PhD Progress Report
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Realtime interpolation of parametric curves with chord-error and feedrate constraints.

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1 Introduction

1.1 Introduction

A majority of CNC systems today supports parametric interpolation because of its various advantages over traditional linear and circular interpolations. The linear interpolator (G01) and the circular interpolator (G02, G03) are the only interpolators defined in the RS274 standard. [1]

There are many merits of parametric interpolation over the traditional linear and circular interpolation, specifically in terms of the model representation, feedrate smoothness and application range. One of the major difficulties of parametric interpolation is the feedrate determination that satisfies geometrical constraints and kinematical characteristics of machine tools.

Parametric interpolation is conducted as a point-to-point (PTP) movement in a CNC machine. At the end of each motion it is important that the positional accuracy of the tool relative to the workpiece is achieved, that is, within a specified tolerance. At the end of each motion, the tool performs its required task after which the next motion begins and the cycle repeats until all machining is completed.

In CNC machine operation, the function of interpolation is to generate coordinated movements to drive the separate axis-of-motions in order to achieve the desired path of the CNC cutting tool relative to the workpiece. Essentially, interpolation generates commands that moves motor drives to follow the desired path or trajectory to produce the physical part that is to be machined.

Generally, the contour error is a measure of how close the actual tool path is to the desired tool path. For a two-dimensional parametric curve, the point-to-point movement turns the contour error (positional accuracy) to become the chord-error. This is the error between two interpolated points.

[1] The RS274/NGC Interpreter is a software system that reads numerical control code in the "NGC" dialect of the RS274 numerical control language and produces calls to a set of canonical machining functions. The output of the Interpreter can be used to drive machining centers with three to six axes.

1.2 Progress Report

This work covers the realtime interpolation of parametric curves with chord-error and feedrate constraints. An interpolation algorithm was developed and tested for ten(10) 2D parametric curves of diverse characteristics.

The curves were selected based on characteristics like closed loop curves, open ended curves, symmetric or non-symmetric about the x-axis and y-axis. Some curves have concave or convex turns. In addition, the x and y dimensions (overall sizes) vary among the different curves.

The parametric equations for the curves are described in terms of x(u), and y(u) where u is an independent parameter limited to

$$u \in [0.0, 1.0]$$

The parametric curves covered in this work are:

1. Teardrop
2. Butterfly
3. Ellipse
4. Skewed-Astroid
5. Circle
6. AstEpi = Astroid + Epicycloid combination
7. Snailshell
8. SnaHyp = Snailshell + Hypotrocoid combination
9. Ribbon-10L
10. Ribbon-100L = 10 times scaleup of Ribbon-10L

1.3 Constraints

The parametric curve interpolation algorithm was develop to obey the following constraints:

1. **(C1)** Absolute constraint not to exceed the user feedrate command, example FC20 (20 mm/s),
2. **(C2)** Constrain the feedrate to stay within the velocity range (min, max) allowable for the CNC machine,
3. **(C3)** Constrain the feedrate to have chord error eps(u) absolutely below tolerance (1E-6) mm, as it tracks the curve trajectory.
4. **(C4)** Constrain the feedrate such that the normal acceleration (not tangential) stay within the acceleration range (min, max) allowable for the CNC machine.

1.4 Brief of the algorithm design

This interpolation algorithm is executed as a point-to-point traversal of the parametric curve.

1. **Step 1** Start at u = 0.0
2. **Step 2** Based on the parametric equations, calculate the following functions:
   (a) x(u), y(u), dx(u)/du, dy(u)/du, d2x(u)/du2, d2y(u)/du2
   (b) evaluate the functions at current point u
   (c) ensure that the second order derivatives exist and are non zero
3. **Step 3** Calculate the next interpolated point (u-next), using the second order Taylor's approximation
4. **Step 4** Calculate the feedrate limit (the minimum of 4 limits below):
   (a) limit 1 based on user provided feedrate command (FC)
   (b) limit 2 based on the allowable velocity range (min, max) allowable for the CNC machine
   (c) limit 3 based on the allowable acceleration (min, max) and jerk allowable for the CNC machine
   (d) limit 4 based on chord-error constraint (epsilon, max error tolerance) for the curve trajectory
5. **Step 5** Calculate the current feedrate
6. **Step 6** If current feedrate is below feedrate limit,
   (a) (6.1) increase current feedrate by adjusting delta, to very near but below feedrate limit
   (b) (6.2) calculate the chord-error (epsilon) with the newly adjusted current feedrate
   (c) (6.3) iterate delta in steps 6.1 and 6.2 until current feedrate is just below feedrate limit and chord-error is below tolerance
   (d) (6.4) on convergence of step 6.3, get the current feedrate
   (e) (6.5) on convergence of step 6.3, use the current feedrate to calculate the next interpolated point (u-next)
7. **Step 7** If current feedrate is above feedrate limit,
   (a) (7.1) decrease current feedrate by adjusting delta, to very near and below feedrate limit
   (b) (7.2) calculate the chord-error (epsilon) with the newly adjusted current feedrate
   (c) (7.3) iterate delta in steps 7.1 and 7.2 until current feedrate is just below feedrate limit and chord-error is below tolerance
   (d) (7.4) on convergence of step 7.3, get the current feedrate
   (e) (7.5) on convergence of step 7.3, use the current feedrate to calculate the next interpolated point (u-next)

8. **Step 8** Upon completion of either one of Step 6 or Step 7, we now have the next interpolated point (u-next). This value must be positive to move forward.

9. **Step 9** Recheck that the epsilon for the new chord (u, u + u-next) is below the chord error tolerance.

10. **Step 10** Increment u by u-next, that is (u = u + u-next). Go back to Step 2 with the new value of u.

11. **Step 11** Repeat Step 2 through Step 10, for the next interpolated point until the value of u = 1.0.

1.5 Important notes

It is important to note that for tracking and monitoring of correct algorithm execution, various important calculated values are captured and stored into appropriate data files. These text data files are used in generating important plots, for example, the feedrate profile smoothness, the x-axis and y-axis feedrates, x and y positions, the chord-error values, and so on across the entire u parameter range.

In addition, not described in the algorithm design above, the RS274/NGC g-code was generated on the fly during the algorithm execution. This provides visual validation that the algorithm executes correctly. The software application LinuxCNC ver 2.8.0 was used to run this g-code.

The same algorithm was applied to the 10 different parametric curves of different complexities. The results and findings in this report verify the robustness and effectiveness of the parametric curve interpolation algorithm for the entire spectrum of selected parametric curves.

1.6 Report Summary

1. Section 2.1 briefly summarizes the characteristics of selected parametric curves in this work. In subsequent pages, the top region provides the parametric representation of the curves in terms of x(u) and y(u). The lower left figures give the overall x and y sizes of curves. The lower right figures provide the color-coded feedrate profiles as the machine traces the path of the parametric curves.

2. Section 2.2 briefly describes the results on the total number of interpolated points generated by the algorithm for five(5) different user specified feedrate commands (FC10, FC20, FC25, FC30 and FC40). The total number of interpolated points generally decreases as the User Feedrate Command (FC) increases. This means that as you increase the feedrate, you have less interpolated points to cover for the same distance traversed. The algorithm executes in a manner that strictly abides by the four(4) defined constraints, C1, C2, C3 and C4 mentioned earlier.

3. Section 2.3 presents tabular results of important variables captured in the execution of the interpolation algorithm. The data are provided in the subsequent pages. The significance of each row item in the data table are explained. For example, Row item 30 - Percentage (Total curve error / Total distance traversed). This percentage is a more meaningful performance parameter for the algorithm. For the different parametric curves, the total curve error varies. For the different user specified feedrate commands FCs, the total curve error varies. However, in both cases the total distance traversed should not vary much (it is the same size parametric curve). This percentage can be expressed in performance like, "the amount of curve error per unit of distance traveled". It is like running speed in meters per second. The running speed is independent of how much distance we run.

4. Section 2.4 describes the results of the feedrate profiles for the ten(10) different parametric curves generated by the algorithm. The color-coded feedrate profile is one of the most important validations of the algorithm for parametric curves. Never exceeding the user specified feedrate command - In all cases of parametric curves, the User Feedrate Command FC, was never exceeded by the current feedrate for the entire range of u-points. Smoothness in feedrates - the x-axis and y-axis feedrates displayed smoothness for the entire range of u-points. Color-coded feedrate transitions - When the parametric curve is being displayed using the feedrate color code spectrum for its (x(u),y(u)) points, the path traced shows smoothness in feedrate color transitions

5. Section 2.5 covers the chord-error per unit length traversed in the point-to-point parametric curve interpolation algorithm. The error/length - The rightmost column in data tables presented in subsequent pages shows the calculated chord-error per unit length for all parametric curves in this work. User specified Feedrate Command FC - In subsequent pages, five(5) data tables are presented, one each for FC10, FC20, FC25, FC30 and FC40. Also calculated are the chord-error per unit length of all parametric curves in this work. Order magnitude error/length - Notice that the calculated chord-error per unit length for all parametric curves in this work is in the order of 1E-6 and 1E-5, where 1E-6 is the specified chord-error tolerance. This fact is true irrespective of the user specified feedrate command. Histogram for all parametric curves - Also in subsequent pages, five(5) histograms for the calculated chord-error per unit length are presented, one each for the parametric curves in this work.

6. Section 2.6 briefly describes the histogram of the total interpolated points for FC10, FC20, FC25, FC30 and FC40, in the point-to-point parametric curve interpolation algorithm for all parametric curves in this work. The histogram are presented in subsequent pages. Histogram x-axis display - The x-axis for the histogram provide the 10-bins for the entire range of u-points from u=0.0 to u=1.0. Histogram y-axis display - The y-axis for the histogram provide the total interpolated points for the five(5) user specified feedrate commands (FC10, FC20, FC25, FC30 and FC40). Histogram for all parametric curves - In subsequent pages, ten(10) histograms for the total interpolated points are presented, one each for the parametric curves in this work. Correct expectation - As expected, the general trend is that the total interpolated points decreases as the user specified feedrate command increases. Essentially the chord length grew longer as the feedrate is increased. Constraints - It is important to note that in all cases, the four(4) mandatory algorithm constraints C1, C2, C3 and C4 specified in Section 1.3 Constraints, are never compromised.

2 Simulation Experiment

2.1 Parametric Curves

This section briefly summarizes the characteristics of selected parametric curves in this work. The details are provided in the subsequent pages.

1. **Teardrop** - Closed loop, Overall Single loop, Reflection x-axis: non-symmetrical, Reflection y-axis: symmetrical
2. **Butterfly** - Closed loop, Overall Multiple loops, Reflection x-axis: non-symmetrical, Reflection y-axis: symmetrical
3. **Ellipse** - Closed loop, Overall Single loop, smooth convex curves, Reflection x-axis: symmetrical, Reflection y-axis: symmetrical
4. **Skewed-Astroid** - Closed loop, Overall Single loop, 4 cusps and 4 concave curves, Reflection x-axis: symmetrical, Reflection y-axis: symmetrical
5. **Circle** - Closed loop, Overall Single loop, smooth convex curves, Reflection x-axis: symmetrical, Reflection y-axis: symmetrical
6. **AstEpi** - Closed loop, Overall Three loops, all convex curves, Reflection x-axis: non-symmetrical, Reflection y-axis: non-symmetrical
7. **Snailshell** - Open ended curve, Overall No loop, smooth and continuous convex curves, Reflection x-axis: non-symmetrical, Reflection y-axis: non-symmetrical
8. **SnaHyp** - Open ended curve, Overall 1 loop, except for 1 concave curve, the rest are convex curves, Reflection x-axis: non-symmetrical, Reflection y-axis: non-symmetrical
9. **Ribbon-10L** - Open ended curve, Overall Single loop, smooth convex curves, Reflection x-axis: non-symmetrical, Reflection y-axis: non-symmetrical
10. **Ribbon-100L** - Open ended curve, Overall Single loop, smooth convex curves, Reflection x-axis: non-symmetrical, Reflection y-axis: non-symmetrical

In subsequent pages,

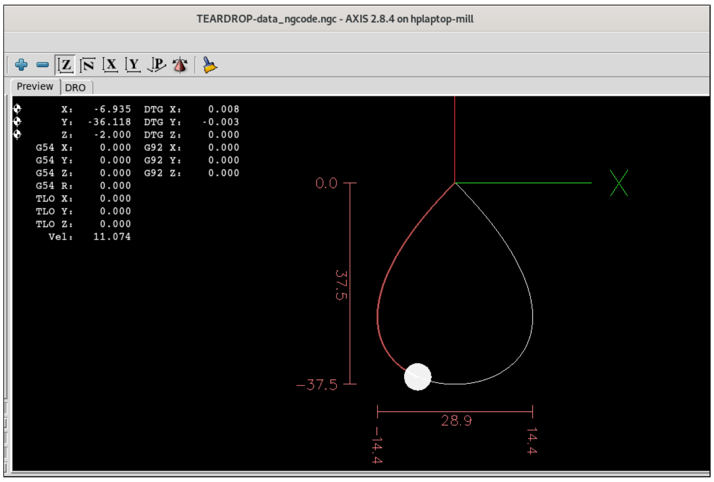
1. the top region provides the parametric representation of the curves in terms of x(u) and y(u).
2. the lower left figures give the overall x and y sizes of the curves.
3. the lower right figures provide the color-coded feedrate profiles as the machine traces (cuts) through the parametric curves.

2.1.1 Teardrop parametric equation

No. 1 - Teardrop parametric curve

$$
\begin{aligned}
x(u) &= -150u + 450u^2 - 300u^3 \\
y(u) &= -150u + 150u^2 \\
u &\in [0.0, 1.0]
\end{aligned}
$$

Closed loop
Overall Single loop
Reflection x-axis: non-symmetrical
Reflection y-axis: symmetrical



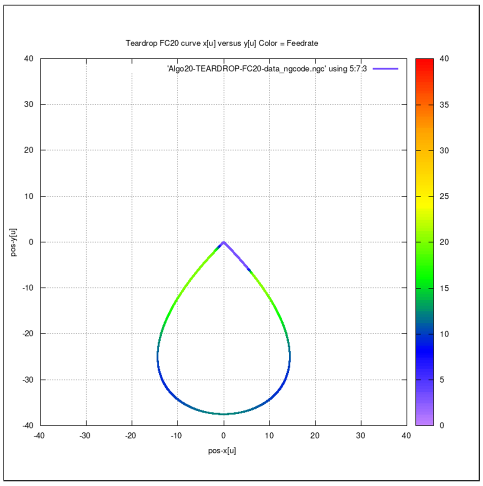


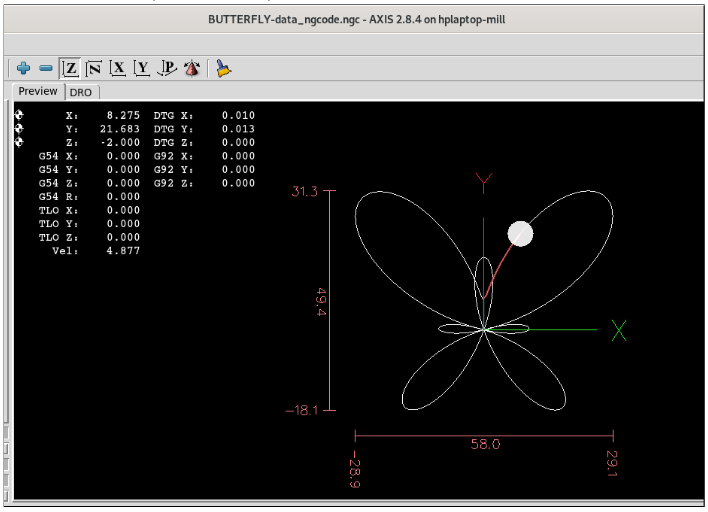
Table 2.1: Teardrop parametric equation and dimensions

2.1.2 Butterfly parametric equation

No. 2 - Butterfly parametric curve

$$
\begin{aligned}
x(u) &= \sin(2\pi u)\left[e^{\cos(2\pi u)} - 2\cos(8\pi u) - (\sin(2\pi u/12))^5\right]\\
y(u) &= \cos(2\pi u)\left[e^{\cos(2\pi u)} - 2\cos(8\pi u) - (\sin(2\pi u/12))^5\right]\\
u &\in [0.0, 1.0]
\end{aligned}
$$

Closed loop
Overall Multiple loops
Reflection x-axis: non-symmetrical
Reflection y-axis: symmetrical



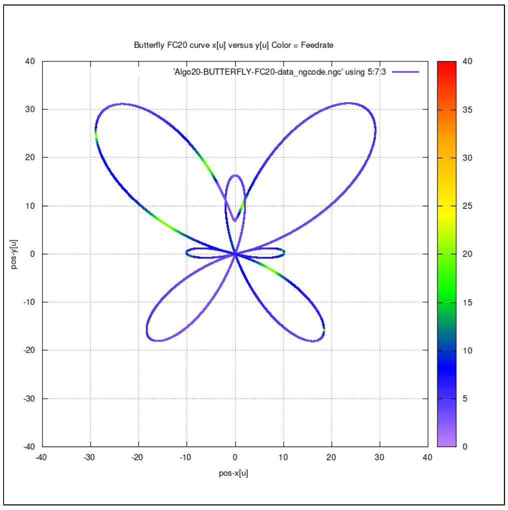


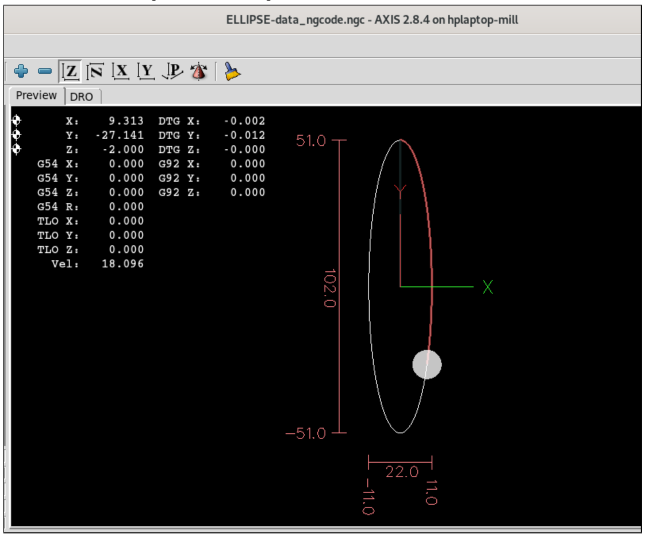
Table 2.2: Butterfly parametric equation and dimensions

2.1.3 Ellipse parametric equation

No. 3 - Ellipse parametric curve

$$\begin{aligned} x(u) &= 11\sin(2\pi u) \\ y(u) &= 51\cos(2\pi u) \\ &\in [0.0, 1.0] \end{aligned}$$

Closed loop
Overall Single loop, smooth convex curves
Reflection x-axis: symmetrical
Reflection y-axis: symmetrical



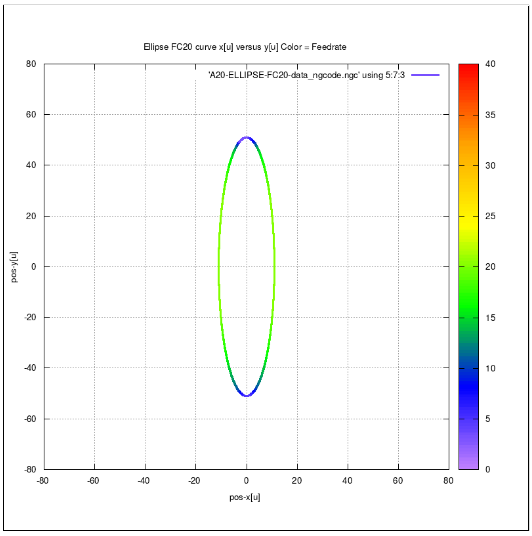


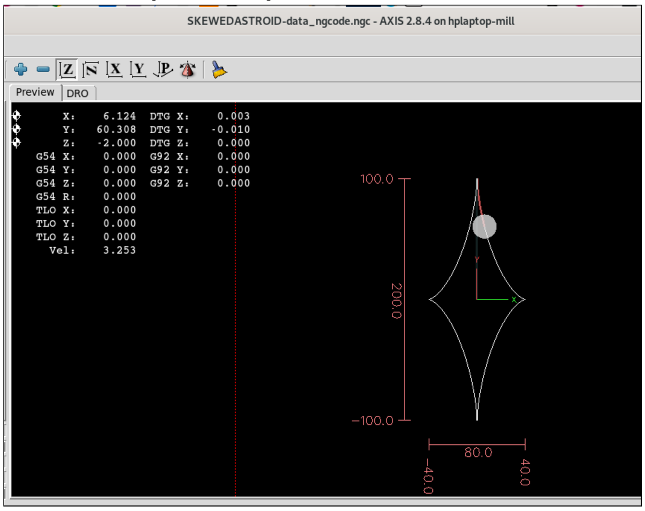
Table 2.3: Ellipse equation and dimensions

2.1.4 Skewed-Astroid parametric equation

No. 4 - Skewed-Astroid parametric curve

$$\begin{aligned} x(u) &= 40[\sin(2\pi u)]^3 \\ y(u) &= 100[\cos(2\pi u)]^3 \\ u &\in [0.0, 1.0] \end{aligned}$$

Closed loop
Overall Single loop, 4 cusps and 4 concave curves
Reflection x-axis: symmetrical
Reflection y-axis: symmetrical



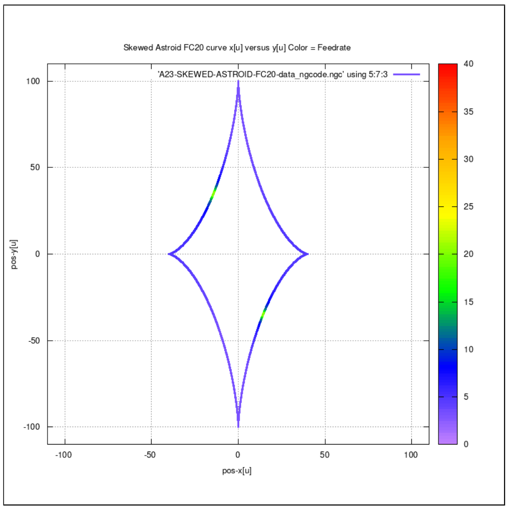


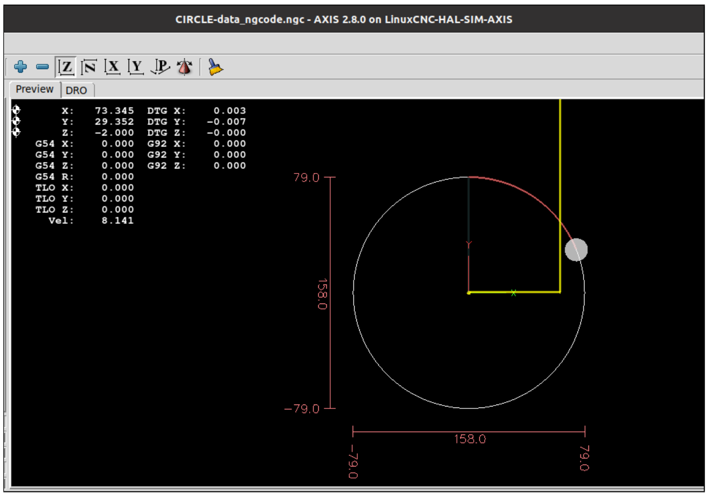
Table 2.4: Skewed-Astroid and dimensions

2.1.5 Circle parametric equation

No. 5 - Circle parametric curve

$$
\begin{aligned}
x(u) &= 79 \sin(2\pi u) \\
y(u) &= 79 \cos(2\pi u) \\
u &\in [0.0, 1.0]
\end{aligned}
$$

Closed loop
Overall Single loop, smooth convex curves
Reflection x-axis: symmetrical
Reflection y-axis: symmetrical



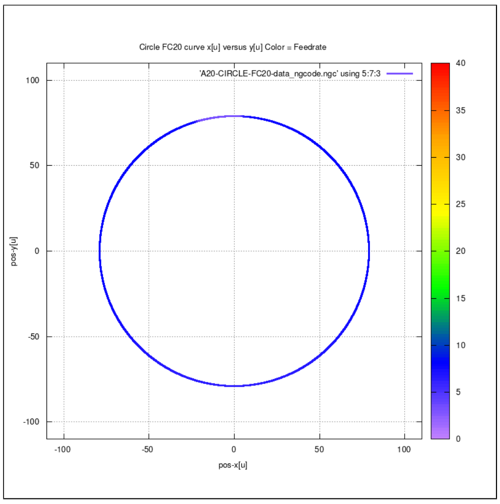


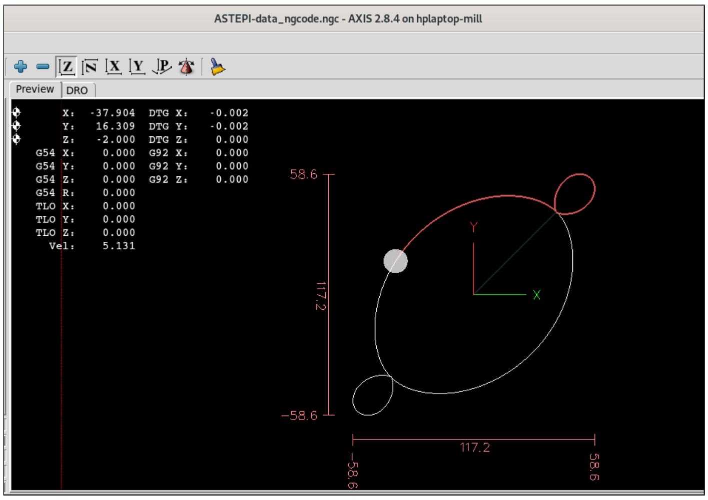
Table 2.5: Circle equation and dimensions

2.1.6 AstEpi parametric equation

No. 6 - AstEpi = Sum of (Astroid + Epicycloid) parametric curves

$$
\begin{aligned}
tiny &= 1.0\times 10^{-10} \\
x(u) &= 40[\sin(2\pi u)]^3 + 50\cos(2\pi u + tiny) - 10\cos(10\pi u - tiny) \\
y(u) &= 40[\cos(2\pi u)]^3 + 50\sin(2\pi u + tiny) - 10\sin(10\pi u - tiny) \\
u &\in [0.0, 1.0]
\end{aligned}
$$

Closed loop
Overall Three loops, all convex curves
Reflection x-axis: non-symmetrical
Reflection y-axis: non-symmetrical



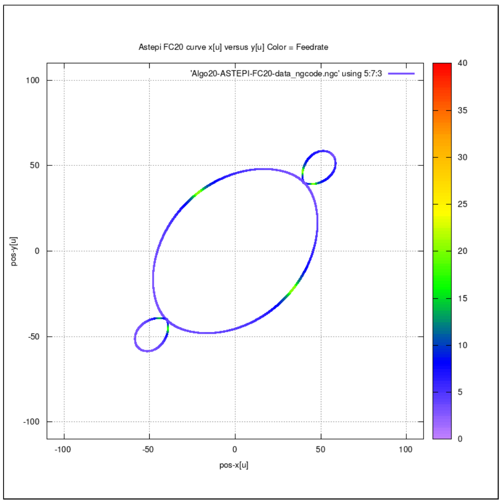


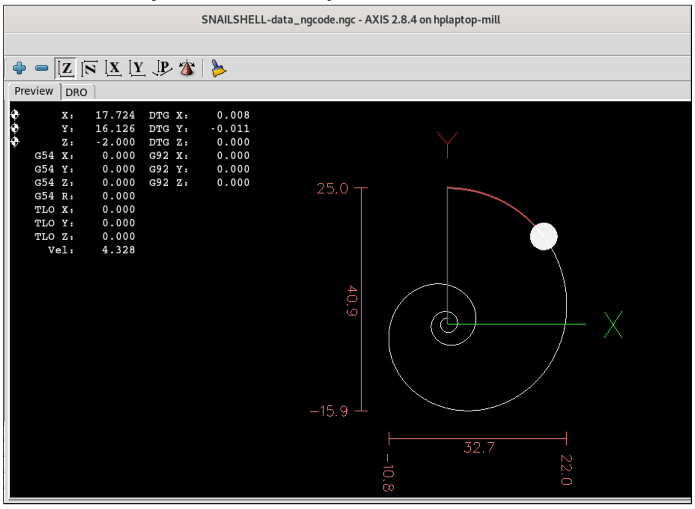
Table 2.6: Astepi equation and dimensions

2.1.7 Snailshell parametric equation

No. 7 - Snailshell parametric curve

$$
\begin{aligned}
x(u) &= 100\sin(6\pi u)/[9(\pi u)^2+4] \\
y(u) &= 100\cos(6\pi u)/[9(\pi u)^2+4] \\
u &\in [0.0, 1.0]
\end{aligned}
$$

Open ended curve
Overall No loop, smooth and continuous convex curves
Reflection x-axis: non-symmetrical
Reflection y-axis: non-symmetrical



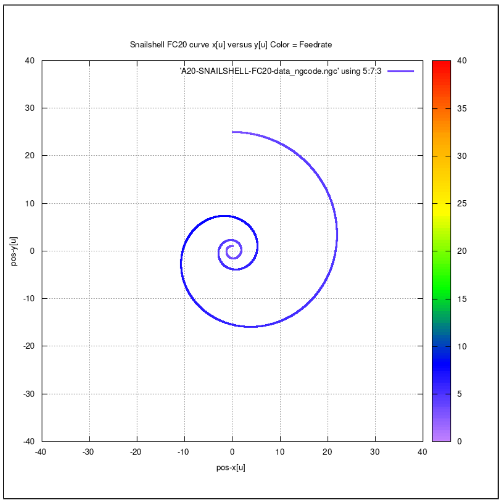


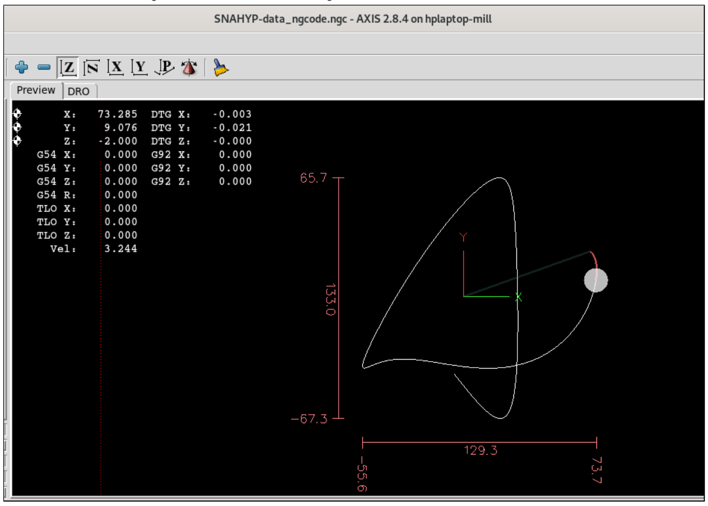
Table 2.7: Snailshell equation and dimensions

2.1.8 SnaHyp parametric equation

No. 8 - SnaHyp = Sum of (Snailshell + Hypotrocoid) parametric curves

$$
\begin{aligned}
xsna(u) &= [4\sin(8\pi u)]/[16(\pi u)^2 + 4] \\
xhyp(u) &= [2\cos(4\pi u) + 5\cos(8\pi u/3)] \\
x(u) &= 10[xsna(u) + xhyp(u)] \\
ysna(u) &= [10\cos(8\pi u)]/[16(\pi u)^2 + 4] \\
yhyp(u) &= [2\sin(8\pi u) - 5\sin(8\pi u/3)] \\
y(u) &= 10[ysna(u) + yhyp(u)] \\
u &\in [0.0, 1.0]
\end{aligned}
$$

