

Instruction

Show your work and how you derive an answer step by step. Feel free to discuss your work with your classmates but do not copy solutions. Each student has to submit their own solutions in Canvas. Your scanned submission should be high-quality and professionally presented. Please remember that late submissions will not be accepted. The total score is 100 points. You will get 40 points just by submitting your solutions on time.

1 Spence's Job-Market Signaling Model (15 points)

There are two types of workers, A type and B type, considering whether to get an Economics PhD. Getting an Econ PhD increases a worker's productivity by 2. Specifically, a A -type worker's productivity is 3 if no Econ PhD, and 5 if they get an Econ PhD. A B -type worker's productivity is 1 if no Econ PhD, and 3 if they get an Econ PhD. The mental costs of an Econ PhD are 1 for A type and 3 for B type. Each worker's payoff (utility) equals their income minus the cost of education. Companies that seek to hire these workers are perfectly competitive in the labor market.

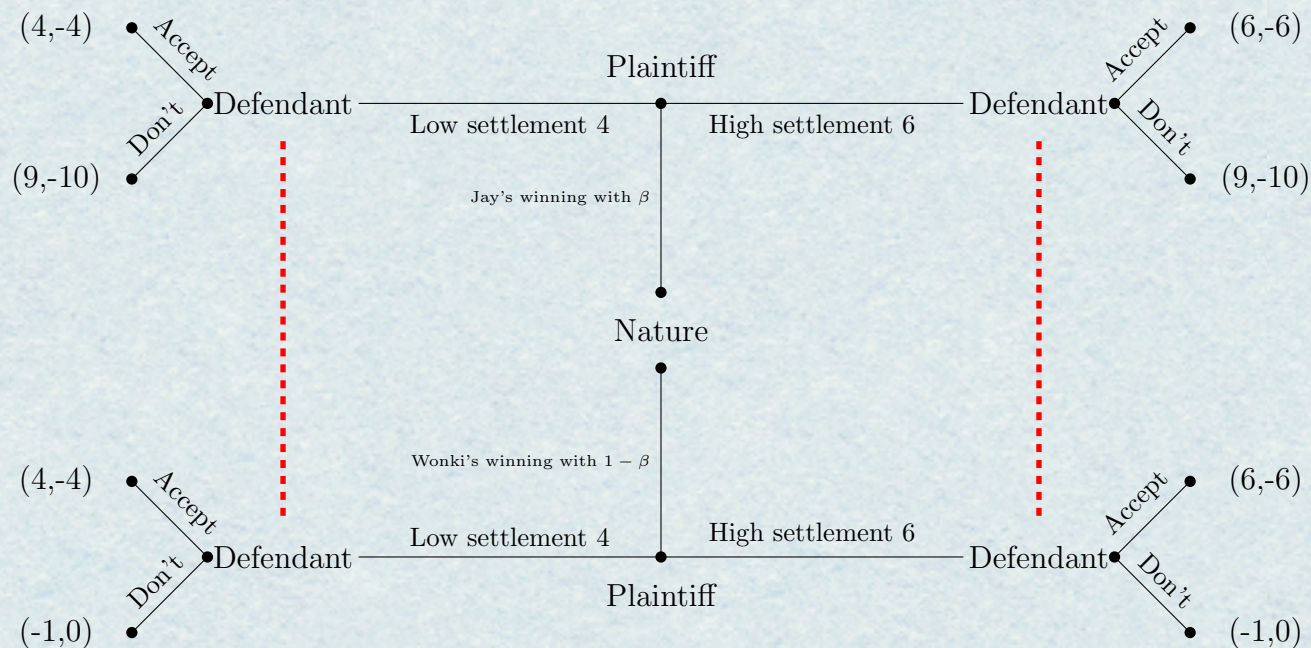
- (a) If types are public information (observable and verifiable), find expressions for the levels of education, incomes, and utilities of the two types of workers.

Now suppose each worker's type is their private information.

- (b) Suppose employers offer two kinds of employment packages: anyone having an econ PhD will get salary 5, and anyone not having an econ PhD will get salary 1. Verify that type A workers strictly prefer to get an econ PhD, and type B 's also do prefer to get an econ PhD. That is, verify that separation by self-selection cannot achieve the outcome that one would obtain under public information about types.
- (c) If we leave the wage level for non-PhD as 1, what is the range of wages for econ PhD that can achieve separation? (If needed, you can assume that type B workers do not get an econ PhD if their utilities, with an econ PhD and without it, are the same.)
- (d) Of the possible separating contracts, which one do you expect to prevail? Give a verbal, but not a formal, explanation for your answer.
- (e) Who gains or loses from the information asymmetry? How much?

