A Concentrated Portfolio Selection Model

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December 17, 2021

Executive Summary

In practice, concentrated portfolios are useful under the following situations:

1. When the transaction cost is high, or when holding a large number of securities with small quantities is not possible. For example, the households.

2. When an enough amount of good stocks are not available, or when the investors prefer to hold a few stocks with superior performance.

3. When the risk is high enough that downturn of any sector may lead to bankruptcy.

However, classic portfolio selection models such as the mean-variance model, mean-semivariance model, mean-CVaR model choose well-diversified portfolios, while the cardinality constrained investment models are in general very hard to solve. In order to meet the needs for an easily computed concentrated portfolio selection model, the mean-greedy-matrix model (MG) is developed. The mean-greeady matrix model uses the risk measure $\rho(\omega) = \omega^T G \omega$, where G is the greedy matrix, and ω denotes the proportion of each asset in the portfolio. G is computed similarly as we compute the covariance matrix of the selectable assets, but with the diagonal entries replaced by the downside deviation of the corresponding asset minus the upside deviation. G is called the greedy matrix because it uses the greedy psychology of investment: the investors would want returns above 0 to be as large as possible, and returns below 0 to be as small as possible. A direct advantage of this greedy psychology is that it chooses well-performed concentrated portfolios.

Based on the risk measure of $\rho(\omega)$, a simple model and a realistic model is developed. The simple model ignores tax, transaction costs, and dividends, and it minimizes the portfolio's risk measure while maintaining a required level of portfolio return called the target return. The realistic model takes tax, transaction costs, and dividends into consideration, it minimizes the portfolio's risk measure given constraints on the target return, the total transaction costs, and the portfolio's concentration.

Experiments on evaluating the model performance are done on the data sets of American stock market and Chinese stock market. Different time periods are used so that the data sets come from different market regimes, thus the results have generality. The experiments compare the performance of optimal portfolios selected by MG and MV, and the following conclusions were made:

1. Optimal portfolios selected by MG are concentrated and greatly outperform those selected by MV.

2. Adjusting the target return and unit transaction cost does not affect the performance of optimal portfolios selected by realistic MG.

3. Performance of simple MG is sentitive to trading frenquency. In American stock market, investors should trade less often; In Chinese stock market, investors should trade more often.

In conclusion, MG selects concentrated and well-performed portfolios, and it could be solved easily using non-convex optimizers such as Gurobi. Thus, investors should consider using MG if they want to hold concentrated portfolios.

1 Introduction

In financial investment, high returns are always associated with high risks. Based on this risk-expected-return relationship, Markowitz introduced the modern portfolio theory in 1952: it assumes that an investor wants to maximize a portfolio's expected return contingent on any given amount of risk, or in other words, an investor wants to minimize a portfolio's risk contingent on any given amount of expected return.

The most important issue in financial investment is a portfolio's risk measure. Markowitz introduced the classic mean-variance model [2], and later, risk measures such as semi-variance (M-SV) [3], value-at-risk (M-VaR) [4], and conditional value-at-risk (M-CVaR) [5] are brought up. Usually, portfolio selection models will select well-diversified portfolios, since concentrated portfolios are usually considered to associated with a high risk and a poor performance. However, in practice, investors tend to hold concentrated portfolios because of the following three reasons [1]:

1. Transaction cost makes it uneconomic for investors to hold a large number of securities with small quantities. [6]

2. Investors with limited investment experiences could not find many good stocks, while advanced investors would focus on a few stocks with superior performance.

3. Diversified portfolios might result in increasing competition with others, making the investment strategy less attractive.

Considering the above, Chen, Li, and Wang [1] introduced a new risk measure (MG) based on the greedy psychology of investment: investors want the upside deviation as large as possible while the downside deviation as small as possible. A direct consequence of this psychology is that the portfolios chosen by this model would be highly concentrated. The subject paper first introduces a simple and a realistic model based on this new risk measure, then introduces several indicators to comprehensively evaluate the chosen portfolios' performance, and finally compares the optimal portfolios' performance chosen by the MG, to those chosen by MV, M-SV and M-CVaR. The empirical results are done on the data sets of American stock market and Chinese stock market. Both data sets have the same sample size and are taken from a different period of time, so that they do not come from the same market regime. Results indicate that the concentrated portfolios chosen by MG model outperform all portfolios chosen by MV, M-SV, and M-CVaR, and that the optimal portfolios of MG have a steady performance regardless the target returns and transaction costs.