Open ended curve
Overall 1 loop, except for 1 concave curve, the rest are convex curves
Reflection x-axis: non-symmetrical
Reflection y-axis: non-symmetrical



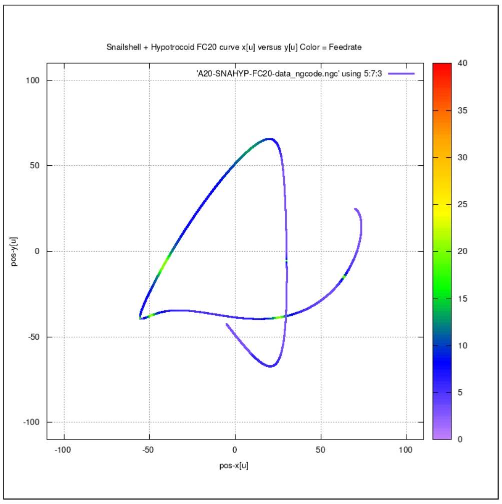


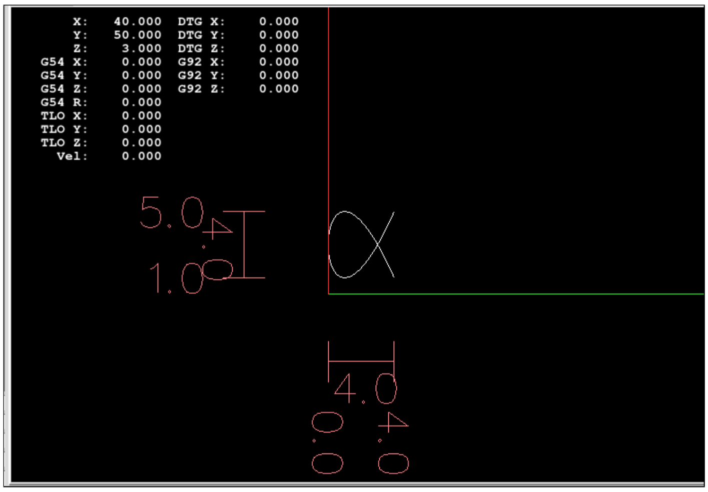
Table 2.8: SnaHyp equation and dimensions

2.1.9 Ribbon-10L parametric equation

No. 9 - Ribbon-10L parametric curve

$$
\begin{aligned}
t(u) &= 4(u-0.50) \\
x(u) &= t^2 \\
y(u) &= t^3 - 3t + 3 \\
u &\in [0.0, 1.0]
\end{aligned}
$$

Open ended curve
Overall Single loop, smooth convex curves
Reflection x-axis: non-symmetrical
Reflection y-axis: non-symmetrical



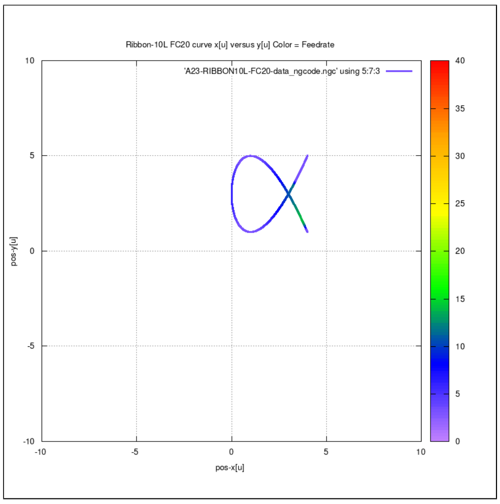


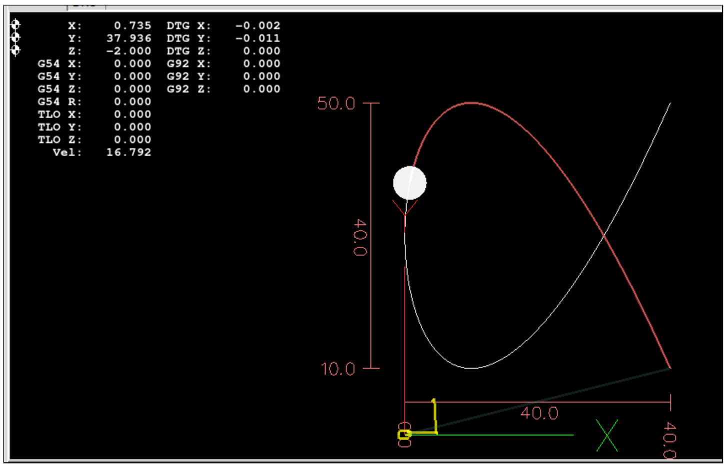
Table 2.9: Ribbon-10L equations and dimensions

2.1.10 Ribbon-100L parametric equation

No. 10 - Ribbon-100L parametric curve

$$
\begin{aligned}
t(u) &= 4(u - 0.50) \\
x(u) &= 10t^2 \\
y(u) &= 10t^3 - 30t + 30 \\
u &\in [0.0, 1.0]
\end{aligned}
$$

Open ended curve (10 times larger than RIBBON-10L)
Overall Single loop, smooth convex curves
Reflection x-axis: non-symmetrical
Reflection y-axis: non-symmetrical



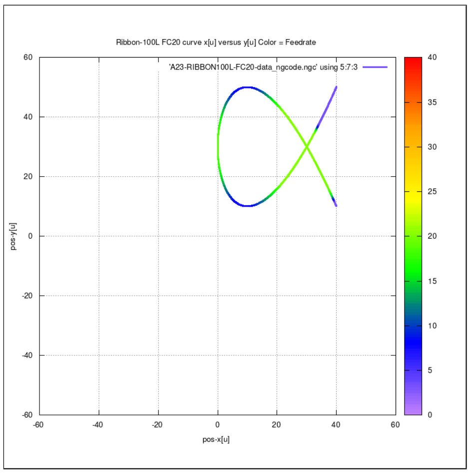


Table 2.10: Ribbon-100L equation and dimensions

2.2 Total Interpolated Points

This section briefly describes the results on the total number of interpolated points generated by the algorithm for five(5) different user specified feedrate commands (FCs).

1. **FC10** - User Feedrate Command = 10 mm/s
2. **FC20** - User Feedrate Command = 20 mm/s
3. **FC25** - User Feedrate Command = 25 mm/s
4. **FC30** - User Feedrate Command = 30 mm/s
5. **FC40** - User Feedrate Command = 40 mm/s

The results are as follows:

1. The total number of interpolated points generally decreases as the User Feedrate Command (FC) increases.
2. This means that as you increase the feedrate, you have less interpolated points to cover for the same distance traversed.
3. The algorithm executes in a manner that strictly abides by the four(4) defined constraints, mentioned earlier.

