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Kaon–nucleon scattering lengths from kaonic deuterium experiments revisited

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ABSTRACT

We analyse the impact of the recent measurement of kaonic hydrogen X rays by the SIDDHARTA collaboration on the allowed ranges for the kaon–deuteron scattering length in the framework of non-relativistic effective field theory. Based on data from $\bar{K}N$ scattering only, we predict the kaon–deuteron scattering length $A_{Kd} = (-1.46 + i1.08)$ fm, with an estimated uncertainty of about 25% in both the real and the imaginary part.

1. Recently, the SIDDHARTA collaboration at LNF-INFN has performed a measurement of the energy level shift (ϵ_{1s}) and width (Γ_{1s}) of the kaonic hydrogen ground-state [1]

$$\epsilon_{1s} = -283 \pm 36\,(\text{stat}) \pm 6\,(\text{syst})\ \text{eV},$$
$$\Gamma_{1s} = 541 \pm 89\,(\text{stat}) \pm 22\,(\text{syst})\ \text{eV}, \tag{1}$$

which allows to extract the fundamental antikaon–proton (K^-p) scattering length based on an improved Deser-type formula developed in [2]. This measurement resolved the long-standing puzzle of the discrepancy between the earlier DEAR [3] and the less accurate KpX experiment at KEK [4]. The DEAR data have been puzzling the community for a long time. As first pointed out in Ref. [2], the energy shift and width of kaonic hydrogen measured by DEAR is incompatible with the predicted values taking the underlying $\bar{K}N$ scattering lengths from scattering data only. This issue was studied and exposed in more detail in a series of papers by various groups, see, e.g., Refs. [5–8]. In addition, based on the framework of non-relativistic effective field theory (for a recent comprehensive review with many applications to hadronic atoms, see Ref. [9]), it was shown in [10] that with the DEAR central values for the kaonic hydrogen ground-state energy and width, a solution for the isoscalar (a_0) and the isovector (a_1) kaon–nucleon scattering lengths exists only in a very restricted domain of input values for the kaon–deuteron scattering length. Consequently, it was concluded that the anticipated measurement of kaonic deuterium by the SIDDHARTA collaboration [11,12] would pose stringent constraints on the kaon–deuteron interaction at low energies. Presently, an upgrade of the detector at LNF-INFN, called SIDDHARTA2, is being considered to perform measurements of X rays in kaonic deuterium in 2012 [13]. It is therefore of high interest to reanalyse the predictions for kaonic deuterium in the light of the new kaonic hydrogen measurements. This is exactly what will be done in this Letter.

2. To reanalyse the predictions for kaonic deuterium, the elementary antikaon–nucleon ($\bar{K}N$) scattering lengths $a_{\bar{K}N}$ have to be related to the K^-d scattering length A_{Kd}. As the $a_{\bar{K}N}$ are comparable in size to the average distance of the nucleons in the deuteron, the multi-scattering series is non-perturbative and needs to be resummed. The resummed fixed-centre-approximation (FCA) to the Kd problem has been formulated in Ref. [14] (see also Ref. [10]), although a very similar formula has been derived in potential scattering theory long time ago [15]. A three-body calculation beyond the FCA has been performed, e.g., in Ref. [16] where the loop momentum integration is retained. This allows to take account of the strong energy dependence of the elementary scattering processes near threshold. Recoil corrections beyond the FCA can in principle be included in a controlled way in a systematic expansion in integer and half-integer powers of the parameter $\xi = M_K/m_N \simeq 1/2$ with M_K (m_N) the kaon (nucleon) mass. For double scattering this has been shown in Ref. [17], see also Ref. [18]. A consistent

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inclusion of these corrections in the multiple scattering series is still an open issue. Here, we will rely on the resummed FCA approximation without recoil corrections (except for trivial kinematical factors) of Refs. [10,14],

$$\hat{a}_{Kd}(r) = \frac{\tilde{a}_p + \tilde{a}_n + (2\tilde{a}_p\tilde{a}_n - b_x^2)/r - 2b_x^2\tilde{a}_n/r^2}{1 - \tilde{a}_p\tilde{a}_n/r^2 + b_x^2\tilde{a}_n/r^3} + \delta\hat{a}_{Kd}, \tag{2}$$

with $b_x^2 = \tilde{a}_x^2/(1+\tilde{a}_u/r)$. The K^-d scattering length A_{Kd} is obtained from $\hat{a}_{Kd}$ via a folding with the deuteron wave function and the $\tilde{a}$ on the right-hand side are related to the elementary scattering lengths $a_{\bar{K}N}$ [10,14]. Here, we employ for simplicity the CD-Bonn potential for the S- and D-wave parts of the wave function. In Eq. (2), the indices p, n, x, and u refer to the processes $K^-p \to K^-p$, $K^-n \to K^-n$, $K^-p \to \bar{K}^0 n$, and $\bar{K}^0 n \to \bar{K}^0 n$, in order. The quantity $\delta\hat{a}_{Kd}$ indicates a genuine three-body piece which is not determined but argued to be small in Ref. [10], and thus $\delta\hat{a}_{Kd} = 0$ in the present study. This issue deserves further investigations in the future.