Subject to the work of Chen, Li, and Wang [1], we studied the MG model and ran the same experiments on the data sets of American stock market and Chinese stock market. However, since the data used are from 10 years ago, some stocks' data are partially missing. So, the data we used are the same 600 historical returns of Chinese stock market, and the last 137 historical returns of American stock market. Another difference is that we used a more efficient optimizer called Gurobi to solve the MG model, while the original paper uses the algorithm introduced by Chen and Burer [7]. Due to these differences, our results are numerically different from that in the paper, but the conclusions are similar.

In this report, we will introduce the methodology in section 2, including the new risk measure of greedy matrix, the simple and realistic MG model, and the indicators used for performance evaluation. In section 3, we will briefly introduce the implement of the simple and realistic MG models. And data and results are stated in section 4.

2 Methodology

2.1 A new model: mean-greedy-matrix

Chen, Li, and Wang[1] introduced a new portfolio selecting model that is concentrated and based fully on the historical data. It uses a new risk measure that is inspired by the greedy psychology of investment: the investors want returns above 0 to be as large as possible, and returns below 0 to be as small as possible. All content in this section refers to the work of subject paper [1].

2.1.1 New risk measure

Definition 1. *For any portfolio ω, the new risk measure of greedy matrix is defined as $\rho(\omega) = \omega^T G \omega$, with G computed as follows:*
For non-diagonal elements, $i \neq j, i, j = 1, ..., N$,

$$g_{ij} = \frac{1}{T-1} \sum_{t=1}^{T} (x_{it} - \bar{x}_i)(x_{jt} - \bar{x}_j).$$

For diagonal elements,

$$g_{ii} = \sigma_{ii}^{-} - \sigma_{ii}^{+},$$

where σ_{ii}^{-} denotes the below zero deviation of asset S_i, σ_{ii}^{+} denotes the above zero deviation of asset S_i.

Portfolios selected using this risk measure is highly concentrated. This is because of the greedy psychology: the return rate of an asset with good intrinsic value will become higher and higher, because more capital is pumped into it. Thus, investors will only invest in a few assets that perform outstandingly well.

Another property of this risk measure is that $\rho(\omega)$ could be negative. This brings some computational difficulty, but brings also the following advantages:

1. The range $\mathbb{R}$ of $\rho(\omega)$ can describe the rule of minimizing risk more naturally in practice: the smaller the risk value is, the better the portfolio performs.

2. The possible negative values of $\rho\omega$ comes from the diagonal elements, which transforms a bi-objective measure into a single measure: it minimizes the downside deviation and maximizes the upside deviation at the same time.

3. Negative values of $\rho(\omega)$ could reflect the potential extreme losses and gains more efficiently. This is verified using the following example:

   Example 1. *Consider two assets A and B, each with 5 different possible outcomes with the same probabilities:*

   $$A : (-0.006, -0.002, 0.002, 0.007, 0.009),$$

   $$B : (-0.005, -0.003, 0.002, 0.006, 0.010).$$

   Clearly, A has larger extreme loss and smaller extreme gain compared to B, so B is what we prefer. However, A and B have the same mean and variance. So, we use $\rho\omega = \omega^T G\omega$ to reflect this difference:

   $$\rho(A) = 1.20 \times 10^{-6}, \rho(B) = -1.20 \times 10^{-6}.$$

Based on this new risk measure, a simple and a realistic MG model are developed.

2.1.2 Simple MG model

The simple MG model defined in equation (1) to (4) contains only three constraints: target return, a sum of weight of 1, and no short-selling.

$$
\begin{aligned}
\min_{\omega} \quad & \rho(\omega) = \omega^T G \omega && (1)\\
\text{s.t.} \quad & \sum_{i=1}^{n} \omega_i \overline{r}_i = e && (2)\\
& \sum_{i=1}^{n} \omega_i = 1 && (3)\\
& \omega_i \geq 0, \quad i = 1, \cdots, n. && (4)
\end{aligned}
$$

where the target return rate e is decided by investors in advance, the mean return of the ith asset is calculated by $\overline{r}_i = \sum_{t=1}^{T} r_{it}/T$, and a total number of n assets including the risk-free asset are considered.

2.1.3 Realistic MG model

Apart from the constraints in the simple MG model, the realistic MG model considers more market frictions such as transaction costs, lower and upper bounds for investment, and taxes and dividends.