2.2.1 Teardrop profile of interpolated points

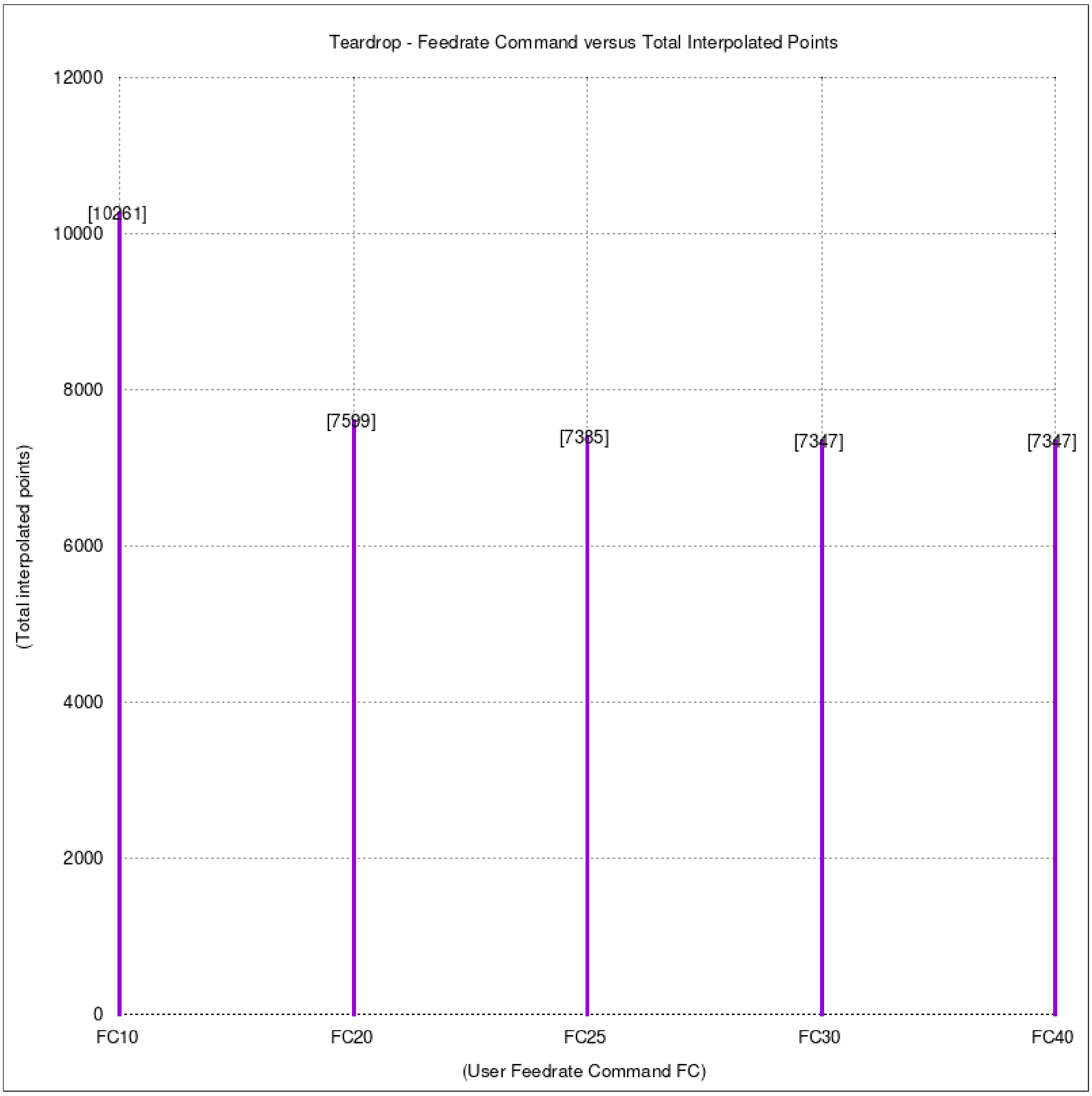


Table 2.11: Teardrop profile of interpolated points

2.2.2 Butterfly profile of interpolated points

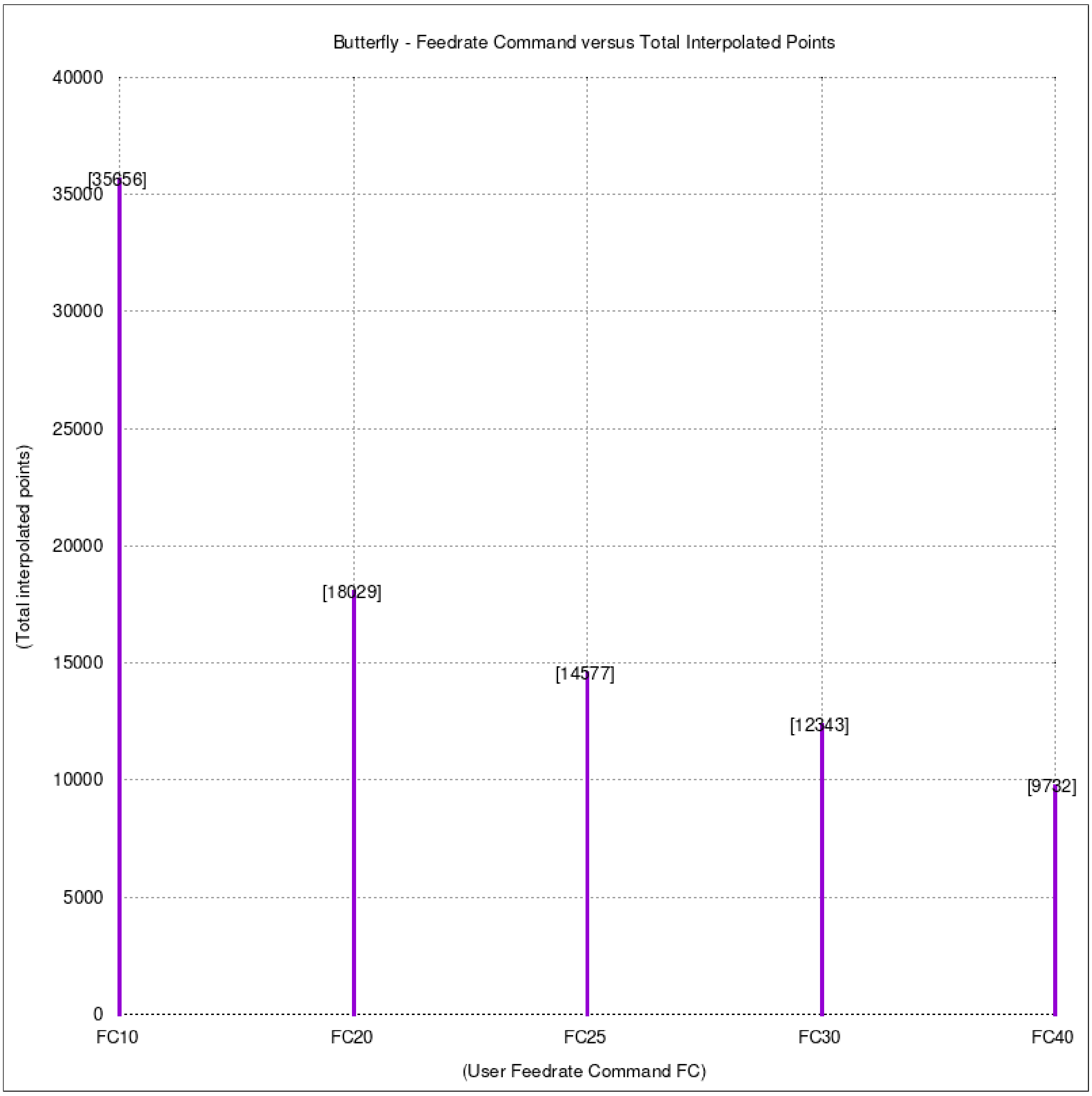


Table 2.12: Butterfly profile of interpolated points

2.2.3 Ellipse profile of interpolated points

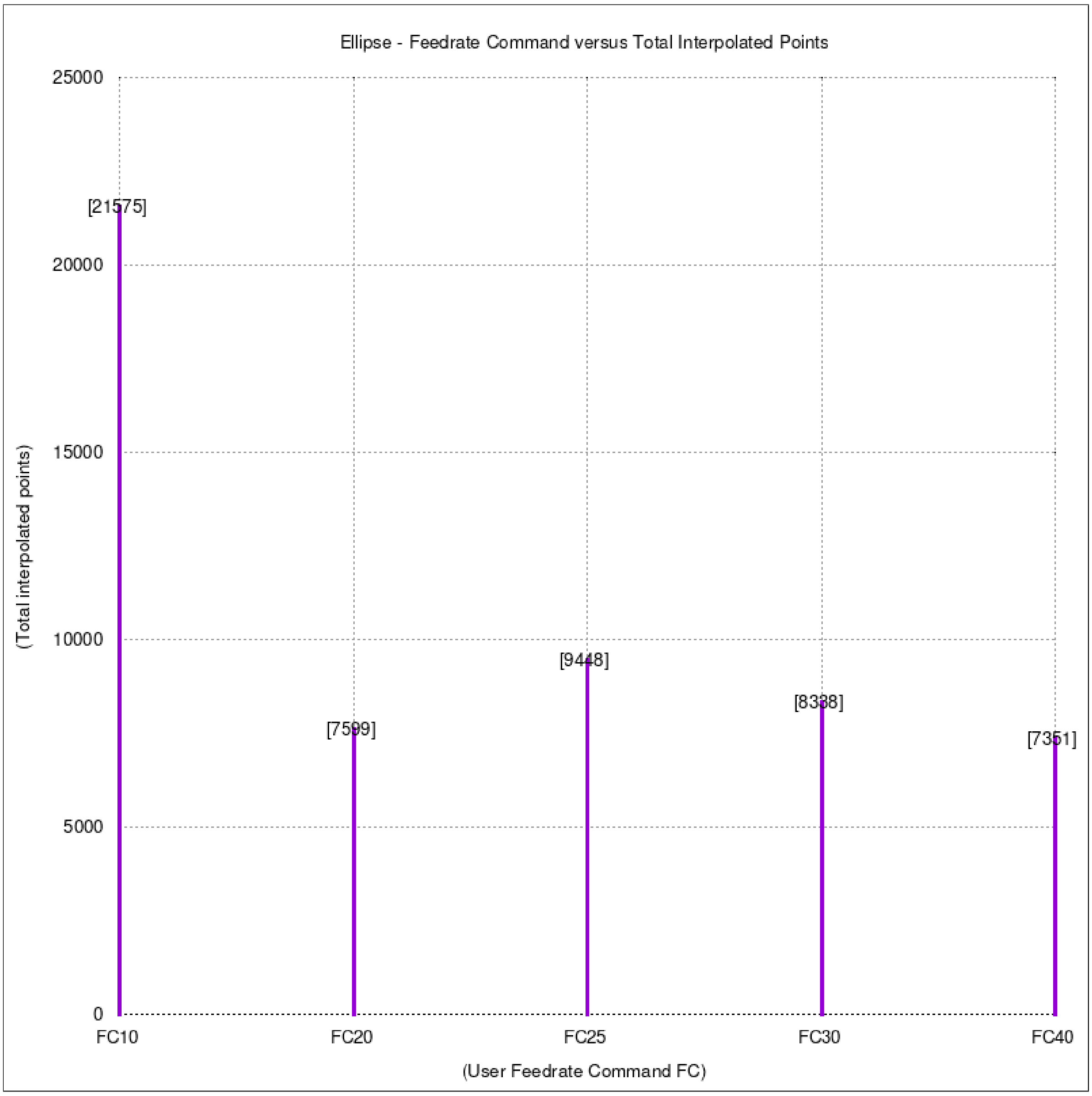


Table 2.13: Ellipse profile of interpolated points

2.2.4 Skewed-Astroid profile of interpolated points

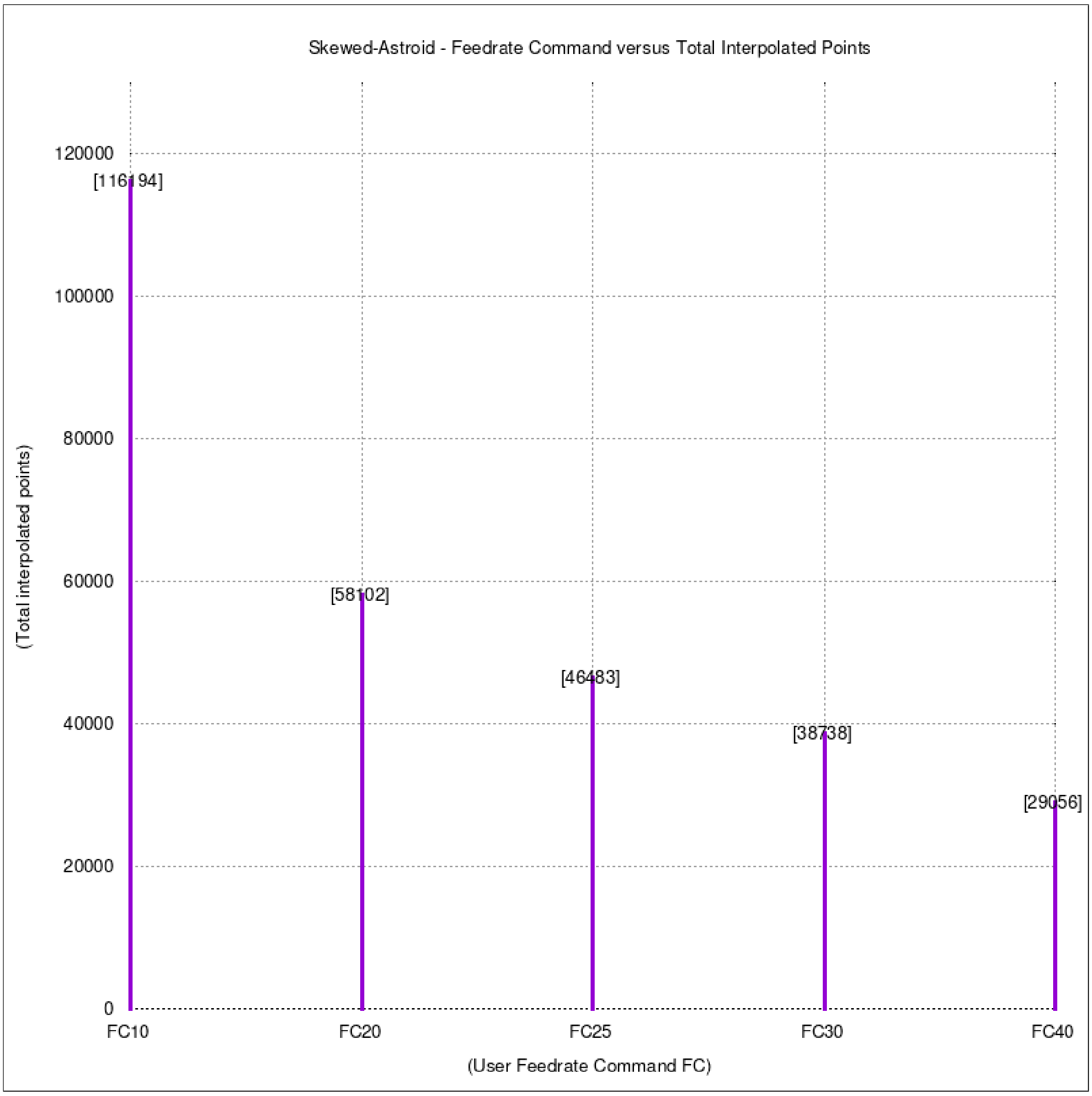


Table 2.14: Skewed-Astroid profile of interpolated points

2.2.5 Circle profile of interpolated points

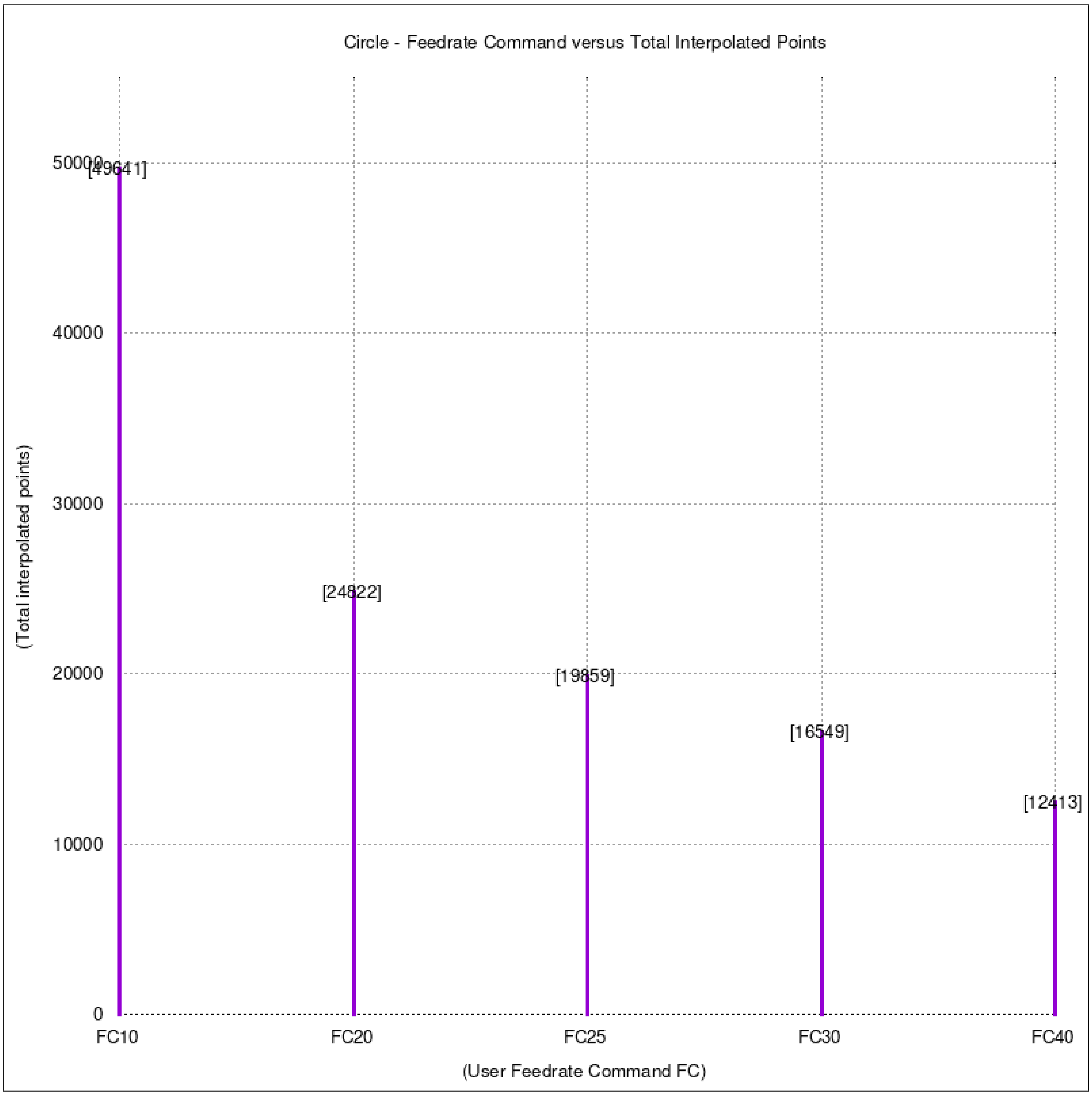


Table 2.15: Circle profile of interpolated points

2.2.6 Astroid-Epicycloid profile of interpolated points

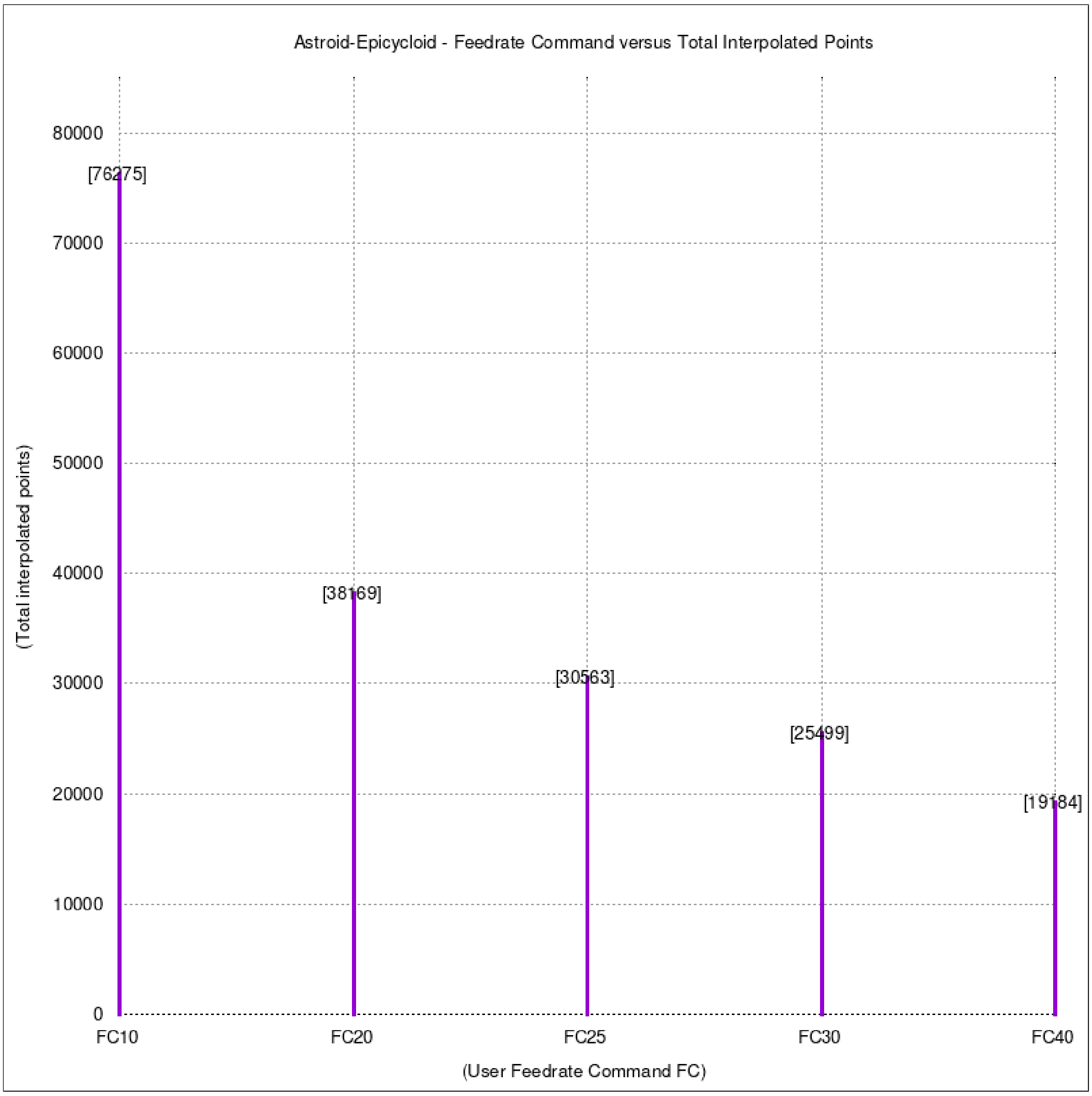


Table 2.16: AstEpi profile of interpolated points

2.2.7 Snailshell-Hypotrocoid profile of interpolated points

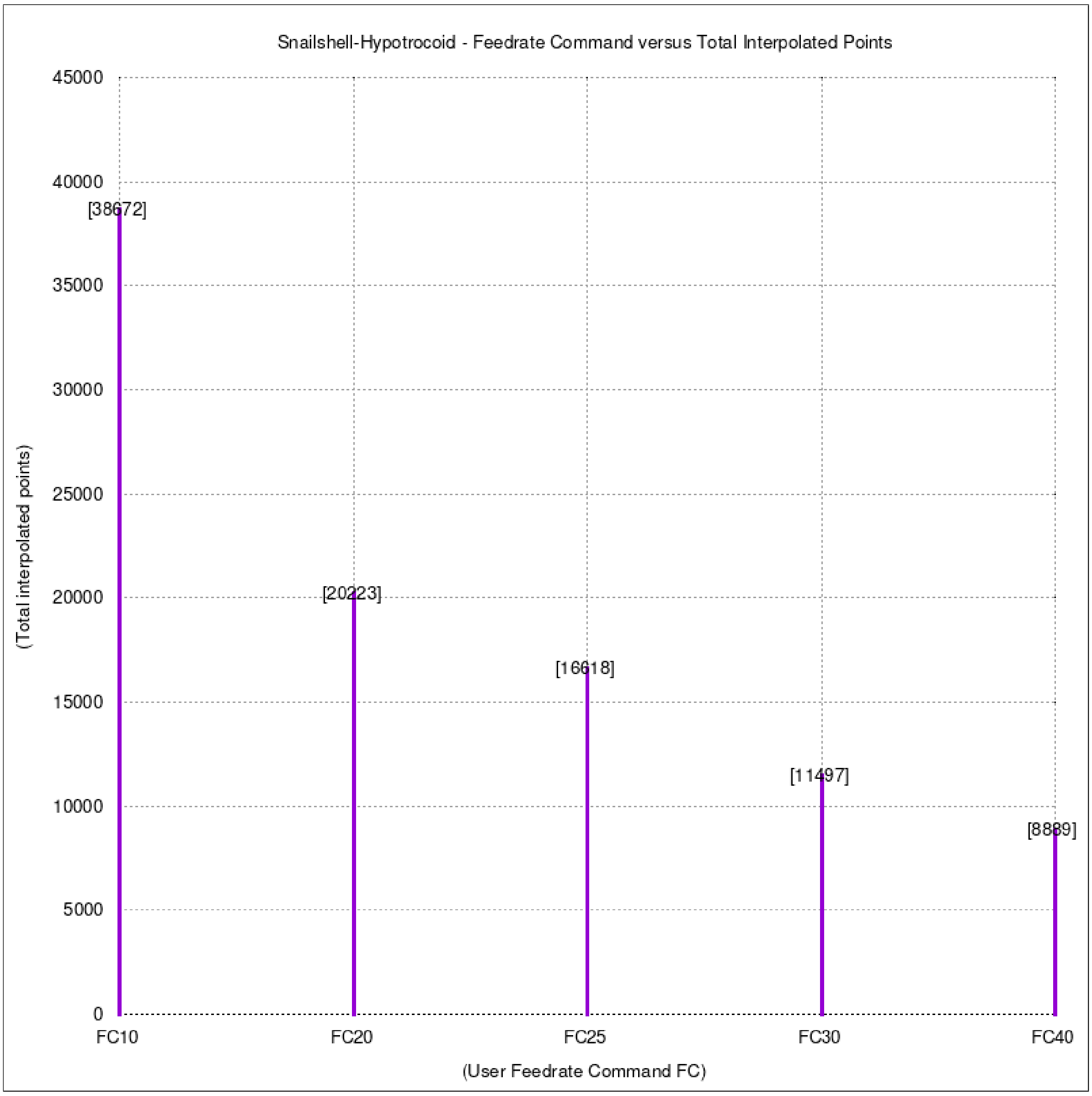


Table 2.17: SnaHyp profile of interpolated points

2.2.8 Snailshell profile of interpolated points

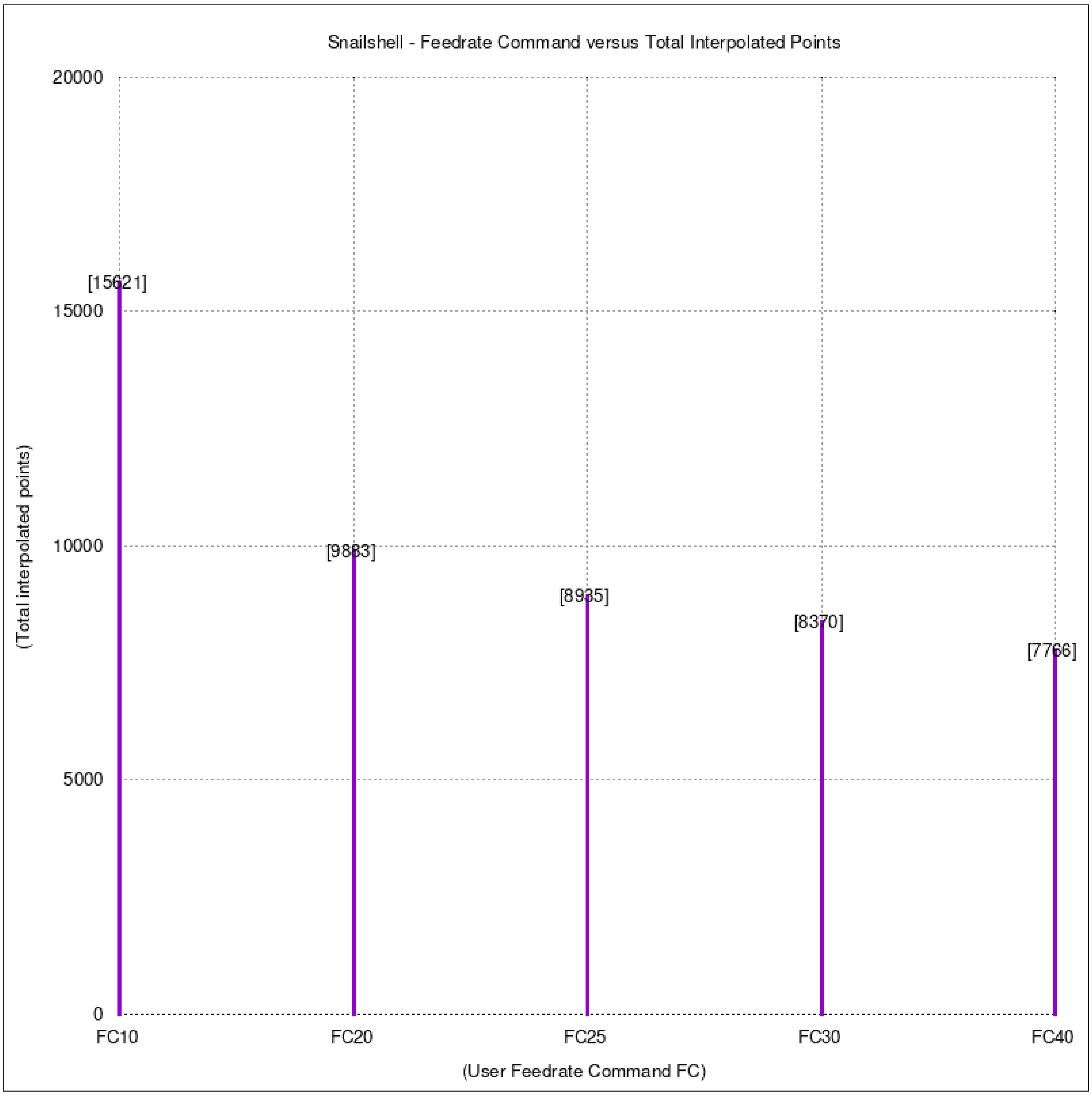


Table 2.18: Snailshell profile of interpolated points

2.2.9 Ribbon-10L profile of interpolated points

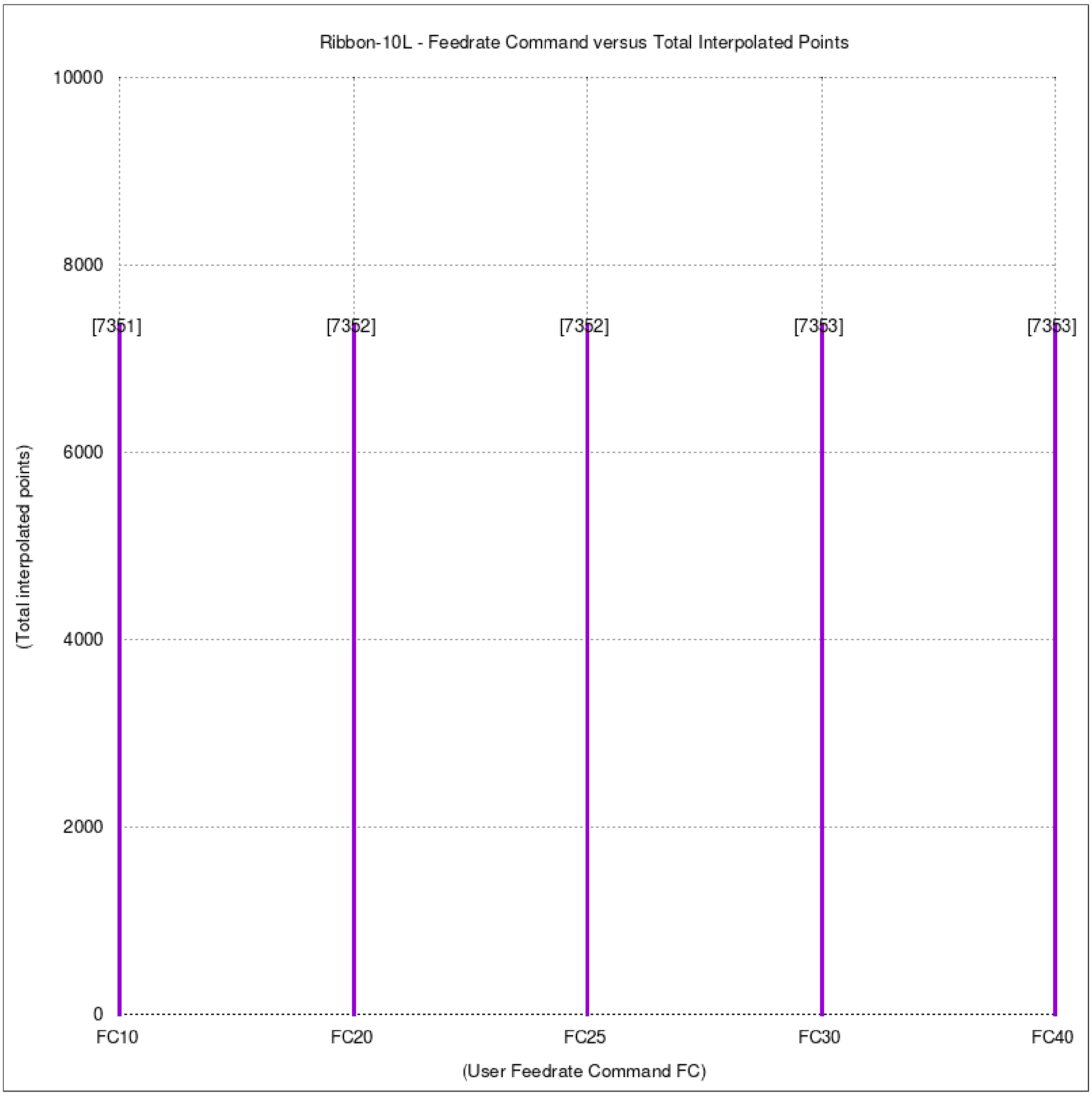


Table 2.19: Ribbon-10L profile of interpolated points

2.2.10 Ribbon-100L profile of interpolated points

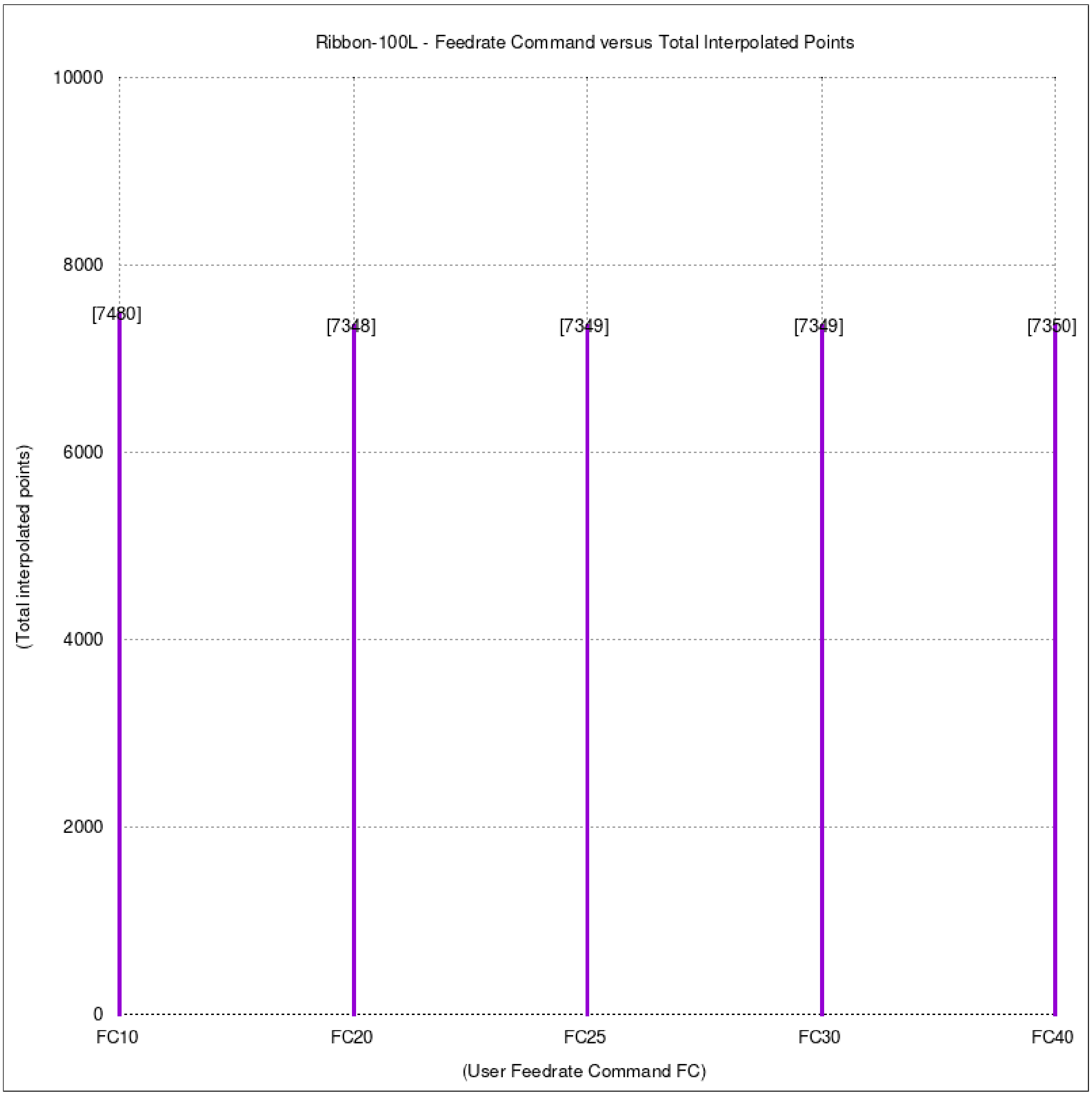


Table 2.20: Ribbon-100L profile of interpolated points

2.3 Experimental Run Results

This section covers the tabular results of important variables captured in the execution of the algorithm. The data are provided in the subsequent pages. We explain below the significance of row items in the data table.

1. **Row item 1** - Run user feedrate command (mm/s).
   This is the user specified feedrate command (FC10, FC20, FCC25, FC30, FC40). The current feedrate will never exceed this specified feedrate command.

2. **Row item 2** - Total interpolated u-points.
   This is the result of the execution of the algorithm execution.

3. **Row item 3** - Parameter completion (reached u-end).
   If at the end of execution u reaches u = 1.0, then the algorithm completed normally.

4. **Row item 4** - Count before pushdown, eps(u) is already below (1E-6).
   This is the number of interpolated points counted where the chord-error (epsilon at u) is already below the error tolerance (1E-6) and does not need to be reduced further.

5. **Row item 5** - Count chord-error pushdown points, eps(u) to below (1E-6).
   This is the number of interpolated points counted where the chord-error (epsilon at u) is above error tolerance (1E-6) and must be reduced to just below tolerance. See the convergence criteria in Step (6.3) and Step (7.3) in the Section 1.3 Brief of algorithm design. As a check, note that the sum of Row item 4 and Row item 5 equals the total interpolated u-points (Row item 2).

6. **Row item 6 through Row item 11** - This set of rows displays the distribution of interpolated points that fall within the chord-error range (epsilon at u) after completion of the algorithm execution. Similarly, the sum of row item 6 through row item 11 also equals the total interpolated u-points (Row item 2).

7. **Row item 12** - Count_rising_S_curve u-points
   This is the number of interpolated points that falls in the region of the gradual rising feedrate S_curve specified by the user. The feedrate does not increase and jump instantly.

8. **Row item 13 through Row item 15** - This set of rows displays the distribution of interpolated points where current feedrate is compared to the feedrate limit evaluated at the specific u-point.
   When the current feedrate is lower than the feedrate limit, the current feedrate is pushed up (See Step 6 in the Section 1.3 Brief of algorithm design).When the current feedrate is higher than the feedrate limit, the current feedrate is pushed down (See Step 7 in the Section 1.3 Brief of algorithm design). Note that by design, at algorithm execution completion, all current feedrate values are below their respective feedrate limits.

9. **Row item 14** - There are no interpolated points at u where the calculated current feedrate limit is exactly matching the calculated feedrate limit. This is expected because in computation when values are declared in fixed scientific "doubles" representation, calculated values cannot be compared in "exact equality" terms because of precision. The standard practice is to only compare using "less than" (in the case of Row item 13) or "greater than" (in the case of Row item 15)".

10. **Row item 16** - Count_falling_S_curve u-points.
    This is the number of interpolated points that falls in the region of the gradual falling feedrate S_curve specified by the user. The feedrate does not decrease instantly.

11. **Row item 17 through Row item 26** - This set of rows displays the histogram distribution of u-points divided into 10-equal bin sizes from u=0.0 to u=1.0. The histogram provides the distribution comparisons when the user specified feedrate command varies, that is, for FC10, FC20, FC25, FC30 and FC40. We can see that as expected, when FC increases the number of u-points decreases in the respective bins.

12. **Row item 27** - Check Total u-points.
    Again, this is a sum check that the total u-points in the histogram is equal to the Total interpolated u-points, as in Row item 2.

13. **Row item 28 through Row item 30** - This is considered the algorithm performance criteria.

14. **Row item 28** - Total curve error (sum of epsilon(u)).
    The total curve chord-error is the sum of all computed epsilon(u) for the entire u-point range. Note that the total number of u-points (total interpolated points) varies for the different user specified feedrate commands, FC10, FC20, FC25, FC30 and FC40.

15. **Row item 29** - Total distance traversed (sum of chord lengths).
    The total curve distance traversed is the sum of all computed chord lengths at each u-point for the entire u-point range. Note that the total curve distance traversed varies for the different parametric curves (different dimension x and y curve sizes, number of loops, and so on).

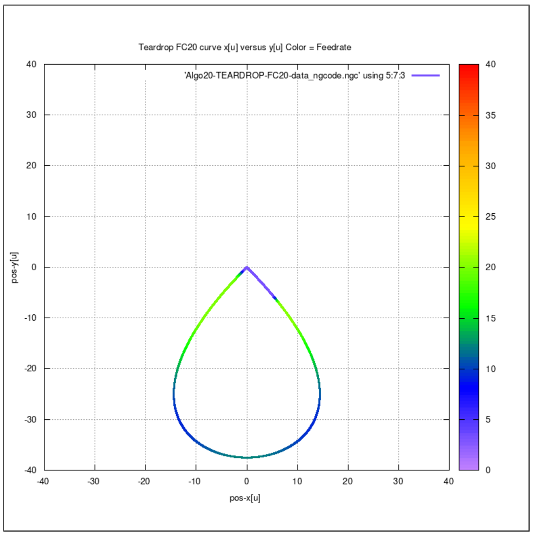
16. **Row item 30** - Percentage (Total curve error / Total distance traversed).
    This percentage is a more meaningful performance parameter for the algorithm. For the different parametric curves, the total curve error varies. For the different user specified feedrate commands, the total curve error varies. However, in both cases the total distance traversed should not vary much (it is the same parametric curve size). This percentage can be expressed in performance like, "the amount of curve error per unit of distance traveled". It is like running speed in meters per second. The running speed is independent of how much distance we run.

2.3.1 Teardrop and Butterfly Run Data

Date: 2023-06-06
Author: wruslandr@gmail.com

Report CNC Parametric Curve Interpolation and Trajectory Tracking
Part 1 of 5 Teardrop and Butterfly (x-y) parametric curves

ITEM	DESCRIPTION	TEARDROP CURVE					BUTTERFLY CURVE				
		FC10	FC20	FC25	FC30	FC40	FC10	FC20	FC25	FC30	FC40
1	Run user feedrate command (mm/s)	FC10	FC20	FC25	FC30	FC40	FC10	FC20	FC25	FC30	FC40
2	Total interpolated u-points	10261	7599	7385	7347	7347	35656	18029	14577	12343	9732
3	Parameter completion (reached u-end)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	Pushdown epsilon eps(u) algorithm										
4	Count before pushdown, eps(u) is below (1E-6)	8498	1427	527	0	0	35465	17421	13602	11010	7256
5	Count pushdown points, eps(u) to below (1E-6)	1763	6172	6858	7347	7347	191	608	975	1333	2476
	Epsilon eps(u) chord error										
6	Count eps(u) above (1E-6)	0	0	0	0	0	0	0	0	0	0
7	Count eps(u) in (1E-7, 1E-6)	10261	7599	7385	7347	7347	2995	12494	13794	12343	9732
8	Count eps(u) in (1E-8, 1E-7)	0	0	0	0	0	32661	5535	783	0	0
9	Count eps(u) in (1E-9, 1E-8)	0	0	0	0	0	0	0	0	0	0
10	Count eps(u) in (1E-10, 1E-9)	0	0	0	0	0	0	0	0	0	0
11	Count eps(u) below (1E-10)	0	0	0	0	0	0	0	0	0	0
	Count interpolated u-points										
12	Count_rising_S_curve u-points	960	480	389	370	370	1323	693	575	500	418
13	Count_frate is_lower than fratelimit	4734	4342	4260	4202	4049	17751	8968	7255	6129	4772
14	Count_frate is_equal to fratelimit	0	0	0	0	0	0	0	0	0	0
15	Count_frate is_higher than fratelimit	3608	2298	2348	2406	2559	15254	7673	6171	5213	4124
16	Count_falling_S_curve u-points	959	479	388	369	369	1328	695	576	501	418
	Count u-points histogram (G01 codes)										
17	Count u-points [0.00 <= u < 0.10)	1734	875	768	748	748	3463	1763	1431	1214	952
18	Count u-points [0.10 <= u < 0.20)	1120	791	791	791	791	4332	2167	1733	1444	1112
19	Count u-points [0.20 <= u < 0.30)	809	794	794	794	794	2983	1554	1287	1117	927
20	Count u-points [0.30 <= u < 0.40)	726	710	710	711	711	3220	1611	1293	1098	877
21	Count u-points [0.40 <= u < 0.50)	741	629	629	629	629	3832	1920	1545	1299	998
22	Count u-points [0.50 <= u < 0.60)	742	629	629	628	629	3829	1919	1544	1298	997
23	Count u-points [0.60 <= u < 0.70)	726	710	711	711	711	3222	1612	1294	1098	878
24	Count u-points [0.70 <= u < 0.80)	809	794	793	794	793	2981	1553	1286	1117	926
25	Count u-points [0.80 <= u < 0.90)	1120	791	791	791	792	4323	2162	1730	1441	1110
26	Count u-points [0.90 <= u <= 1.00]	1734	876	769	750	749	3471	1768	1434	1217	955
27	Check Total u-points	10261	7599	7385	7347	7347	35656	18029	14577	12343	9732
	Performance										
28	Total curve error (sum of epsilon(u))	0.005809	0.007141	0.007301	0.007337	0.007335	0.001939	0.003534	0.004231	0.004847	0.005851
29	Total dist traversed (sum of chord lengths)	101.8357	101.8419	101.8348	101.8596	101.8356	356.0747	356.0737	356.0723	356.0728	356.0732
30	Percentage (Tot curve error / Tot dist traversed)	0.005704	0.007012	0.00717	0.007203	0.007203	0.000545	0.000993	0.001188	0.001361	0.001643



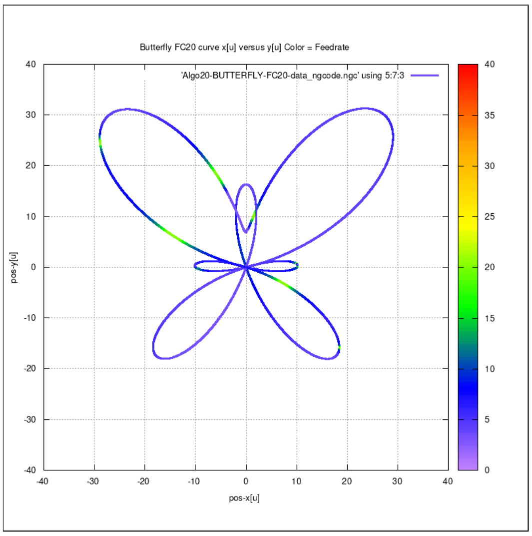


Table 2.21: Teardrop and Butterfly Run Data

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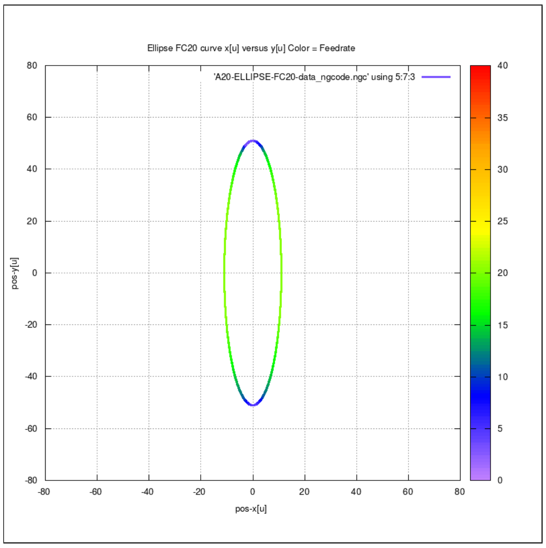
2.3.2 Ellipse and Skewed-Astroid Run Data

Date: 2023-06-06
Author: wruslandr@gmail.com

Report CNC Parametric Curve Interpolation and Trajectory Tracking
Part 2 of 5 Ellipse and Skewed-Astroid (x-y) parametric curves

ITEM	DESCRIPTION	ELLIPSE CURVE					SKEWED-ASTROID CURVE				
1	Run user feedrate command (mm/s)	FC10	FC20	FC25	FC30	FC40	FC10	FC20	FC25	FC30	FC4
2	Total interpolated u-points	21575	7599	9448	8338	7351	116194	58102	46483	38738	2905
3	Parameter completion (reached u-end)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.0
	Pushdown epsilon eps(u) algorithm										
4	Count before pushdown, eps(u) is below (1E-6)	21396	1427	7046	5149	928	116194	58102	46483	38738	2905
5	Count pushdown points, eps(u) to below (1E-6)	179	6172	2402	3189	6423	0	0	0	0	
	Epsilon eps(u) chord error										
6	Count eps(u) above (1E-6)	0	0	0	0	0	0	0	0	0	
7	Count eps(u) in (1E-7, 1E-6)	7058	7599	9448	8338	7351	0	0	0	1455	843
8	Count eps(u) in (1E-8, 1E-7)	14517	0	0	0	0	9483	33004	46483	37283	2062
9	Count eps(u) in (1E-9, 1E-8)	0	0	0	0	0	106711	25098	0	0	
10	Count eps(u) in (1E-10, 1E-9)	0	0	0	0	0	0	0	0	0	
11	Count eps(u) below (1E-10)	0	0	0	0	0	0	0	0	0	
	Count interpolated u-points										
12	Count_rising_S_curve u-points	443	480	370	370	370	3673	1837	1470	1225	91
13	Count_frate is_lower than fratelimit	14764	4342	6120	5270	3308	75069	39802	31837	26531	1993
14	Count_frate is_equal to fratelimit	0	0	0	0	0	0	0	0	0	
15	Count_frate is_higher than fratelimit	5926	2298	2589	2329	3304	33779	14626	11707	9757	727
16	Count_falling_S_curve u-points	442	479	369	369	369	3673	1837	1469	1225	91
	Count u-points histogram (G01 codes)										
17	Count u-points [0.00 <= u < 0.10)	1231	779	748	748	748	7322	3661	2929	2441	183
18	Count u-points [0.10 <= u < 0.20)	2586	1292	1035	878	791	12603	6302	5042	4202	315
19	Count u-points [0.20 <= u < 0.30)	3154	1578	1262	1051	797	18245	9123	7298	6081	456
20	Count u-points [0.30 <= u < 0.40)	2586	1293	1034	863	710	12604	6302	5042	4202	315
21	Count u-points [0.40 <= u < 0.50)	1230	704	644	628	629	7322	3662	2930	2442	183
22	Count u-points [0.50 <= u < 0.60)	1228	704	644	629	629	7323	3662	2930	2442	183
23	Count u-points [0.60 <= u < 0.70)	2586	1293	1034	862	710	12603	6302	5041	4201	315
24	Count u-points [0.70 <= u < 0.80)	3155	1577	1262	1051	796	18244	9122	7298	6082	456
25	Count u-points [0.80 <= u < 0.90)	2586	1293	1035	878	791	12604	6303	5043	4202	315
26	Count u-points [0.90 <= u <= 1.00]	1233	782	750	750	750	7324	3663	2930	2443	183
27	Check Total u-points	21575	11295	9448	8338	7351	116194	58102	46483	38738	2905
	Performance										
28	Total curve error (sum of epsilon(u))	0.002991	0.007141	0.005928	0.006562	0.007332	0.000516	0.001033	0.001291	0.001549	0.0020
29	Total dist traversed (sum of chord lengths)	215.6437	215.6499	215.644	215.6478	215.6439	445.7143	445.7143	445.7143	445.7143	445.714
30	Percent (Tot curve error / Tot dist traversed)	0.001387	0.003311	0.002749	0.003043	0.0034	0.000116	0.000232	0.00029	0.000347	0.00046



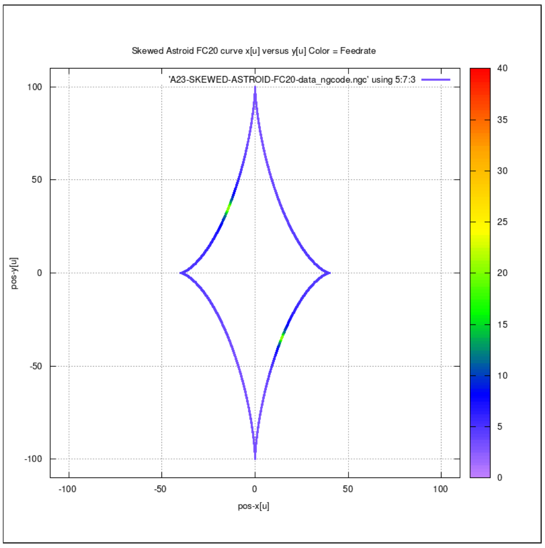


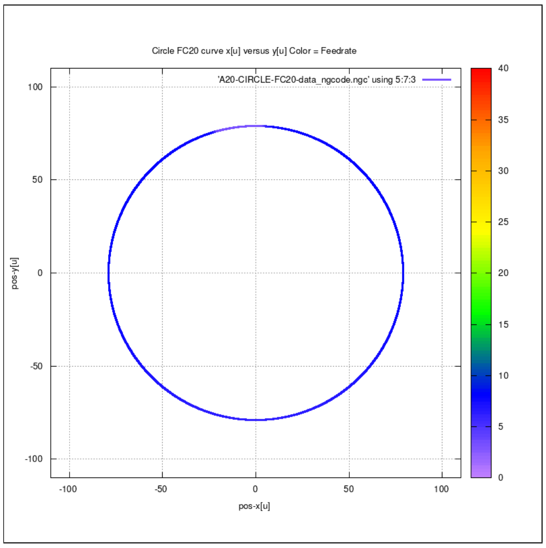
Table 2.22: Ellipse and Skewed-Astroid Run Data

