ECO 4400 | **Game Theory**

Assignment 5 | Instructor: Jay Lee

Due Date: 3/2/22 | jeongwoo@ufl.edu

Instruction

Show your work and how you derive an answer step by step. Feel free to discuss your work with your classmates but do not copy solutions. Each student has to submit their own solutions in Canvas. Your scanned submission should be high-quality and professionally presented. Please remember that late submissions will not be accepted. The total score is 100 points. You will get 40 points just by submitting your solutions on time.

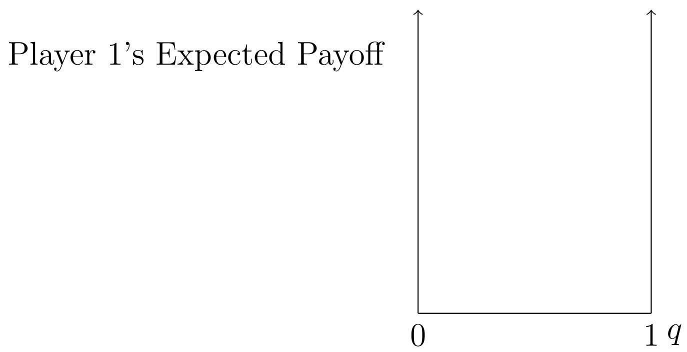
1 Mixed Strategies in 3 by 3 (10 points)

Consider the following game.

		Player 2		
		Left	Center	Right
Player 1	Top	$(3, 0)$	$(0, 1)$	$(0, 0)$
	Middle	$(1, 1)$	$(1, 1)$	$(5, 0)$
	Bottom	$(0, 2)$	$(4, 1)$	$(0, 0)$

(a) Find all strictly dominated strategies.

(b) After eliminating all the strictly dominated strategies found in (a), draw the graphs for the expected payoffs for player 1 depending on player 2's strategy, q.



(c) After eliminating all the strictly dominated strategies found in (a), does player 1 have a strategy that is strictly dominated by a mixed strategy? Explain.

(d) Find all the Nash equilibrium of this game, including any equilibria in mixed strategies.

2 Mixed Strategies in 4 by 2 (15 points)

After committing a serious crime, Wonki is attempting to flee, potentially abroad. He can choose to take either a plane, a train, a car, or a motorcycle in the order of mode speed. Jay, a member of Interpol, has two choices, either to put him in the international most wanted list or not. If he does, then it is more dangerous for Wonki to take a faster mode. But for Jay, it is more difficult to find Wonki when he takes a relatively slower mode. The payoffs to this game are as follows.

		Jay	
		Listing	Not
Wonki	Plane	$(0, 6)$	$(12, 0)$
	Train	$(1, 5)$	$(11, 1)$
	Car	$(4, 2)$	$(10, 2)$
	Motor	$(6, 0)$	$(9, 3)$

(a) Suppose Wonki believes that Jay is going to choose "Listing" with the probability of $\frac{1}{3}$. Find the expected payoffs for Wonki to take a plane, a train, a car and a motor, respectively.

(b) Let q be the probability assigned by Wonki's belief to Jay's playing "Listing". Explain what Wonki should do if $q > \frac{1}{3}$, $q < \frac{1}{3}$, and $q = \frac{1}{3}$, respectively? (Hint: expected payoffs are weighted averages. If you look at Wonki's payoffs carefully and use (a), you do not need any graphs.)

(c) Suppose you are an accomplice in the crime. Are there any modes you would advice Wonki certainly not to take? Explain your answer.

(d) Find a Nash equilibrium in which one player plays a pure strategy and the other player plays a mixed strategy.

(e) Find a different mixed-strategy equilibrium in which that the pure strategy in the above question is assigned zero weight.

(f) (Bonus Point: 5 points only if you provide a perfect answer. No partial credit.) Find all the Nash equilibria of this game.

