
4-element Array{CartesianIndex{2},1}:
    CartesianIndex(1, 4)
    CartesianIndex(4, 7)
    CartesianIndex(5, 9)
    CartesianIndex(3, 18)

julia> findnz(R)
([1, 3, 4, 5], [1, -5, 2, 3])
```

```
julia> findall(!iszero, R)
4-element Array{Int64,1}:
1
3
4
5
```

Another way to create a sparse array is to convert a dense array into a sparse array using the sparse function:

```
julia> sparse(Matrix(1.0I, 5, 5))
5×5 SparseMatrixCSC{Float64,Int64} with 5 stored entries:
    [1, 1] = 1.0
    [2, 2] = 1.0
    [3, 3] = 1.0
    [4, 4] = 1.0
    [5, 5] = 1.0

julia> sparse([1.0, 0.0, 1.0])
3-element SparseVector{Float64,Int64} with 2 stored entries:
    [1] = 1.0
    [3] = 1.0
```

You can go in the other direction using the Array constructor. The issparse function can be used to query if a matrix is sparse.

```
julia> issparse(spzeros(5))
true
```

Sparse matrix operations

Arithmetic operations on sparse matrices also work as they do on dense matrices. Indexing of, assignment into, and concatenation of sparse matrices work in the same way as dense matrices. Indexing operations, especially assignment, are expensive, when carried out one element at a time. In many cases it may be better to convert the sparse matrix into (I, J, V) format using findnz, manipulate the values or the structure in the dense vectors (I, J, V), and then reconstruct the sparse matrix.

Correspondence of dense and sparse methods

The following table gives a correspondence between built-in methods on sparse matrices and their

corresponding methods on dense matrix types. In general, methods that generate sparse matrices differ from their dense counterparts in that the resulting matrix follows the same sparsity pattern as a given sparse matrix S, or that the resulting sparse matrix has density d, i.e. each matrix element has a probability d of being non-zero.

Details can be found in the Sparse Vectors and Matrices section of the standard library reference.

Sparse	Dense	Description
spzeros(m,n)	zeros(m,n)	Creates a m -by- n matrix of zeros. (spzeros(m, n) is empty.)
sparse(I, n, n)	Matrix(I,n,n)	Creates a <i>n</i> -by- <i>n</i> identity matrix.
Array(S)	sparse(A)	Interconverts between dense and sparse formats.
sprand(m,n,d)	rand(m,n)	Creates a m -by- n random matrix (of density d) with iid non-zero elements distributed uniformly on the half-open interval $[0,1)$.
sprandn(m,n,d)	randn(m,n)	Creates a <i>m</i> -by- <i>n</i> random matrix (of density <i>d</i>) with iid non-zero elements distributed according to the standard normal (Gaussian) distribution.
sprandn(rng,m,n,d)	randn(rng,m,n)	Creates a <i>m</i> -by- <i>n</i> random matrix (of density <i>d</i>) with iid non-zero elements generated with the rng random number generator

Sparse Arrays

SparseArrays.AbstractSparseArray - Type

AbstractSparseArray{Tv,Ti,N}

Supertype for N-dimensional sparse arrays (or array-like types) with elements of type Tv and index type Ti. SparseMatrixCSC, SparseVector and SuiteSparse.CHOLMOD.Sparse are subtypes of this.

SparseArrays.AbstractSparseVector - Type

AbstractSparseVector{Tv,Ti}

Supertype for one-dimensional sparse arrays (or array-like types) with elements of type Tv and index type Ti. Alias for AbstractSparseArray{Tv, Ti, 1}.

SparseArrays.AbstractSparseMatrix - Type

AbstractSparseMatrix{Tv,Ti}

Supertype for two-dimensional sparse arrays (or array-like types) with elements of type Tv and index type Ti. Alias for AbstractSparseArray{Tv, Ti, 2}.

SparseArrays.SparseVector - Type

SparseVector{Tv,Ti<:Integer} <: AbstractSparseVector{Tv,Ti}</pre>

Vector type for storing sparse vectors.

SparseArrays.SparseMatrixCSC — Type

SparseMatrixCSC{Tv,Ti<:Integer} <: AbstractSparseMatrixCSC{Tv,Ti}</pre>

Matrix type for storing sparse matrices in the Compressed Sparse Column format. The standard way of constructing SparseMatrixCSC is through the sparse function. See also spzeros, spdiagm and sprand.

SparseArrays.sparse — Function

sparse(A)

Convert an AbstractMatrix A into a sparse matrix.

Examples

```
julia> A = Matrix(1.0I, 3, 3)
3×3 Array{Float64,2}:
1.0  0.0  0.0
0.0  1.0  0.0
0.0  0.0  1.0

julia> sparse(A)