To determine the influence of the isoscalar and isovector scattering lengths, a_0 and a_1, on the Kd scattering length, one has to relate them to the elementary processes of Eq. (2). This has been achieved in Ref. [2] by means of effective field theory up to next-to-leading order in isospin breaking. There, it has been shown that the large leading order isospin correction is provided by the unitary cusp. By resumming neutral kaon loops, the elementary scattering lengths in the particle basis can be related to a_0 and a_1 as shown in detail in Ref. [2]. The result for a_p as a function of a_0, a_1 can be rewritten as

$$a_p = \frac{(a_0 + a_1)/2 + q_0 a_0 a_1}{1 + q_0(a_0 + a_1)/2}. \tag{3}$$

This condition, together with the requirement from unitarity, $\operatorname{Im} a_I \geqslant 0$, $i = 0, 1$, leads to a restriction for the possible values of a_0, a_1 [10] in form of circles in the complex a_I planes, i.e. values of a_I inside these circles are excluded. The circle is given by

$$a_I^{\lim} = C + R\exp(i\phi), \quad \phi, R \in \mathbb{R},$$
$$\frac{1}{R} = -4q_0 \operatorname{Im} \frac{1}{1 + a_p q_0}, \quad C = -\frac{1}{q_0} + iR, \tag{4}$$

for both $I = 0$ and $I = 1$. In Eqs. (3) and (4), the parameter q_0 determines the strength of the cusp, $q_0 = \sqrt{2\mu_0\Delta}$, where μ_0 is the reduced mass of the $\bar{K}^0$ and the n, and $\Delta = m_n + M_{\bar{K}^0} - m_p - M_K$ as derived in Ref. [2].

Within this framework, we can now study the K^-d scattering length A_{Kd}, using the constraints provided by a_p. The ground-state energy shift and width of kaonic hydrogen can be related to the K^-p scattering length a_p at next-to-leading order in isospin breaking as derived in Ref. [2],

$$\epsilon_{1s} - \frac{i}{2}\Gamma_{1s} = -2\alpha^3\mu_c^2 a_p\big(1 - 2\alpha\mu_c(\ln\alpha - 1)a_p\big) \tag{5}$$

where μ_c is the reduced mass of the K^- and the p. In Table 1 values for a_p extracted from different experiments are shown. Scattering lengths have also been extracted in Ref. [8] considering the meson–baryon interaction up to next-to-leading order in a coupled-channel chiral SU(3) unitary approach. In a combined fit to K^-p induced reactions, different interaction kernels within the unitarisation scheme are considered and, using statistical criteria, non-linear errors on the values of a_0 and a_1 are provided (confidence regions in the complex a_I planes). Here, we use the extracted a_p scattering length of the full approach of Ref. [8], $a_p = (-1.05 + i0.75)$ fm. This value can be combined with the scattering length extracted from the latest experiment on kaonic atoms, SIDDHARTA [1]. The average is shown in the last line of Table 1, and we consider this number as the most reliable available value of a_p. In the following section, we will determine the restrictions on a_0, a_1 and also A_{Kd} from these measurements.

Table 1
Values of the K^-p scattering length a_p extracted from different experiments/analyses by using Eq. (5).

a_p [fm]	Experiment
$-0.82 + i0.64$	KpX [4]
$-0.48 + i0.35$	DEAR [3]
$-0.66 + i0.81$	SIDDHARTA [1]
$-0.85 + i0.78$	Average SIDDHARTA [1] & scattering [8]

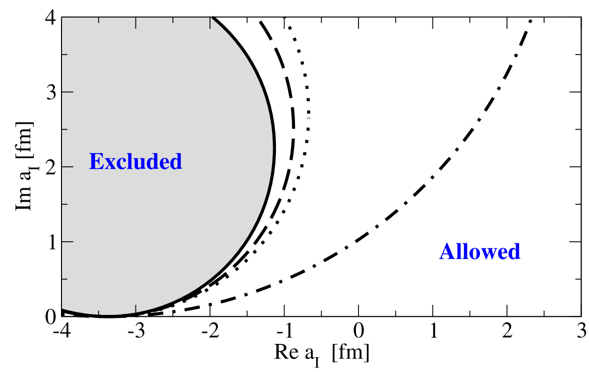

Fig. 1. Solid line: Restrictions on the values of the scattering lengths a_0 and a_1 set by the SIDDHARTA data [1] combined with the K^-p scattering length obtained from scattering data [8]. For comparison, we also display the restrictions from SIDDHARTA only (dashed line), DEAR [3] (dot-dashed line) and KpX [4] (dotted line).

