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Accommodating binary and count variables in mediation: A case for conditional indirect effects

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Abstract

The existence of several accessible sources has led to a proliferation of mediation models in the applied research literature. Most of these sources assume endogenous variables (e.g., M, and Y) have normally distributed residuals, precluding models of binary and/or count data. Although a growing body of literature has expanded mediation models to include more diverse data types, the nonlinearity of these models presents a substantial hurdle to their implementation and interpretation. The present study extends the existing literature (e.g., Hayes & Preacher, 2010; Stolzenberg, 1980) to propose conditional indirect effects as a useful tool for understanding mediation models that include paths estimated using the Generalized Linear Model (e.g., logistic regression, Poisson regression). We briefly review the relevant literature, culminating in a discussion of conditional indirect effects and their importance when examining nonlinear associations. We present a simple extension of the equations presented by Hayes and Preacher (2010) and provide an applied example of the technique.

Keywords

generalized linear model, logistic, mediation, poisson

Theories of human behavior tend to specify longitudinal associations through which predictors lead to theoretically specified outcomes through a chain of potential intermediaries. Mediation models provide one approach for testing such indirect associations, and the relative accessibility of the mediation literature (e.g., Hayes, 2013; Preacher & Hayes, 2004) has spurred widespread adoption of mediation analyses in the social and behavioral sciences. The application of mediation models has, however, remained somewhat constrained by the ubiquity of linear models in applied research. Mediation models typically consider the restricted case in which key endogenous variables (i.e., the mediator and outcome) are sufficiently continuous to warrant reliance on linear regression. Researchers interested in extending their mediation hypotheses to include more complex associations are faced with a relative paucity of accessible resources. Of the resources that are available for such analyses, most are highly technical and specify methods that may be beyond the reach of applied researchers (e.g., causal mediation; see Muthén & Asparouhov, 2015).

Hayes and Preacher (2010) provide succinct advice for testing indirect effects in the presence of nonlinear associations, but that paper specifically applies to models that assume a normally distributed residual. The current article illustrates how the same nonlinear modeling framework readily applies to common generalized linear models that accommodate non-continuous data, thus providing applied researchers with a useful alternative to causal mediation models. Specifically, we illustrate how Hayes and Preacher's (2010) technique applies to logistic and loglinear (e.g., Poisson) regression approaches. We provide a brief review of the relevant literature, culminating in a discussion of conditional indirect effects and their importance when examining nonlinear associations. We present a simple extension of the equations presented by Hayes and Preacher (2010) followed by an applied example.

Mediation under the general linear model

Mediation analyses typically investigate whether the association between two variables, X and Y, can reasonably be accounted for by an intervening variable, M (Hayes & Preacher, 2010). Researchers have traditionally explored such associations using three-variable multiple linear regression models. For instance, Baron and Kenny's (1986) widely cited model for mediation analysis consists of four key paths, which we label a_1, b_1, c_1, and $c_1{}'$ in Figures 1 and 2. Figure 1 summarizes the unmediated association between X and Y:

$$Y = c_o + c_1X + r_Y \tag{1}$$

where c_o and c_1 represent the intercept and regression slope for variable X, respectively, for the prediction of outcome Y. r_Y represents the residual of variable Y. Figure 2 similarly summarizes the two equations required to model the indirect association mediated by M:

$$M = a_0 + a_1X + r_M \tag{2}$$

$$Y = b_0 + b_1M + c_1{}'X + r_{Y'} \tag{3}$$

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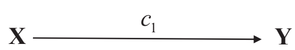

Figure 1. The unmediated model.

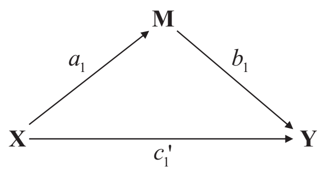

Figure 2. The mediated model.

In these equations, Y is the ultimate outcome, X is the focal predictor, and M is the mediator. a_i, b_i, and c_i, represent regression estimates, where coefficients with 0 subscripts represent intercepts and coefficients with 1 subscripts represent regression slopes. Subscripted r values represent equation-specific residuals.

In the above equations, the difference between c_1 and $c_1{}'$ indicates the degree that the target association can be attributed to the mediator (i.e., the indirect association between X and Y, as mediated by M). Researchers commonly compute the indirect effect by taking the product of a_1 and b_1, but the difference in coefficients method (i.e., $c_1 - c_1{}'$) and the product of coefficients method (i.e., $a_1 * b_1$) produce identical results for linear regression models (MacKinnon, Warsi, & Dwyer, 1995).

Mediation with generalized linear models

Mediation analyses derived from linear regression models exclude many types of dependent variables, limiting tests of theoretically predicated research questions that specify non-normally distributed endogenous variables. For instance, traditional regression techniques do not readily apply to nonlinear models for binary or count outcomes. Such variables violate the model's assumptions of normality and homoscedasticity. Mediation models should therefore be extended to accommodate the generalized linear model (GzLM; see Agresti, 2007; Imai, Keele, & Tingley, 2010) that can appropriately handle these diverse data types.

