ECO 4400 | **Game Theory**
Assignment 6 | Instructor: Jay Lee
Due Date: 3/28/22 | jeongwoo@ufl.edu

Instruction

Show your work and how you derive an answer step by step. Feel free to discuss your work with your classmates but do not copy solutions. Each student has to submit their own solutions in Canvas. Your scanned submission should be high-quality and professionally presented. Please remember that late submissions will not be accepted. The total score is 100 points. You will get 40 points just by submitting your solutions on time.

1 Strategy in Repeated Games (10 points)

Consider a repeated game with two players. Each player has two options, *Cooperate* and *Defect*.

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	$(2, 2)$	$(0, 3)$
	Defect	$(3, 0)$	$(1, 1)$

(a) Suppose that this game is played twice. List at least 10 pure strategies player 1 has. You can use abbreviations. For example, "C-CDCD" can be a shortened form of "*Play 'Cooperate' in the first stage. Then play 'Cooperate' in the second stage if (C,C) has been played in the first stage, 'Defect' if (C,D), 'Cooperate' if (D,C), and 'Defect' if (D,D)*". (Hint: A strategy is a complete plan of action. A player's strategy is a complete set of instructions that tell the player what actions to pick at every conceivable situation in the game.)

(b) Suppose that this game is played twice. How many pure strategies does each player have? If you wanted to draw a payoff matrix to find a NE in this game, how big would it be? (Do not actually attempt to draw the payoff matrix... it is too big. Also, this problem has nothing to do with equilibria. Do not find any equilibrium.)

(c) Suppose that this game is played THREE times. How many strategies does each player have? (Hint: for each scenario in the previous stage, 4 possible scenarios are followed in the next stage. The answer is greater than 1 million.)

(d) Briefly explain why you think we are focusing only on contingent strategies and what the benefit of the one-shot deviation principle is?

2 Finitely Repeated Game (15 points)

Consider a variant of the extra point game we studied in class.

		Bomi		
		Left	Center	Right
Jay	Top	$(1, 1)$	$(5, 0)$	$(0, 0)$
	Middle	$(0, 5)$	$(4, 4)$	$(0, 0)$
	Bottom	$(0, 0)$	$(0, 0)$	$(3, 3)$

(a) Find all the pure strategy NE of this game.

(b) Suppose that this game is played twice. Explain why the following strategies constitute a pure strategy SPNE in which $((Middle, Center), (Bottom, Right))$ is an outcome.

Jay

- Period 1: Play Middle.
- Period 2:
  - Play Bottom if (Middle, Center) has been played in period 1.
  - Play Top otherwise.

Bomi

- Period 1: Play Center.
- Period 2:
  - Play Right if (Middle, Center) has been played in period 1.
  - Play Left otherwise.

(c) Suppose that this game is played twice. Can $((Middle, Left), (Bottom, Right))$ be an outcome of a SPNE in pure strategies? If so, construct a pure strategy SPNE in which (Middle, Left) is played in the first stage and (Bottom, Right) is played in the second stage. If not, why not? (Hint: Make sure you have specified actions for all information sets. If an equilibrium exists, it looks similar to the above one. You can use each of the multiple NE as a reward or a punishment.)

(d) Suppose that this game is played twice. Can $((Middle, Center), (Middle, Center))$ be an outcome of a SPNE in pure strategies? If so, construct a pure strategy SPNE. If not, why not?

(e) Suppose that this game is played twice. Can $((Middle, Center), (Top, Left))$ be an outcome of a SPNE in pure strategies? If so, construct a pure strategy SPNE. If not, why not?

(f) Suppose that this game is played twice. Can $((Bottom, Left), (Bottom, Right))$ be an outcome of a SPNE in pure strategies? If so, construct a pure strategy SPNE. If not, why not?

(g) Suppose that this game is played THREE times.

Can $((Bottom, Left), (Middle, Center), (Bottom, Right))$ be an outcome of a SPNE in pure strategies? If so, construct a pure strategy SPNE. If not, why not?

3 Finitely Repeated Game (10 points)

Consider a variant of the extra point game we studied in class.

		Bomi			
		a	b	c	d
Jay	A	(3, 1)	(0, 0)	(0, 0)	(5, 0)
	B	(0, 0)	(1, 3)	(0, 0)	(0, 0)
	C	(0, 0)	(0, 0)	(2, 2)	(0, 0)
	D	(0, 0)	(0, 5)	(0, 0)	(4, 4)

(a) Find all the pure strategy NE of this game.

(b) Suppose that this game is played twice. Construct a pure strategy SPNE in which (D, d) is played in the first stage. (Hint: you can use each of the multiple NE as a reward or a punishment. Be careful. The payoffs are not exactly symmetric between the two players. First, build the equilibrium strategies for the second period. Make sure you have specified actions for all information sets. Then, think about what each player would choose in the first stage given your second-stage strategy combination.)

