ECO 4400 **Game Theory**
Assignment 4
Due Date: 2/23/22

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Instruction

Show your work and how you derive an answer step by step. Feel free to discuss your work with your classmates but do not copy solutions. Each student has to submit their own solutions in Canvas. Your scanned submission should be high-quality and professionally presented. Please remember that late submissions will not be accepted. The total score is 100 points. You will get 40 points just by submitting your solutions on time.

1 Equilibrium in Mixed Strategies (15 points)

Find all equilibria (including mixed strategies) for the following games. Each equilibrium should look: $((p, 1-p), (q, 1-q))$.

(a)	C1	C2
R1	$(3,3)$	$(1,1)$
R2	$(0,1)$	$(2,2)$

(b)	C1	C2
R1	$(3,1)$	$(1,3)$
R2	$(0,2)$	$(2,1)$

(c)	C1	C2
R1	$(1,1)$	$(3,3)$
R2	$(0,2)$	$(2,1)$

2 Florida Gators: the Kyle-to-Kyle (15 points)

The Florida Gators having finished 8-4 were ranked No. 7 in the 2020 final ranking. Let's recall how they played in the 2020 season. (Not 2021. Forget about the 2021 season.) Before the season, we did not know exactly how strong our offense was. (also how weak our defense was...) The following payoff matrix represents the pre-season probabilities of the Florida gators' success on third down with one yard to go.

		Opponent (Defense)	
		Run	Pass
Gators	Run	$(40, 60)$	$(70, 30)$
	Pass	$(60, 40)$	$(20, 80)$

Before season opening

(a) Find all Nash equilibria in the preseason.

In the season opening game, the Kyle-to-Kyle connection has been revealed, and it led Gators to the big season-opening win against Rebels. After then, it started to be considered as the college football's most potent passing combination. This K2K changed the payoff matrix in the following way.

		Opponent	
		Run	Pass
Gators	Run	$(40, 60)$	$(70, 30)$
	Pass	$(80, 20)$	$(20, 80)$

After season opening

(b) Since there is no game theorist in South Carolina Gamecocks, they plan to use the preseason equilibrium q-mix. On the other hand, Florida Gators in the more prestigious school knowing that sports performance analysts are important, has hired a game theorist, you. You now know the modified payoff matrix exactly. How would you advise the offensive coach and the quarterback? Derive the in-season best response to the Gamecocks' preseason equilibrium q-mix.

(c) After the consecutive big wins of Florida Gators, Texas A&M Aggies has hired a game theorist ahead of their game with Gators. Find all Nash equilibria in the in-season game.

(d) Compare the mixed-strategy equilibria in the preseason and the in-season. Does the K2K change Gators' equilibrium mixed strategy? Does it change Opponent's mix? Should Gators use passing more often? Give some intuition.

(e) What are the Gators' expected payoffs in NE, in the preseason and the in-season, respectively? How does the K2K affect Gators?

3 SEC Championship (15 points)

After their tremendous season in 2020, the Florida Gators has finally advanced to the SEC championship game. As in the previous problem, the following payoff matrix represents the in-season probabilities of the Florida gators' success.

		Opponent	
		Run	Pass
Gators	Run	$(40, 60)$	$(70, 30)$
	Pass	$(80, 20)$	$(20, 80)$

Regular Season Game

Before the game, you, the Gator sports performance analyst, are in the final meeting with the offensive coach. Since this game is so important...SEC championship!, the values for each success and fail are much greater. (say, 100 times.)

		Crimson Tide	
		Run	Pass
Gators	Run	$(4000, 6000)$	$(7000, 3000)$
	Pass	$(8000, 2000)$	$(2000, 8000)$

SEC Championship

(a) Find all Nash equilibria in the SEC championship game.

(b) The offensive coach wants to discuss their p-mix with you. The risky play (Pass) is more risky in this big game. Specifically, the riskier play (Pass) could give you only 2000, but the safer play (Run) secures at least 4000. The coach looks worried about playing risky in this big game. How would you advise the coach? Give some intuition.

4 Gator Nation and Covid-19 (15 points)

Six UF graduates, Andrew, Brandon, Caleb, David, Ethan, Joseph, who took Game theory in Spring 2022 are going to the Florida gators' national championship game together. They all are so into economics that they have become Homo economicus, meaning they only care about their own utility and never care about others. Also, they are risk-neutral. [1] Since they all received As from the course, they know well what to do in a game situation. They are excited because it is the first game after their graduation, and a vaccine for COVID-19 is developed. However, the virus is still around. The value for the football game to a graduate who does not contract the virus is 6. The value for the game to a graduate who contracts the virus is 0.

The vaccine costs different amounts for different graduates because they have now different health plans, and their values of times are different. The vaccine costs 1 for Andrew, it costs 2 for Brandon, 3 for Caleb, 4 for David, 5 for Ethan, and 6 for Joseph.

If a graduate gets vaccinated, they will not contract the virus. But, if they are not vaccinated, then their probability of contracting the virus depends on the total number in the group who are not vaccinated. If Caleb is the only person not to get vaccinated, then the probability that he contracts the virus is $\frac{1}{6}$. If there is one other person who is not vaccinated (i.e., two in all including him), then the probability that he contracts the virus is $\frac{2}{6}$. If there are two other people who are not vaccinated (i.e., three including him) then the probability that he contracts the virus is $\frac{3}{6}$, and so on.

For example, if only Caleb (3) and Ethan (5) get vaccinated, then Caleb (3)'s expected payoff is $6-3$ where 3 is the cost of the vaccination. Ethan (5)'s expected payoff in this case is $6-5$. Joseph (6)'s expected payoff in this case is $\frac{2}{6}\times 6+\frac{4}{6}\times 0$ where the fraction $\frac{4}{6}$ is the probability that he contracts the Covid-19.

Before the game day, they do not communicate each other regarding getting vaccinations. In other words, the graduates simultaneously decide whether or not to get vaccinated.

(a) Explain concisely whether or not it is a NE for Andrew, Brandon, Caleb, and David to get vaccinated and Ethan and Joseph not to get vaccinated.

(b) Explain concisely whether or not it is a NE for Andrew, Brandon and Caleb to get vaccinated and David, Ethan and Joseph not to get vaccinated.

(c) Is it possible to have an equilibrium with only one of them getting vaccinated? How about two?

(d) To find 'an' equilibrium, eliminate weakly dominated strategies if any.[2] Keep eliminating weakly dominated strategies and find a NE in this game. Explain your answer carefully.

[1] meaning you can calculate their expected payoffs as they are. Yeah, "normal" people are risk averse. However, they are much more risk loving than normal people so that they are planning to go to the stadium where a stadium crowd is around without masks.

[2] To find all NE, we shouldn't eliminate weakly dominated strategies. But this game is too complicated, so let's just try it to find an equilibrium.