2.3.3 Circle and Astepi Run Data

Date: 2023-06-06
Author: wruslandr@gmail.com

Report CNC Parametric Curve Interpolation and Trajectory Tracking
Part 3 of 5 Circle and Astepi (x-y) parametric curves

ITEM	DESCRIPTION	CIRCLE CURVE					ASTEROID + EPICYCLOID CURVE				
1	Run user feedrate command (mm/s)	FC10	FC20	FC25	FC30	FC40	FC10	FC20	FC25	FC30	FC40
2	Total interpolated u-points	49641	24822	19859	16549	12413	76275	38169	30563	25499	19184
3	Parameter completion (reached u-end)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	Pushdown epsilon eps(u) algorithm										
4	Count before pushdown, eps(u) is below (1E-6)	49641	24822	19859	16549	12413	76275	38061	30413	25309	18917
5	Count pushdown points, eps(u) to below (1E-6)	0	0	0	0	0	0	108	150	190	267
	Epsilon eps(u) chord error										
6	Count eps(u) above (1E-6)	0	0	0	0	0	0	0	0	0	0
7	Count eps(u) in (1E-7, 1E-6)	0	6959	17603	16549	12413	407	820	4011	6679	13976
8	Count eps(u) in (1E-8, 1E-7)	49641	17863	2256	0	0	23373	37349	26552	18820	5208
9	Count eps(u) in (1E-9, 1E-8)	0	0	0	0	0	52495	0	0	0	0
10	Count eps(u) in (1E-10, 1E-9)	0	0	0	0	0	0	0	0	0	0
11	Count eps(u) below (1E-10)	0	0	0	0	0	0	0	0	0	0
	Count interpolated u-points										
12	Count_rising_S_curve u-points	2483	1242	993	828	621	4437	2219	1775	1479	1110
13	Count_frate is_lower than fratelimit	22363	11171	8936	7449	5586	33349	16686	13352	11144	8085
14	Count_frate is_equal to fratelimit	0	0	0	0	0	0	0	0	0	0
15	Count_frate is_higher than fratelimit	22313	11168	8937	7445	5585	35536	17787	14254	11891	9250
16	Count_falling_S_curve u-points	2482	1241	993	827	621	2953	1477	1182	985	739
	Count u-points histogram (G01 codes)										
17	Count u-points [0.00 <= u < 0.10)	4964	2482	1985	1654	1241	8110	4055	3244	2704	2028
18	Count u-points [0.10 <= u < 0.20)	4964	2482	1986	1655	1241	4901	2478	2009	1702	1334
19	Count u-points [0.20 <= u < 0.30)	4964	2482	1986	1655	1241	7391	3696	2957	2464	1848
20	Count u-points [0.30 <= u < 0.40)	4964	2482	1986	1655	1241	7234	3617	2894	2412	1809
21	Count u-points [0.40 <= u < 0.50)	4964	2482	1986	1655	1242	9182	4592	3673	3061	2297
22	Count u-points [0.50 <= u < 0.60)	4964	2483	1985	1655	1241	7216	3608	2887	2406	1804
23	Count u-points [0.60 <= u < 0.70)	4964	2482	1986	1655	1241	9831	4916	3933	3278	2458
24	Count u-points [0.70 <= u < 0.80)	4964	2482	1986	1655	1241	7525	3763	3010	2508	1882
25	Count u-points [0.80 <= u < 0.90)	4964	2482	1986	1654	1242	8801	4401	3521	2935	2201
26	Count u-points [0.90 <= u <= 1.00]	4965	2483	1987	1656	1242	6084	3043	2435	2029	1523
27	Check Total u-points	49641	24822	19859	16549	12413	76275	38169	30563	25499	19184
	Performance										
28	Total curve error (sum of epsilon(u))	0.001094	0.001094	0.002188	0.002735	0.004375	0.00084	0.001642	0.00202	0.00239	0.003111
29	Total dist traversed (sum of chord lengths)	496.3786	496.3772	496.3964	496.3757	496.3943	426.2622	426.2622	426.2622	426.2622	426.2622
30	Percent (Tot curve error / Tot dist traversed)	0.00022	0.00022	0.000441	0.000551	0.000881	0.000197	0.000385	0.000474	0.000561	0.00073



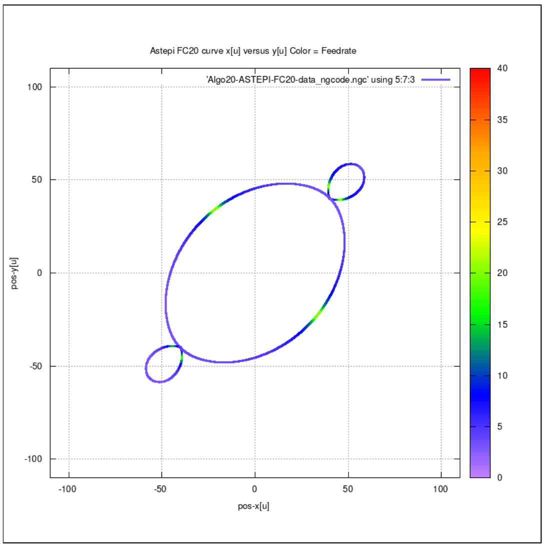


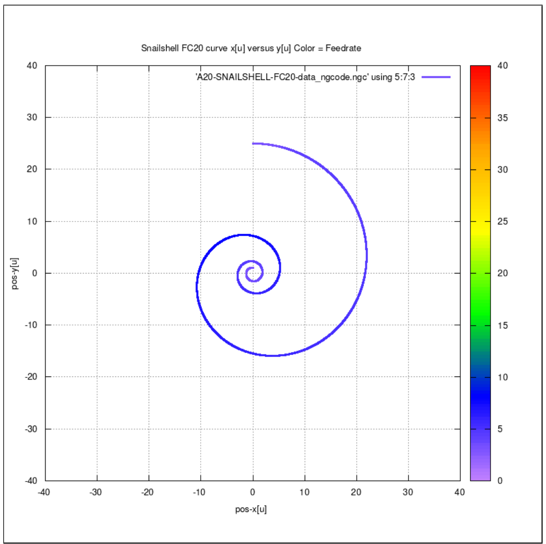
Table 2.23: Circle and Astepi Run Data

2.3.4 Snailshell and SnaHyp Run Data

Date: 2023-06-06
Author: wruslandr@gmail.com

Report CNC Parametric Curve Interpolation and Trajectory Tracking
Part 4 of 5 Snailshell and SnaHyp (x-y) parametric curves

ITEM	DESCRIPTION	SNAILSHELL CURVE					SNAILSHELL + HYPOTROCOID CURVE				
1	Run user feedrate command (mm/s)	FC10	FC20	FC25	FC30	FC40	FC10	FC20	FC25	FC30	FC40
2	Total interpolated u-points	15621	9883	8935	8370	7766	38672	20223	16618	11497	8889
3	Parameter completion (reached u-end = 1.00)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.706104	0.703536
	Pushdown epsilon eps(u) algorithm									***	***
4	Count before pushdown, eps(u) is below (1E-6)	12245	5010	3569	2671	1592	37227	18167	14266	0	0
5	Count pushdown points, eps(u) to below (1E-6)	3376	4873	5366	5699	6174	1445	2056	2352	0	0
	Epsilon eps(u) chord error										
6	Count eps(u) above (1E-6)	0	0	0	0	0	0	0	0	0	0
7	Count eps(u) in (1E-7, 1E-6)	7893	9323	8935	8370	7766	4106	6971	9900	0	0
8	Count eps(u) in (1E-8, 1E-7)	7728	560	0	0	0	24178	13252	6718	0	0
9	Count eps(u) in (1E-9, 1E-8)	0	0	0	0	0	10388	0	0	0	0
10	Count eps(u) in (1E-10, 1E-9)	0	0	0	0	0	0	0	0	0	0
11	Count eps(u) below (1E-10)	0	0	0	0	0	0	0	0	0	0
	Count interpolated u-points										
12	Count_rising_S_curve u-points	2320	1161	929	774	581	3177	1589	1272	0	0
13	Count_frate is_lower than fratelimit	11722	6605	5826	5386	4860	18305	9773	8142	0	0
14	Count_frate is_equal to fratelimit	0	0	0	0	0	0	0	0	0	0
15	Count_frate is_higher than fratelimit	1210	1747	1811	1859	1956	14859	7695	6271	0	0
16	Count_falling_S_curve u-points	369	370	359	369	369	2331	1166	933	0	0
	Count u-points histogram (G01 codes)									***	***
17	Count u-points [0.00 <= u < 0.10)	4435	2218	1774	1479	1109	4631	2317	1856	1563	1217
18	Count u-points [0.10 <= u < 0.20)	3237	1619	1296	1080	849	8961	4480	3584	2987	2240
19	Count u-points [0.20 <= u < 0.30)	2054	1028	851	796	793	6140	3074	2470	2072	1586
20	Count u-points [0.30 <= u < 0.40)	1312	714	710	711	711	2960	1526	1257	1086	885
21	Count u-points [0.40 <= u < 0.50)	881	629	629	629	629	4860	2431	1945	1620	1216
22	Count u-points [0.50 <= u < 0.60)	657	628	628	629	628	3973	1987	1589	1325	994
23	Count u-points [0.60 <= u < 0.70)	710	711	711	710	711	1324	841	769	732	710
24	Count u-points [0.70 <= u < 0.80)	794	794	794	794	794	794	794	794	112	41
25	Count u-points [0.80 <= u < 0.90)	791	791	792	792	792	1141	828	798	0	0
26	Count u-points [0.90 <= u <= 1.00]	750	751	750	750	750	3888	1945	1556	0	0
27	Check Total u-points	15621	9883	8935	8370	7766	38672	20223	16618	11497	8889
	Performance										
28	Total curve error (sum of epsilon(u))	0.005115	0.00627	0.006558	0.006764	0.007046	0.002847	0.004003	0.004459	0	0
29	Total dist traversed (sum of chord lengths)	138.5595	138.5614	138.5607	138.5602	138.5599	478.9871	478.9987	479.0064	0	0
30	Percent (Tot curve error / Tot dist traversed)	0.003692	0.004525	0.004733	0.004882	0.005085	0.000594	0.000836	0.000931	0	0



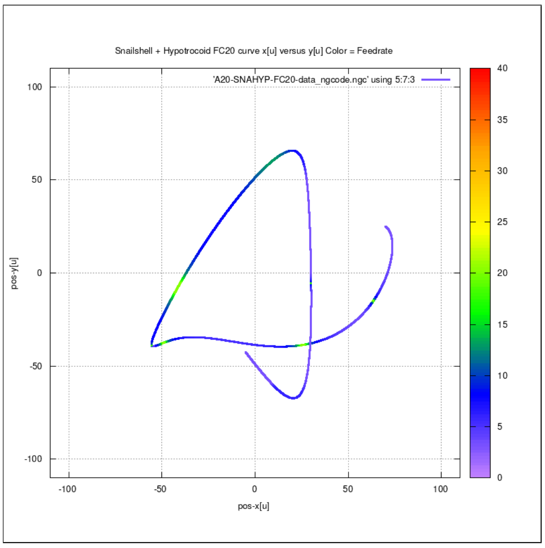


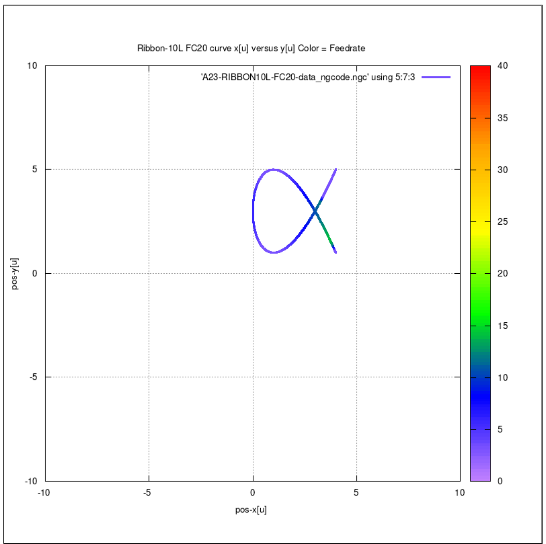
Table 2.24: Snailshell and SnaHyp Run Data

2.3.5 Ribbon-10L and Ribbon-100L Run Data

Date: 2023-06-06
Author: wruslandr@gmail.com

Report CNC Parametric Curve Interpolation and Trajectory Tracking
Part 5 of 5 Ribbon-10L and Ribbon-100L (x-y) parametric curves

ITEM	DESCRIPTION	RIBBON-10L CURVE					RIBBON-100L CURVE				
1	Run user feedrate command (mm/s)	FC10	FC20	FC25	FC30	FC40	FC10	FC20	FC25	FC30	FC40
2	Total interpolated u-points	7351	7352	7352	7353	7353	7480	7348	7349	7349	7350
3	Parameter completion (reached u-end)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	Pushdown epsilon eps(u) algorithm										
4	Count before pushdown, eps(u) is below (1E-6)	0	0	0	0	0	919	0	0	0	0
5	Count pushdown points, eps(u) to below (1E-6)	7351	7352	7352	7353	7353	6561	7348	7349	7349	7350
	Epsilon eps(u) chord error										
6	Count eps(u) above (1E-6)	0	0	0	0	0	0	0	0	0	0
7	Count eps(u) in (1E-7, 1E-6)	7351	7352	7352	7353	7353	7480	7348	7349	7349	7350
8	Count eps(u) in (1E-8, 1E-7)	0	0	0	0	0	0	0	0	0	0
9	Count eps(u) in (1E-9, 1E-8)	0	0	0	0	0	0	0	0	0	0
10	Count eps(u) in (1E-10, 1E-9)	0	0	0	0	0	0	0	0	0	0
11	Count eps(u) below (1E-10)	0	0	0	0	0	0	0	0	0	0
	Count interpolated u-points										
12	Count_rising_S_curve u-points	370	370	370	370	370	436	370	370	370	370
13	Count_frate is_lower than fratelimit	3491	3310	3310	3308	3309	5562	3939	3733	3545	3307
14	Count_frate is_equal to fratelimit	0	0	0	0	0	0	0	0	0	0
15	Count_frate is_higher than fratelimit	3121	3303	3303	3305	3305	1047	2670	2877	3065	3303
16	Count_falling_S_curve u-points	369	369	369	370	369	435	369	369	369	370
	Count u-points histogram (G01 codes)										
17	Count u-points [0.00 <= u < 0.10)	749	749	749	749	749	815	748	748	748	749
18	Count u-points [0.10 <= u < 0.20)	791	792	792	792	792	791	792	792	792	791
19	Count u-points [0.20 <= u < 0.30)	795	794	794	794	794	794	794	794	794	794
20	Count u-points [0.30 <= u < 0.40)	711	711	711	711	712	711	711	711	711	711
21	Count u-points [0.40 <= u < 0.50)	629	629	629	630	629	629	628	629	629	629
22	Count u-points [0.50 <= u < 0.60)	629	630	630	629	629	628	629	628	629	629
23	Count u-points [0.60 <= u < 0.70)	711	711	711	711	711	711	711	712	711	711
24	Count u-points [0.70 <= u < 0.80)	794	794	794	795	795	794	794	794	794	794
25	Count u-points [0.80 <= u < 0.90)	792	792	792	791	792	791	791	791	791	792
26	Count u-points [0.90 <= u <= 1.00]	750	750	750	751	750	816	750	750	750	750
27	Check Total u-points	7351	7352	7352	7353	7353	7480	7348	7349	7349	7350
	Performance										
28	Total curve error (sum of epsilon(u))	0.007332	0.007331	0.00733	0.007331	0.00733	0.007227	0.007334	0.007335	0.007334	0.007334
29	Total dist traversed (sum of chord lengths)	15.2108	15.21069	15.20945	15.21391	15.21192	152.0974	152.1029	152.1321	152.1103	152.1394
30	Percent (Tot curve error / Tot dist traversed)	0.048201	0.048194	0.048195	0.048185	0.048187	0.004752	0.004822	0.004821	0.004821	0.00482



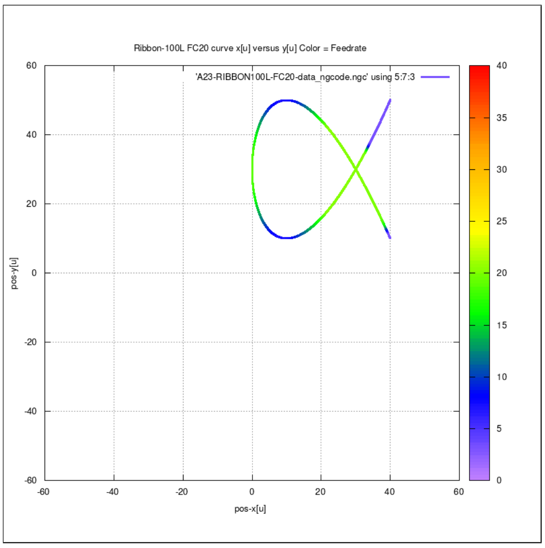


Table 2.25: Ribbon-10L and Ribbon-100L Run Data

2.4 Results Feedrate Profile

This section briefly describes the results of the feedrate profiles for the ten(10) different parametric curves generated by the algorithm. The color-coded feedrate profile is one of the most important validations of the algorithm for parametric curves.

1. **Never exceeding the user specified feedrate command** - In all cases of parametric curves, the User Feedrate Command FC was never exceeded by the current feedrate for the entire range of u-points.

2. **Smoothness in feedrates** - the x-axis and y-axis feedrates displayed smoothness for the entire range of u-points.

3. **Color-coded feedrate transitions** - When the parametric curve is being displayed using the feedrate color codes for its (x(u),y(u)) points, the path trace shows smoothness in feedrate color transitions.