Definition 2. *Let ω_0 and ω each denotes the original and final proportion of assets in the portfolio, k^s and k^b each denotes the unit transaction cost for selling and buying, then the transaction cost of an asset is defined as*

$$
c = \begin{cases} k^s(\omega^0 - \omega) & \omega < \omega_0 \\ \\ k^b(\omega - \omega^0) & \omega \geq \omega_0 \end{cases}
$$

Apparently, a too high transaction cost would be uneconimic, we thus impose a limitation of γ on the total transaction cost:

$$
\sum_{i=1}^{n} c_i \leq \gamma
$$

Combining the above using the technique in [8], we have the constraints on transaction cost:

$$\begin{cases} c_i \geq k_i^s(\omega_i^0 - \omega_i), \quad i = 1, ..., n \\ c_i \geq k_i^b(\omega_i - \omega_i^0) \quad i = 1, ..., n \\ \sum_{i=1}^n c_i \leq \gamma \end{cases}$$

We then limit the weights of individual assets to guarantee a specific level of portfolio's diversity:

$$\underline{\omega_i} \leq \omega_i \leq \overline{\omega_i}, i = 1, \cdots, n$$

where $\underline{\omega_i}$ and $\overline{\omega_i}$ are the upper and lower bounds for the weight of the ith asset.

The assets' returns are calculated differently in the realistic MG model, since we need to consider tax and dividends. The net return of the portfolio is defined as

$$r(\omega) = \sum_{i=1}^{n} R_i\omega_i - \sum_{i=1}^{n} c_i$$

where $R_i = (1 - t_g)r_i + (1 - t_0)d_i$ denotes the after-tax return rate, with t_g and t_0 each represents the marginal capital gains tax rate and marginal ordinary income tax rate, and r_i and d_i each denotes the rate of return and dividend yield. The target return constraint is thus defined as

$$E(r(\omega)) = \sum_{i=1}^{n} \overline{R}_i\omega_i - \sum_{i=1}^{n} c_i = e$$

where $\overline{R}_i$ is the sample mean of the after-tax returns.

According to the above, we establish a realistic MG model in equation

(5) to (11):

$$\begin{aligned}
\min_{\omega} \quad & \rho(\omega) = \omega^T G \omega && (5)\\
\text{s.t.} \quad & \sum_{i=1}^{n} \overline{R_i}\omega_i - \sum_{i=1}^{n} c_i = e && (6)\\
& \sum_{i=1}^{n} \omega_i = 1 && (7)\\
& c_i \geq k_i^s(\omega_i^0 - \omega_i) && (8)\\
& c_i \geq k_i^b(\omega_i - \omega_i^0) && (9)\\
& \sum_{i=1}^{n} c_i \leq \gamma && (10)\\
& \underline{\omega_i} \leq \omega_i \leq \overline{\omega_i}, i = 1, \cdots, n && (11)
\end{aligned}$$

Note that for simplification, we set tax $t_0 = t_g = 0$, and $\omega_i^0 = 0$ when computing the empirical results, as the subject paper did. This would reduce the after tax return to $R_i = r_i + d_i$ and eliminate constraint (8).

2.2 Performance Evaluation

Chen, Li and Wang [1] also provided several indicators that are used to comprehensively evaluate the out-of-sample performance of the portfolios. We will use them to compare MG and MV later in section .

Firstly, we compare the return rate of the portfolios. The expected rate of return

$$R = \frac{1}{T}\sum_{t=1}^{T} x_t,$$

where $x_t = \sum_{i=1}^{n} \omega_i x_{it}$ is the portfolio's return at time t. Clearly, the higher the expected return, the better the portfolio's performance.

Secondly, we compare the portfolios' risks using appropriate risk measures. The smaller the risk is, the better the portfolio performs. Suitable risk measures are standard deviation (Std) and CVaR:

$$Std = \frac{1}{T-1}\sum_{t=1}^{T}(x_t - R)^2,$$

where x_t and R is defined same as above. And sample CVaR is estimated as follows:

1. For each data point, compute the portfolio loss $L_t = -x_{it}\omega_i$.
2. Put the T's loss in order: $L_{(1)}, L_{(2)}, ..., L_{(T)}$.
3. $C\hat{VaR}_\alpha = \frac{1}{(1-\alpha)T}\sum_{j=T\alpha+1}^{T} L_{(j)}$.

Thirdly, we compare the portfolios' diversifications. Usually, the more diversified the portfolio is, the less risk the portfolio has. A portfolio's diversification is evaluated using zero-norm and Herfindahl index:

Definition 3. *For a portfolio with asset weights $\omega = (\omega_1, ..., \omega_n)$:*

$$ZN(\omega) = \#\{i|\omega_i \neq 0, i = 1, ..., n\}$$

$$HI(\omega) = \sum_{i=1}^{n} \omega_i^2$$

And most importantly, we need to compare the portfolios' reward-to-risk ratios, which reflects how much the investor is compensated for unit risk bearing. The higher this ratio is, the better the portfolio performs. We use Return/std, Return/CVaR, and the Farnelli-Tibiletti ratio introduced by[9]:

Definition 4. *Suppose random variable X represents the portfolio's return, p is the right order, q is the left order, b is the benchmark, then its Farnelli-Tibiletti ratio is defined as:*

$$FT(X) = \frac{E^{1/p}[\{(X-b)^+\}^p]}{E^{1/q}[\{(X-b)^-\}^q]}.$$

The Farnelli-Tibiletti ratio treats upside deviation as reward, and downside deviation as risk. p and q reflect the investor's attitude to extreme events: the larger the p or q, the more important the fluctuation of corresponding tail events. Since investors usually consider extreme losses as more important, we have $p \leq q$ in general.

