ECO 4400 **Game Theory**
Assignment 1
Due Date: 01/24/22

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Instruction

Show your work and how you derive an answer step by step. Feel free to discuss your work with your classmates but do not copy solutions. Each student has to submit their own solutions in Canvas. Your scanned submission should be high-quality and professionally presented. Please remember that late submissions will not be accepted. The total score is 100 points. You will get 30 points just by submitting your solutions on time.

1 Pure Strategy Nash Equilibrium (10 points)

Find all (pure strategy) equilibria for the following games. If you use the IESDS, give the order in which the eliminations occur. If you use the best response analysis, you do not need to give the order, but mark the circles clearly.

(a)	C1	C2
R1	(0,0)	(0,0)
R2	(0,0)	(1,0)

(b)	C1	C2
R1	(0,0)	(1,0)
R2	(0,1)	(1,1)

(c)	C1	C2
R1	(0,1)	(1,0)
R2	(1,0)	(0,1)

(d)	C1	C2	C3
R1	(3,6)	(9,6)	(0,7)
R2	(5,9)	(10,8)	(1,6)
R3	(6,8)	(2,9)	(7,9)

(e)	C1	C2	C3
R1	(3,10)	(3,9)	(2,5)
R2	(6,4)	(10,4)	(1,10)
R3	(5,9)	(3,5)	(7,7)

2 4×4 (10 points)

Consider the following game.

		Hyesoo			
		w	x	y	z
Bomi	a	$(15, 15)$	$(20, 6)$	$(15, 8)$	$(12, 9)$
	b	$(22, 6)$	$(18, 10)$	$(16, 12)$	$(17, 12)$
	c	$(8, 3)$	$(19, 25)$	$(6, 3)$	$(15, 11)$
	d	$(10, 14)$	$(21, 15)$	$(11, 18)$	$(20, 5)$

(a) Find all dominant strategies and dominated strategies in the original game.

(b) Use iterated elimination of dominated strategies to reduce the game as much as possible. Give the order in which the eliminations occur and give the reduced form of the game.

(c) Is this game dominance solvable? Explain why or why not.

(d) State the (pure strategy) Nash equilibrium (or equilibria) of this game.

3 Prisoner's Dilemma (10 points)

Consider the prisoner's dilemma of Bomi and Hyesoo. After being arrested several times, each has realized they may hire a lawyer.

		Hyesoo		
		Confess	Don't Confess	Hire Lawyer
Bomi	Confess	$(-10, -10)$	$(0, -15)$	$(-10, -1-x)$
	Don't Confess	$(-15, 0)$	$(-1, -1)$	$(-1, -x)$
	Hire Lawyer	$(-1-x, -10)$	$(-x, -1)$	$(-x, -x)$

x represents years of income needed to hire an effective attorney. (Do not assume that x is an integer. It can be any real number.)

(a) For what range of values for x (if any), is (Hire,Hire) a (pure strategy) NE? (Hint: use the definition of N.E.)

Answer: x should be **(greater than / greater than or equal to / less than / less than or equal to)** _____.

(b) For what range of values of x (if any), is (Confess,Confess) not an equilibrium?

Answer: x should be **(greater than / greater than or equal to / less than / less than or equal to)** _____.

4 Group Project (10 points)

Consider 2 students working on a group project, Wonki and Jay. They decide simultaneously whether to exert high effort or low effort. Both students know each will receive a 100 if they both exert high effort, an 85 if one student exerts high effort and another exerts low effort, and a 55 if both students exert low effort. However, both students incur a cost of c for exerting high effort. (Do not assume that c is an integer.) Both students' payoffs are their own grades, minus any cost from high effort.

(a) Write down this game in strategic form.

(b) Suppose (high effort, high effort) is a (pure strategy) Nash equilibrium. What range of values for c is this possible for?

(c) Suppose (low effort, low effort) is a (pure strategy) NE. What range of values for c is this possible for?

(d) Is it possible that both of the above (pure strategy) NE exist simultaneously? If so, for what range of values for c?

(e) Suppose both (high effort, low effort) and (low effort, high effort) are (pure strategy) NE. What range of values for c is this possible for?

5 Oligopoly (10 points)

Ferrari and Lamborghini are in an oligopoly where the price is determined by the total number of high end luxury cars produced by both. Both firms decide whether to produce 300 or 400 cars simultaneously. Their payoff functions are as follows.

$$\text{Ferrari Profit} = (1200 - (F + L)) \cdot F - 100000$$
$$\text{Lamborghini} = (1200 - (F + L)) \cdot L - 100000$$

where F denotes the number of cars Ferrari produces, and L denotes that of Lamborghini.

(a) Construct the strategic form game.

(b) Identify all dominated strategies.

(c) Identify all (pure strategy) NE.

6 Congressional District Election (10 points)

Consider a liberal congressional district with 3 candidates, Alan, Anita, and Michael. They have a chance of running in the Left, Moderate, or Conservative "Lanes", with their payoffs represented by their probability of winning. Running to the left increases a candidate's chance of winning, but sharing a lane with another candidate decreases the chance of winning. As per usual the row player's payoffs are first, column player's are second, and the panel player's(Michael's) payoffs are third.