3×3 SparseMatrixCSC{Float64,Int64} with 3 stored entries:
[1, 1] = 1.0
[2, 2] = 1.0
[3, 3] = 1.0
```

```
sparse(I, J, V,[ m, n, combine])
```

Create a sparse matrix S of dimensions m x n such that S[I[k], J[k]] = V[k]. The combine function is used to combine duplicates. If m and n are not specified, they are set to maximum(I) and maximum(J) respectively. If the combine function is not supplied, combine defaults to + unless the elements of V are Booleans in which case combine defaults to |. All elements of I must satisfy 1 <= I[k] <= m, and all elements of J must satisfy 1 <= J[k] <= n. Numerical zeros in (I, J, V) are retained as structural nonzeros; to drop numerical zeros, use dropzeros!

For additional documentation and an expert driver, see SparseArrays.sparse!.

Examples

```
julia> Is = [1; 2; 3];

julia> Js = [1; 2; 3];

julia> Vs = [1; 2; 3];

julia> sparse(Is, Js, Vs)

3×3 SparseMatrixCSC{Int64, Int64} with 3 stored entries:
    [1, 1] = 1
    [2, 2] = 2
    [3, 3] = 3
```

SparseArrays.sparsevec — Function

```
sparsevec(I, V, [m, combine])
```

Create a sparse vector S of length m such that S[I[k]] = V[k]. Duplicates are combined using the combine function, which defaults to + if no combine argument is provided, unless the elements of V are Booleans in which case combine defaults to |.

Examples

```
julia> II = [1, 3, 3, 5]; V = [0.1, 0.2, 0.3, 0.2];
julia> sparsevec(II, V)
5-element SparseVector{Float64, Int64} with 3 stored entries:
  [1] = 0.1
  [3] = 0.5
  [5] = 0.2
julia> sparsevec(II, V, 8, -)
8-element SparseVector{Float64, Int64} with 3 stored entries:
  [1] = 0.1
  [3] = -0.1
  [5] = 0.2
julia> sparsevec([1, 3, 1, 2, 2], [true, true, false, false, false])
3-element SparseVector{Bool, Int64} with 3 stored entries:
  [1] = 1
  [2] = 0
  [3] = 1
```

```
sparsevec(d::Dict, [m])
```

Create a sparse vector of length m where the nonzero indices are keys from the dictionary, and the nonzero values are the values from the dictionary.

Examples

```
julia> sparsevec(Dict(1 => 3, 2 => 2))
2-element SparseVector{Int64, Int64} with 2 stored entries:
  [1] = 3
```

```
[2] = 2
```

```
sparsevec(A)
```

Convert a vector A into a sparse vector of length m.

Examples

```
julia> sparsevec([1.0, 2.0, 0.0, 0.0, 3.0, 0.0])
6-element SparseVector{Float64,Int64} with 3 stored entries:
  [1] = 1.0
  [2] = 2.0
  [5] = 3.0
```

SparseArrays.issparse - Function

```
issparse(S)
```

Returns true if S is sparse, and false otherwise.

Examples

```
julia> sv = sparsevec([1, 4], [2.3, 2.2], 10)
10-element SparseVector{Float64, Int64} with 2 stored entries:
    [1 ] = 2.3
    [4 ] = 2.2

julia> issparse(sv)
true

julia> issparse(Array(sv))
false
```

```
SparseArrays.nnz — Function
```

```
nnz(A)
```

Returns the number of stored (filled) elements in a sparse array.

Examples

```
julia> A = sparse(2I, 3, 3)
3×3 SparseMatrixCSC{Int64,Int64} with 3 stored entries:
   [1, 1] = 2
   [2, 2] = 2
   [3, 3] = 2
julia> nnz(A)
3
```

SparseArrays.findnz — Function

```
findnz(A)
```

Return a tuple (I, J, V) where I and J are the row and column indices of the stored ("structurally non-zero") values in sparse matrix A, and V is a vector of the values.

Examples

```
julia> A = sparse([1 2 0; 0 0 3; 0 4 0])
3×3 SparseMatrixCSC{Int64,Int64} with 4 stored entries:
    [1, 1] = 1
    [1, 2] = 2
    [3, 2] = 4
    [2, 3] = 3

julia> findnz(A)
([1, 1, 3, 2], [1, 2, 2, 3], [1, 2, 4, 3])
```

SparseArrays.spzeros — Function

```
spzeros([type,]m[,n])
```

Create a sparse vector of length m or sparse matrix of size m \times n. This sparse array will not contain any nonzero values. No storage will be allocated for nonzero values during construction.

The type defaults to Float64 if not specified.

Examples

```
julia> spzeros(3, 3)
3×3 SparseMatrixCSC{Float64,Int64} with 0 stored entries

julia> spzeros(Float32, 4)
4-element SparseVector{Float32,Int64} with 0 stored entries
```

SparseArrays.spdiagm — Function