Using above indicators, we compare the performance of MG to MV in section 4

3 Software and Tools

Since the Greedy matrix G is not positive definite, the risk measure $\rho(\omega) = \omega^T G\omega$ is not convex, and thus the MG model is a non-convex quadratic optimization problem. The subject paper proves that MG is a DC quadratic program (proposition 1), and it uses the algorithm introduced by Chen and Burer [7] to solve it. While in this project, we uses a more convenient optimizer called Gurobi. Gurobi supports R, Python and matlab, and it solves all kinds of quadratic programmings. In this project, we only use a simple non-convex quadratic model with only linear constraints and bound constraints presented as follows:

$$\begin{aligned} \min_{x} \quad & x^T Q x \\ \text{s.t.} \quad & Ax = b \\ & l \leq x \leq u \end{aligned}$$

Below is a sample R code on how to build the model and specify the parameters, more information could be found on www.gurobi.com/documentation/9.1.

```
# Build model
model = list()
model$Q = Q #the quadratic objective Q
model$A = A #the linear constraint matrix A
model$sense = c('=','<','>') #the linear constraint sense
model$rhs = b #the right hand side of linear constraint
model$lb = l #lower bound for vector x
model$ub = u #upper bound for vector x

# Set up parameters
library(Gurobi)
params <- list()
params$method = 2 #solving algorithm to use
params$NonConvex = 2 #programming type

# Compute optimized solution x
result <- gurobi(model, params)
weight = result$x
```

We then introduce in details about how to build the simple and realistic MG models, especially on how the quadratic objective Q and linear constraint $Ax = b$ are specified.

For a simple MG model, we find the global minimum with respect to weight vector ω, with each entry of ω bounded by 0 and 1. So we have $Q = G$, $l = \mathbf{0}$, $u = \mathbf{1}$, while the linear constraints of equation (2) and (3) are built as follows:

$$\begin{pmatrix} \bar{r_1} & \cdots & \cdots & \bar{r_n} \\ 1 & \cdots & \cdots & 1 \end{pmatrix} \begin{pmatrix} \omega_1 \\ \vdots \\ \vdots \\ \omega_n \end{pmatrix} = \begin{pmatrix} e \\ 1 \end{pmatrix}$$

For a realistic MG model, we find the global minimum with respect to weight vector w and transaction cost vector c, with each entry of w bounded by 0 and an upper bound of u. So we have:

$$Q = \left(\begin{array}{c|c} G & 0 \\ \hline 0 & 0 \end{array} \right)$$

and linear constraint $Ax = b$ as:

$$\left(\begin{array}{cccccccc} \bar{R}_1 & \cdots & \cdots & \bar{R}_n & -1 & \cdots & \cdots & -1 \\ 1 & \cdots & \cdots & 1 & 0 & \cdots & \cdots & 0 \\ 0 & \cdots & \cdots & 0 & 1 & \cdots & \cdots & 1 \\ \hline -k_1 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ 0 & -k_2 & \cdots & 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & & & & \vdots \\ 0 & \cdots & \cdots & -k_n & 0 & \cdots & \cdots & 1 \end{array} \right) \begin{pmatrix} \omega \\ c \end{pmatrix} \begin{array}{c} = \\ = \\ \leq \\ \hline \geq \\ \vdots \\ \vdots \\ \geq \end{array} \left(\begin{array}{c} 1 \\ e \\ \gamma \\ \hline 0 \\ \vdots \\ \vdots \\ 0 \end{array} \right)$$

with the first three rows of matrix A represent constraints (6),(7),(10) respectively, while the last n rows represent constraint (9).

4 Empirical Result

4.1 Data structure

In this project, we use the same data sets of the American stock market and Chinese stock market as in the subject paper, but with some modifications. The stocks we used are listed in the two tables in figure 1.

Table I. Stock pool of American stock markets (in alphabetical order).

No.	Name	No.	Name	No.	Name
1	AAPL	11	FDX	21	NKE
2	AEP	12	GS	22	NWS
3	APOL	13	HD	23	PG
4	BA	14	JNJ	24	PTR
5	BIDU	15	JPM	25	RIO
6	CBS	16	KO	26	SCEI
7	CHL	17	LFC	27	SNP
8	DUK	18	MCD	28	WMT
9	EL	19	MDLZ	29	XOM
10	F	20	MSFT	30	ZNH

Table II. Stock pool of Chinese stock markets (in numerical order).