3. In Fig. 1, we show the restrictions on the values of the isoscalar and isovector scattering lengths corresponding to the condition defined in Eq. (4). We only show one quadrant of the whole circle. The new determination of the K^-p scattering length is less restrictive than the one based on the DEAR results and not very different from the one based on the older KpX measurement. This is another way of demonstrating that now the determinations of the K^-p scattering length from kaonic hydrogen and from scattering data are consistent, see also Fig. 1 of Ref. [10].

In Fig. 2, we analyse the values the complex-valued kaon–deuteron scattering length can take where solutions for a_0 and a_1 exist at all. For that, we have scanned the region -2 fm $<$ $\operatorname{Re} A_{Kd} < 0$ and $0 < \operatorname{Im} A_{Kd} < 2$ fm and tried to find solutions, using the input data collected in Table 1. Using our best determination of a_p or directly the one extracted from SIDDHARTA, the allowed region is again increased as compared to the one based on the DEAR data. For the central values extracted from scattering data, $a_0 = (-1.64 + i0.75)$ fm and $a_1 = (-0.06 + i0.57)$ fm [8], we predict the kaon–deuteron scattering length as:

$$A_{Kd} = (-1.46 + i1.08)\ \text{fm}, \tag{6}$$

which is also displayed as a red circle in Fig. 2, together with other predictions from the literature. It is interesting to observe that our prediction agrees with the one of Ref. [19] which is based on a ground-breaking Faddeev calculation. We note in this context the (somewhat model-dependent) limit on the K^-d scattering length extracted from the $\bar{K}^0 d$ mass spectrum obtained from the reaction $pp \to d\bar{K}^0 K^+$ measured at the Cooler Synchrotron COSY at Jülich, namely $\operatorname{Im} A_{Kd} \leqslant 1.3$ fm and $|\operatorname{Re} A_{Kd}| \leqslant 1.3$ fm [23]. Our prediction for the real part in Eq. (6) is not in contradiction with this bound, because the uncertainties in the determination of a_0 and a_1

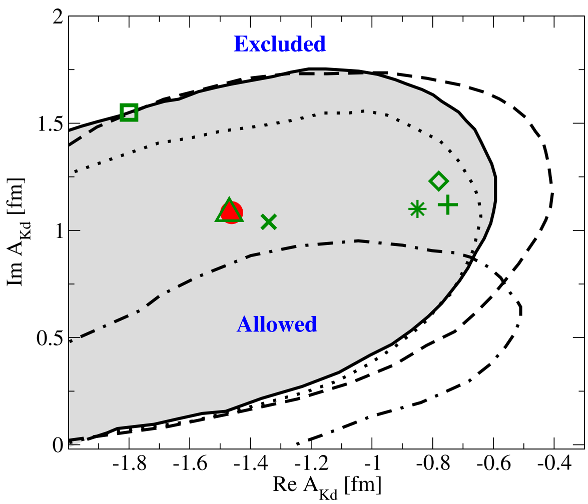

Fig. 2. The region in the $(\mathrm{Re}\, A_{Kd}, \mathrm{Im}\, A_{Kd})$-plane where solutions for a_0 and a_1 exist. The grey area bounded by the solid line is our central prediction using the average of the K^-p scattering length from SIDDHARTA [1] and Ref. [8]. The dashed, dot-dashed and dotted lines are generated using the experimental input from SIDDHARTA, DEAR [3] and KpX [4], respectively. The filled (red) circle is the prediction for A_{Kd} based on the $\bar{K}N$ S-wave scattering lengths from Ref. [8], cf. Eq. (6). Older predictions for A_{Kd} are from: [19] (triangle), [20] (cross), [21] (star: Faddeev equations, plus sign: FCA), [16] (square), [22] (diamond). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