2 Civil Lawsuit (15 points)

Consider a game between Jay (plaintiff) and Wonki (defendant). Jay knows exactly whether he will win the lawsuit or not. Wonki only knows that the probability of Jay's winning is β . Now, Jay and Wonki are negotiating for a settlement. Jay can ask either a high or a low amount. Wonki has two choices: either to accept the out-of-court settlement or to take the case to trial. The game tree for this game is as follows.



- In this game, a separating equilibrium does not exist. Explain why. (Hint: Wonki's strategy should be a complete plan of action. When Jay plays a separating strategy, Wonki can conjecture which node he is in. Thus, Wonki's actions will be different between two information sets.)
- Suppose that $\beta = 0.3$. Find all (pooling) perfect Bayesian Nash equilibria.

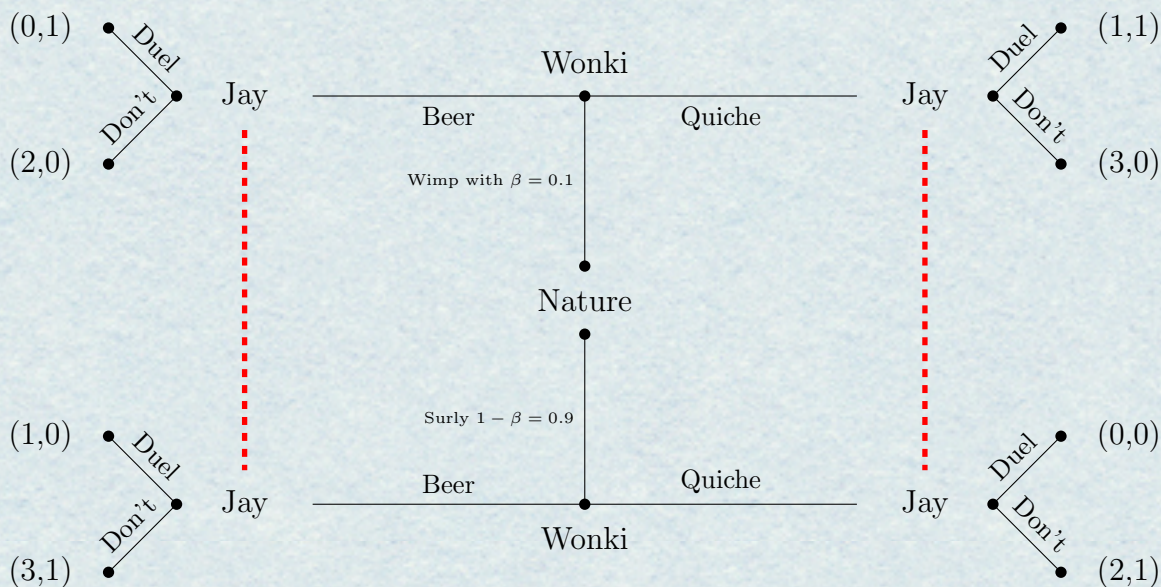
3 Signaling Game and Intuitive Criterion (15 points)

Consider a game between Wonki and Jay.¹ Wonki is either a wimp or surly. Nature has selected the disposition of Wonki, with probability 0.9 that the Wonki selected is surly.

At the start of the game, Wonki knows his own disposition or type, and Wonki is faced with the choice of what breakfast to have, before setting off for the day. The choices available are quiche and beer. Wonki's preferences concerning breakfast depends on his type: if Wonki is a wimp, he derives incremental payoff 1 for having quiche and 0 from beer; if Wonki is surly, beer is worth 1, and quiche is worth 0.

After breakfast, Wonki meets with Jay. There are four conceivable circumstances under which the meeting could take place, corresponding to the two types of Wonki and the two types of breakfast; Jay, at this meeting, chooses whether to duel Wonki. When Jay chooses whether to duel, he does so knowing what Wonki had for breakfast, but not knowing for sure what is Wonki's type.

Jay's choice whether to duel effectively ends the game. Wonki, whether surly or not, wishes that Jay choose not to duel. Wonki gets incremental payoff 2 if Jay chooses not to duel, and 0 if Jay does duel. Jay wishes to duel with Wonki if and only if Wonki is a wimp. The game tree for this game is as follows.



- In this game, a separating equilibrium does not exist. Explain why.
- Find a perfect Bayesian Nash equilibria in which Wonki, regardless of type, has beer for breakfast.
- Find a perfect Bayesian Nash equilibria in which Wonki, regardless of type, has quiche for breakfast.
- Between the above two equilibria, one of them does not satisfy Cho and Krep's *Intuitive Criterion*. What is it? Explain why.