No.	Name	No.	Name	No.	Name
1	000002	11	600125	21	600887
2	000063	12	600138	22	600970
3	000951	13	600150	23	601006
4	002024	14	600298	24	601088
5	600011	15	600388	25	601111
6	600019	16	600519	26	601318
7	600028	17	600598	27	601398
8	600030	18	600601	28	601600
9	600085	19	600832	29	601857
10	600118	20	600859	30	601919

Figure 1: The stock pool of American and Chinese stock markets.

For the American stock market, data for CHL, NWS, and SCEI could not be found, while data for APOL is only partially available. So, instead of using data from June 1, 2009 to October 13, 2011 as the subject paper does, we use the data period from March 30, 2011 to October 13, 2011.

For the Chinese stock market, only data for stock 600832 is missing, and all other stocks' data are complete. So, we use the same 600 historical returns from January 13, 2009 to July 1, 2011, as in the subject paper.

The historical returns are calculated using adjusted close prices as follows:

$$\frac{\text{Price}_{\text{today}} - \text{Price}_{\text{yesterday}}}{\text{Price}_{\text{yesterday}}}$$

4.2 Results for the simple MG model

In this section, we study the impact of rising the target level in equation (2) on the optimal portfolio's performance. The experiments are performed on the data sets of American stock market and Chinese stock market. We rise the target return from 0.0007 to 0.001, with step-size 0.0001, and compare the portfolios chosen by MG (labelled as MG) and portfolios chosen by MV (labelled as MV), using the indicators introduced in section 2.2. The performance is tested using an out-of-sample size of 5 trading days (OS-5) and 10 trading days (OS-10).

4.2.1 American stock market

In figure 2 and 3, we compare the characteristics of the optimal portfolios with different target return rates. Note that there are some infinities in the

	0.0007		0.0008		0.0009		0.001	
	MG	MV	MG	MV	MG	MV	MG	MV
Return	0.004	0.0191	0.004	0.0175	0.004	0.0164	0.004	0.0153
Std	0.0176	0.0057	0.0176	0.0058	0.0176	0.0058	0.0177	0.0058
0.8-CVaR	0.0058	0.0025	0.0058	0.0022	0.0058	0.0019	0.0059	0.0017
R/Std	0.2263	3.3166	0.2271	3.0205	0.228	2.8258	0.2288	2.6265
R/CVaR	0.6897	7.6174	0.6887	8.0874	0.6877	8.4329	0.6867	8.866
FT(1,1)	1.1121	Inf	1.0966	Inf	1.0812	Inf	1.0661	Inf
FT(1,3)	0.3803	Inf	0.375	Inf	0.3698	Inf	0.3646	Inf
ZN	6	28	8	28	7	28	6	28
HI	0.9423	0.2971	0.9409	0.2735	0.9395	0.259	0.9382	0.2477

Figure 2: The characteristics of optimal portfolios (OS-5) under simple MG and MV and different target return rates - American stocks

	0.0007		0.0008		0.0009		0.001	
	MG	MV	MG	MV	MG	MV	MG	MV
Return	0.0248	0.0192	0.0249	0.0179	0.025	0.017	0.025	0.0161
Std	0.0292	0.0088	0.0293	0.0088	0.0294	0.0087	0.0295	0.0086
0.8-CVaR	0.0024	0.0018	0.0025	0.0016	0.0025	0.0015	0.0025	0.0014
R/Std	0.8493	2.1742	0.849	2.0426	0.8484	1.9589	0.8475	1.8674
R/CVaR	10.184	10.913	10.14	11.488	10.097	11.668	10.054	11.877
FT(1,1)	3.5468	Inf	3.5126	Inf	3.4787	Inf	3.4453	Inf
FT(1,3)	0.7641	Inf	0.7568	Inf	0.7495	Inf	0.7423	Inf
ZN	6	28	8	28	7	28	6	28
HI	0.9423	0.2971	0.9409	0.2735	0.9395	0.259	0.9382	0.2477

Figure 3: The characteristics of optimal portfolios (OS-10) under simple MG and MV and different target return rates - American stocks

$FT(1,3)$ ratios, because we used a benchmark of $b = 0$ and some portfolios have a positive return every day, leading $E^{1/q}[((X-b)^{-})^{q}] = 0$.

Firstly, let us look at the 4 optimal portfolios selected by MG in the same figure, they have very similar performance. While for the portfolios selected by MV, increasing the target return would decrease the expected returns and reward-to-risk ratios. Same results follow as in the case of OS-10. Thus, we conclude that rising the target return could not improve the portfolio's performance.