```
spdiagm(kv::Pair{<:Integer,<:AbstractVector}...)
spdiagm(m::Integer, n::Ingeger, kv::Pair{<:Integer,<:AbstractVector}...)</pre>
```

Construct a sparse diagonal matrix from Pairs of vectors and diagonals. Each vector kv. second will be placed on the kv. first diagonal. By default (if size=nothing), the matrix is square and its size is inferred from kv, but a non-square size $m \times n$ (padded with zeros as needed) can be specified by passing m, n as the first arguments.

Examples

```
julia> spdiagm(-1 => [1,2,3,4], 1 => [4,3,2,1])
5×5 SparseMatrixCSC{Int64,Int64} with 8 stored entries:
    [2, 1] = 1
    [1, 2] = 4
    [3, 2] = 2
    [2, 3] = 3
    [4, 3] = 3
    [3, 4] = 2
    [5, 4] = 4
    [4, 5] = 1
```

```
SparseArrays.blockdiag — Function
```

```
blockdiag(A...)
```

Concatenate matrices block-diagonally. Currently only implemented for sparse matrices.

Examples

```
julia> blockdiag(sparse(2I, 3, 3), sparse(4I, 2, 2))
5×5 SparseMatrixCSC{Int64,Int64} with 5 stored entries:
  [1, 1] = 2
  [2, 2] = 2
  [3, 3] = 2
  [4, 4] = 4
  [5, 5] = 4
```

SparseArrays.sprand — Function

```
sprand([rng],[type],m,[n],p::AbstractFloat,[rfn])
```

Create a random length m sparse vector or m by n sparse matrix, in which the probability of any element being nonzero is independently given by p (and hence the mean density of nonzeros is also exactly p). Nonzero values are sampled from the distribution specified by rfn and have the type type. The uniform distribution is used in case rfn is not specified. The optional rng argument specifies a random number generator, see Random Numbers.

Examples

```
julia> sprand(Bool, 2, 2, 0.5)
2×2 SparseMatrixCSC{Bool,Int64} with 1 stored entry:
   [2, 2] = 1

julia> sprand(Float64, 3, 0.75)
3-element SparseVector{Float64,Int64} with 1 stored entry:
   [3] = 0.298614
```

SparseArrays.sprandn — Function

```
sprandn([rng][,Type],m[,n],p::AbstractFloat)
```

Create a random sparse vector of length m or sparse matrix of size m by n with the specified (independent) probability p of any entry being nonzero, where nonzero values are sampled from the normal distribution. The optional rng argument specifies a random number generator, see

Random Numbers.



• Julia 1.1

Specifying the output element type Type requires at least Julia 1.1.

Examples

```
julia> sprandn(2, 2, 0.75)
2×2 SparseMatrixCSC{Float64,Int64} with 2 stored entries:
  [1, 2] = 0.586617
  [2, 2] = 0.297336
```

SparseArrays.nonzeros — Function

```
nonzeros(A)
```

Return a vector of the structural nonzero values in sparse array A. This includes zeros that are explicitly stored in the sparse array. The returned vector points directly to the internal nonzero storage of A, and any modifications to the returned vector will mutate A as well. See rowvals and nzrange.

Examples

```
julia> A = sparse(2I, 3, 3)
3×3 SparseMatrixCSC{Int64,Int64} with 3 stored entries:
  [1, 1] = 2
  [2, 2] = 2
  [3, 3] = 2
julia> nonzeros(A)
3-element Array{Int64,1}:
2
2
2
```

```
SparseArrays.rowvals — Function
```

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```
rowvals(A::AbstractSparseMatrixCSC)
```

Return a vector of the row indices of A. Any modifications to the returned vector will mutate A as well. Providing access to how the row indices are stored internally can be useful in conjunction with iterating over structural nonzero values. See also nonzeros and nzrange.

Examples

```
julia> A = sparse(2I, 3, 3)
3×3 SparseMatrixCSC{Int64,Int64} with 3 stored entries:
    [1, 1] = 2
    [2, 2] = 2
    [3, 3] = 2

julia> rowvals(A)
3-element Array{Int64,1}:
    1
    2
    3
```

SparseArrays.nzrange — Function

```
nzrange(A::AbstractSparseMatrixCSC, col::Integer)
```

Return the range of indices to the structural nonzero values of a sparse matrix column. In conjunction with nonzeros and rowvals, this allows for convenient iterating over a sparse matrix :

```
A = sparse(I,J,V)
rows = rowvals(A)
vals = nonzeros(A)
m, n = size(A)
for j = 1:n
   for i in nzrange(A, j)
      row = rows[i]
      val = vals[i]
      # perform sparse wizardry...
   end
end
```

SparseArrays.droptol! - Function

```
droptol!(A::AbstractSparseMatrixCSC, tol)
```

Removes stored values from A whose absolute value is less than or equal to tol.

```
droptol!(x::SparseVector, tol)
```

Removes stored values from x whose absolute value is less than or equal to to1.

```
SparseArrays.dropzeros! - Function
```

```
dropzeros!(A::AbstractSparseMatrixCSC;)
```

Removes stored numerical zeros from A.