Readers familiar with traditional regression models (i.e., the general linear model) already have an intuitive exposure to the GzLM, as the GzLM represents a simple extension of the general linear model. Specifically, the GzLM allows flexibility in three aspects of the model: the systematic component, the random component, and the link function (Agresti, 2007). The systematic component describes how the outcome systematically varies as a function of one of more predictors. The systematic component is underlined in the following model, where $f(\hat{Y})$ represents a function of the expected value of the outcome variable Y, X_1 and X_2 are predictors and bs represent regression weights:

$$f(\hat{Y}) = b_0 + \underline{b_1X_1 + b_2X_2} \tag{4}$$

Predictors enter the systematic component of GzLMs linearly (i.e., additively), thus giving Generalized *Linear* Models their name. It is important to note, however, that GzLMs do not necessarily describe equations that are linear in their observed associations. A readily understood example of this is the standard regression equation for a quadratic association: $\hat{Y} = b_0 + b_1X_1 + b_2X_1^2$. Here, all predictors enter the equation linearly even though the overall model describes a curvilinear association between Y and X_1.

The random component of GzLMs should also be familiar to readers accustomed to traditional regression models, as the random component describes the response distribution of Y. If Equation (4) represents a traditional regression model, we would assume that, after conditioning on predictors and the intercept, Y is normally distributed with zero mean and some estimated variance (i.e., residuals $\sim N\ [0, \sigma_r^2]$). If Equation (4) instead represents a GzLM, the random component can be altered to better match the outcome variable's expected distribution. For example, the random component might specify a binomial distribution if Y is binary or a Poisson distribution if Y represents a count.

The third component of GzLMs, link functions, describe the function of Y predicted in the model. For instance, traditional regression approaches assume an identity link such that $f(Y) = Y$. A one-unit change in predictor X_1 therefore leads to a b_1 unit change in Y. When the association between Y and the set of predictors is nonlinear in form, the identity link is not appropriate. Instead, GzLMs typically specify a logit link for binary outcomes or a log link for count outcomes. The logit link specifies that a one-unit change in predictor X_1 leads to a b_1-unit change in the logit (i.e., log odds) of Y. Similarly, a log link specifies that a one-unit change in X_1 leads to a b_1-unit change in the natural log of Y. For these reasons, we henceforth refer to GzLMs with a logit link as logistic regression models and refer to GzLMs that use a log link as loglinear regression models. These terms encompass models that specify multiple possible random components (e.g., Poisson or negative binomial distributions for count data).

Assessing mediation within the framework of GzLMs not only allows the inclusion of non-normally distributed endogenous variables, but GzLMs can also accommodate different types of endogenous variables within a single model. Any form of GzLM can be specified for M and Y, and these forms do not necessarily need to match. This flexibility makes GzLMs especially useful for testing indirect associations using already-available software (e.g., Mplus or Stata's gsem routine).

Due to their potentially nonlinear nature, calculating indirect effects that include GzLMs requires slightly different procedures from those described above for Ordinary Least Squares (OLS) regression models. In the following sections, we first review common approaches to mediation models for binary and/or count data. We then describe and illustrate the usefulness of applying conditional indirect effects to such models. Because our focus centers on the utility of conditional indirect effects, readers are encouraged to reference the cited literature for more thorough discussions of each topic.

Binary data

MacKinnon and colleagues (e.g., MacKinon & Dwyer, 1993; MacKinnon, Lockwood, Brown, Wang, & Hoffman, 2007) discussed the prevalence of mediation models that include dichotomous outcomes (e.g., presence vs. absence of a disorder) but noted difficulties associated with estimating indirect effects using logistic or probit regression. For instance, MacKinnon, Warsi, and Dwyer (1995) showed that the product and difference in coefficients methods for assessing mediation produce equivalent results under the assumptions of the general linear model. However, the same does not hold when Y is binary. Two approaches are therefore commonly used to circumvent this and related problems.

First, MacKinnon et al. (2007) noted that logistic and probit regression models normalize residual variances (to $\pi^2/3$ or 1, respectively). The raw estimates of the c_1 and c_1' paths are therefore not comparable due to differences in scaling. MacKinnon and colleagues then performed a data simulation to show the benefit of standardizing parameter estimates to a common metric before computing $c_1 - c_1'$ (formulas for standardization provided in MacKinnon et al., 2007). Their results suggest the product of coefficients approach produces relatively unbiased estimates of the indirect effect, with the standardized version of the difference in coefficients method performing nearly as well in most cases. The unstandardized differences in coefficients approach performed poorly.

A second approach to modeling mediation for binary data is to treat the binary variable(s) as dichotomous representations of continuous latent responses (e.g., Muthén, 2011, see also Muthén, 1984). That is, researchers can assume each binary endogenous variable represents an item-specific continuous latent factor. This approach rescales all endogenous binary variables into a normally distributed latent metric, meaning the mediation analysis can proceed as for traditional regression models. The caveat, of course, is that the product of a_1 and b_1 only provides a valid estimate of the indirect effect as it pertains to the continuous latent response variable(s). Such an approach may be particularly useful when observed binary variables provide coarse estimates of continuous underlying latent variables, but modeling a continuous latent construct may be less defensible when the underlying construct truly is binary.

Because an indirect effect involving truly binary endogenous variables is nonlinear, the actual magnitude of the indirect effect varies as a function of the predictors. The indirect effect depends fully on "where on the probability curve the change takes place" (Muthén & Asparouhov, 2015, p. 14). This nonlinearity presents a challenge for the above models.

Count data

Substantially fewer sources have considered the case of endogenous count variables in an indirect effects equation. In a rare examination of this topic, Coxe and MacKinnon (2010) noted that Poisson regression (i.e., loglinear regression with a Poisson-distributed residual) places constraints on the residual variance of Poisson-distributed outcomes, implying similar limitations to those described by MacKinnon and colleagues (2007) for binary outcomes. In their simulation, Coxe and MacKinnon (2010) showed that both the product and difference of coefficients methods result in biased estimates of the true indirect effect, although the product of coefficients method performed best. Most subsequent papers discussing count variables in the framework of indirect effects have come from the causal indirect effects literature, however.