We than compare the performance of the same portfolio (in the same column) in case OS-5 and OS-10. We conclude that as we increase the out-of-sample size from 5 to 10, the portfolio performance improves for both model, while the improvement is very significant for portfolios chosen by MG. This leads to the follow two findings: the American stock market is steady, so over trading leads to a poor performance, while MG model is more sensitive to this compared to the MV model.

Finally, we compare between MG and MV. MG portfolios are less diversified, while they greatly outperform MV portfolios in the OS-10 case. This means a low diversification is not necessarily associated with a poor performance, an efficient portfolio that is concentrated and performs well is possible. Note that in the subject paper, MG outperforms MV in both OS-5 and OS-10 cases. This is because MV has a much poorer perform under all situations in the subject paper, compared to the results in this project.

4.2.2 Chinese stock market

We then run the same experiment on the Chinese stock market. Results are listed in figure 4 and 5, and the conclusions are different from what we have for American stocks.

Rising the target returns improves the portfolio performance for both models and in both cases. For MG, the optimal portfolio is obtained at a target return of 0.0009, while for MV, the optimal portfolio is obtained at a target return of 0.001. Also, the portfolios have a worse performance in OS-10 case than in OS-5 case, and this impact is greater for MG models. This is because the Chinese stock market is seriously subject to the national policies and sudden events, and the stock prices fluctuate more often. As a result, an optimal portfolio will not remain optimal for a long time, so investors should trade more often. Another thing is that MV greatly outperforms MG in all situations, and this is because MV in this project has a much better

	0.0007		0.0008		0.0009		0.001	
	MG	MV	MG	MV	MG	MV	MG	MV
Return	-0.0016	0.0138	0.0032	0.018	0.0071	0.02	0.0045	0.0212
Std	0.01	0.0088	0.0079	0.0074	0.0226	0.0064	0.01	0.0058
0.8-CVaR	0.0034	0.0009	0.0014	0.0024	0.0052	0.0034	0.0013	0.0041
R/Std	-0.1628	1.5801	0.4077	2.4317	0.3157	3.1273	0.4485	3.6844
R/CVaR	-0.4827	15.985	2.2777	7.4402	1.3695	5.868	3.4454	5.1983
FT(1,1)	0.7112	Inf	2.6264	Inf	2.0227	Inf	2.8923	Inf
FT(1,3)	0.2432	Inf	0.8982	Inf	0.6917	Inf	0.9892	Inf
ZN	2	29	2	29	2	29	2	29
HI	0.9572	0.2381	0.997	0.2231	0.9916	0.2113	0.9106	0.2041

Figure 4: The characteristics of optimal portfolios (OS-5) under simple MG and MV and different target return rates - Chinese stocks

	0.0007		0.0008		0.0009		0.001	
	MG	MV	MG	MV	MG	MV	MG	MV
Return	0.0007	0.0127	-0.003	0.0172	0.0047	0.0195	0.0013	0.021
Std	0.0091	0.0081	0.0133	0.0072	0.0214	0.0066	0.013	0.0062
0.8-CVaR	0.0028	0.0001	0.0058	0.0016	0.0065	0.0025	0.0044	0.0031
R/Std	0.0758	1.5701	-0.213	2.3871	0.2185	2.9682	0.1009	3.396
R/CVaR	0.2434	86.913	-0.493	10.628	0.7229	7.7958	0.2953	6.7544
FT(1,1)	0.9118	10.816	0.5221	Inf	1.3609	Inf	0.5824	Inf
FT(1,3)	0.1964	2.3303	0.1125	Inf	0.2932	Inf	0.1255	Inf
ZN	2	29	2	29	2	29	2	29
HI	0.9572	0.2381	0.997	0.2231	0.9916	0.2113	0.9106	0.2041

Figure 5: The characteristics of optimal portfolios (OS-10) under simple MG and MV and different target return rates - Chinese stocks