		Anita		
		Left	Moderate	Conservative
Alan	Left	$(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$	$(0, 1, 0)$	$(\frac{1}{4}, \frac{1}{4}, \frac{1}{2})$
	Moderate	$(1, 0, 0)$	$(0, 0, 1)$	$(\frac{1}{3}, 0, \frac{2}{3})$
	Conservative	$(\frac{1}{2}, \frac{1}{4}, \frac{1}{4})$	$(0, \frac{1}{3}, \frac{2}{3})$	$(0, 0, 1)$

Table 1: Michael Runs Left

		Anita		
		Left	Moderate	Conservative
Alan	Left	$(0, 0, 1)$	$(1, 0, 0)$	$(\frac{2}{3}, 0, \frac{1}{3})$
	Moderate	$(0, 1, 0)$	$(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$	$(\frac{1}{4}, \frac{1}{2}, \frac{1}{4})$
	Conservative	$(0, \frac{2}{3}, \frac{1}{3})$	$(\frac{1}{2}, \frac{1}{4}, \frac{1}{4})$	$(0, 0, 1)$

Table 2: Michael Runs Moderate

		Anita		
		Left	Moderate	Conservative
Alan	Left	$(\frac{1}{4}, \frac{1}{4}, \frac{1}{2})$	$(\frac{2}{3}, \frac{1}{3}, 0)$	$(1, 0, 0)$
	Moderate	$(\frac{1}{3}, \frac{2}{3}, 0)$	$(\frac{1}{4}, \frac{1}{4}, \frac{1}{2})$	$(1, 0, 0)$
	Conservative	$(0, 1, 0)$	$(0, 1, 0)$	$(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$

Table 3: Michael Runs Conservative

(a) Find all dominant strategies and dominated strategies. (in the original game.)

(b) State the (pure strategy) Nash equilibrium (or equilibria) of this game. (Hint: there are 11 NE.)

(c) Suppose that a candidate will win the election if he/she has the highest winning probability among the candidates. For each equilibrium, state the lane (strategy) the winning candidate runs to.

(d) How do you think the practice of nominating candidates in primaries and the relative weakness of 3rd parties in US politics might be related to this zero-sum (constant-sum) coordination game?

7 Strategic Voting (10 points)

A company has three shareholders. Ankita controls 25% of the shares, Brandon controls 35%, and Cade controls 40%. The company has offers from two other companies, denoted A and B, to purchase it. The company also has a third option, which is to decline both offers. Ankita ranks the three choices, from the most to least preferred, as follows: accept A's offer, accept B's offer, and accept neither offer (which we will denote option C). Brandon's ranking is B, then C, then A; and Cade's ranking is C, then B, then A. The rankings are summarized in the following.

	Shareholder		
	Ankita	Brandon	Cade
1st choice	A	B	C
2nd choice	B	C	B
3rd choice	C	A	A

Assume that a shareholder gets a payoff of 2 if their most preferred choice is implemented, a payoff of 1 for their second choice, and a payoff of 0 for their third choice. The three shareholders cast their votes simultaneously. There are 100 votes, allocated according to share ownership, so Ankita has 25 votes, Brandon has 35 votes, and Cade has 40 votes. Shareholders are required to allocate their votes as a bloc. For example, Ankita has to cast all of her 25 votes for A, B, or C; she cannot divvy them up among the projects. The strategy set for a player is then composed of A, B, and C. Plurality voting applies, which means that the alternative with the most votes is implemented.

(a) Suppose that Brandon casts his 35 votes for B, and Cade casts his 40 votes for C. What is the best response for Ankita? Would she cast her vote for her first choice?

(b) Fill out the payoffs in the matrix in the next page.

(c) Consider Brandon's strategy A. Is A strictly or weakly dominated for him?

(d) Consider Cade's strategy A. Is A strictly or weakly dominated for him?

(e) Consider a strategy combination: (A, A, A), i.e., everyone votes for A. Given the other players' choices, is it optimal for Brandon to vote for A, which is his least preferred alternative? Does Cade has an incentive to switch his choice given the other players' choices?

(f) State all the (pure strategy) Nash equilibria of this game. (Hint: there are 5 NE.)

		Brandon		
		A	B	C
Ankita	A	(, ,)	(, ,)	(, ,)
	B	(, ,)	(, ,)	(, ,)
	C	(, ,)	(, ,)	(, ,)

Table 4: Cade votes for A

		Brandon		
		A	B	C
Ankita	A	(, ,)	(, ,)	(, ,)
	B	(, ,)	(, ,)	(, ,)
	C	(, ,)	(, ,)	(, ,)

Table 5: Cade votes for B

		Brandon		
		A	B	C
Ankita	A	(, ,)	(, ,)	(, ,)
	B	(, ,)	(, ,)	(, ,)
	C	(, ,)	(, ,)	(, ,)

Table 6: Cade votes for C