Causal indirect effects

Whereas the literature described to this point applies specifically to models designed for traditional definitions of direct and indirect effects, the causal mediation literature (e.g., Muthén, 2011; Muthén & Asparouhov, 2015; Pearl, 2012; Valeri & Vanderweele, 2013) has recently expanded mediation modeling beyond these simple assessments. Drawing on the logic of counterfactuals, this literature offers parametrically agnostic definitions of mediation (e.g., Pearl's, 2012, Mediation Formula) that allow applied researchers to test hypotheses about indirect effects for binary and/or count variables. For instance, Muthén (2011; see also Muthén & Asparouhov, 2015) linked the causal indirect effects literature to methods commonly implemented by users of structural equation modeling (SEM). Formula 20 from that paper, drawing on VanderWeele and Vansteelandt (2010), approximated the indirect effect for a model with a continuous mediator and a binary outcome with a simple odds ratio: e^{a1b1}. As specified above, a_1 represents the linear regression of M on X, and b_1 represents the logistic regression of Y on M (assuming no interaction between X and M). Additional models are provided for probit regression and models that include endogenous count variables. The causal mediation literature therefore provides one approach to dealing with the nonlinearity inherent in mediation models that include GzLMs.

The nonlinearity problem

Although GzLMs are linear when thought about in one metric (e.g., logits), they are nonlinear when approached using metrics more commonly considered by applied researchers (e.g., probabilities, counts). This nonlinearity results in conditional indirect effects whose overall magnitudes depend strongly on the actual level of the predictors (Muthén & Asparouhov, 2015; see also marginal effects). For illustrative purposes, consider the direct effects shown in Figure 3. We simulated these data to illustrate a negative association between high-school students' academic motivation and two related outcomes. Panel (A) illustrates a logistic regression in which the probability of a student dropping out of high school decreases as academic motivation increases. Panel (B) similarly illustrates a negative association between academic motivation and the number of unexcused school absences (obtained using Poisson regression).

The figure shows a clear association between academic motivation and both outcomes, although the overall effects of a one-unit increase in the predictor are not constant (see also Figure 3 in Muthén & Asparpuhov, 2015). In the logistic regression, the slope at Academic Motivation = 45 is quite steep. A 1-unit increase in motivation substantially decreases the odds of high school dropout. The slope at Academic Motivation = 55 is much more shallow. Thus, an increase or decrease in academic motivation will impact the predicted probability of dropout more strongly when scores are between 40 and 50 than when scores are more extreme (in either direction). The count model similarly shows an exponentially decreasing association between academic motivation and the number of recorded absences. Changing academic motivation does little to impact the expected number of absences when motivation is relatively high (e.g., greater than 50) but has an exponentially stronger impact as motivation decreases.

Because direct associations between predictors and binary/count endogenous variables are non-constant, the same necessarily applies to related indirect effects. Readers familiar with generalized linear models will note that interpreting such nonlinear effects is not an insurmountable task, however, because the nonlinearity can easily be summarized using single statistics (e.g., odds ratios). The overall impact of this nonlinearity can be easily obfuscated when considering indirect effects, however. The implicitly causal nature of indirect effects models therefore demands further qualification of the relative magnitude of effects at various levels of the predictors.

For instance, we might be interested in understanding how enrollment in an experimental program decreases students' probability of dropping out of high school, as mediated through increases in academic motivation. Finding a significant indirect association between program participation and high school

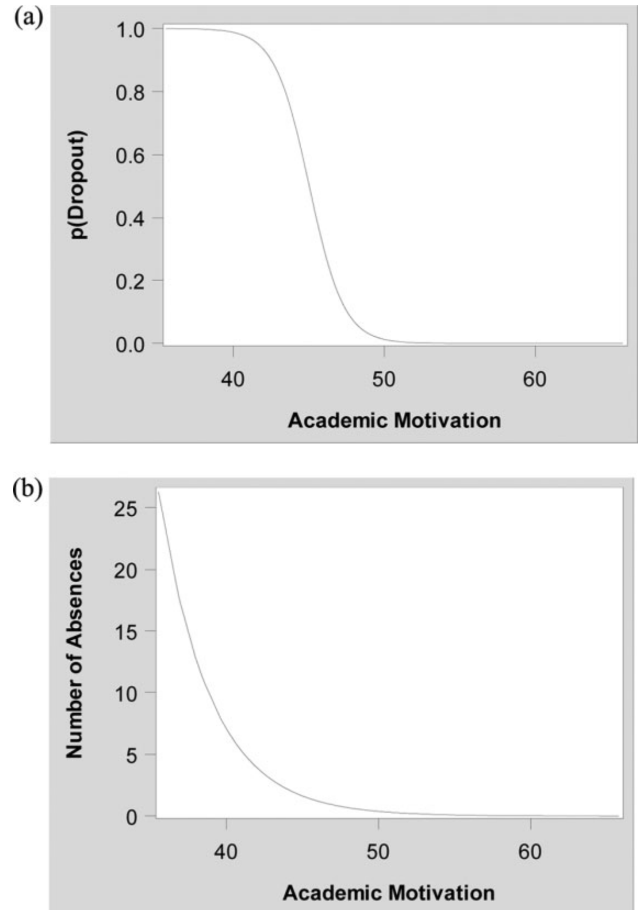

Figure 3. Illustration of (a) logistic and (b) loglinear associations.

completion, as mediated by academic motivation, will do little to guide school administrators because the summary p value cannot inform which students benefit most from the program. Administrators may need to work within the confines of narrow budgets that preclude delivery of new programs to all students. A quick examination of Figure 3(a) suggests that increasing academic motivation will substantially improve the chances of graduating high school among students low on the factor but will have relatively little impact on students with an already sufficient level of motivation (e.g., those above a score of 50). Extrapolating such conditional effects to mediation models can similarly provide researchers deeper insight into their indirect associations than p values alone.

