

Verification of SE Formulas for Mediation Analysis

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There is a short exchange in the American Journal of Epidemiology

1 The Samoilenko and Lefebvre group

This group does essentially what B&B are proposing with exact formulas and δ -method standard errors. In particular, they avoid any requirement for rare events. They also do direct and indirect effects. However, I haven't seen any mention of random-effects/multilevel-models.

1.1 Samoilenko et al. [2018]

Gives exact formulas for direct and indirect mediation effects, as well as δ -method standard errors. Binary outcome, binary mediator. No analytical SE when exposure-mediator interaction is present.

1.2 Samoilenko and Lefebvre [2021]

Extends analytical results of Samoilenko et al. [2018] to handle exposure-mediator interaction term in model for outcome.

1.3 Samoilenko and Lefebvre [2023]

As Samoilenko et al. [2018], but with continuous mediator and binary response. No mixed-effects. Very nice simulation study.

1.4 Caubet et al. [2024]

Extends the work of Samoilenko and Lefebvre [2023] to case-control data.

2 Derivative-Based

Defines mediation effects in terms of derivatives. E.g. They define the indirect effect of X on Y as $(\partial EY/\partial EM) \cdot (\partial EM/\partial X)$, although that's not quite how they state it in the paper. This approach has had a small, but consistent following.

2.1 Maria et al. [2024]

Incorporates random-effects in the derivative-based approach. Mostly does Bayesian inference. Mentions bootstrap.

3 GLMM UQ

3.1 Zheng and Cadigan [2021]

Does joint uncertainty quantification for the parameters and random effects. Based on maximizing the Laplace Approximation to the marginal likelihood. Gives δ -method SEs with gradients from automatic differentiation

4 Applications

4.1 Not Relevant

Brown [2022] talks about mediation analysis and includes models with mixed-effects, but the latter are only used for imputation.

References

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