For an out-of-place version, see dropzeros. For algorithmic information, see fkeep!.

```
dropzeros!(x::SparseVector)
```

Removes stored numerical zeros from x.

For an out-of-place version, see dropzeros. For algorithmic information, see fkeep!.

SparseArrays.dropzeros — Function

```
dropzeros(A::AbstractSparseMatrixCSC;)
```

Generates a copy of A and removes stored numerical zeros from that copy.

For an in-place version and algorithmic information, see dropzeros!.

Examples

```
julia> A = sparse([1, 2, 3], [1, 2, 3], [1.0, 0.0, 1.0])
3×3 SparseMatrixCSC{Float64,Int64} with 3 stored entries:
    [1, 1] = 1.0
    [2, 2] = 0.0
    [3, 3] = 1.0

julia> dropzeros(A)
3×3 SparseMatrixCSC{Float64,Int64} with 2 stored entries:
    [1, 1] = 1.0
    [3, 3] = 1.0
```

```
dropzeros(x::SparseVector)
```

Generates a copy of x and removes numerical zeros from that copy.

For an in-place version and algorithmic information, see dropzeros!.

Examples

```
julia> A = sparsevec([1, 2, 3], [1.0, 0.0, 1.0])
3-element SparseVector{Float64, Int64} with 3 stored entries:
  [1] = 1.0
  [2] = 0.0
```

```
[3] = 1.0

julia> dropzeros(A)
3-element SparseVector{Float64, Int64} with 2 stored entries:
 [1] = 1.0
 [3] = 1.0
```

SparseArrays.permute — Function

```
permute(A::AbstractSparseMatrixCSC{Tv,Ti}, p::AbstractVector{<:Integer},
    q::AbstractVector{<:Integer}) where {Tv,Ti}</pre>
```

Bilaterally permute A, returning PAQ (A[p,q]). Column-permutation q's length must match A's column count (length(q) == size(A, 2)). Row-permutation p's length must match A's row count (length(p) == size(A, 1)).

For expert drivers and additional information, see permute!.

Examples

```
julia> A = spdiagm(0 => [1, 2, 3, 4], 1 => [5, 6, 7])
4×4 SparseMatrixCSC{Int64,Int64} with 7 stored entries:
  [1, 1] = 1
  [1, 2] = 5
  [2, 2] = 2
  [2, 3] = 6
 [3, 3] = 3
  [3, 4] = 7
  [4, 4] = 4
julia> permute(A, [4, 3, 2, 1], [1, 2, 3, 4])
4×4 SparseMatrixCSC{Int64,Int64} with 7 stored entries:
  [4, 1] = 1
  [3, 2] = 2
  [4, 2] = 5
  [2, 3] = 3
  [3, 3] = 6
  [1, 4] = 4
 [2, 4] = 7
julia> permute(A, [1, 2, 3, 4], [4, 3, 2, 1])
4×4 SparseMatrixCSC{Int64,Int64} with 7 stored entries:
```

```
[3, 1] = 7

[4, 1] = 4

[2, 2] = 6

[3, 2] = 3

[1, 3] = 5

[2, 3] = 2

[1, 4] = 1
```

Base.permute! - Method

Bilaterally permute A, storing result PAQ (A[p,q]) in X. Stores intermediate result (AQ)^T (transpose(A[:,q])) in optional argument C if present. Requires that none of X, A, and, if present, C alias each other; to store result PAQ back into A, use the following method lacking X:

X's dimensions must match those of A (size(X, 1) == size(A, 1) and size(X, 2) == size(A, 2)), and X must have enough storage to accommodate all allocated entries in A (length(rowvals(X)) >= nnz(A) and length(nonzeros(X)) >= nnz(A)). Column-permutation q's length must match A's column count (length(q) == size(A, 2)). Rowpermutation p's length must match A's row count (length(p) == size(A, 1)).

C's dimensions must match those of transpose(A) (size(C, 1) == size(A, 2) and size(C, 2) == size(A, 1)), and C must have enough storage to accommodate all allocated entries in A (length(rowvals(C)) >= nnz(A) and length(nonzeros(C)) >= <math>nnz(A)).

For additional (algorithmic) information, and for versions of these methods that forgo argument checking, see (unexported) parent methods unchecked_noalias_permute! and unchecked_aliasing_permute!.

See also: permute.

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