As noted above, the causal mediation literature provides one approach for addressing the nonlinearity problem. That literature remains mathematically technical, however, and the analytic approaches described therein may be beyond the reach of applied researchers. As such, applied research demands alternative approaches to address nonlinearity. The conditional indirect effects literature provides one such solution.

Conditional indirect effects

In an attempt to demystify nonlinear models like those discussed above, Stolzenberg (1980) provided an elegant solution for models that are linear in their parameters, regardless of whether or not the models are linear in their associations (e.g., GzLMs). That paper noted such nonlinear associations can be described as a series of conditional linear associations. The nonlinear association between X and Y is really nothing more than a series of conditionally linear associations whose magnitudes vary as a function of X.

More specifically, the conditional linear association between X and Y is defined by the line tangent to the nonlinear curve linking X and Y at a pre-specified value of X. That is, this line represents the linear association between X and Y at a specific level of X. We illustrate this concept in Figure 4, where we have added tangent lines to Figure 3(a) The dotted line illustrates the conditional association when Academic Motivation $= 45$ and the dashed line represents the conditional association when Academic Motivation $=$ 55. The slopes for these lines are obtained by taking the first partial derivative of the logistic regression equation with respect to the predictor. Here, the first partial derivatives give conditional associations of $-.22$ and -0.00, respectively. Note that because the overall trend is monotonic and negative, the conditional association will always be negative no matter how extreme the academic motivation score is. Conditional slopes can asymptotically approach, but never reach, zero in this example.

Conditional associations like the ones described above are already widely used by applied researchers. Take, for instance, a researcher who wishes to plot simple slopes in order to understand an interaction. The first partial derivative of the regression equation $\hat{Y} = b_0 + b_1X + b_2Z + b_3XZ$, taken with respect to X, gives the often-used simple slope formula: $b_1 + b_3Z$. Thus, simple slopes are just tangent lines. First partial derivatives are also implicitly used in standard OLS regression models, where the effect of any predictor is constant. The first partial derivative of the regression equation $\hat{Y} = b_0 + b_1X$, taken with respect to X, simply provides the slope for X: b_1.

Stolzenberg (1980) applied the concept of first partial derivatives to modeling indirect effects, noting that an indirect effect can be described as the product of two first partial derivatives. One derivative comes from each equation involved in the mediation model. For linear regression, these two equations (as specified above) are:

$$M = a_0 + a_1X + r_M \tag{2}$$

$$Y = b_0 + b_1M + c_1{}'X + r_Y \tag{3}$$

The first partial derivative of Equation (2) taken with respect to X, is a_1. The first partial derivative of Equation (3) is then taken with respect to M, giving b_1. The product of these derivatives produces the common formula for an indirect effect: $a_1 * b_1$. Although Stolzenberg (1980) left open the possibility of modeling nonlinear associations as part of mediation models, Hayes and Preacher (2010) clarified the application of Stolzenberg's approach and introduced several examples in which a nonlinear transformation was applied to one or both predictors (e.g., $a_1 \ln(X), b_1e^M$). They provided example Mplus and SPSS syntax for fitting these models.

It is important to note that Hayes and Preacher labeled the product of first partial derivatives an "instantaneous indirect effect" (e.g., 2010, p. 630). We instead use the term *conditional* indirect effect, as this term underscores the fact that such associations are conditional on a set level of X. Although we do not consider the extended case of moderated mediation in this article, we also note that use of the term *conditional* highlights similarity between the models used for conditional slopes in nonlinear mediation and for those used for plotting moderated associations.

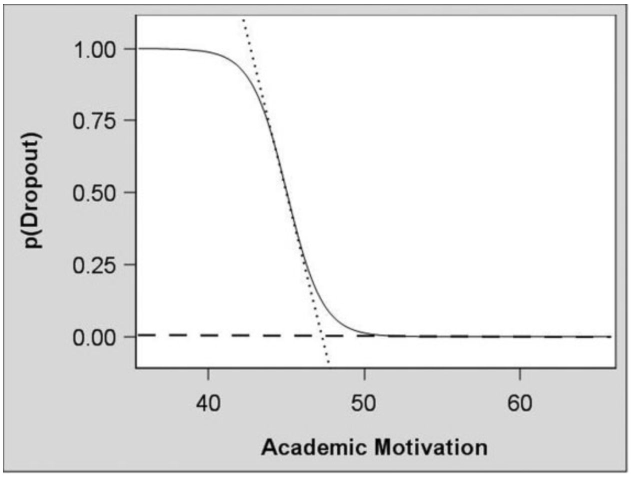

Figure 4. Logistic example with tangent lines for Academic motivation = 45 (dotted line) and Academic motivation = 55 (dashed line).

Table 1. First partial derivatives for three link functions: a_1 paths.