(c) In the above equilibrium, what are the values of "temptation to defect today", "reward tomorrow", and "punishment tomorrow", respectively?

(d) What if the two players can renegotiate after the first stage before the second stage? Do you think your equilibrium has the renegotiation issue? Why?

(e) If the payoffs for (C, c) changed to (1.9,1.9), would the above SPNE still be an equilibrium? Explain.

4 Pay for Play in the Prisoners' Dilemma (10 points)

James Andreoni and Hal Varian (1999)[1] introduce a mechanism in which each player can offer to pay the other player to cooperate so that they can achieve the Pareto efficient outcome in the prisoner's dilemma.

A neutral TV game show host runs the game. There are two players, Wonki and Jay, who are advanced to the final episode. The host gives two cards to each: 2 and 7 to Wonki and 4 and 8 to Jay. Who gets what cards is common knowledge. Then, playing simultaneously and independently, each player is asked to hand over to the host either his High card or his Low card. The host hands out payoffs-which come from the show's sponsor, not from the players' pockets-that are measured in dollars and depend on the cards that he collects. If Wonki chooses his Low card, 2, then Wonki gets \$2; if he chooses his High card, 7, then Jay gets \$7. If Jay chooses his Low card, 4, then Jay gets \$4; if he chooses his High card, 8, then Wonki gets \$8. The payoff matrix for this game is as follows.

		Jay	
		Low	High
Wonki	Low	$(2,4)$	$(10,0)$
	High	$(0,11)$	$(8,7)$

(a) What is the NE? Is this game an example of the Prisoner's Dilemma? Explain.

(b) Now suppose that the game has the following stages. The host hands out cards as before; who gets what cards is common knowledge. At stage 1, each player, out of his own pocket, can hand over a sum of money, which the host is to hold in an escrow account. This amount can be zero but cannot be negative. When both players have made their stage 1 choices of sums to hand over, these choices are publicly disclosed. Then, at stage 2, the two players make their choices of cards, again simultaneously and independently. The host hands out payoffs from the show's sponsor in the same way as in the single-stage game before, but in addition, he disposes of the escrow account as follows: If Jay chooses his High card, the host hands over to Jay the sum that Wonki put into the escrow account; if Jay chooses his Low card, Wonki's sum reverts back to Wonki. The disposition of the sum that Jay deposited depends similarly on Wonki's card choice. All these rules are common knowledge. Find the SPNE of this two-stage game. Does it resolve the prisoners' dilemma? If necessary, you can assume that players choose the Pareto efficient outcome when there are multiple equilibria. (Hint: Call the amount that Wonki puts in the escrow account (to reward Jay for cooperative behavior) r, and the amount that Jay puts in the escrow account (to reward Wonki for cooperative behavior) c. First, draw a payoff matrix in the second stage. Then, find the minimum r and c that make (High, High) an equilibrium. In the first stage, each player has two choices, and these values are one of the choices each player has in the first stage.)

(c) What is the role of the escrow account in the above stage game?

[1] `https://www.pnas.org/content/96/19/10933`

5 Intermediate Punishment Strategies (15 points)

Consider the following game.

		Firm 2	
		Collude	Defect
Firm 1	Collude	$(2, 1)$	$(-1, 4)$
	Defect	$(3, -3)$	$(0, 0)$

Assume that this game is repeated an infinite number of times. Both firms discount the future with the same discount factor δ.

(a) Suppose that both players follow the following grim-trigger strategy: "play Collude as long as no one has ever played Defect; otherwise play Defect." Find the minimum value of δ such that this is a subgame-perfect Nash equilibrium.

(b) Suppose that both players follow the following tit-for-tat strategy: "play Collude in the first period. Then, choose the action chosen by the rival in the previous period." Find the value of δ (if exists) such that this is a subgame-perfect Nash equilibrium. If not, explain.

(c) Now suppose that the players follow a strategy in which they both start in "phase C" (described below) and then switch "phases" as the instructions dictate:

- phase C: play Collude provided that, in each period since the (re)start of the phase, either both players chose Collude or both players chose Defect. Stay in phase C until one player chooses Defect while the other chooses Collude. In this event, if it is Firm 1 who chose Defect go to phase P_1; if it is Firm 2 who chose Defect go to phase P_2.
- phase P_1: play Defect for T_1 periods (regardless of what happens during those periods) then revert to phase C.
- phase P_2: play Defect for T_2 periods (regardless of what happens during those periods) then revert to phase C.

Write down expressions in terms of δ for the smallest T_1 and T_2 such that this is a SPNE. (These expressions might be quite ugly so do not attempt to simplify them much.) For any given δ, which of T_1 and T_2 is larger? What is the intuition?