	MG						MV					
	0.2	0.3	0.4	0.5	0.6	1	0.2	0.3	0.4	0.5	0.6	1
Return	0.0267	0.0102	0.0143	0.0181	0.0209	0.0304	0.0223	0.0171	0.0118	0.0063	0.0038	0.0038
Std	0.0134	0.0153	0.019	0.0229	0.0243	0.0308	0.0101	0.0107	0.0116	0.0125	0.0125	0.0125
0.8-CVaR	0.0024	0.0023	0.0021	0.0017	0.0008	0.0013	0.0023	0.0008	0.0008	0.0026	0.0033	0.0033
R/Std	1.9881	0.6661	0.7517	0.7922	0.8585	0.9848	2.2032	1.5944	1.02	0.5081	0.3024	0.305
R/CVaR	11.223	4.3901	6.915	10.754	27.627	24.125	9.5958	21.679	14.954	2.4736	1.1375	1.1497
FT(1,1)	Inf	2.2178	4.5418	9.3799	17.402	Inf	Inf	6.1989	2.2462	1.1596	0.882	0.8846
FT(1,3)	Inf	0.4778	0.9785	2.0208	3.7491	Inf	Inf	1.3355	0.4839	0.2498	0.19	0.1906
ZN	6	5	4	3	3	3	28	28	28	28	28	28
HI	0.1897	0.2752	0.3458	0.4561	0.485	0.897	0.1585	0.1967	0.2619	0.3554	0.4003	0.4002

Figure 6: The characteristics of optimal portfolios (OS-10) under realistic MG and MV and different upper bounds of weights - American stocks

performance than MV in the subject paper.

4.3 Results for the realistic case

In this section, we study the impacts of target return, transaction cost, and bound of weights on the performance of optimal portfolio. Note that all results are computed in the out-of-sample size of 10 trading days, and we assume 0 tax and an initial weight of 0 for each stock.

4.3.1 American stock market

We first compare the optimal portfolios selected by limiting the upper bound of weights to 0.2, 0.3, 0.4, 0.5, 0.6, and unbounded (which is 1). We fix target return $e = 0.0006$, unit transaction cost $k = 0.001$, and total transaction cost $\gamma = 0.005$. Results are shown in figure 6.

We conclude that for MG portfolios, limiting the upper bound of weights generally worsen the performance. The best optimal portfolio is achieved by not bounding weights. While for MV portfolios, the situation is quite the opposite: a high concentration leads to a poor performance. We also conclude that MG generally outperforms MV.

We then investigate the impact of target returns. We leave the upper bound of weights unbounded, fix the unit transaction cost $k = 0.001$, total transaction cost $\gamma = 0.005$ and increase the target return e from 0.0004 to 0.0007. We compare the portfolio performance using the expected returns, which are plotted in figure 7. Note that the subject paper also plots the

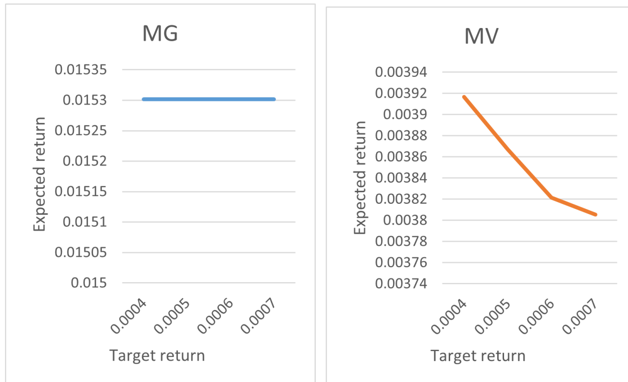

Figure 7: The expected returns of optimal portfolios (OS-10) under MG and MV and different target returns - American stock

$FT(1,3)$ ratios, while we did not because some $FT(1,3)$ ratios are infinity due to the reason explained before.

We conclude that rising target returns will not affect the performance of MG portfolios at all, while it causes a decrease on the expected return of MV portfolios.

Similarly, we investigate the impact of transaction cost. We leave the upper bound of weights unbounded, fix the target return $e = 0.0006$, total transaction cost $\gamma = 0.005$ and increase the unit transaction cost k from 0.0008 to 0.0011. Results are plotted in figure 8. We conclude that for MG portfolios, increasing transaction cost will not affect the portfolio performance, while it causes the MV portfolios' performance to fluctuate.

4.3.2 Chinese stock market

We repeat the same experiments on the Chinese stock market. The impacts on portfolio performance of upper bound, target return, and transaction cost are investigated in figure 9, figure 10 and figure 11, respectively.

From figure 9, we conclude that for MG, the performance of optimal portfolios will first decreases then increases as the upper bound of weights increases. The best optimal portfolio is obtained at an upper bound of 0.2, while the worst is obtained at 0.4. While for MV, high concentration again leads to poor performance. Note also that if we compare this to what we have in figure 5, we find out that the portfolios chosen by the realistic MG greatly outperform the portfolios chosen by the simple MG. Note also that