Function	Prediction equation	First partial derivative with respect to X
Identity	$\hat{M} = a_0 + a_1X$	a_1
Log	$\hat{M} = e^{a_0+a_1X}$	$a_1 e^{a_0+a_1X}$
Logit	$E(p(M)) = \hat{M} = \frac{e^{a_0+a_1X}}{1+e^{a_0+a_1X}}$	$\frac{a_1 e^{a_0+a_1X}}{(1+e^{a_0+a_1X})^2}$

Table 2. First partial derivatives for three link functions: b_1 paths.

Function	Prediction equation	First partial derivative with respect to M
Identity	$\hat{Y} = b_0 + b_1M + c_1'X$	b_1
Log	$\hat{Y} = e^{b_0+b_1M+c_1'X}$	$b_1 e^{b_0+b_1M+c_1'X}$
Logit	$E(p(Y)) = \hat{Y} = \frac{e^{b_0+b_1M+c_1'X}}{1+e^{b_0+b_1M+c_1'X}}$	$\frac{b_1 e^{b_0+b_1M+c_1'X}}{(1+e^{b_0+b_1M+c_1'X})^2}$

Because the parameters for generalized linear models (e.g., logistic and loglinear regression) enter their respective equations linearly, Stolzenberg's (1980) logic indicates that conditional indirect effects provide a useful approach for describing indirect associations that include endogenous binary and/or count variables. Paralleling the need to plot interactions, plotting multiple conditional indirect effects would allow researchers to better understand for whom the magnitude of an indirect association is relatively stronger versus relatively weaker.

Conditional indirect effects that include logistic and/or loglinear components

We next discuss conditional indirect associations as they apply to two specific forms of the generalized linear model: logistic and loglinear regression. We describe the general approach, providing a table of derivatives that can be multiplied to determine indirect effects of varying kinds. We then discuss the interpretation of conditional indirect associations and methods for statistical inference.

The model

Conditional indirect effects are easily computed by multiplying the first partial derivative of the prediction equation for M (taken with respect to X) by the first partial derivative of the prediction equation for Y (taken with respect to M) (Hayes & Preacher, 2010; Stolzenberg, 1980). For OLS regression, these partial derivatives give a_1 and b_1 (see first row of Table 1 and Table 2), and the indirect effect is taken as their product.

Although nonlinear link functions allow systematic equations to follow the same linear form as traditional regression, link functions can muddy the meaning of a one-unit change in Y. For instance, using a log link function may result in the following regression equation:

$$\ln(\hat{M}) = a_0 + a_1X \tag{5}$$

In other words, a one-unit change in X produces an a_1 change in the natural log of $\hat{M}$. This situation would be easily interpretable were M only an outcome, but handling the natural log of M when M (in its raw metric) also serves as a predictor can be cumbersome. M would have a different meaning when estimating a_1 versus when estimating b_1. As such, we note that researchers can enhance interpretability by first re-arranging loglinear and logistic equations to model changes in endogenous variables' natural metrics before taking first partial derivatives. For instance, the second line of the "Prediction equation" column in Table 1 transforms equation (5) by exponentiating both sides of the equation. The third line of the table similarly parameterizes change in the probability of a binary M, allowing $\hat{M}$ to take its natural metric (i.e., a probability between 0 and 1). Table 2 provides parallel equations for b_1.

After appropriately parameterizing the logistic and/or loglinear components of one's mediation model, conditional indirect effects can be obtained by multiplying the appropriate first partial derivatives. For instance, the conditional indirect effect for a model that contains a binary M (using logistic regression for the a_1 path) and continuous Y (modeled using standard regression) is simply:

$$\frac{a_1 e^{a_0+a_1X}}{(1+e^{a_0+a_1X})^2} * b_1 \tag{6}$$

Equation (6) is the same as Li, Schneider, and Bennett's (2007) formula (17) for indirect effects when a_1 is estimated using logistic regression, b_1 is estimated using linear regression, and no covariates are included in the model.

The magnitude of conditional indirect associations depends on both the level of X and on the level of M. For instance, the equations in Table 2 include both X and M as predictors. That said, it is unlikely researchers will be interested in conditional indirect associations for values of M discordant with a chosen value of X (i.e., a level of M different from $\hat{M}|X$, or the expected value of M given a set level of X). Following Hayes and Preacher (2010), we therefore suggest that researchers can replace M with $\hat{M}|X$ when calculating conditional indirect associations. $\hat{M}|X$ can be obtained using the equations provided in the "Prediction Equation" column of Table 1. This table assumes no covariates in the model, although covariates can (and oftentimes should) be considered when calculating $\hat{M}|X$ because the actual level of X matters when estimating these associations. If applicable, covariates should therefore be set to reasonable levels (e.g., their respective means) and included in the equations (see Hayes & Preacher, 2010).

Conditional total associations

Researchers may also wish to compare conditional indirect associations to the total conditional association between X and Y. If desired, conditional total effects can be quantified by replacing M in the "Prediction Equation" column of Table 2 with the appropriate function used to predict $\hat{M}$ in Table 1. Taking the first partial derivative of the resulting equation (with respect to X) provides a formula for the conditional total effect. For linear mediation models that assume normally distributed data and identity links (e.g., OLS regression), this approach yields the well-known formula for total effects: $\frac{d}{dx}\left(b_0 + b_1 * (a_0 + a_1X) + c_1{}'X\right) = a_1 * b_1 + c_1{}'$. These derivatives become substantially more complex for alternative models, however, limiting their applicability among applied researchers.