2.4.1 Teardrop FC20 u versus x-y-curr feedrate profile

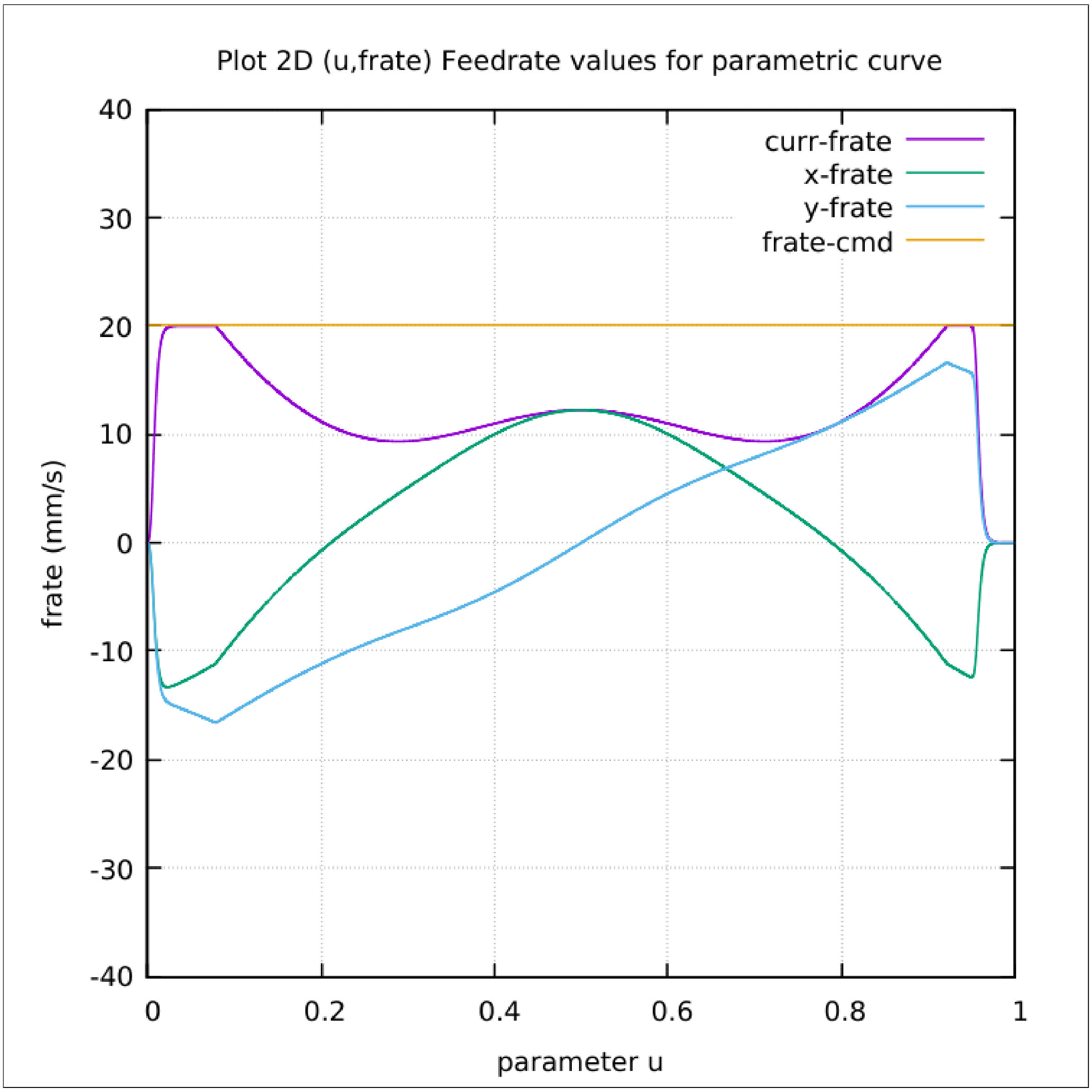


Table 2.26: Teardrop FC20 u versus x-y-curr feedrate profile

2.4.2 Teardrop FC20 x-y and colored feedrate profile

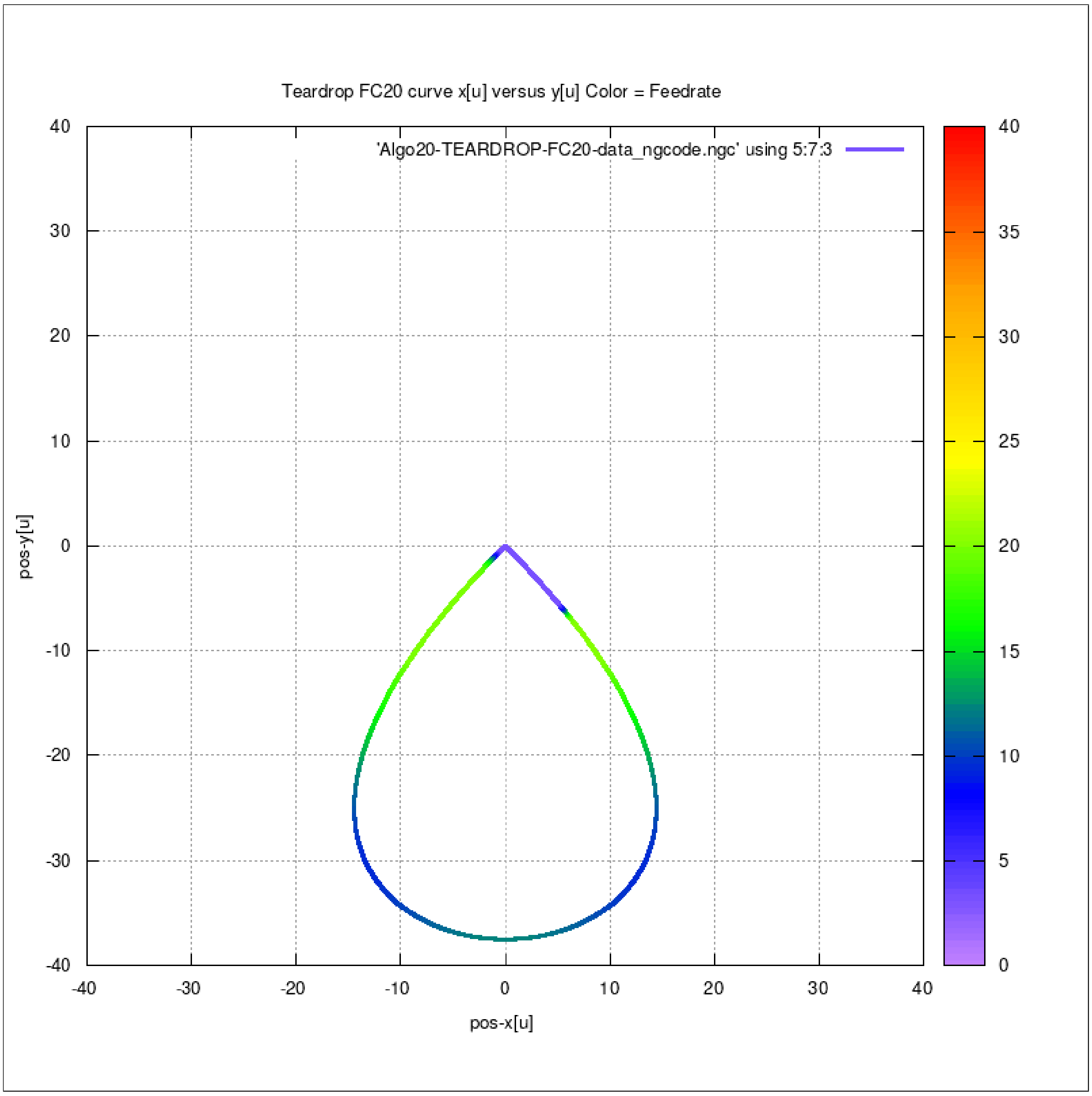


Table 2.27: Teardrop FC20 x-y and colored feedrate profile

2.4.3 Butterfly FC20 u versus x-y-curr feedrate profile

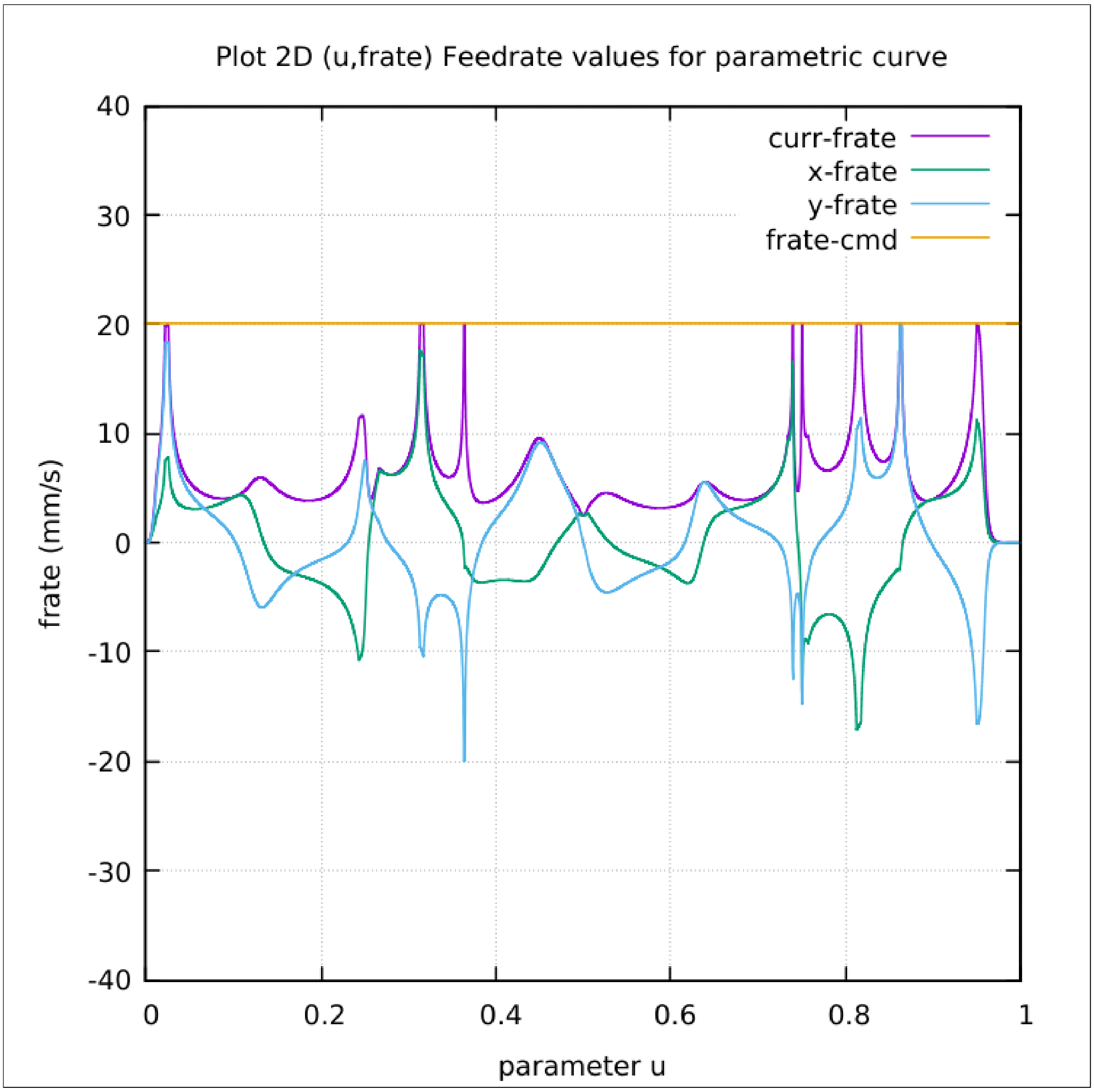


Table 2.28: Butterfly FC20 u versus x-y-curr feedrate profile

2.4.4 Butterfly FC20 x-y and colored feedrate profile

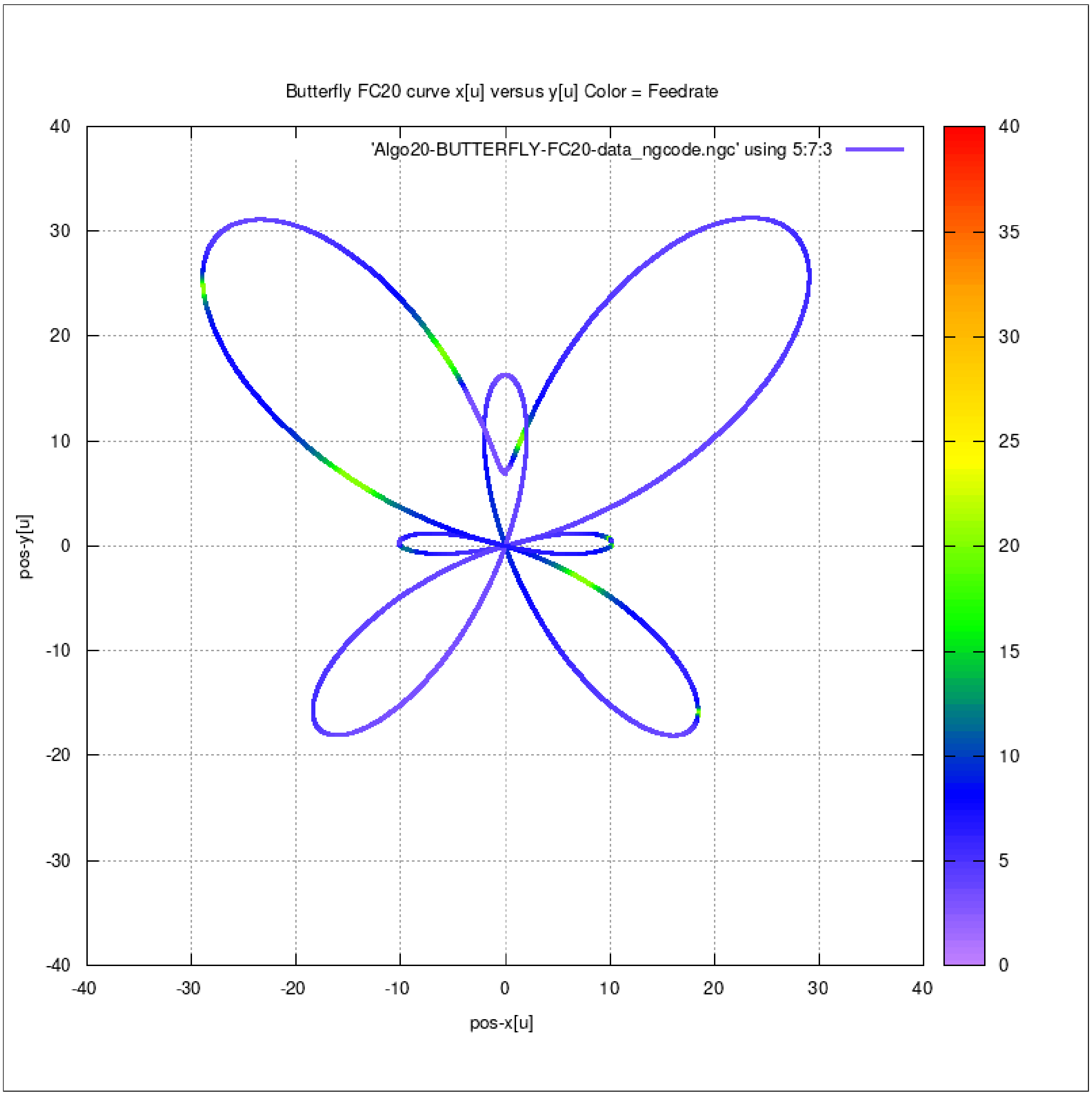


Table 2.29: Butterfly FC20 x-y and colored feedrate profile

2.4.5 Ellipse FC20 u versus x-y-curr feedrate profile

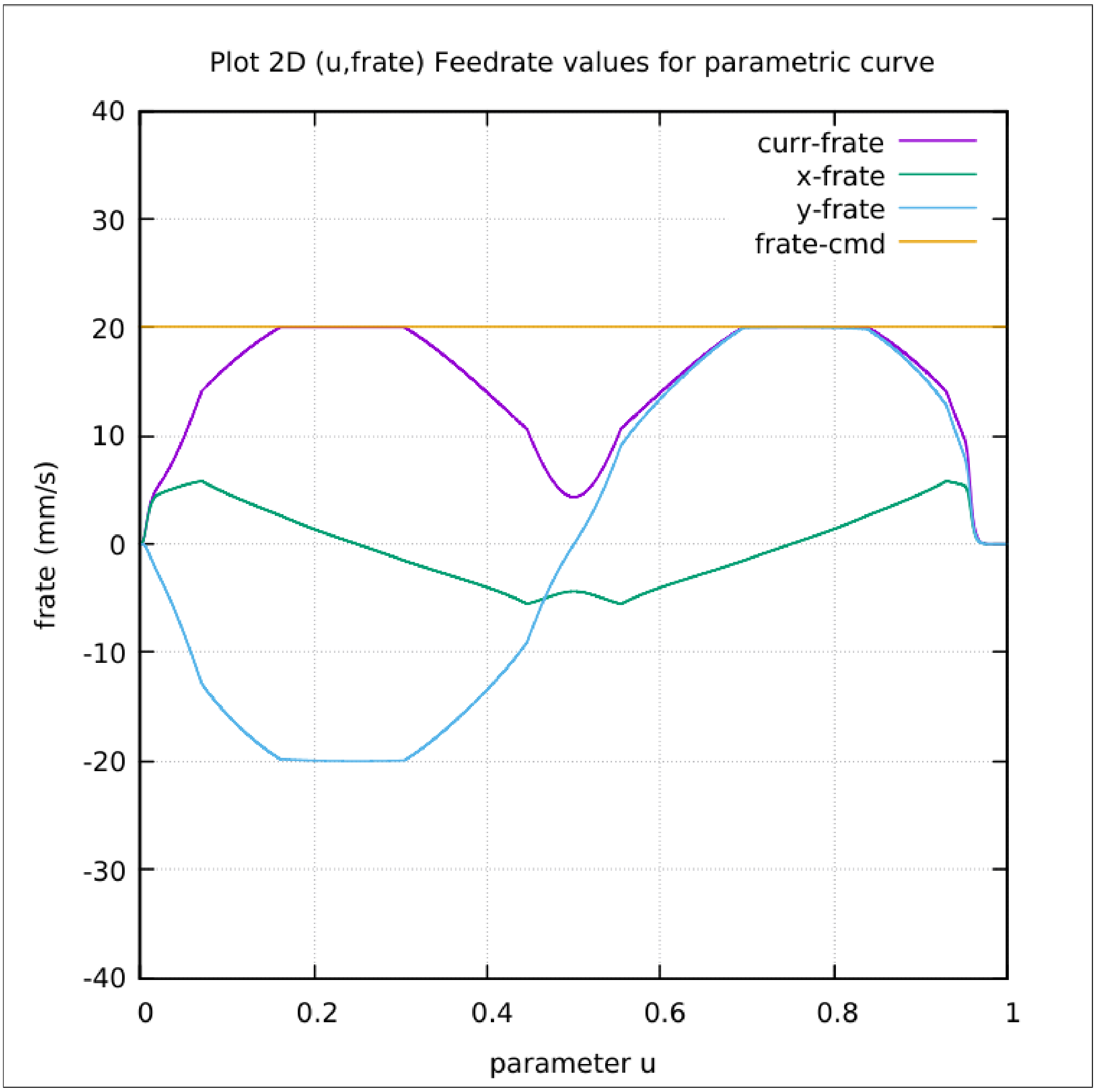


Table 2.30: Ellipse FC20 u versus x-y-curr feedrate profile

2.4.6 Ellipse FC20 x-y and colored feedrate profile

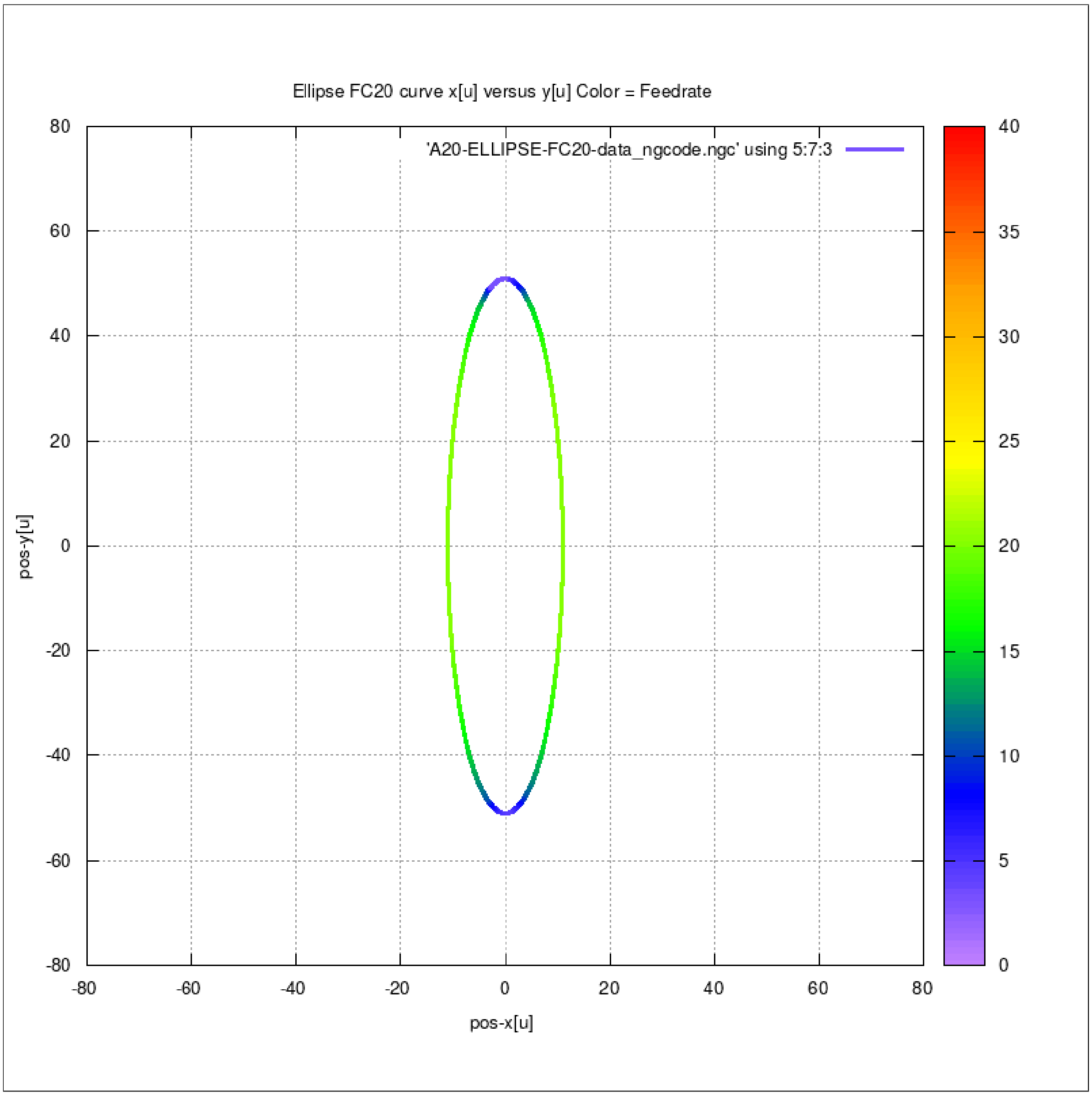


Table 2.31: Ellipse FC20 x-y and colored feedrate profile

2.4.7 Skewed-Astroid FC20 u versus x-y-curr feedrate profile

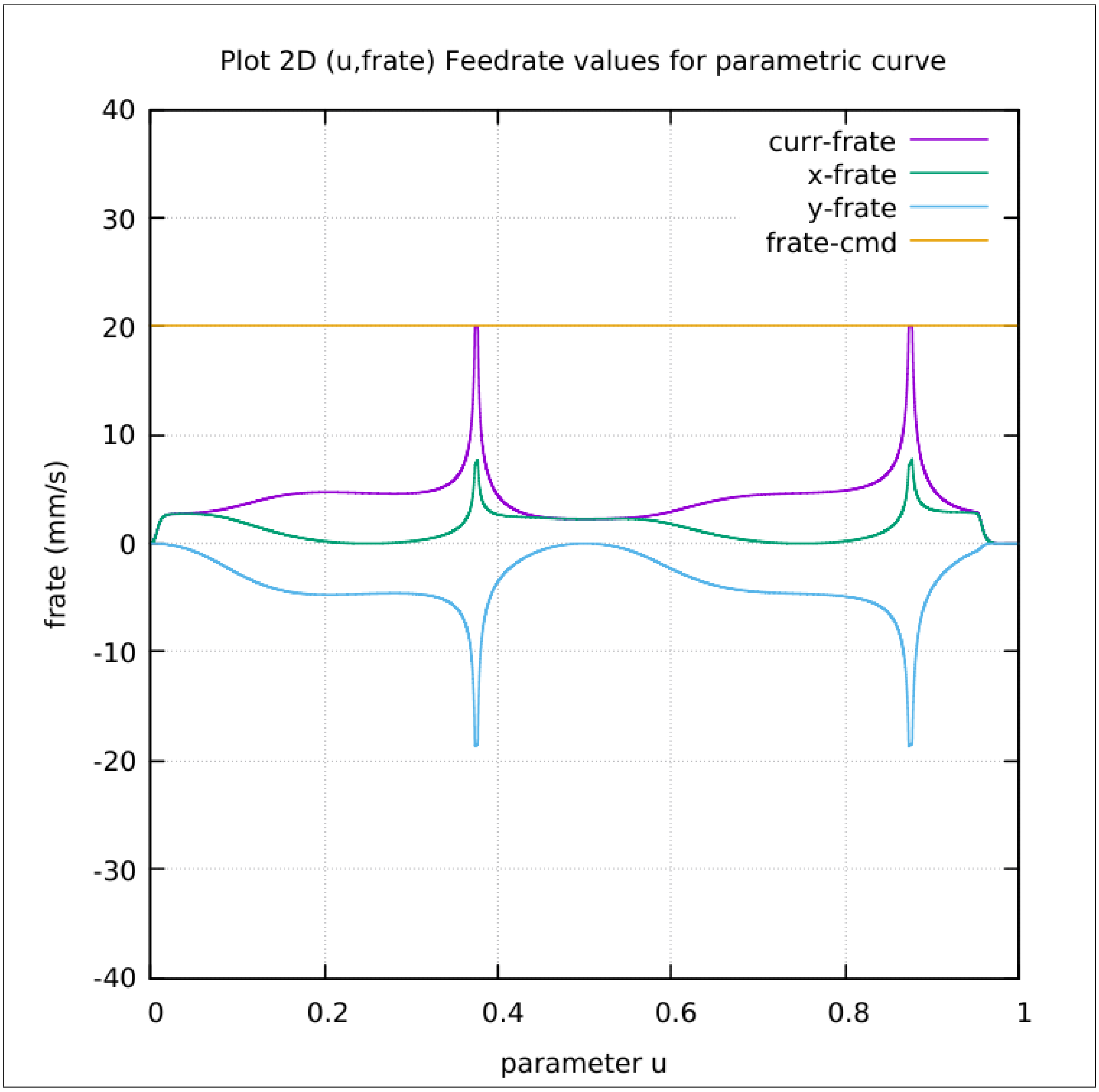


Table 2.32: Skewed-Astroid FC20 u versus x-y-curr feedrate profile

2.4.8 Skewed-Astroid FC20 x-y and colored feedrate profile

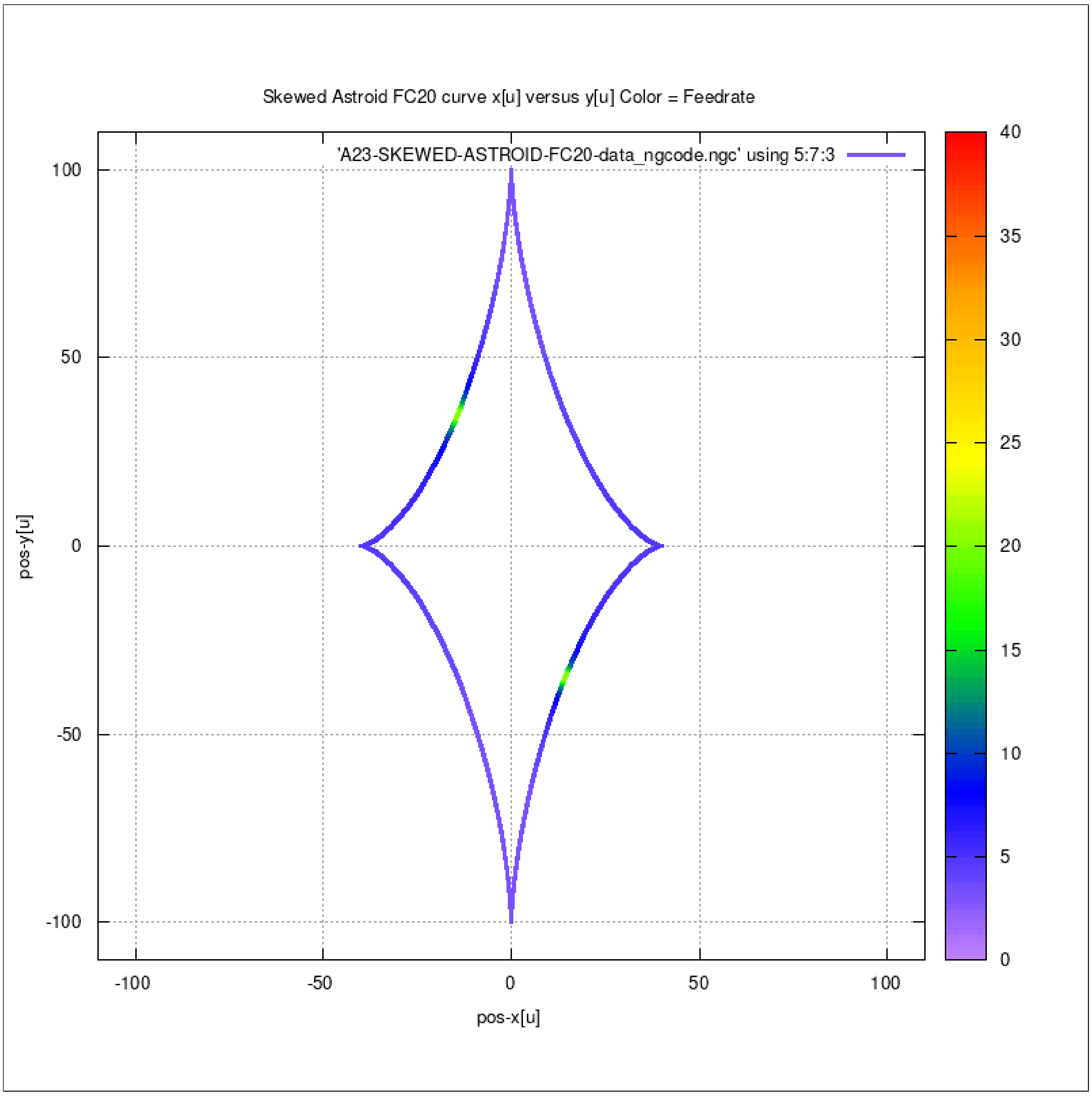


Table 2.33: Skewed-Astroid FC20 x-y and colored feedrate profile

2.4.9 Circle FC20 u versus x-y-curr feedrate profile

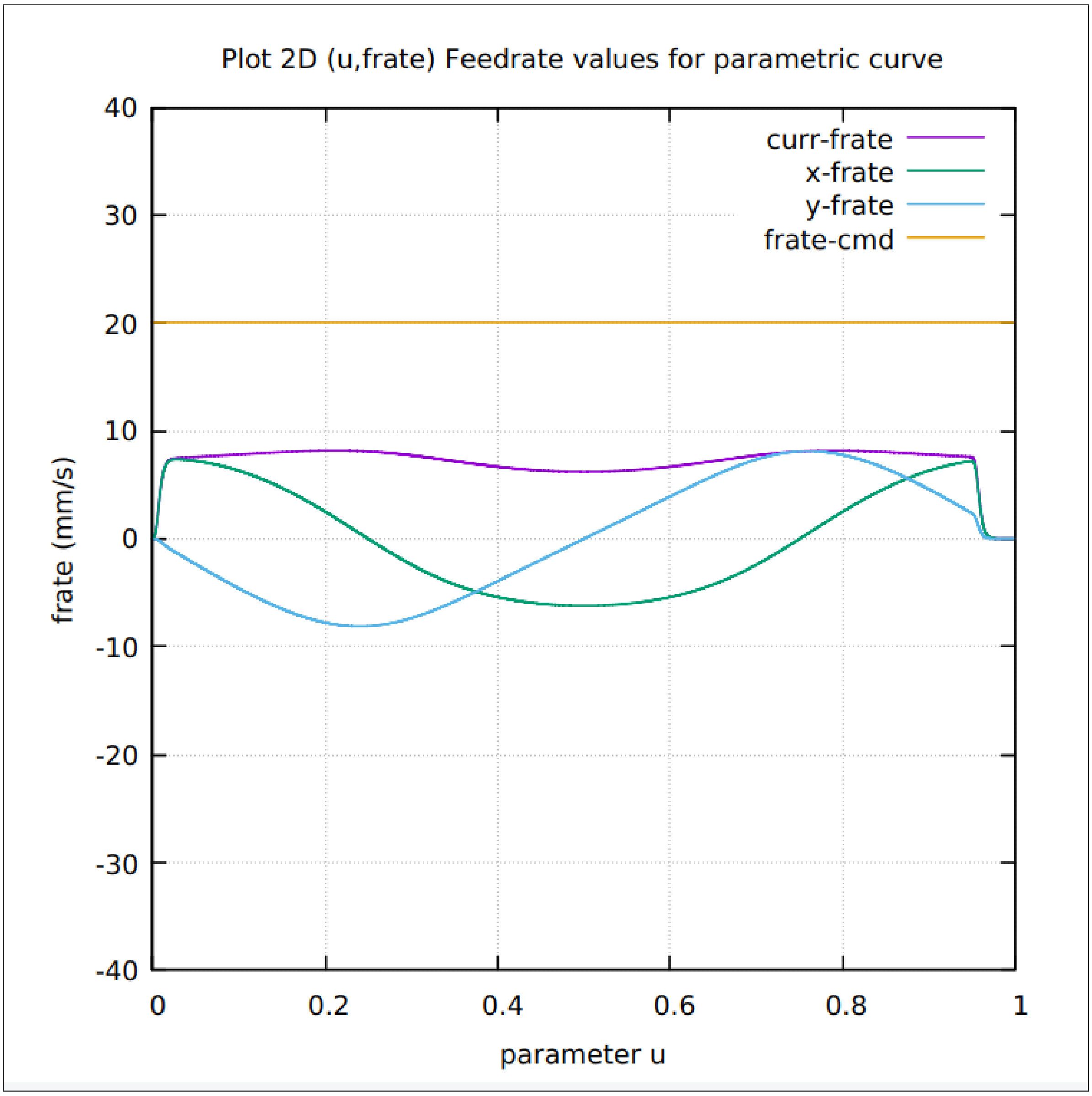


Table 2.34: Circle FC20 u versus x-y-curr feedrate profile

2.4.10 Circle FC20 x-y and colored feedrate profile

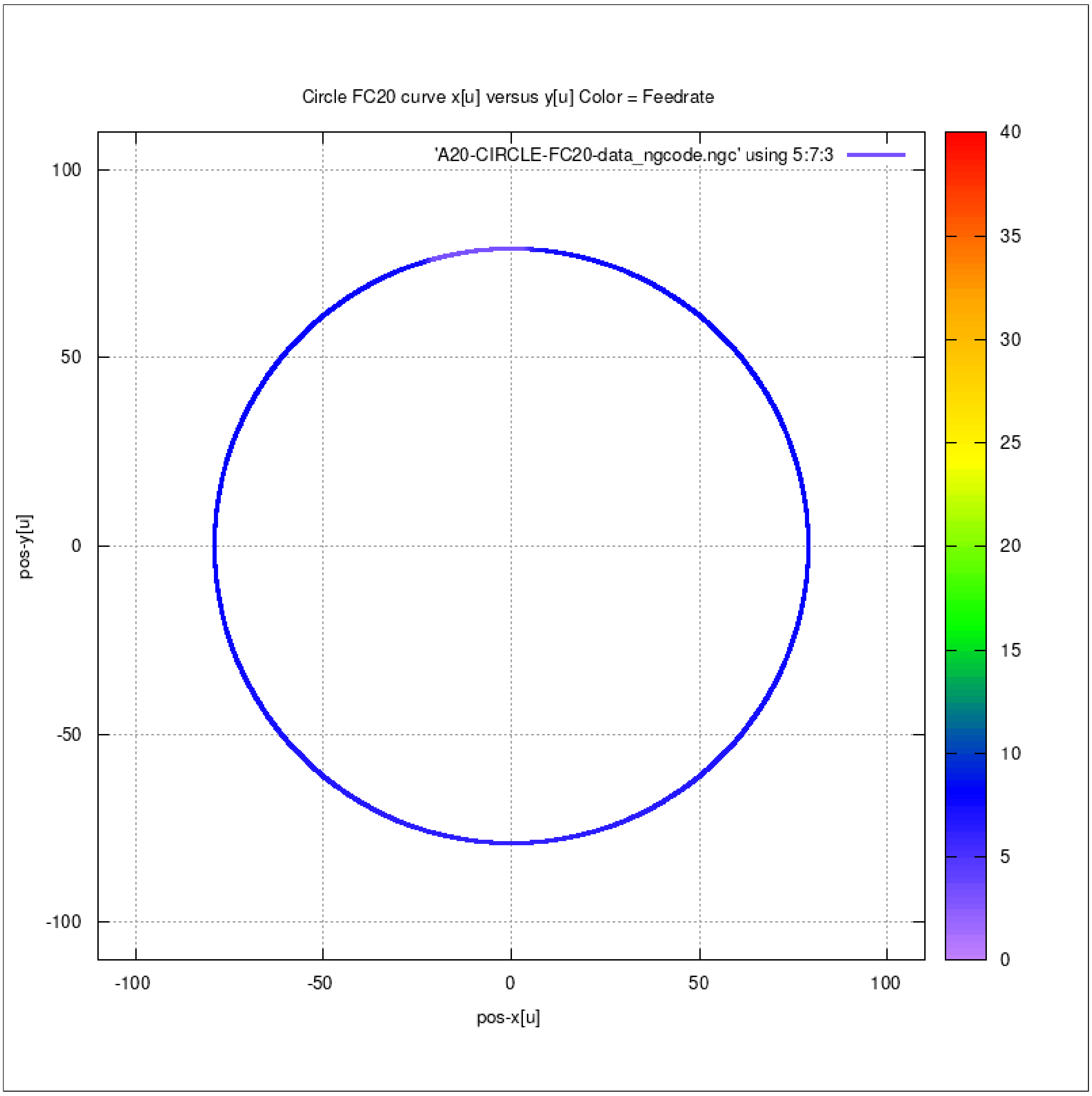


Table 2.35: Circle FC20 x-y and colored feedrate profile

2.4.11 AstEpi FC20 u versus x-y-curr feedrate profile

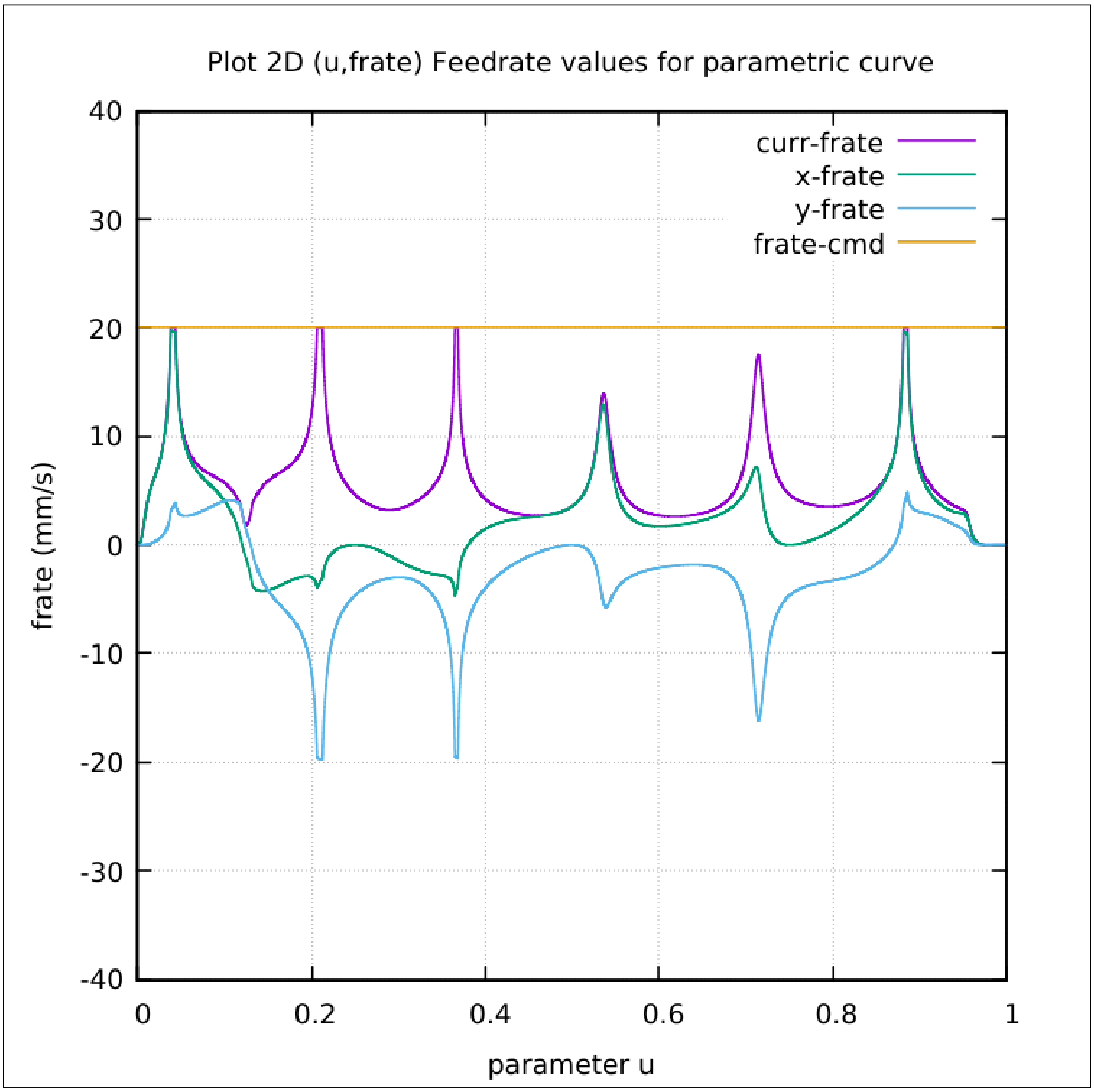


