ECO 4400 | **Game Theory**
Assignment 3 | Instructor: Jay Lee
Due Date: 2/16/22 | jeongwoo@ufl.edu

Instruction

Show your work and how you derive an answer step by step. Feel free to discuss your work with your classmates but do not copy solutions. Each student has to submit their own solutions in Canvas. Your scanned submission should be high-quality and professionally presented. Please remember that late submissions will not be accepted. The total score is 100 points. You will get 30 points just by submitting your solutions on time.

1 Guessing a Number (10 points)

Consider the following game, a variant of the guessing the half game in class. Suppose that we have 10 students in our class. Each student simultaneously and privately messages a number between 0 and 100. (no decimal numbers.) The game theory professor computes the mean of these numbers and calls it A. This time, the student who messages the number closest to $\frac{2}{3} \times (A + 9)$ wins, and gets an extra credit. If multiple students win, only one student is randomly picked (probability of $\frac{1}{10}$) and gets the extra credit.

(a) Find a symmetric NE in which all students message the same number. (Hint: let's say you message x. For everyone messaging the same number to be a NE, your number should also be the winning number. Then, what would be A?)

(b) Show that choosing the number 5 is a dominated strategy. (Hint: what should be A for 5 to be the winning number?)

(c) Show that choosing the number 90 is a dominated strategy.

(d) Is there any equilibrium that is different from part (a)? Find all (pure strategy) NE.

(e) (Bonus Point: 5 points only if you provide a perfect answer. No partial credit.) Now suppose that if multiple students win, they all get the extra credit. Find all (pure strategy) NE.

2 Bertrand Model (10 points)

Consider a variant of the Bertrand model we studied in class. Two firms sell a homogeneous product with market demand as below:

$$Q = 110 - P$$

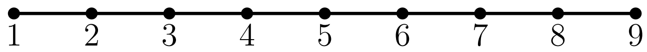
Consumers buy everything from a firm with a lower price. If the prices are the same, each firm takes 50% of the total consumers. Now, two firms have different marginal costs. Specifically, firm 1's marginal cost is \$5 per unit, firm 2's is \$10 per unit. The firms compete on price, not quantity. Firms can price their goods to one decimal place, such as \$0.1, \$0.2, \$0.3, $\cdots$, \$4.9, \$5.0, \$5.1, $\cdots$, \$9.9, \$10.0, \$10.1, $\cdots$.

(a) Find all pure strategy NE prices. What is each firm's profit in the NE? (Hint: there are two NE.)

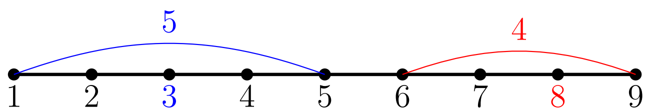
(b) Suppose firm 2 has succeeded in R&D, so their marginal cost is now reduced. Specifically, firm 1's marginal cost is still \$5 per unit, but firm 2's is now \$5 per unit. Find all pure strategy NE prices. What is each firm's profit in the NE? (Hint: there are two NE.)

3 Candidate-Voter Game (15 points)

Consider a variant of the election game we studied in class. There are **NINE** voters, spread along a political spectrum from left to right. Represent this spread by a horizontal line whose extreme points are 1 (left) and 9 (right).



Now, any voter can become a candidate in an election, at a cost $c = 10$. Each voter has two strategies, to run or not to run the election. Each voter votes for the candidate whose political position is closest to the voter's own position. For example, if both 3 and 8 run, then 3 takes 5 votes and 8 takes 4 votes. If a voter is at the center of two candidates, a half vote will be given to each candidate. (You can regard this as a situation in which half the population votes for each candidate.)



A voter located x who does not run the election receives payoff $-|x-y|$ if the winner is at y. (the negative distance from the winner.) The value of winning in this election is 30. Thus, a voter located x who runs the election receives payoff $(30-c)$ if she wins, $(-c-|x-y|)$ if she loses against other candidate at y. $(c = 10)$. For example, 3 receives $(30-10) = 20$, and 8 receives $(-10-5) = -15$ in the above case. Also, 1 receives -2, 2 receives -1, and so on. If candidates tie, they flip a coin to determine the winner. If no candidates at all, a voter will be elected with probability of $\frac{1}{9}$.