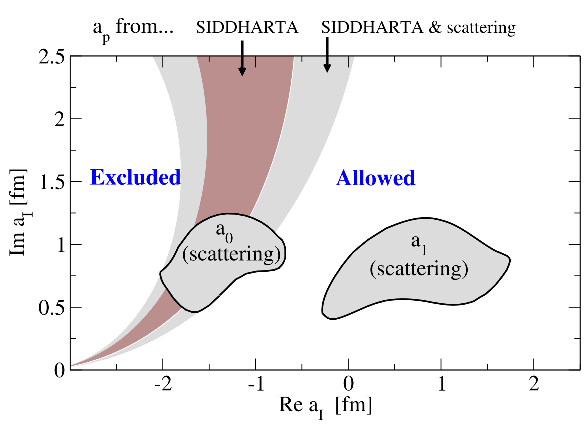

Fig. 3. Error estimates. Shaded areas with borders: allowed values from scattering data, result from Ref. [8]. Dark shaded area: uncertainty of the border between allowed and excluded values, using SIDDHARTA uncertainties [1]. Light shaded area: same, using combined uncertainties from SIDDHARTA and scattering [8] data.

given in Ref. [8] lead to an approximate uncertainty of ± 0.35 fm in $\mathrm{Re}\, A_{Kd}$ (and a similar uncertainty in $\mathrm{Im}\, A_{Kd}$).

So far, we have worked with the central values of the input quantities. To estimate the error of our predictions, the statistical and systematic errors of the SIDDHARTA measurement have been added in quadrature, and the error of the extracted a_p has been determined to be approximately $\Delta\, \mathrm{Re}\, a_p = 0.10$ fm, $\Delta\, \mathrm{Im}\, a_p = 0.13$ fm. These values lead to an uncertainty of the boundary between allowed and excluded values for a_0 and a_1, as indicated with the dark shaded area in Fig. 3. The uncertainties of a_p determined from scattering data [8] are estimated to be $\Delta\, \mathrm{Re}\, a_p \sim \Delta\, \mathrm{Im}\, a_p \sim 0.2$ fm. If the errors from SIDDHARTA and scattering are added in quadrature, one obtains the light shaded areas in Fig. 3.

Note that scattering provides restrictions on (a_0, a_1) that are not fully encoded in the single value of a_p from Ref. [8]; indeed, as indicated in Fig. 3, scattering provides the irregularly shaped areas

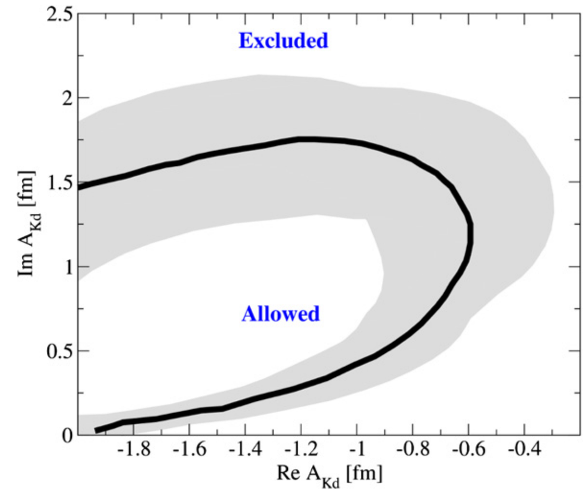

Fig. 4. Uncertainty of the boundary of allowed values for A_{Kd}. The central value (solid line) corresponds to the average of a_p from scattering data and the SIDDHARTA value; uncertainty (shaded area) from the combined errors of these two sources.

for a_0 and a_1 which pin these values down much more precisely. In any case, the figure clearly shows that there is no conflict between scattering data and scattering lengths.

The combined uncertainty of a_p from scattering and kaonic atom data translates into the uncertainty of the boundary of allowed values for the kaon–deuteron scattering length. This is indicated with the shaded area in Fig. 4. Additional uncertainty stems from the static approach to Kd scattering itself, i.e. Eq. (2). Recoil corrections can be sizeable and should be included in future calculations. As for the genuine three-body term $\delta\hat{a}_{Kd}$ in Eq. (2), note that its imaginary part is related to the total two-nucleon absorption rate of the K^- which amounts to $(1.22 \pm 0.09)\%$ [24]; assuming $\mathrm{Re}\, \delta\hat{a}_{Kd}$ to be of similar size as $\mathrm{Im}\, \delta\hat{a}_{Kd}$, the three-body force contributes only with a few percent to A_{Kd}.

4. In summary, we have reanalysed the predictions for the kaon–deuteron scattering length in view of the new kaonic hydrogen experiment from SIDDHARTA. Based on consistent solutions for input values of the K^-p scattering length, we have explored the allowed ranges for the isoscalar and isovector kaon–nucleon scattering lengths and explored the range of the complex-valued kaon–deuteron scattering length that is consistent with these values. In particular, the new SIDDHARTA measurement is shown to resolve inconsistencies for a_0, a_1, and A_{Kd} as they arose from the DEAR data. A precise measurement of the K^-d scattering length from kaonic deuterium would therefore serve as a stringent test of our understanding of the chiral QCD dynamics and is urgently called for.

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