Interpretation

The nonlinearity inherent in many GzLMs makes the interpretation of conditional indirect effects somewhat less intuitive than the interpretation of linear regression coefficients. In linear regression, researchers typically interpret a regression coefficient as the expected change in an outcome, given a one-unit change in the predictor. The total association between X and Y in a linear mediation model (i.e., $a_1 * b_1 + c_1{}'$) represents the expected change in Y given a one-unit change in X. The indirect effect, then, represents the portion of this expected change that can be attributed to M (i.e., $a_1 * b_1$). The same does not hold for nonlinear associations.

When one or more components of the mediation model are nonlinear (e.g., a GzLM), conditional indirect effects instead represent the instantaneous association between X and Y, as mediated by M. Because this effect is instantaneous (i.e., describing an infinitely small increase in the predictor), it is not appropriate to interpret the association as describing the effect of a one-unit change in X. For example, we might consider the following model, where M is binary (modeled using logistic regression) and Y is continuous/normal:

$$\hat{M} = \frac{e^{2X}}{1 + e^{2X}} \tag{7}$$

$$\hat{Y} = -.50 + 1M \tag{8}$$

These equations combine to produce the following formula for a conditional indirect association:

$$\frac{2e^{2X}}{(1 + e^{2X})^2} * 1 \tag{9}$$

Note that for didactic parsimony $c_1{}'X$ was set to zero in Equation (8) and does not factor into Equation (9). The conditional total and indirect associations are therefore equivalent.

In this example, there is a 50% chance that $M = 1$ when X is zero. That is, replacing X with zero in Equation (7) leads to $\hat{M}|(X = 0) = \frac{e^0}{1+e^0} = \frac{1}{2} = .50$. Furthermore, Y is a one-to-one function of M, adjusting for the intercept of $-.50$, meaning $\hat{Y}|(X = 0) = -.50 + 1 * .5 = 0$. Highlighting the didactic equivalence of the total and indirect effects in this example, the conditional indirect association (i.e., Equation [9]) also reduces to $\frac{2e^{2*0}}{(1+e^{2*0})^2} * 1 = \frac{2e^0}{(1+e^0)^2} = .50$. Given common interpretations based on traditional linear models, a researcher might therefore be tempted to interpret the indirect association of .50 as meaning a one-unit increase in X (i.e., increasing from zero to one) will accordingly increase Y by .50 (i.e., from zero to .50). Because of the instantaneous nature of the conditional indirect effect, however, a one-unit increase in X actually only changes Y by .38. That is, replacing X with one in Equation (7) results in $\hat{M}|(X = 1) = \frac{e^{2*1}}{1+e^{2*1}} = .88$, and Equation (8) therefore gives $\hat{Y}|(X = 1) = -.50 + 1 * .88 = .38$.

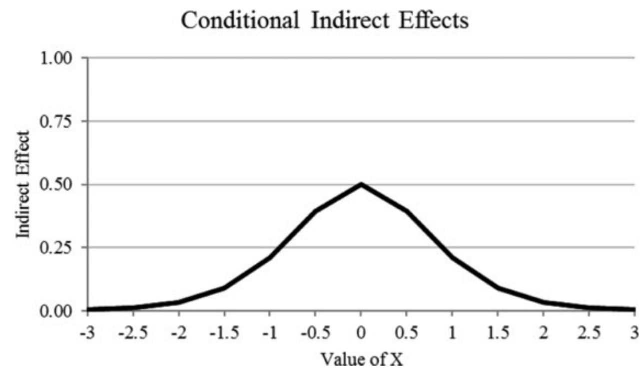

Figure 5. Conditional indirect associations between X and Y. This plot uses Equations (7) and (9), assuming X is unit-normal.

How do we best interpret conditional indirect associations, then? Although a conditional indirect association can technically be interpreted as the slope of a line connecting two expected values of Y obtained for levels of X separated by an infinitely small amount, we instead advocate for interpreting only their relative magnitudes. Thus, a researcher might select a reasonable range for X (e.g., $\pm$ 3 sample *SD*s) and some interval for examining conditional indirect effects within that range (e.g., every .50 *SD*) (see Hayes & Preacher, 2010). These conditional indirect effects can then be presented in a table or figure. For instance, Figure 5 presents the conditional indirect effects for the associations described in Equations (7) through (9). As Figure 5 shows, the association between these variables is stronger when X is near zero and weaker when X is more extreme. Had this figure described two OLS regression models, the line would have been flat (i.e., the indirect effect would have been invariant across levels of X).

Statistical inference

Researchers typically supplement the magnitude of an indirect association with a p value that provides more precise information about whether the indirect effect is statistically different from zero. One approach to estimating p values for indirect effects rely on Wald statistics, t-distributed statistics obtained by dividing an estimate by its standard error. Reliance on Wald statistics closely parallels the use of standard errors in traditional regression models but requires an assumption that the parameter estimates will be normally distributed across repeated sampling. As is now well-known, the product of two normally distributed variables (e.g., $a_1 * b_1$) is not normally distributed (see Preacher & Selig, 2012 for a brief discussion). Methods for estimating statistical significance that rely on standard errors and the associated distributional assumptions may therefore be unreasonable when examining indirect effects.

The literature describes many alternative approaches to assessing statistical significance for indirect associations. For instance, Monte Carlo confidence intervals (MCCIs) and bootstrapping both allow researchers to obtain confidence intervals that appropriately accommodate the indirect effect's non-normal distribution (Preacher & Hayes, 2008; Preacher & Selig, 2012). As explained by Preacher and Kelley (2011), bootstrapping involves simulating the sampling process that led to the original sample by treating the observed sample as an observed population. Multiple new samples are then drawn (with replacement) from this population. A statistic of interest is computed for each resample, and the sampling distribution of these estimates allows one to obtain confidence limits at predetermined percentiles.