Table 2.36: AstEpi FC20 u versus x-y-curr feedrate profile

2.4.12 AstEpi FC20 x-y and colored feedrate profile

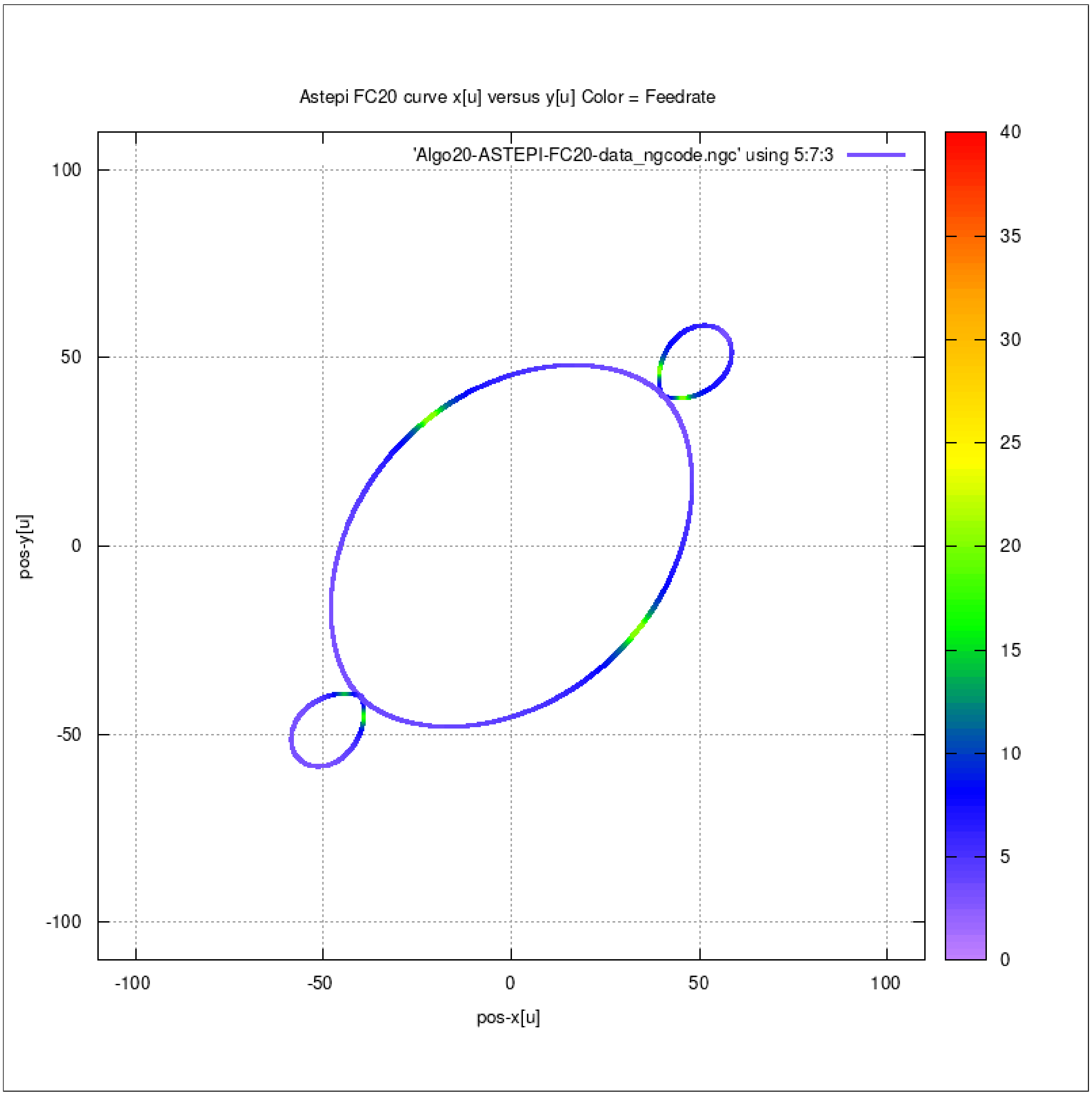


Table 2.37: AstEpi FC20 x-y and colored feedrate profile

2.4.13 Snailshell FC20 u versus x-y-curr feedrate profile

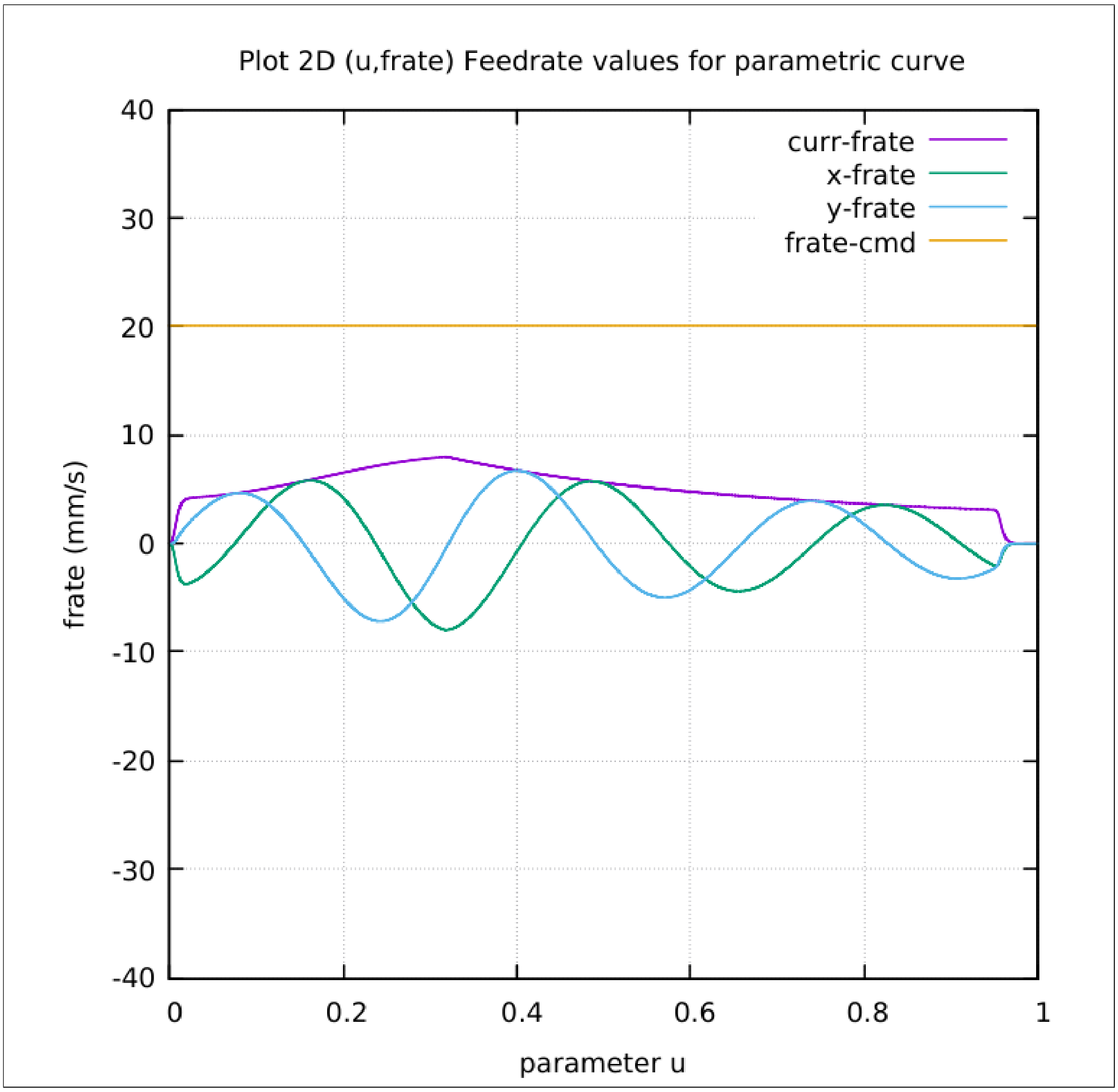


Table 2.38: Snailshell FC20 u versus x-y-curr feedrate profile

2.4.14 Snailshell FC20 x-y and colored feedrate profile

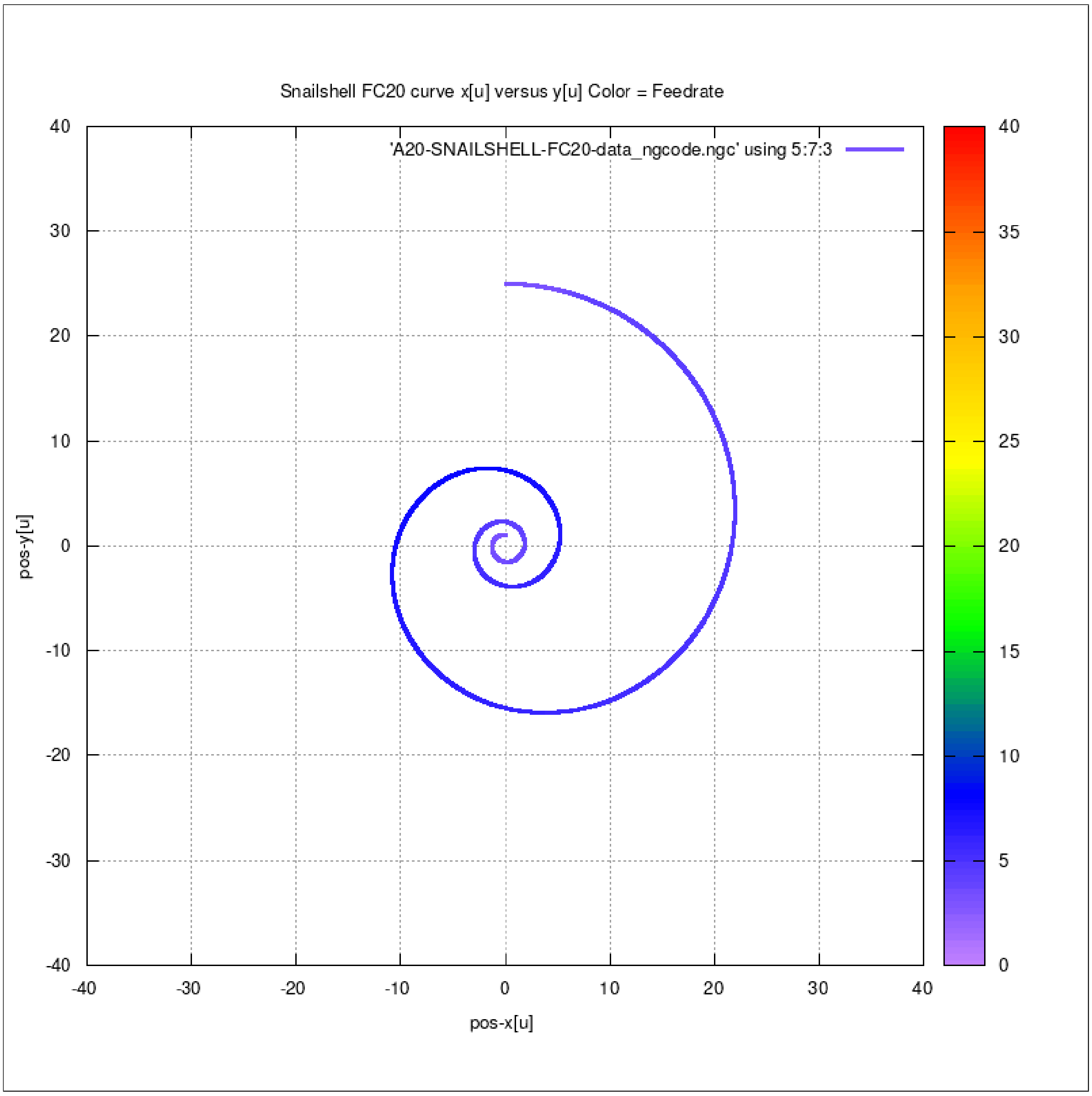


Table 2.39: Snailshell FC20 x-y and colored feedrate profile

2.4.15 SnaHyp FC20 u versus x-y-curr feedrate profile

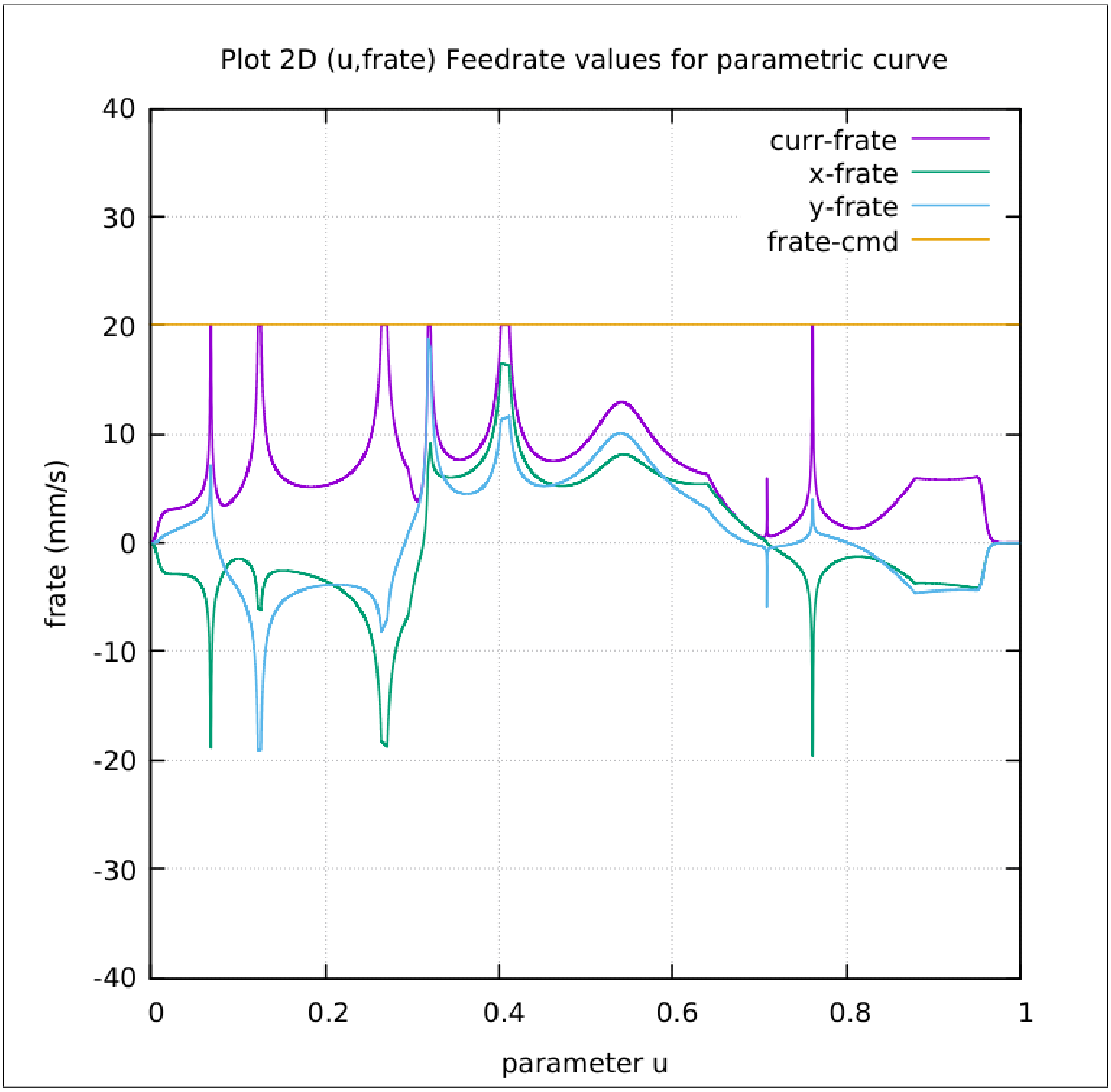


Table 2.40: SnaHyp FC20 u versus x-y-curr feedrate profile

2.4.16 SnaHyp FC20 x-y and colored feedrate profile

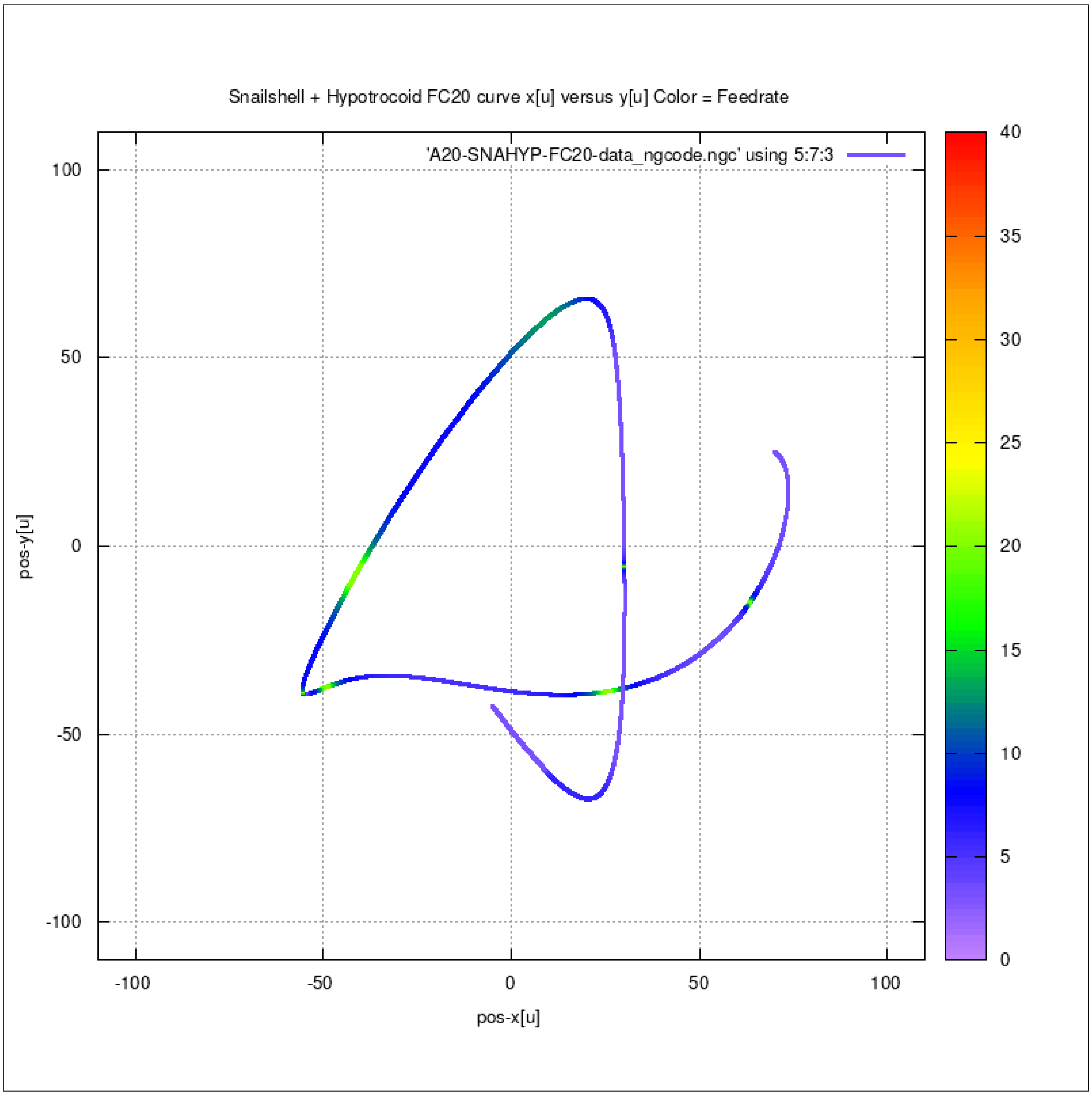


Table 2.41: SnaHyp FC20 x-y and colored feedrate profile

2.4.17 Ribbon-10L FC20 u versus x-y-curr feedrate profile

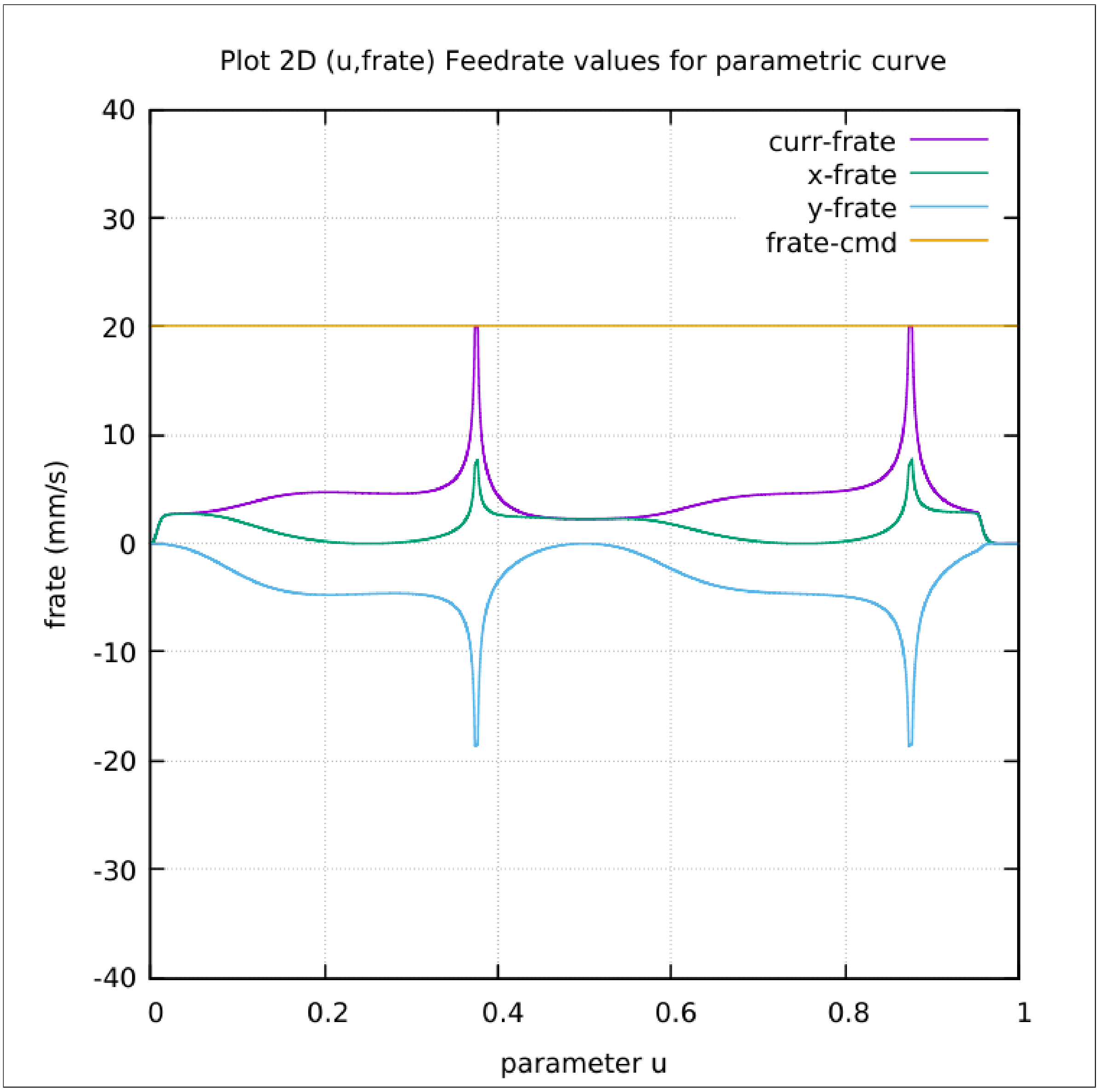


Table 2.42: Ribbon-10L FC20 u versus x-y-curr feedrate profile

2.4.18 Ribbon-10L FC20 x-y and colored feedrate profile

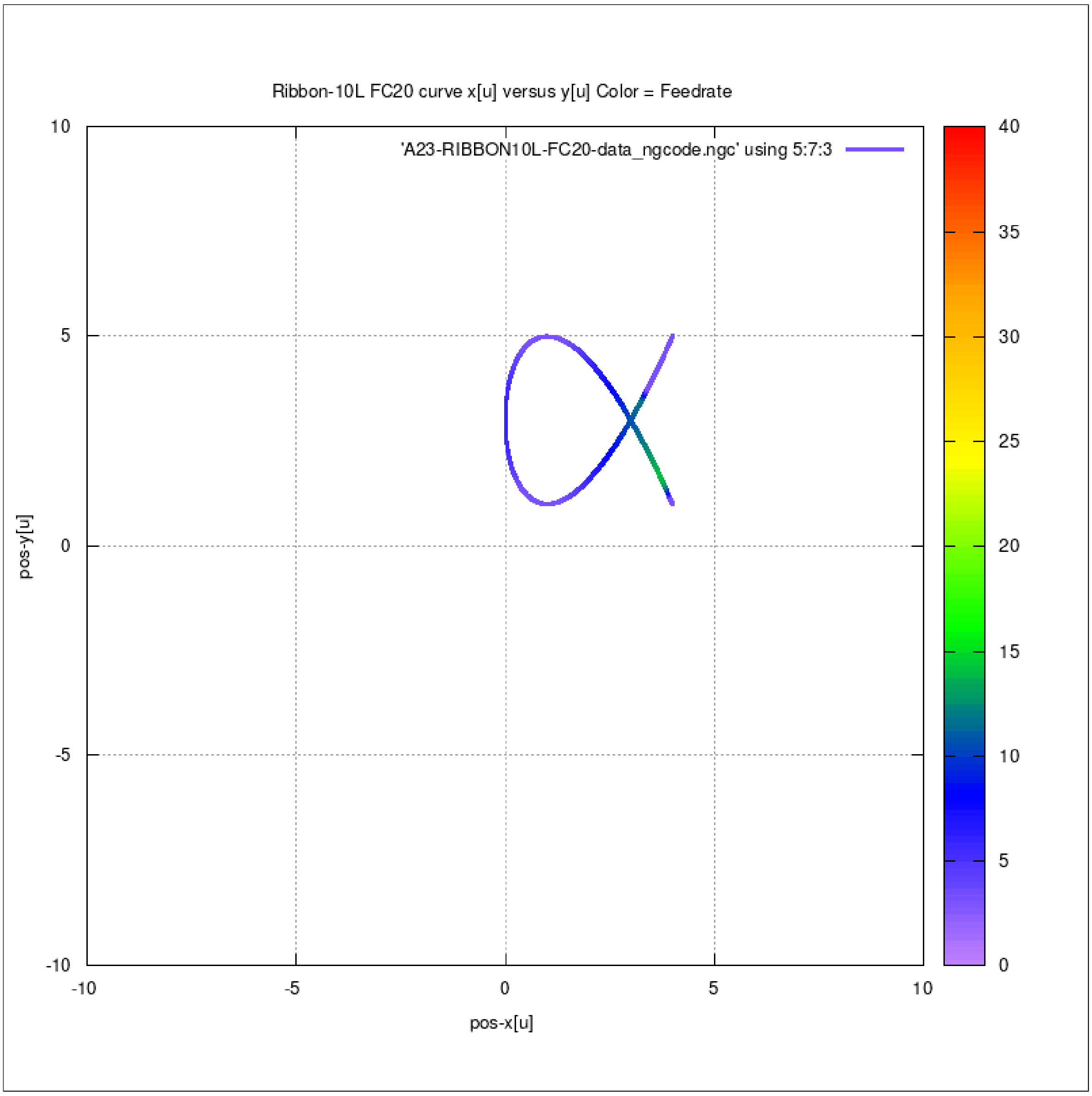


Table 2.43: Ribbon-10L FC20 x-y and colored feedrate profile

2.4.19 Ribbon-100L FC20 u versus x-y-curr feedrate profile

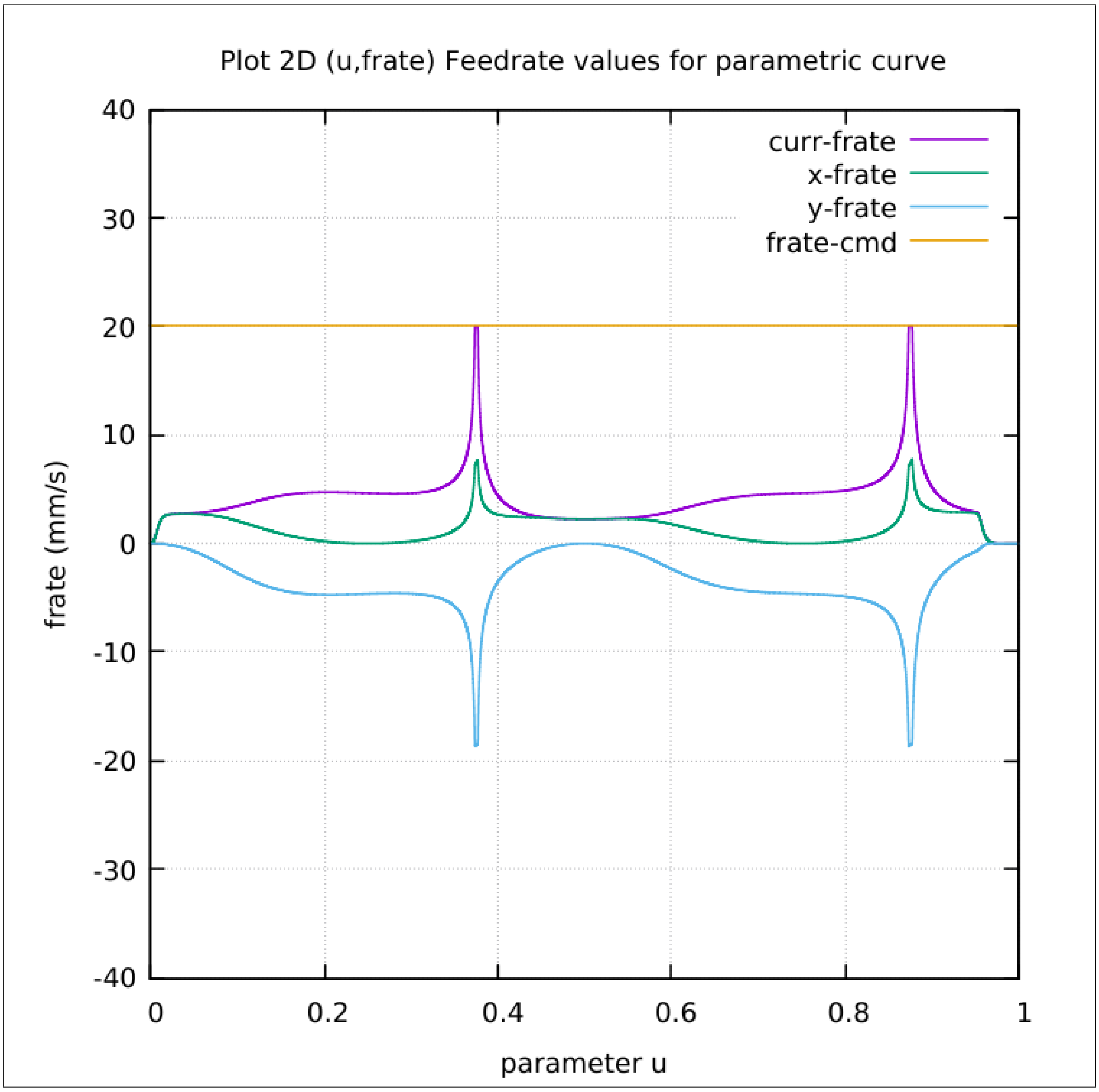


Table 2.44: Ribbon-100L FC20 u versus x-y-curr feedrate profile

2.4.20 Ribbon-100L FC20 x-y and colored feedrate profile

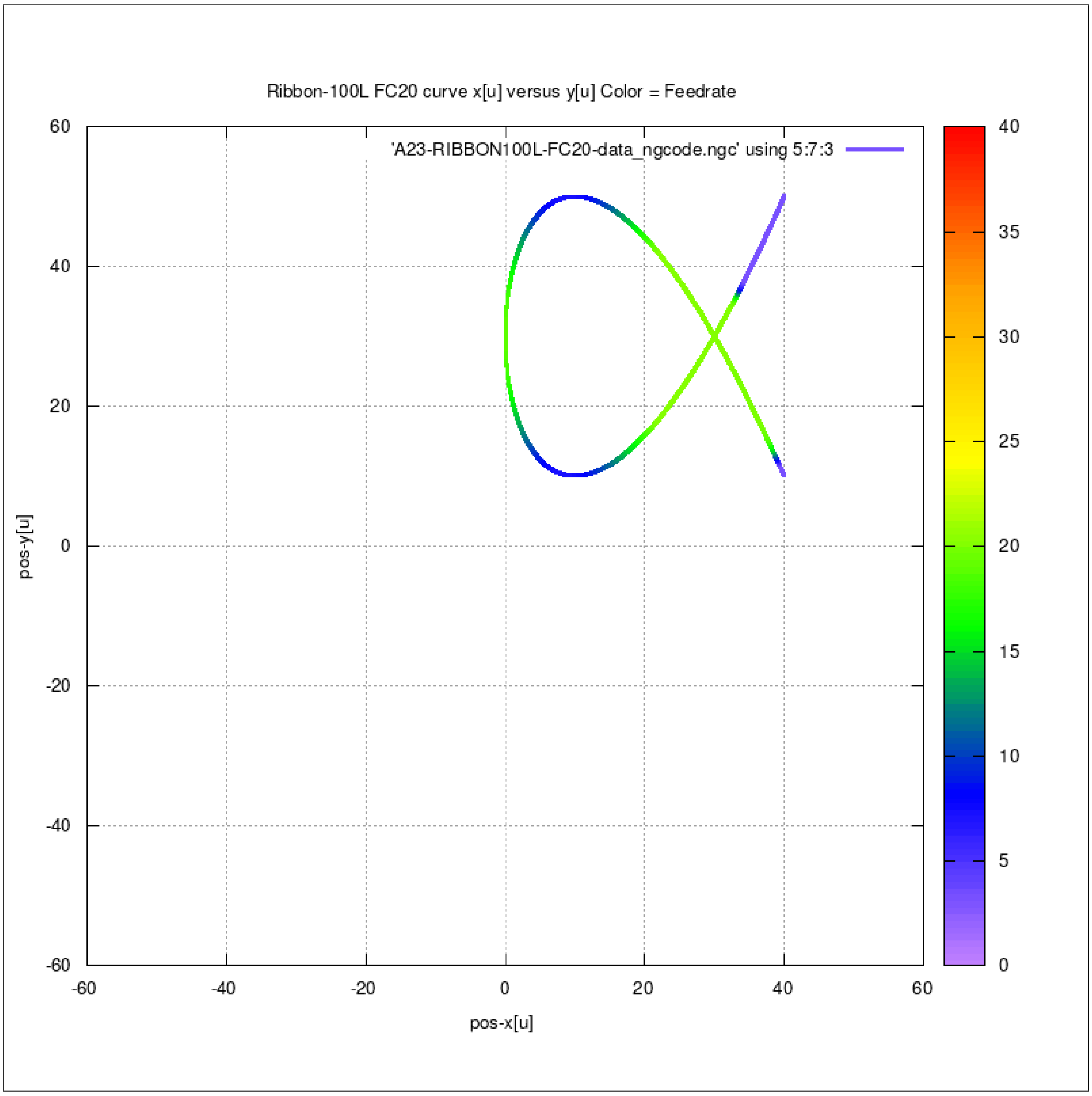


Table 2.45: Ribbon-100L FC20 x-y and colored feedrate profile

2.5 Error per unit length traversed

This section briefly describes the chord-error per unit length traversed in the point-to-point parametric curve interpolation algorithm.