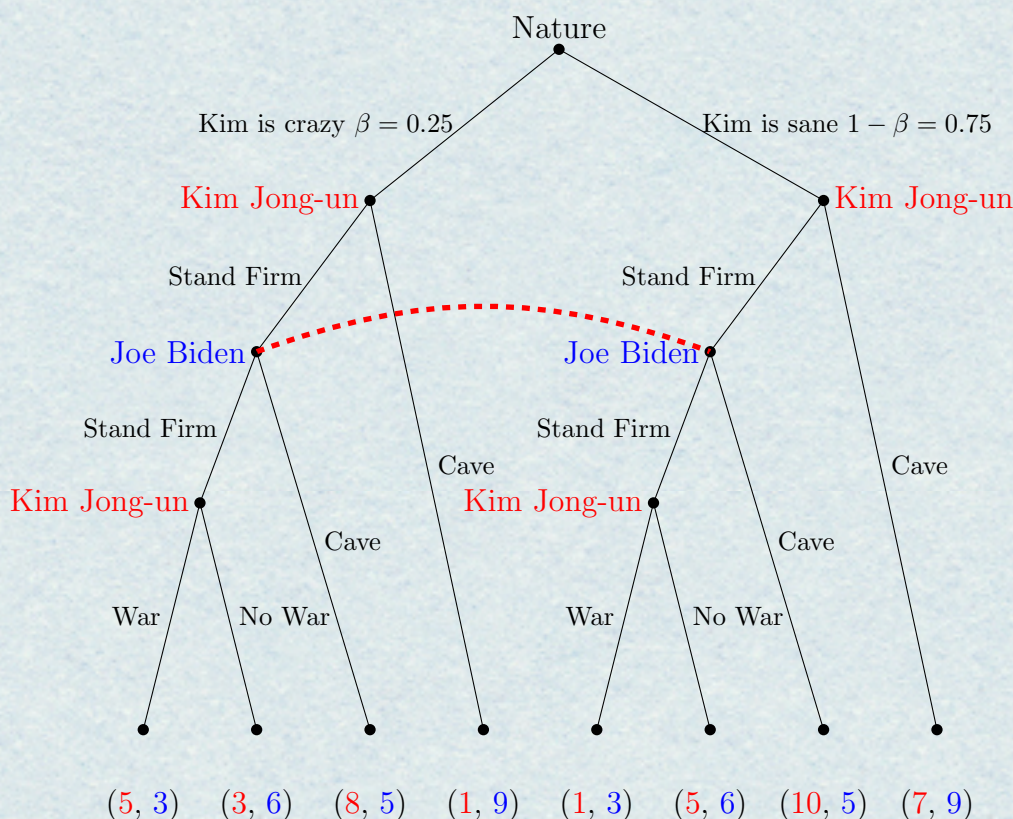
¹You can refer to the original paper: <https://www.jstor.org/stable/1885060>.

4 Brinkmanship (15 points)

Consider a game between the U.S. President Joe Biden and North Korean Supreme leader Kim Jong-un. Nature begins by determining whether Kim is sane or crazy, where the probability that he is crazy is 0.25. After learning his own mental state, Kim decides whether to stand firm in developing nuclear weapons or to cave to the demands of the United States and the United Nations. If he caves, then the game is over. If he stands firm, then the United States must decide whether it will stand firm or cave. If the United States caves, then, again, the game is over. If the United States stands firm, then North Korea decides whether or not to take a hostile action such as launching nuclear weapons.

As just described, Kim is one of two possible types: sane or crazy. If sane, he ranks the four possible outcomes as follows: He stands firm and Biden caves with a payoff of 10, he caves with 7, both he and Biden stand firm and he avoids war with 5, and both he and Biden stand firm and he goes to war with 1. If Kim is insane, then his preference ordering is as follows: He stands firm and Biden caves with 8, both he and Biden stand firm and he goes to war with 5, both he and Biden stand firm and he avoids war with 3, and he caves with 1. The key difference is that a crazy Kim is willing to go to war if someone does not back down.

Note that Biden has only one information set and Kim has four information sets.



- Find all Perfect Bayesian Nash Equilibria in which Biden stands firm.
- Find all Perfect Bayesian Nash Equilibria in which Biden caves.

Now we are going to find a semi-separating equilibrium in which Kim plays (if crazy, stand firm, war; if sane, stands firm with probability k , no war.) and Biden stands firm with probability b . For the following problems, you can assume that the two players play this strategy.

- (c) Suppose Biden observes Kim stands firm. Since $k > 0$, Biden cannot infer Kim's type, because although Kim will stand firm for sure if he is crazy, he will stand firm with probability k even when he is sane. What is the Biden's belief (probability) that Kim is crazy given that Biden observes Kim stood firm? Represent it in terms of k .
- (d) What is the Biden's belief (probability) that Kim is crazy given that Biden observes Kim caved? Represent it in terms of k .
- (e) Given that North Korea has stood firm, Biden will randomize his strategy if he is indifferent between his two strategies. What is the Biden's indifference condition? Derive k from the condition.
- (f) The sane Kim is indifferent between standing firm and caving. Find the Kim's indifference condition, and derive b from the condition.
- (g) Give some intuition about why the equilibrium strategies are optimal for each player.