The MCCI method differs from bootstrapping in that the MCCI approach does not require direct resampling. As Preacher and Selig (2012) explain, MCCIs are instead created by assuming individual parameters (e.g., a_1 and b_1) are multivariate normally distributed. Using the estimated means, variances, and asymptotic covariances for these parameters, researchers then generate a random sample of parameter estimates. A researcher can then compute the statistic of interest for each sample to create a random sample of indirect effects. As with bootstrapping, percentiles from this distribution provide a confidence interval for the observed indirect association. Preacher and Selig (2012) note that some advantages of the MCCI method over bootstrapping include much faster computation for MCCIs and the ability to obtain MCCIs from summary data (although potentially under the assumption of zero asymptotic covariances).

As with linear indirect effects, conditional indirect associations are not likely to follow a normal (or even symmetric) distribution across repeated sampling. As such, we encourage researchers to apply a method that appropriately accommodates the asymptotic non-normality of the indirect effects when examining conditional indirect associations. Hayes and Preacher (2010) advocated for bootstrapping when testing the significance of an indirect effect with nonlinear paths, and we anticipate that MCCIs should provide similarly valid information. Alternatively, plotting conditional indirect associations might complement an indirect effects model in which statistical inference follows from one of the models described in the causal mediation literature.

Although the overall magnitude of conditional indirect effects varies as a function of the predictor and/or mediator, all three models described in Tables 1 and 2 produce monotonically increasing (or decreasing) effects. A positive raw-metric regression coefficient indicates a positive association regardless of whether the predictor is very large or very small. A negative coefficient similarly indicates a negative association regardless of the value of the predictor. As such, the statistical significance of any given predictor should not change as a function of that predictor's value when implementing a resampling-based approach to statistical inference (e.g., bootstrapping or MCCIs). For any given resample, the first partial derivatives of the prediction equation for M (with the derivatives taken with respect to X) will be in the same direction for all values of X. The same percentage of first partial derivatives will therefore be above/below zero across resamples for all levels of X.

To further illustrate this point, we note that the exponentiated portions of the first partial derivatives in Tables 1 and 2 must always be positive. The direction of the first partial derivatives is therefore completely determined by the sign of the raw-metric regression coefficient. Assuming a resampling-based strategy to statistical inference, the p value associated with the simple product of raw-metric coefficients (i.e., $a_1 * b_1$) should accordingly produce the same p value as that associated with any conditional indirect effect.

It is important to note, however, that standard errors for the conditional associations will not remain constant across levels of X, nor will the Wald statistics obtained by dividing a conditional indirect association by its standard error. A standard-error-based approach to statistical inference will therefore suggest heterogeneous p values for the conditional indirect associations for different levels of X. For instance, the conditional indirect effect shown in Figure 5 may be significant when X is near zero but nonsignificant at more extreme values of X. Although beyond the scope of the present article, future simulation studies must therefore elucidate the relative usefulness of resampling- versus standard-error-based approaches to statistical inference in these models. Until then, researchers should weigh the potential pros and cons of each approach as applied to the context of their individual studies before selecting one over another.

Applied example

We next illustrate the above procedures in an applied example drawn from the adult development literature (Anthony, Geldhof, & Mendez-Luck, in press). These data come from 132 Mexican-origin women currently providing care to a family member aged 60 years or older. The variables analyzed in this example represent the total number of illnesses that the care recipient had at the time of data collection, a Likert-type measure of the caregiver's perceived intensity of providing support for the caregiver, and a count of how many different caregiving activities the caregivers viewed as emotionally draining. The mediation model therefore specified perceived intensity (M) as mediating the association between number of caregiver illnesses (X) and the number of emotionally draining activities (Y). For didactic purposes, we modeled intensity using linear regression and modeled emotional drain using loglinear regression with a Poisson-distributed residual. The analysis was performed using *Mplus* 7.4 (Muthén & Muthén, 1998–2012) with Monte Carlo integration.

Results for the mediation model suggested that the total number of care recipient illnesses significantly predicted caregiving intensity ($p = .037$) but not the number of emotionally draining activities ($p = .427$). Caregiving intensity did significantly predict the number of emotionally draining activities, however ($p < .001$). Parameter estimates are provided in the equations below:

$$\text{E(Intensity)} = 16.447 + .58 * \text{Total Illnesses} \tag{10}$$

$$\begin{aligned}&\text{E(Emotion Drain)}\\&\quad = e(.125 + .053 * \text{Intensity} - .013 * \text{Total Illnesses})\end{aligned} \tag{11}$$

The observed number of illnesses ranged from 1 to 14, and we examined conditional indirect associations between the total number of illnesses and emotional drain across this window. To do so, we combined the first partial derivative (with respect to X) for the linear a_1 path with the first partial derivative (with respect to M) for the loglinear b_1 path, replacing M with its expected value. The resulting formula for conditional indirect effects was therefore:

$$a_1 * b_1 e^{b_0 + b_1 \hat{M} + c_1{}' X} \tag{12}$$

Examining the plot of conditional indirect associations in Figure 6 reveals that the indirect association increased slightly as the total number of illnesses increased. For instance, the conditional indirect association is .084 for one illness and .106 for 14 illnesses. We next tested statistical significance of the indirect association using three approaches: Wald statistics, bootstrapping, and MCCIs. Statistical significance was examined for three values of X: the minimum (1), the integer nearest the mean (6), and the maximum

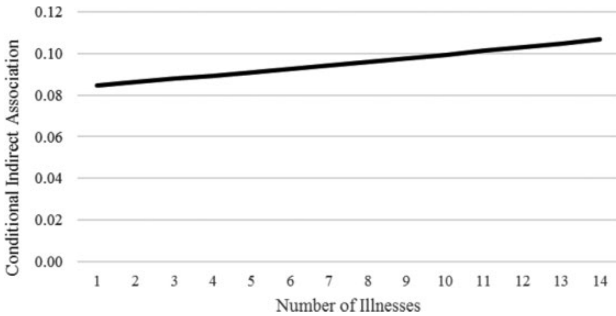

Figure 6. Conditional indirect associations between number of illnesses and emotional drain in the applied example.