(a) If 5 is the only person running, is this a Nash equilibrium? (Hint: does the voter located at 4 have an incentive to run? How about others?)

(b) If 2 and 8 run, is this a Nash equilibrium? (Hint: does the voter located at 1 have an incentive to run? How about 3? How about 5? How about others? Do the candidates have an incentive to not run?)

(c) If 4 and 6 run, is this a Nash equilibrium? (Hint: does the voter located at 3 have an incentive to run? How about 5? How about others? Do the candidates have an incentive to not run?)

(d) Suppose 4 and 7 run. Can the voter 1 change the election result by running in this election? How can she affect this election? Give some intuition. No need to find NE to answer to this question.

(e) Discuss briefly any properties of this game relating with the presence of Green Party candidate Jill Stein in 2016 United States presidential election.[1] Any reasonable answer will be given full credit.

[1] https://www.nbcnews.com/politics/national-security/russians-launched-pro-jill-stein-social-media-blitz-help-trump-n951166

4 Voting (10 points + Bonus Point)

Consider a variant of the voting game we studied in class. (Module 2-1). Two candidates, A and B, compete in an election. Now, k citizens support candidate A and the other m support candidate B. $(2 < k < m)$. Each citizen decides whether to vote for the candidate they support, or to abstain. A citizen who abstains receives payoff 2 if the candidate they support wins, 1 if this candidate ties for first place, and 0 if this candidate loses. A citizen who votes receives payoffs $2 - c$, $1 - c$, and $-c$ in these three cases, where $0 < c < 1$.

(a) Find all pure strategy Nash equilibria. For full credit, you should provide clear explanations.

(b) (Bonus Point: 5 points only if you provide a perfect answer. No partial credit.) Find a mixed strategy Nash equilibrium in which every supporter of candidate A votes with probability $p \in (0, 1)$, k supporters of candidate B vote, and the remaining $m - k$ supporters of candidate B abstain. (all B supporters use pure strategies.)

5 Opponent's Indifference Property (15 points)

A neutral TV game show host runs the game. There are two players, Wonki and Jay, who are advanced to the final episode. The host gives two cards to each: Up and Down to Wonki and Left and Right to Jay. Who gets what cards and payoff structure are common knowledge. Then, playing simultaneously and independently, each player is asked to hand over to the host a card between two. The payoff structure is as follows.

		Jay	
		Left	Right
Wonki	Up	$(1, 16)$	$(4, 6)$
	Down	$(2, 20)$	$(3, 40)$

(a) Find the Nash equilibrium in pure strategies for this game.

(b) Suppose Wonki plays Up with probability $p = 0.00001$. What is Jay's best response? (Hint: you would not need any calculation. Put yourself into Jay's shoes, and think about what you would choose.)

(c) Suppose Wonki plays Up with probability $p = \frac{1}{2}$. What is Jay's best response? (Hint: calculate Jay's expected payoffs from each of his pure strategies.)

(d) What is the Wonki's p-mix (probability p on Up) that keeps Jay indifferent between Left and Right?

(e) Find the Nash equilibrium in mixed strategies for this game.

(f) What are the players' expected payoffs in this equilibrium?

(g) Jay and Wonki jointly get the most money when Wonki plays Down. However, in the equilibrium, he does not always play Down. Why not? Can you think of ways in which a "Pareto efficient" outcome could be sustained? (Hint: since they are close friends, they can secretly give and receive money out of this game. And yes, the best way to get the better outcome is to break the game rule.)

6 Speed Trap and Doughnuts (10 points)

You are traveling south on Interstate 75, heading toward Orlando. Do you travel the legal limit of 70 miles per hour, or do you crank it up to 90 and hope that there is no speed trap? And what about the police? Do they set a speed trap or instead head into town and find out whether the "Hot and Fresh" neon sign is lit up at the Krispy Kreme? The police like to nab speeders, but they do not want to set a speed trap if there will not be any speeders to nab. A strategic form for this setting is shown as follows.

		Police	
		Speed Trap	Krispy Kreme
Driver	90mph	$(10, 100)$	$(70, 50)$
	70mph	$(40, 20)$	$(40, 50)$

(a) Find all Nash equilibria in mixed strategies for this game.