1. **error/length** - The rightmost column in data tables presented in subsequent pages shows the calculated chord-error per unit length for all parametric curves in this work.

2. **User specified Feedrate Command FC** - In subsequent pages, five(5) data tables are presented, one each for FC10, FC20, FC25, FC30 and FC40, also for the calculated chord-error per unit length of all parametric curves in this work.

3. **Order magnitude error/length** - Notice that the calculated chord-error per unit length for all parametric curves in this work is in the order of 1E-6 and 1E-5, where 1E-6 is the specified chord-error tolerance, irrespective of the user specified feedrate command.

4. **Histogram for all parametric curves** - Also in subsequent pages, five(5) histograms for the calculated chord-error per unit length are presented, one each for the parametric curves in this work.

2.5.1 FC10 - Error per unit length traversed

Date: 2023-06-16
Author: wruslandr@gmail.com

Total Interpolated Points for parametric curves

	Total Interpolated Points					FC10	FC10	FC10
CURVE	FC10	FC20	FC25	FC30	FC40	Total curve length	Total error	error/length
Teardrop	10261	7599	7385	7347	7347	101.835673217	0.005809000	5.704288E-05
Butterfly	35656	18029	14577	12343	9732	356.074702570	0.001938860	5.445093E-06
Ellipse	21575	7599	9448	8338	7351	215.649935852	0.002990952	1.386948E-05
Skewed-Astroid	116194	58102	46483	38738	29056	445.714285882	0.000516368	1.158519E-06
Circle	49641	24822	19859	16549	12413	496.378581315	0.001093914	2.203790E-06
AstEpi	76275	38169	30563	25499	19184	426.262247842	0.000840373	1.971493E-06
Snailshell	15621	9883	8935	8370	7766	138.559540611	0.005115139	3.691654E-05
SnaHyp	38672	20223	16618	11497	8889	478.987086578	0.002846873	5.943528E-06
Ribbon-10L	7351	7352	7352	7353	7353	15.210795863	0.007331687	4.820055E-04
Ribbon-100L	7480	7348	7349	7349	7350	152.097354848	0.007227414	4.751834E-05

Table 2.46: FC10 - Error per unit length for all parametric curves

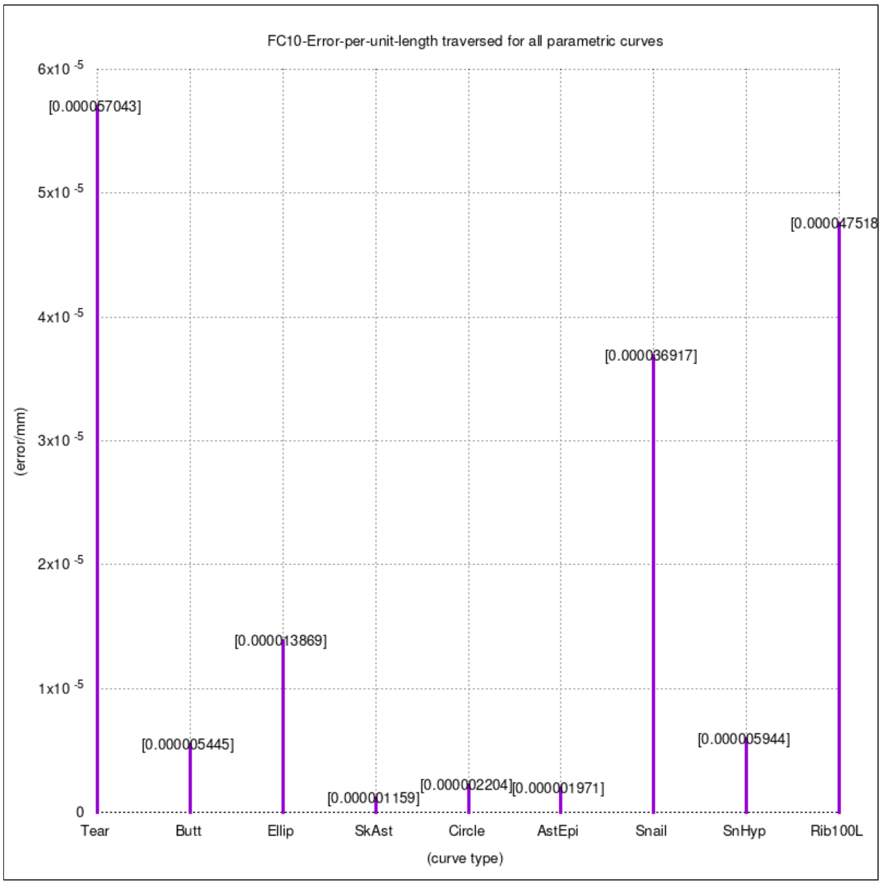


Table 2.47: FC10 - Error per unit length for all parametric curves

2.5.2 FC20 - Error per unit length traversed

Date: 2023-06-16 **Total Interpolated Points for parametric curves**
Author: wruslandr@gmail.com

	Total Interpolated Points					**FC20**	**FC20**	**FC20**
CURVE	**FC10**	**FC20**	**FC25**	**FC30**	**FC40**	**Total curve length**	**Total error**	**error/length**
Teardrop	10261	7599	7385	7347	7347	101.841865570	0.007140807	7.011662E-05
Butterfly	35656	18029	14577	12343	9732	356.073710900	0.003534046	9.925040E-06
Ellipse	21575	7599	9448	8338	7351	215.649935852	0.007140807	3.311296E-05
Skewed-Astroid	116194	58102	46483	38738	29056	445.714285537	0.001032591	2.316711E-06
Circle	49641	24822	19859	16549	12413	496.377158168	0.001093914	2.203796E-06
AstEpi	76275	38169	30563	25499	19184	426.262239690	0.001641843	3.851720E-06
Snailshell	15621	9883	8935	8370	7766	138.561390573	0.006269762	4.524898E-05
SnaHyp	38672	20223	16618	11497	8889	478.998699413	0.004002975	8.356964E-06
Ribbon-10L	7351	7352	7352	7353	7353	15.210689035	0.007330650	4.819407E-04
Ribbon-100L	7480	7348	7349	7349	7350	152.102890678	0.007334489	4.822058E-05

Table 2.48: FC20 - Error per unit length for all parametric curves

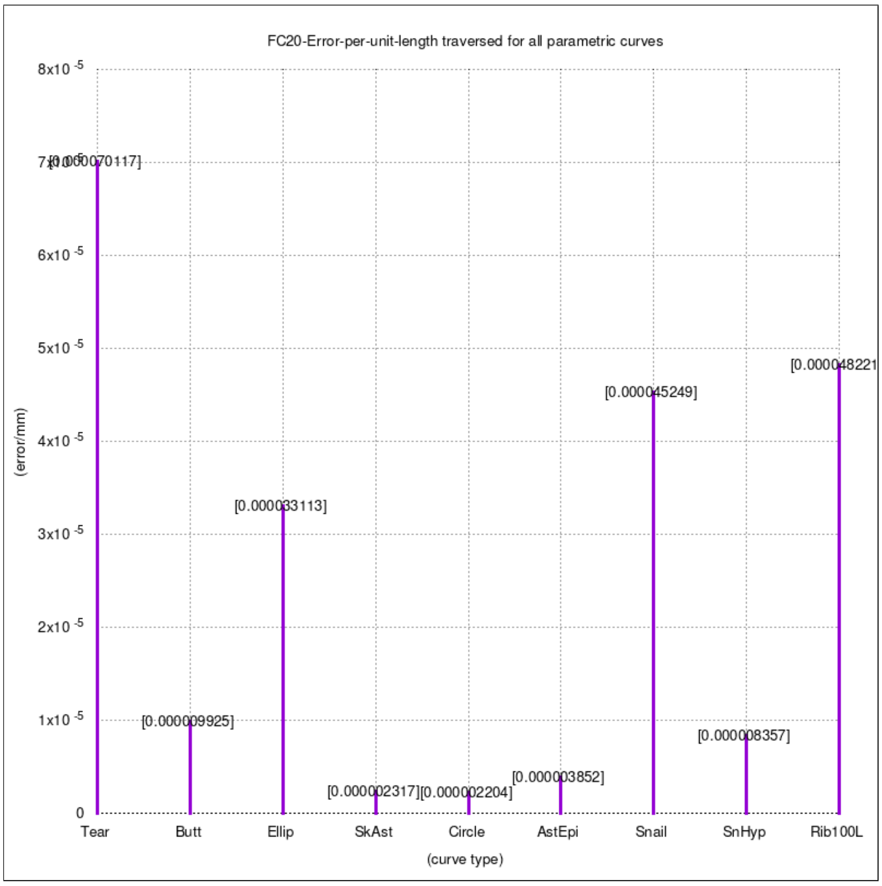


Table 2.49: FC20 - Error per unit length for all parametric curves

2.5.3 FC25 - Error per unit length traversed

Date: 2023-06-16
Author: wruslandr@gmail.com

Total Interpolated Points for parametric curves

CURVE	Total Interpolated Points FC10	FC20	FC25	FC30	FC40	FC25 Total curve length	FC25 Total error	FC25 error/length
Teardrop	10261	7599	7385	7347	7347	101.834772771	0.007301198	7.169651E-05
Butterfly	35656	18029	14577	12343	9732	356.072310198	0.004230857	1.188202E-05
Ellipse	21575	7599	9448	8338	7351	215.644021538	0.005927619	2.748798E-05
Skewed-Astroid	116194	58102	46483	38738	29056	445.714282439	0.001290613	2.895606E-06
Circle	49641	24822	19859	16549	12413	496.396441759	0.002187640	4.407042E-06
AstEpi	76275	38169	30563	25499	19184	426.262236290	0.002020019	4.738911E-06
Snailshell	15621	9883	8935	8370	7766	138.560655481	0.006558063	4.732991E-05
SnaHyp	38672	20223	16618	11497	8889	479.006371543	0.004459255	9.309386E-06
Ribbon-10L	7351	7352	7352	7353	7353	15.209447637	0.007330192	4.819499E-04
Ribbon-100L	7480	7348	7349	7349	7350	152.132129168	0.007334570	4.821184E-05

Table 2.50: FC25 - Error per unit length for all parametric curves

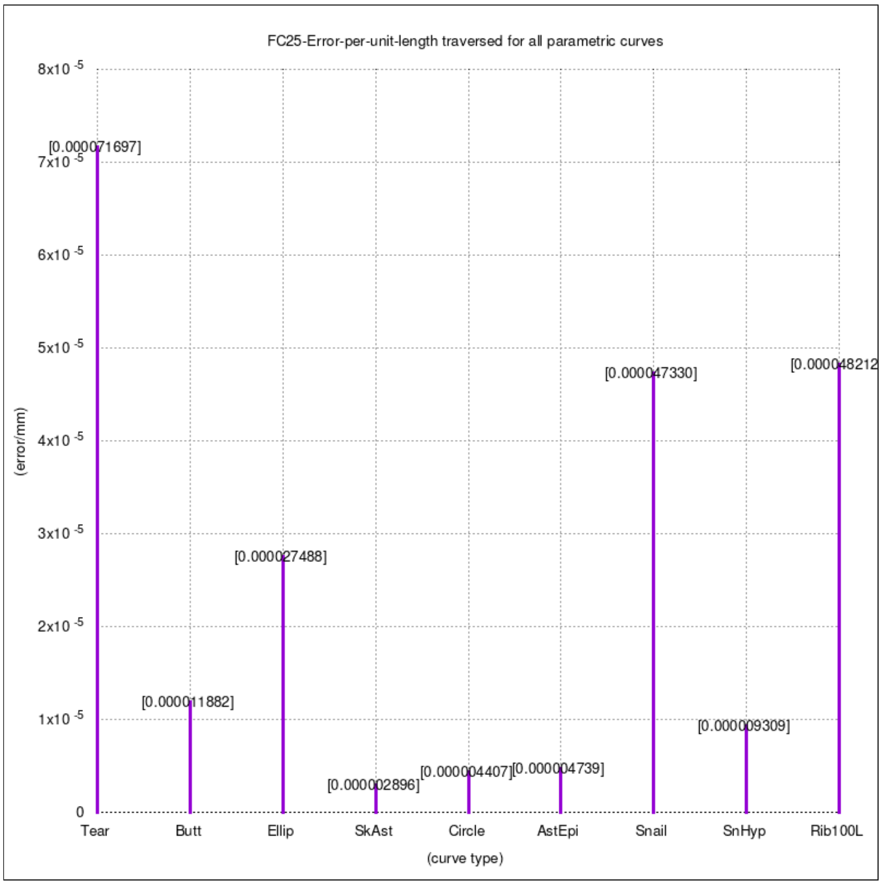


Table 2.51: FC25 - Error per unit length for all parametric curves

2.5.4 FC30 - Error per unit length traversed

Date: 2023-06-16 **Total Interpolated Points for parametric curves**
Author: wruslandr@gmail.com

	Total Interpolated Points					FC30	FC30	FC30
CURVE	**FC10**	**FC20**	**FC25**	**FC30**	**FC40**	**Total curve length**	**Total error**	**error/length**
Teardrop	10261	7599	7385	7347	7347	101.859566601	0.007336794	7.202852E-05
Butterfly	35656	18029	14577	12343	9732	356.072793009	0.004846583	1.361121E-05
Ellipse	21575	7599	9448	8338	7351	215.647842617	0.006561982	3.042916E-05
Skewed-Astroid	116194	58102	46483	38738	29056	445.714284621	0.001548669	3.474578E-06
Circle	49641	24822	19859	16549	12413	496.375730563	0.002734540	5.509012E-06
AstEpi	76275	38169	30563	25499	19184	426.262229552	0.002390002	5.606882E-06
Snailshell	15621	9883	8935	8370	7766	138.560164183	0.006764497	4.881993E-05
SnaHyp	38672	20223	16618	11497	8889	0.000000000	0.000000000	0.000000E+00
Ribbon-10L	7351	7352	7352	7353	7353	15.213913854	0.007330844	4.818513E-04
Ribbon-100L	7480	7348	7349	7349	7350	152.110308628	0.007333765	4.821346E-05

Table 2.52: FC30 - Error per unit length for all parametric curves

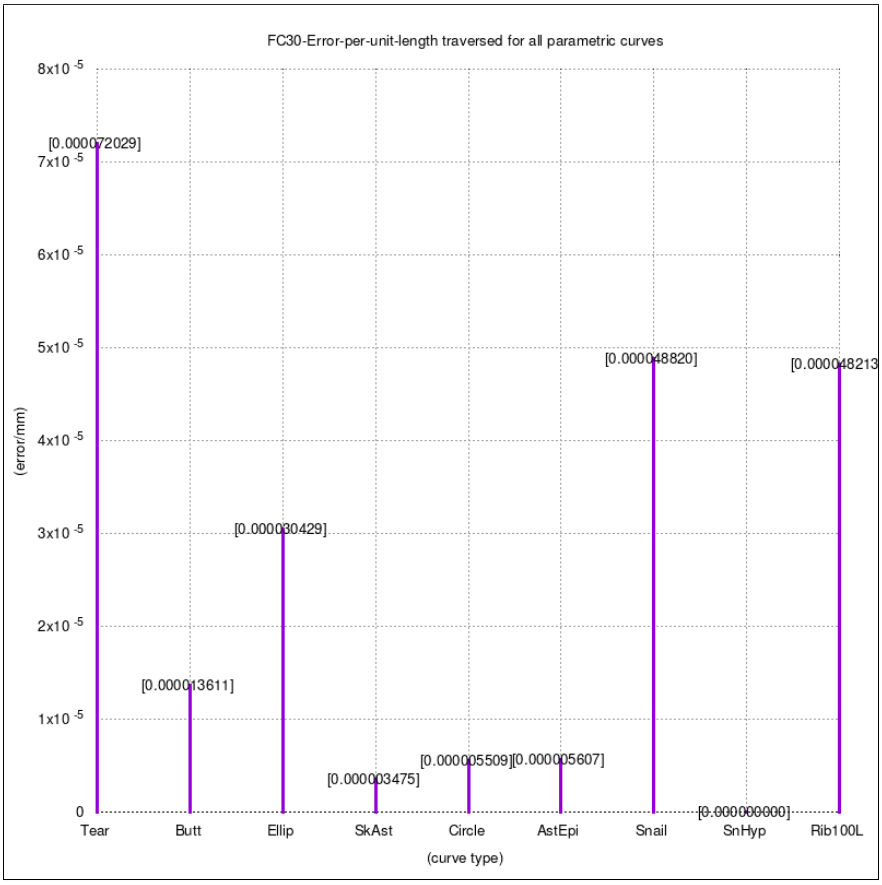


Table 2.53: FC30 - Error per unit length for all parametric curves

2.5.5 FC40 - Error per unit length traversed

Date: 2023-06-16 **Total Interpolated Points for parametric curves**
Author: wruslandr@gmail.com

	Total Interpolated Points					FC40	FC40	FC40
CURVE	**FC10**	**FC20**	**FC25**	**FC30**	**FC40**	**Total curve length**	**Total error**	**error/length**
Teardrop	10261	7599	7385	7347	7347	101.835560839	0.007335147	7.202933E-05
Butterfly	35656	18029	14577	12343	9732	356.073152673	0.005851473	1.643335E-05
Ellipse	21575	7599	9448	8338	7351	215.643855145	0.007331841	3.399977E-05
Skewed-Astroid	116194	58102	46483	38738	29056	445.714283175	0.002060000	4.621795E-06
Circle	49641	24822	19859	16549	12413	496.394293434	0.004374701	8.812956E-06
AstEpi	76275	38169	30563	25499	19184	426.262233355	0.003111370	7.299192E-06
Snailshell	15621	9883	8935	8370	7766	138.559886203	0.007045829	5.085042E-05
SnaHyp	38672	20223	16618	11497	8889	0.000000000	0.000000000	0.000000E+00
Ribbon-10L	7351	7352	7352	7353	7353	15.211915247	0.007330108	4.818662E-04
Ribbon-100L	7480	7348	7349	7349	7350	152.139353248	0.007333840	4.820475E-05

Table 2.54: FC40 - Error per unit length for all parametric curves

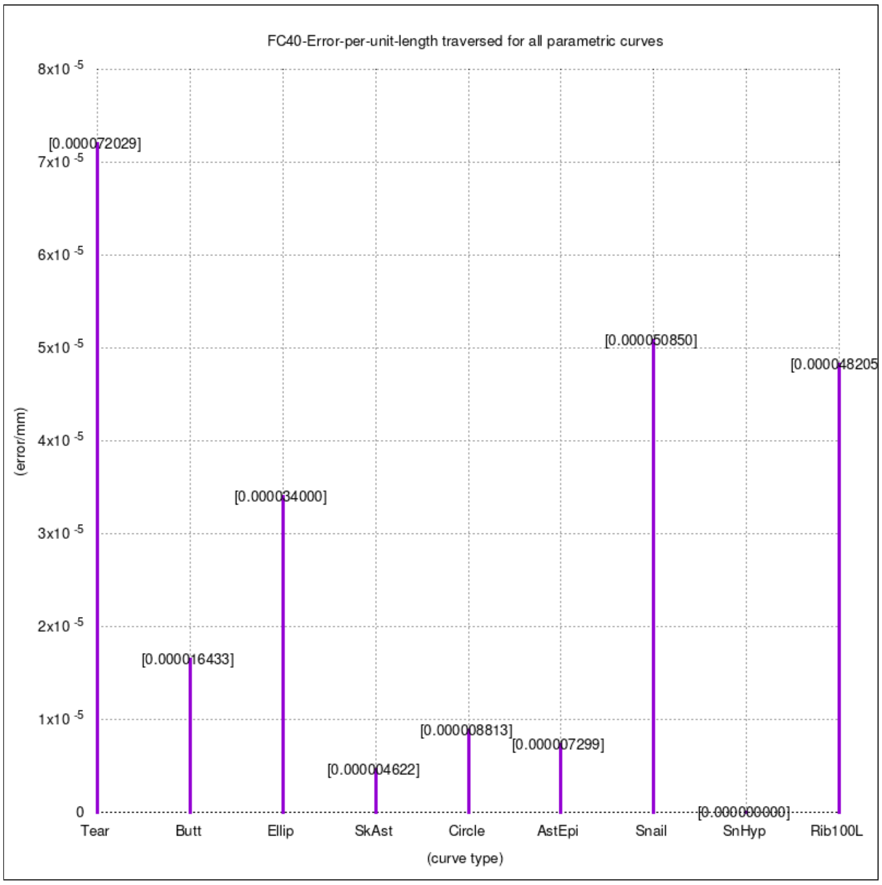


Table 2.55: FC40 - Error per unit length for all parametric curves

2.6 Histogram of Interpolated Points

This section briefly describes the histogram of the total interpolated points for FC10, FC20, FC25, FC30 and FC40, in the point-to-point parametric curve interpolation algorithm for all parametric curves in this work. The histogram are presented in subsequent pages.

1. **Histogram x-axis display** - The x-axis for the histogram provide the 10-bins for the entire range of u-points from u=0.0 to u=1.0.

2. **Histogram y-axis display** - The y-axis for the histogram provide the total interpolated points for the five(5) user specified feedrate commands (FC10, FC20, FC25, FC30 and FC40)

3. **Histogram for all parametric curves** - In subsequent pages, ten(10) histograms for the total interpolated points are presented, one each for the parametric curves in this work.

4. **Correct expectation** - As expected, the general trend is that the total interpolated points decreases as the user specified feedrate command increases. Essentially the chord length grew longer as the feedrate is increased.

5. **Constraints** - It is important to note that in all cases, the four(4) mandatory algorithm constraints C1, C2, C3 and C4 are never compromised.