3 Price Competition on Heterogeneous product (10 points)

Consider two firms selling heterogeneous products with market demand as below:

$$q_1 = 100 - 2p_1 + p_2$$
$$q_2 = 100 - 2p_2 + p_1$$

The marginal cost of each firm is $c = 10$. The firms compete in prices, i.e., the choice variable is its price p_i. Each firm maximizes its own profit. Profit is defined:

$$\pi_i = (p_i - c) \times q_i$$

(a) The given demand system represents demands for differentiated products. How is this different from the demand system for homogeneous goods? What happens to the demand for product 1 if the price for product 2 increases? Explain briefly in 2–4 sentenses. No need to calculate anything.

(b) Find the profit function for each firm in terms of p_1 and p_2. (Hint: This is different from Cournot model. Firms set their prices. You can plug the demand into the given profit formula.)

(c) Derive the best responses and graph them on the $p_1 - p_2$ plane.

(d) Solve for the equilibrium prices of each firm.

(e) What are the quantity demanded in NE? What are the profits for each firm in NE?

(f) Compare the result with the case in which only one of them is in the market, say firm 1. If firm 1 were the monopolist, the demand would be $q_1 = 100 - 2p_1$. (Hint: you can easily find the equilibrium of this case from the best response graph in (c). Or, you can just solve the monopolist problem facing $q_1 = 100 - 2p_1$.)

4 Collusion and Strategic Moves (15 points)

Consider N identical firms serving a market in which the inverse demand function is given by:

$$P = 140 - Q$$

where $Q = \sum q_i$.

The marginal cost of each firm is $c = 20$ per unit. The firms compete in outputs, i.e., the choice variable is its output level, q_i. Each firm maximizes its own profit. Profit is defined:

$$\pi_i = (P - c) \times q_i$$

(a) Suppose that the market is monopolistic, i.e., $N = 1$. Find the monopoly output, the product price, and the profit of the monopolist.

(b) Suppose that the market is duopolistic, i.e., $N = 2$. Draw the best response curves for each firm in a $q_1 - q_2$ plane.

(c) Again, two firms are in the market. Find the equilibrium output, the product price, and the profits of each firm.

(d) The two firms are considering whether to collude each other. If they collude, then the firms behave like one firm. That is, they produce the monopoly output in total, set the monopoly price, and split the monopoly profit into two. However, it is illegal to collude explicitly and create a cartel in the U.S. Thus, this is a game situation for the two firms with each having two strategies, whether to collude (implicitly by each producing half the monopoly output) or to defect (produce a competitive output level derived in (c)). Construct the 2 by 2 payoff matrix for this game situation. Find all the NE in this game.

(e) Now suppose again that the market is duopolistic, i.e., $N = 2$. Instead of the above Collusion game, consider the Stackelberg model. Firm 1 is the market leader and can publicly and credibly commit to a production quantity before firm 2 makes a decision. Firm 2 makes its own production decision after observing firm 1's output level. Find the Subgame Perfect Nash Equilibrium. (SPNE is a strategy combination. The strategy for firm 2 should specify which node the action is for. In other words, the strategy for firm 2 is a function of q_1.)

(f) In the above game, what is the equilibrium production of each firm? What is the market price? What are the profits for each firm? Represent the output levels in the (b) plane. Give some intuition.

5 Cournot Model (10 points)

Consider N identical firms serving a market in which the inverse demand function is given by:

$$P = 140 - Q$$

where $Q = \sum q_i$.

The marginal cost of each firm is $c = 20$ per unit. The firms compete in outputs, i.e., the choice variable is its output level, q_i. Each firm maximizes its own profit. Profit is defined:

$$\pi_i = (P - c) \times q_i$$

(a) Suppose that the market is triopolistic, i.e., $N = 3$. Find the equilibrium output, the market price, and the profits for each firm.

(b) Suppose that $N = 99$. Find the equilibrium outputs for each firm, the product price, and the profits of each firm. (Hint: note that the game is symmetric to all firms, i.e., all firms produce the same amount in NE. First, derive a best response for a firm. In the best response formula, you can use the symmetric property: $q_1 = q_2 = \cdots = q_{99}$.)

(c) How does the equilibrium outputs change if N gets greater? (if $N \to \infty$) How about the total output, the product price and the profit of each firm?

(d) Compare the Cournot model with a monopoly and perfect competition. Briefly discuss any intuition from the result.