Table 3. Significance testing for applied example.

Number of illnesses	Conditional indirect	p for Wald statistic	95% bootstrap CI	95% MCCI	MC > 0*
1	.084	.054	−.009, .165	.006, .151	9815/10000
6	.092	.071	−.009, .192	.006, .185	9815/10000
14	.106	.156	−.008, .286	.005, .277	9815/10000
Product method					
$a_1 * b_1$	.031	.07	−.003, .064	.002, .061	9815/10000

Note. *MC > 0 provides the number of Monte Carlo draws (out of 10,000) for which the conditional indirect association was greater than zero.

value (14). We estimated bootstrapped confidence intervals for all values of X simultaneously, meaning all intervals reflect the same 10,000 bootstrap resamples. Similarly, we generated a single sample of 10,000 randomly-drawn parameter estimates and obtained all MCCIs based on those data. Table 3 presents results from these analyses.

Relying on Wald statistics produced inconsistent p values, with none reaching statistical significance. Statistical significance was equivalent across levels of X for both the bootstrapping and MCCI approaches, although these two approaches differed from each other. Bootstrapping suggested a nonsignificant indirect association, whereas MCCIs suggested a significant indirect effect. Both methods produced lower bounds very close to zero, however, suggesting that bootstrapping and MCCIs provided generally similar estimates. We attribute the discrepancy in statistical significance to minor estimation differences between the approaches but note that the relative efficacy of bootstrap versus Monte Carlo confidence intervals warrants further evaluation for nonlinear models.

In line with our above discussion, the "MC > 0" column of Table 3 also shows that exactly the same number of Monte Carlo samples provided greater-than-zero estimates of the indirect associations for all levels of the predictor. This number also matched the number of $a_1 * b_1$ estimates greater than zero. Taken together, these results suggest that if a researcher is only interested in obtaining p values for the indirect association using a resampling-based approach, significance can be tested at any level of X or even using $a_1 * b_1$.

Discussion

The existence of several accessible sources has led to a proliferation of mediation models in the applied research literature. Most of these sources rely on the general linear model, precluding binary and/or count data. Although a growing body of literature has expanded mediation models to include more diverse data types, the nonlinearity of these models presents a substantial hurdle to their implementation and interpretation.

The present study extended existing literature (Hayes & Preacher, 2010; Stolzenberg, 1980) to propose conditional indirect effects as a useful tool for understanding mediation models that include paths estimated using the Generalized Linear Model (GzLM; e.g., logistic and Poisson regression). By relying on products of partial first derivatives, conditional indirect effects overcome the nonlinearity problem by providing heterogeneous estimates for indirect associations that vary systematically as a function of X.

The primary benefit of estimating conditional indirect associations follows from the logic of computing marginal effects when understanding standard applications of GzLMs. Due to the nonlinearity inherent in these models, it is possible to obtain drastically different indirect effect estimates (in terms of absolute magnitude) for different levels of X. Conditional indirect effects illuminate these potential differences and therefore allow researchers to gain a richer understanding of their nonlinear mediation results.

Although the purpose of this article was not to suggest outright replacement of alternative models that also accommodate non-normally distributed endogenous variables (e.g., those described in the causal mediation literature), we note that standard errors, bootstrapping, or MCCIs can supplement conditional indirect effect estimates for the purpose of statistical inference. Thus, estimation of conditional indirect effects also allows applied researchers to test the statistical significance of their indirect effect estimates using tools they are already familiar with.

We do not intend this article to provide a comprehensive review of all potential applications of instantaneous indirect effects, and additional work is still needed to extend these models to accommodate, for instance, moderated mediation and nonlinear mediation in multilevel frameworks. Because such extensions already exist for linear mediation models, we anticipate that extending the instantaneous indirect effects framework in these ways will be relatively straightforward.

Additional research is also needed to better understand the relative efficacy of different approaches to estimating the statistical significance presented in this article. As noted above, standard-error-based approaches imply potentially different levels of statistical significance for different values of X, whereas resampling-based approaches imply invariant levels of statistical significance across levels of X. Future research should similarly compare multiple valid approaches to statistical inference to each other. For instance, studies comparing bootstrap and Monte Carlo confidence intervals with analogous tests derived from the causal mediation literature could be especially informative. Huang, Sivaganesan, Succop, and Goodman, 2004, provide a relevant article comparing multiple approaches to logistic mediation models, for instance.

Despite the somewhat constrained nature of this article, we presented initial evidence that Stolzenberg's (1980) framework (see also Hayes & Preacher, 2010) may be useful for clarifying nonlinear indirect associations that include pathways estimated using the GzLM. The magnitude of such associations clearly varies as a function of X, yet few other frameworks explicitly address this nonlinearity problem. Expanding nonlinear mediation models to include GzLMs therefore strengthens the applied researcher's tool kit while also providing quantitative methodologists a new direction for exploring indirect associations.

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