2.6.1 Teardrop histogram of interpolated points

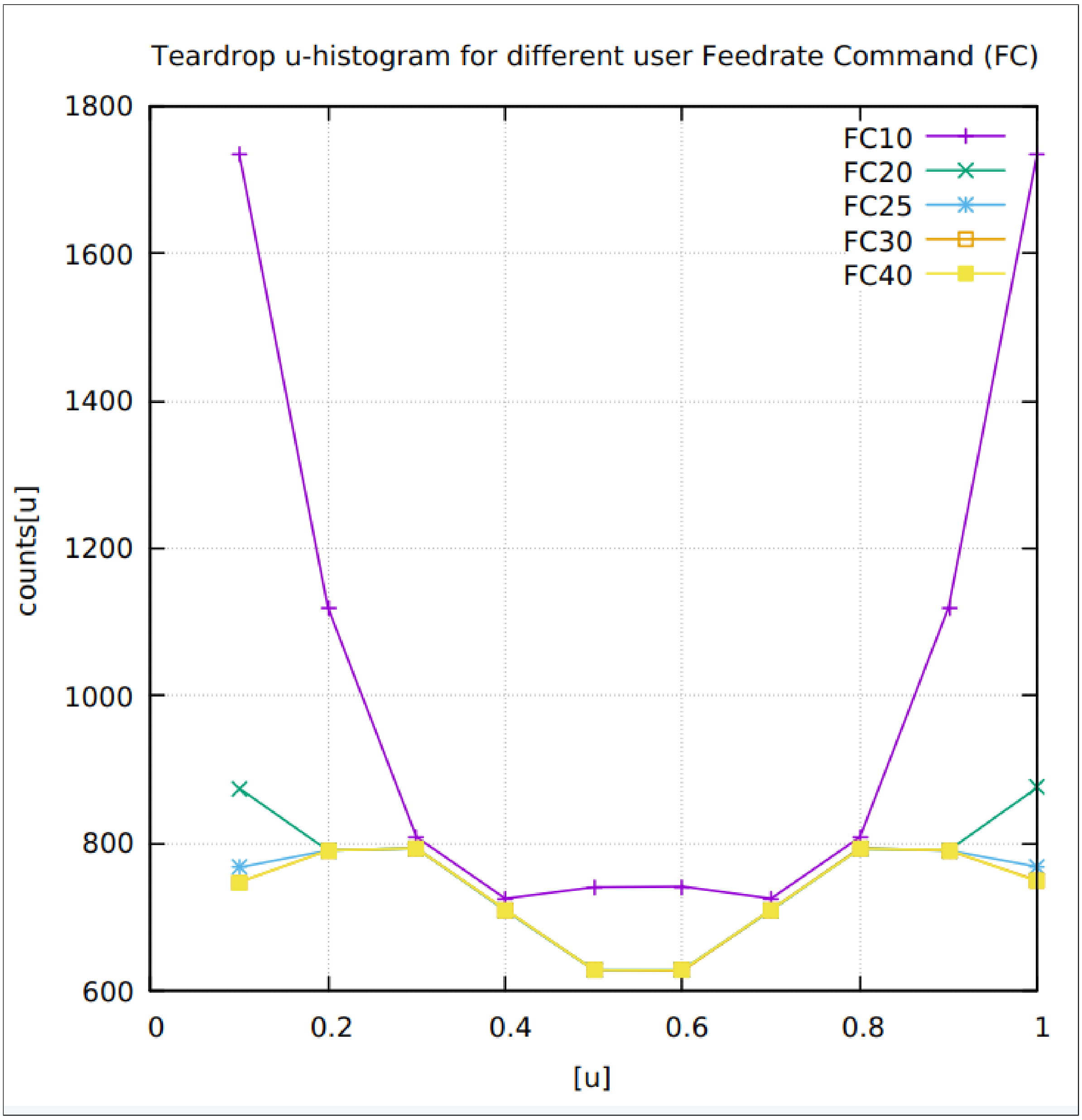


Table 2.56: Teardrop histogram of interpolated points

2.6.2 Butterfly histogram of interpolated points

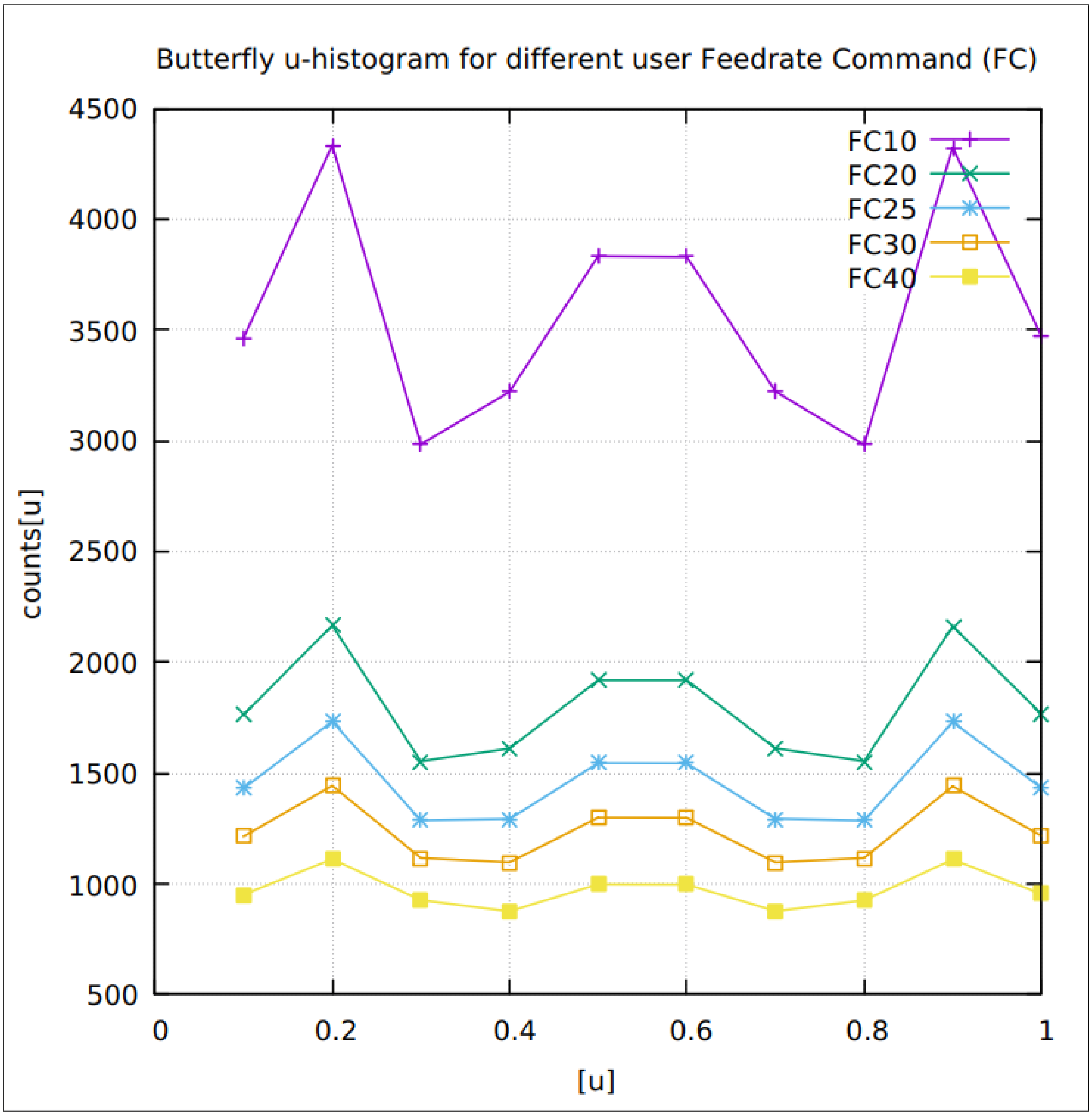


Table 2.57: Butterfly histogram of interpolated points

2.6.3 Ellipse histogram of interpolated points

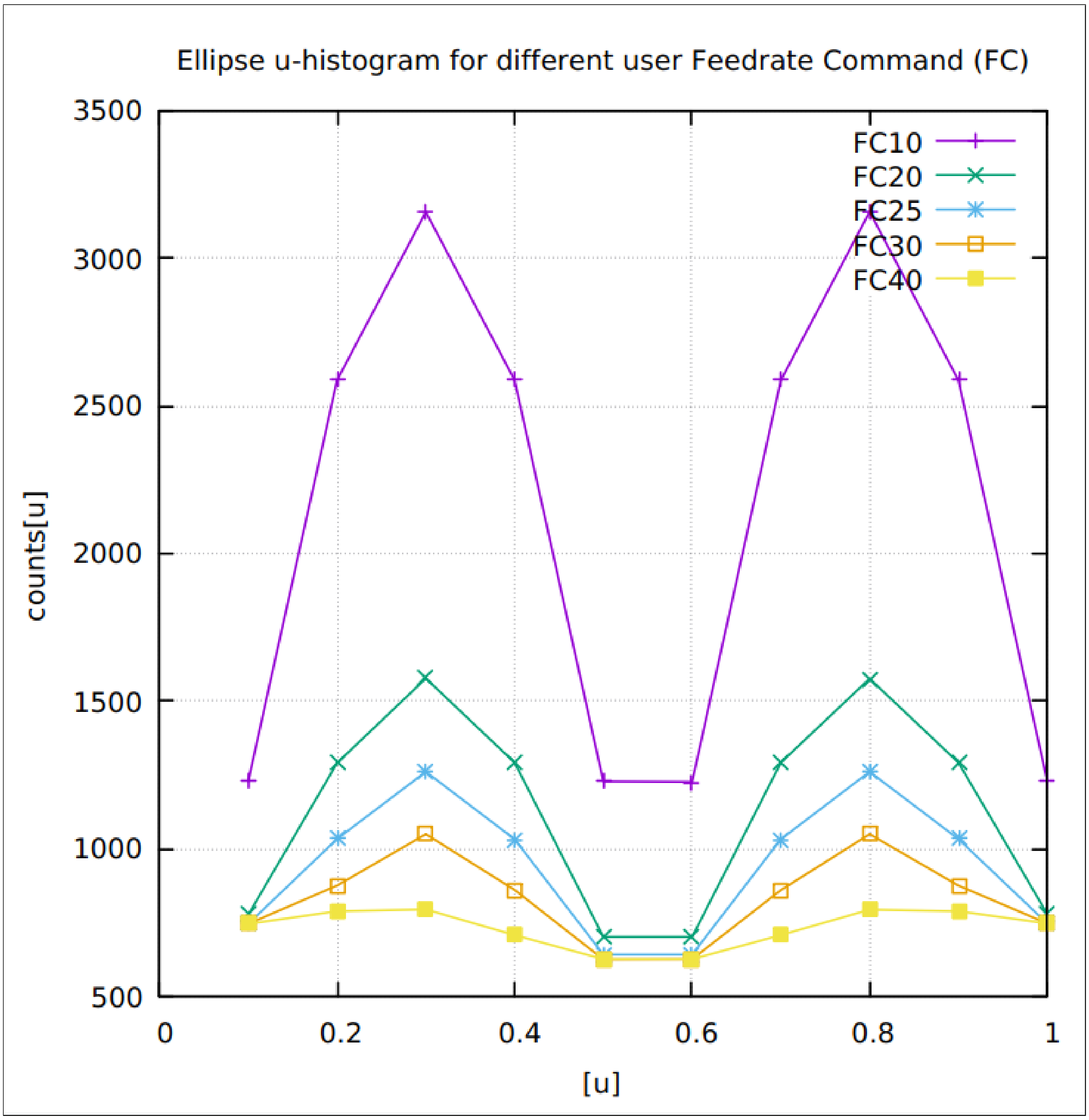


Table 2.58: Ellipse histogram of interpolated points

2.6.4 Skewed-Astroid histogram of interpolated points

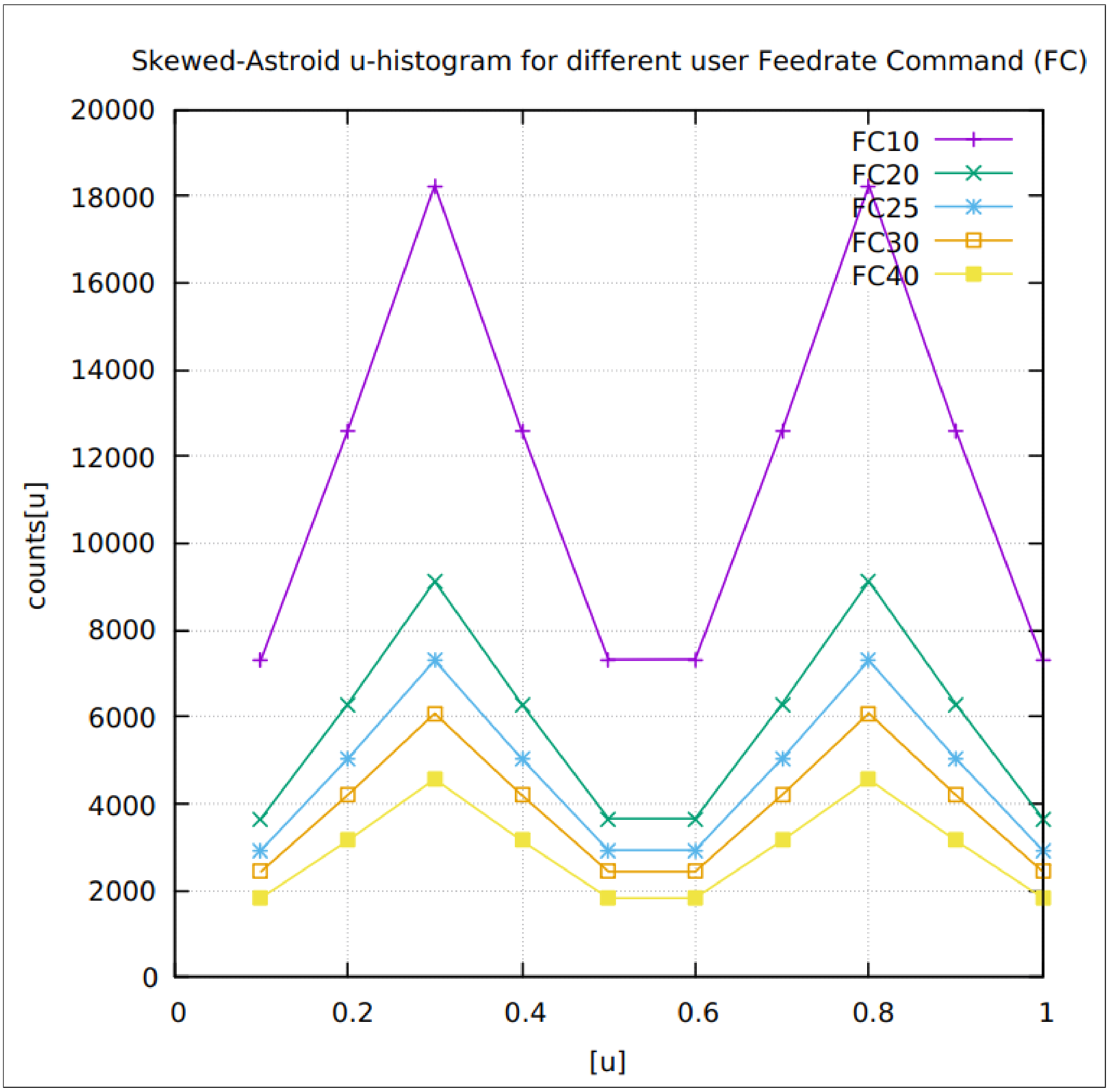


Table 2.59: Skewed-Astroid histogram of interpolated points

2.6.5 Circle histogram of interpolated points

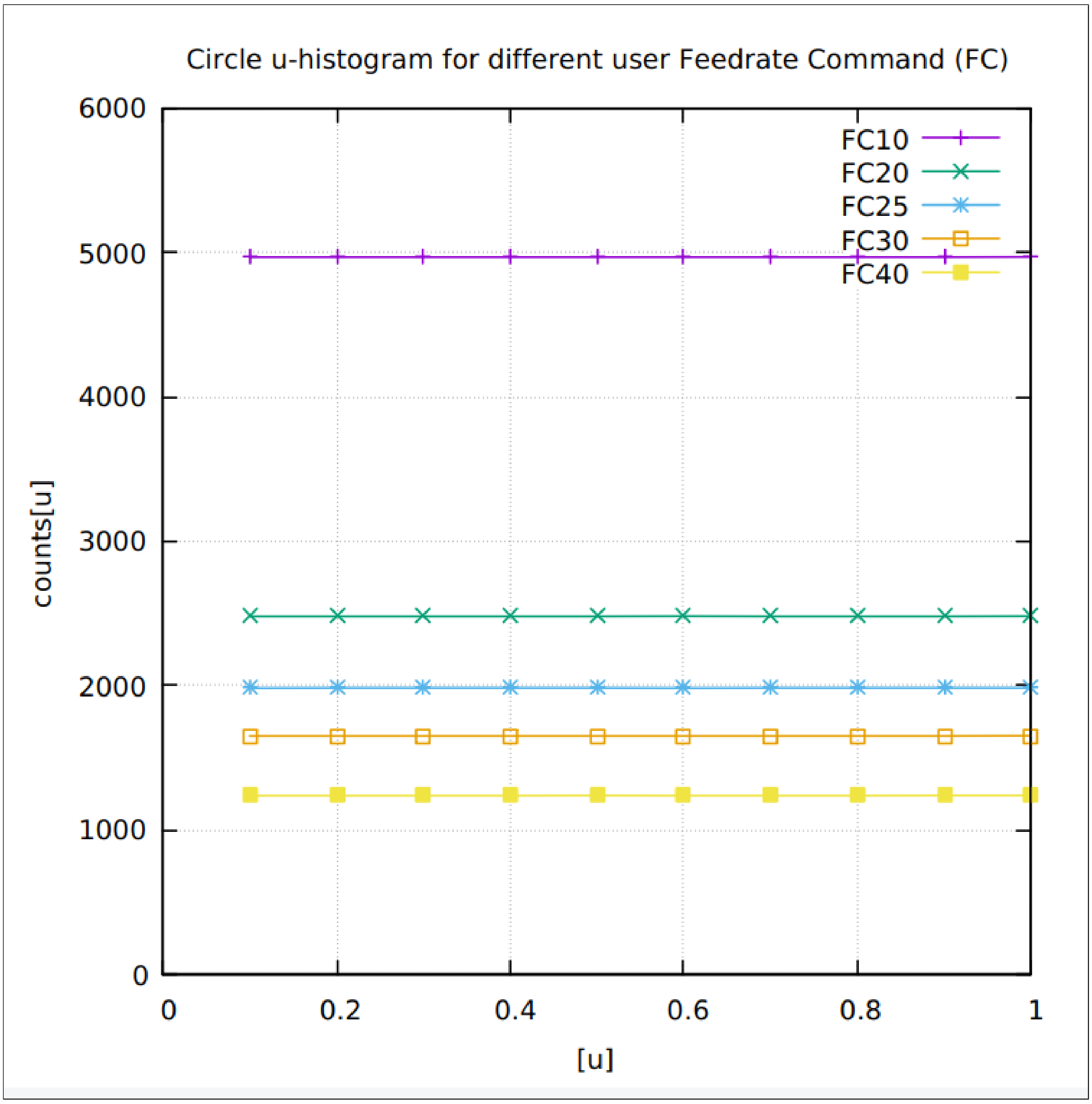


Table 2.60: Circle histogram of interpolated points

2.6.6 AstEpi histogram of interpolated points

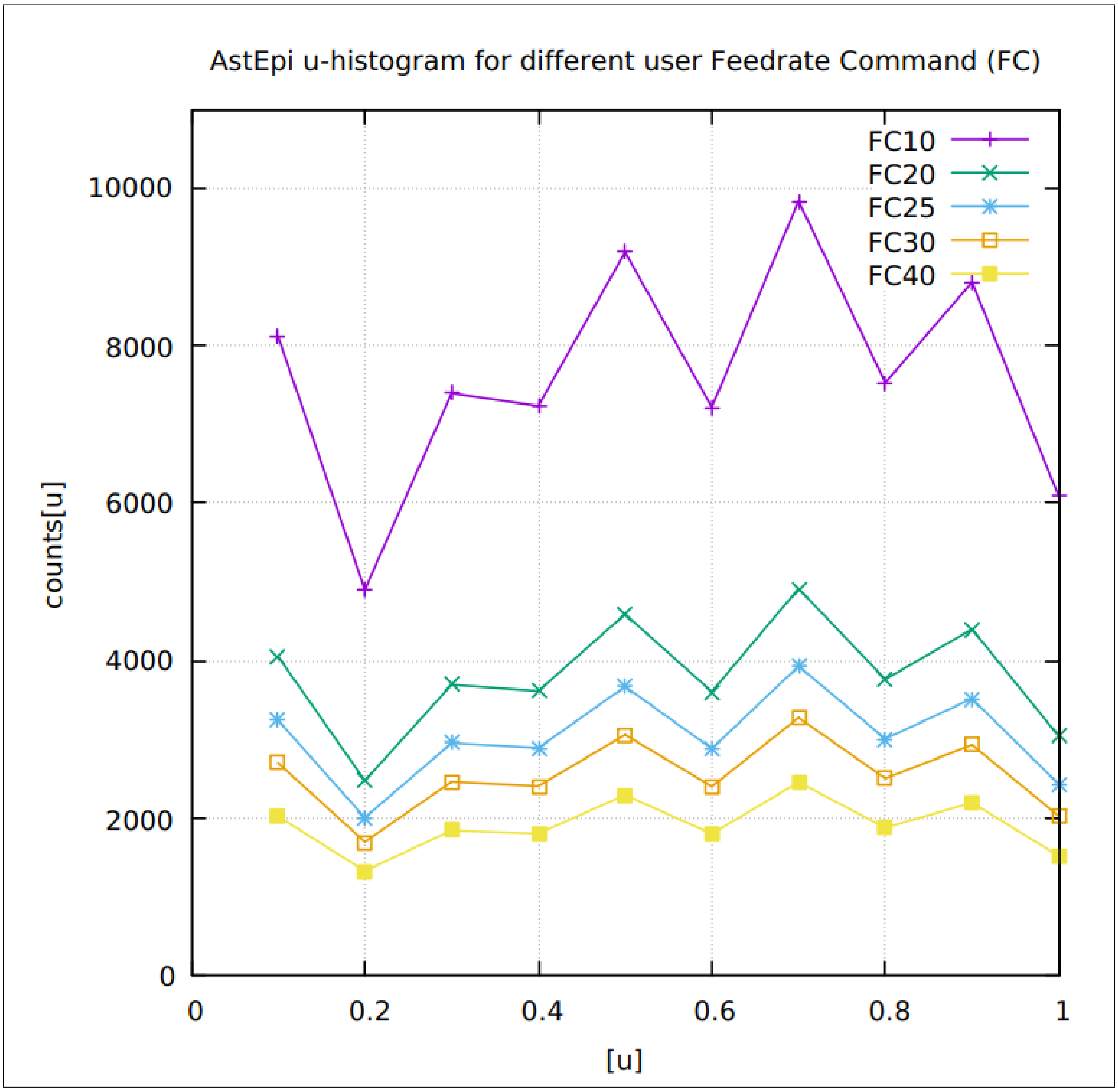


Table 2.61: AstEpi histogram of interpolated points

2.6.7 Snailshell histogram of interpolated points

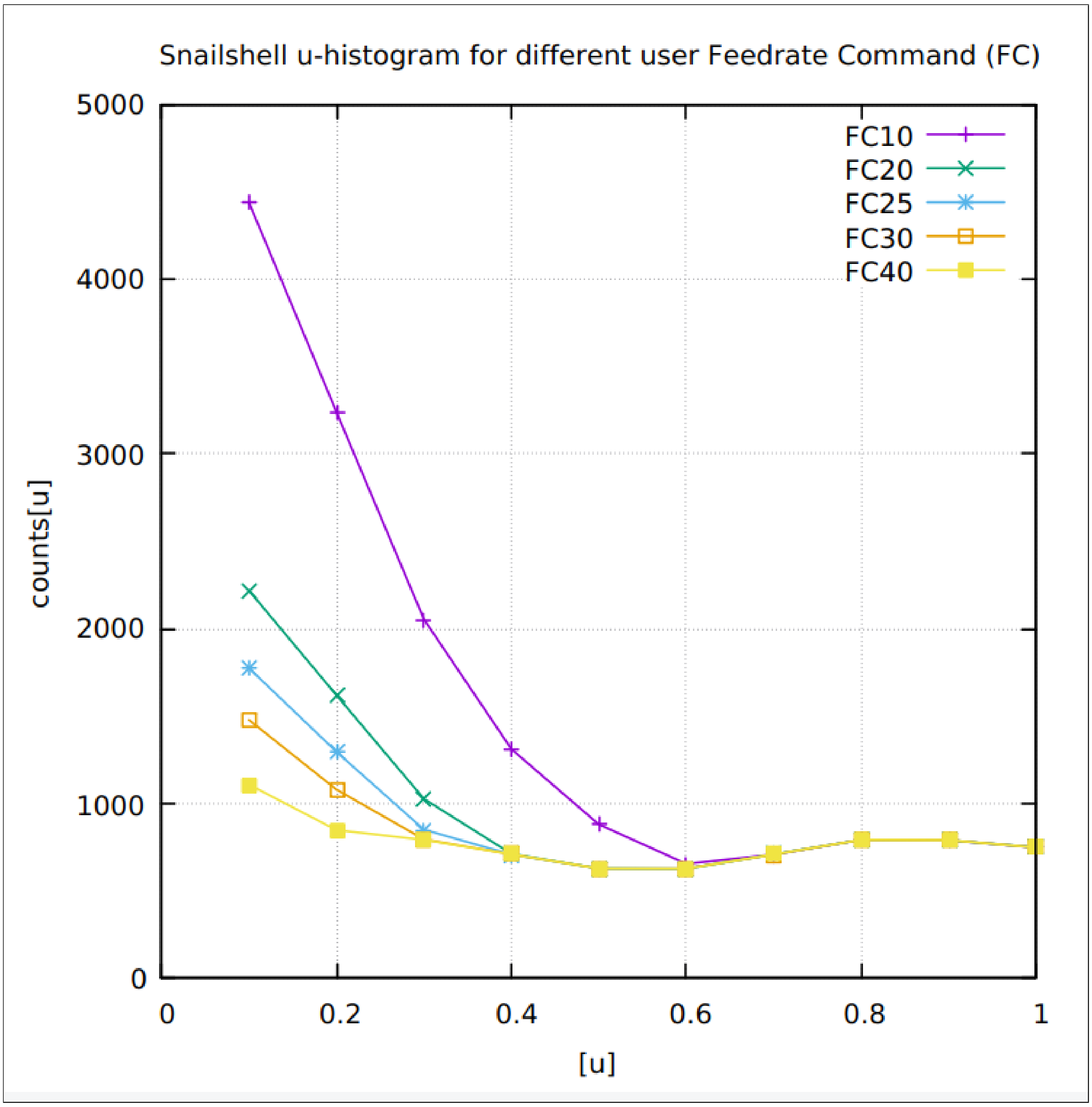


Table 2.62: Snailshell histogram of interpolated points

2.6.8 SnaHyp histogram of interpolated points

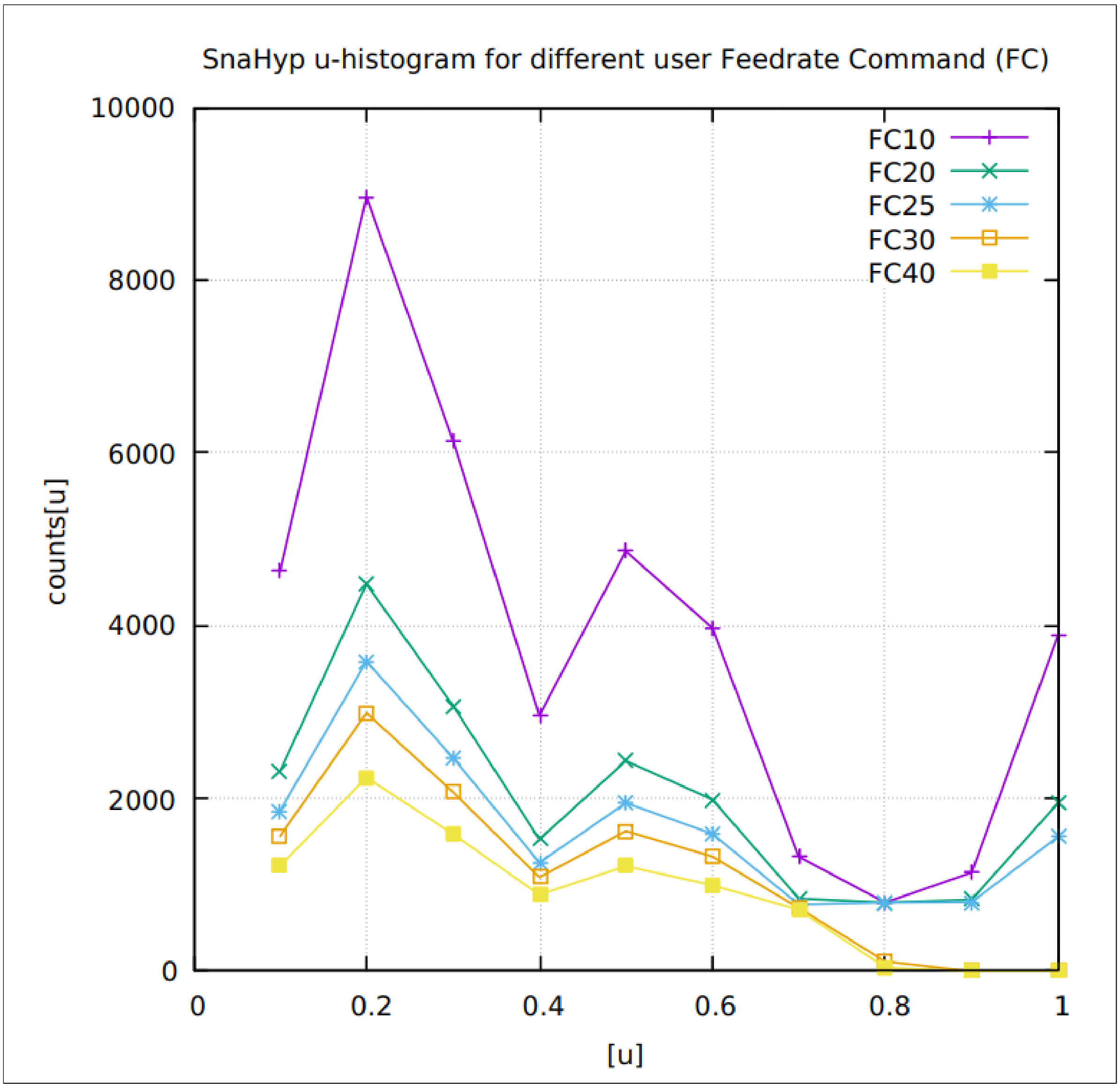


Table 2.63: SnaHyp histogram of interpolated points

2.6.9 Ribbon-10L histogram of interpolated points

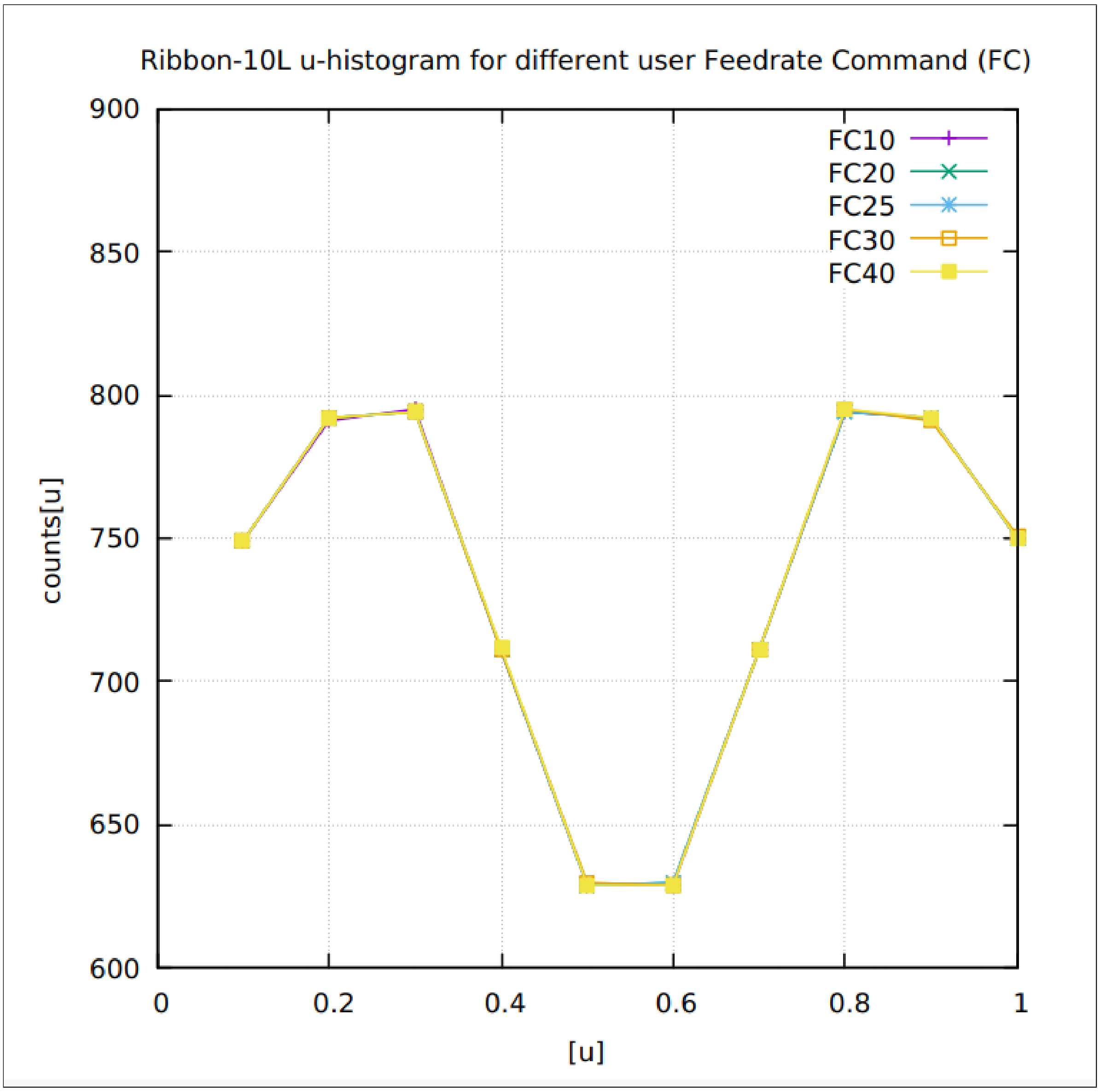


Table 2.64: Ribbon-10L histogram of interpolated points

2.6.10 Ribbon-100L histogram of interpolated points

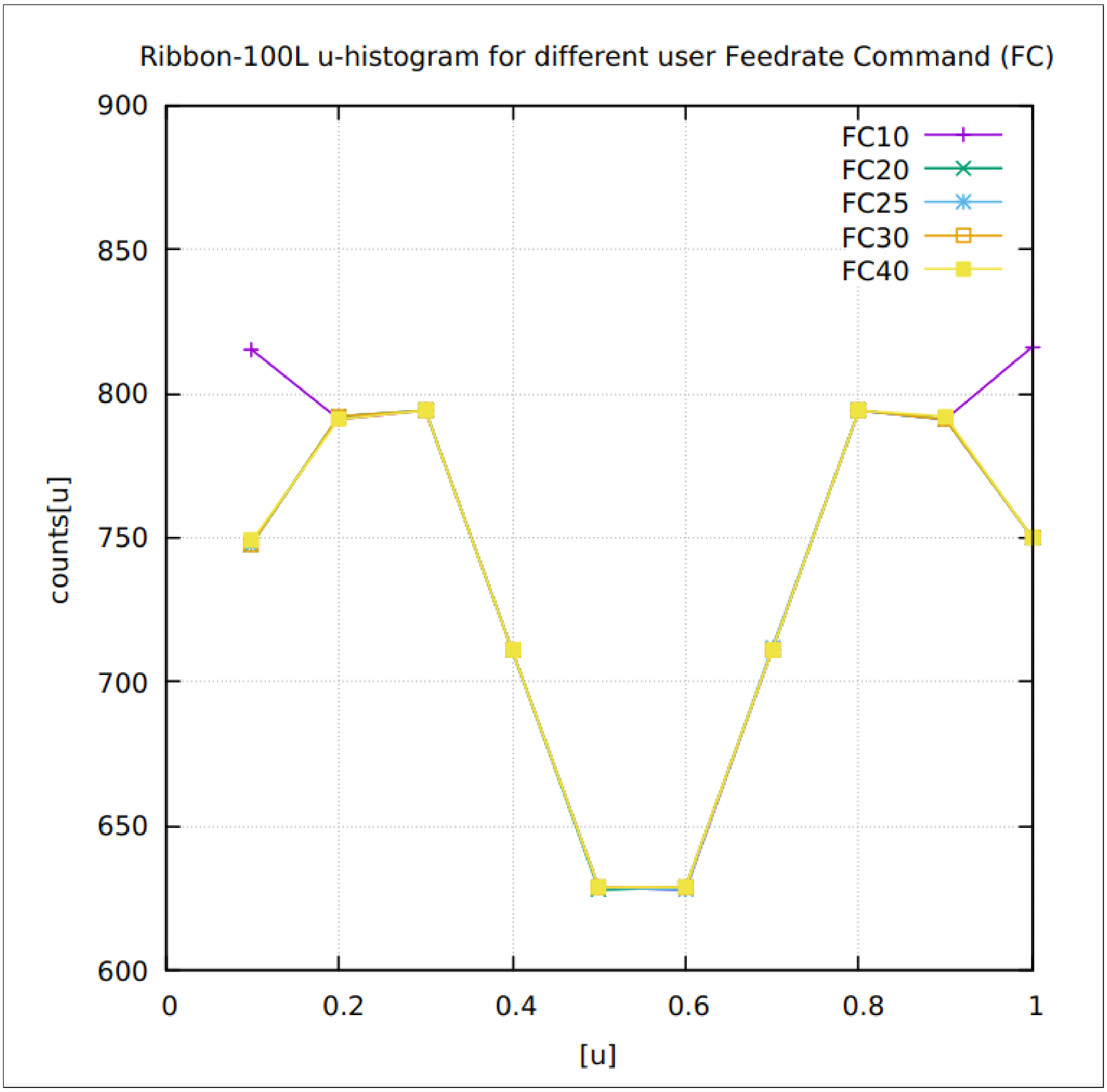


Table 2.65: Ribbon-100L histogram of interpolated points

2.7 Experimental Validation

1. The experimental validation is work-in-progress.

2. This section briefly describes the hardware setup and tests carried out on the hardware for the execution of point-to-point parametric curve interpolation algorithm.

3. The execution of the RS274/NGC g-codes generated on the fly by the parametric curve interpolation algorithm has been conducted on the hardware setup using the LinuxCNC Axis ver 2.8.0 software application. It was successful and displayed all the expected feedrate characteristics.

3 Analyses and Discussion

4 Conclusion

1 Appendices