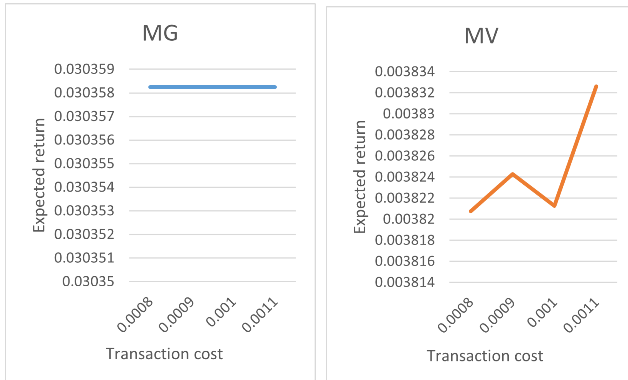

Figure 8: The expected returns of optimal portfolios (OS-10) under MG and MV and different transaction costs - American stock

	MG						MV					
	0.2	0.3	0.4	0.5	0.6	1	0.2	0.3	0.4	0.5	0.6	1
Return	0.0247	0.0145	0.0038	0.005	0.0058	0.0153	0.0245	0.0222	0.0221	0.0221	0.0221	0.0221
Std	0.0071	0.0088	0.0103	0.0122	0.0133	0.0163	0.0062	0.0062	0.0062	0.0062	0.0062	0.0062
0.8-CVaR	0.0035	0.0011	0.0018	0.0023	0.0023	0.0016	0.0041	0.0036	0.0036	0.0036	0.0036	0.0036
R/Std	3.4827	1.655	0.3719	0.4065	0.4395	0.937	3.9337	3.6047	3.5754	3.578	3.5761	3.5787
R/CVaR	7.1563	13.146	2.1145	2.1513	2.5348	9.824	5.9378	6.1512	6.1973	6.1941	6.1967	6.1932
FT(1,1)	Inf	Inf	2.5733	2.42	2.6066	5.5168	Inf	Inf	Inf	Inf	Inf	Inf
FT(1,3)	Inf	Inf	0.5544	0.5214	0.5616	1.1886	Inf	Inf	Inf	Inf	Inf	Inf
ZN	5	4	6	6	4	1	29	29	29	29	29	29
HI	0.2	0.28	0.36	0.5	0.52	1	0.1498	0.1913	0.1961	0.1962	0.1962	0.1963

Figure 9: The characteristics of optimal portfolios (OS-10) under realistic MG and MV and different upper bounds of weights - Chinese stocks

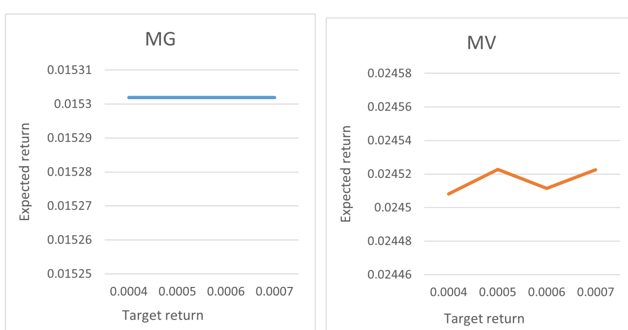

Figure 10: The expected returns of optimal portfolios (OS-10) under MG and MV and different target returns - Chinese stock

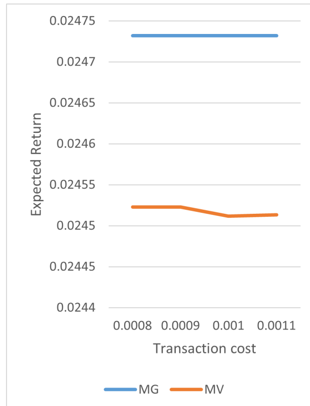

Figure 11: The expected returns of optimal portfolios (OS-10) under MG and MV and different transaction costs - Chinese stock

in the subject paper, the impact of upper bound for both stock markets is the same: the best portfolio is obtained by leaving weights unbounded. And in the subject paper, the performance of realistic MG in the case of OS-10 is as bad as that in the case of OS-5.

From figure 10 and figure 11, we conclude that the target returns and transaction costs do not affect the performance of optimal portfolios chosen by MG, while it causes the MV portfolios' performance to fluctuate. Results are consistent with that for American stock market.

5 Conclusion

In conclusion, although our data sets are not the same as those used in the subject paper, we both achieve similar conclusions. We both conclude that MG selects concentrated well-performed portfolios, and that the realistic MG is very steady: the optimal portfolios' performance is not affected by the level of target return and transaction cost. Yet, due to the differences in the data sets and the optimizing algorithms, our results are numerically different. In general, we obtain a better portfolio performance than the subject paper does, and the improvement is significant in the cases of MV and the cases of realistic MG for Chinese stock market tested in OS-10. A possible source of this improvement is the optimizer Gurobi. The algorithm used in the subject paper was developed 10 years ago and was not widely used (the main package code hasn't been updated since then), while Gurobi is a strong and popular optimizer that solves all kinds of quadratic programmings.

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