ECO 4400 **Game Theory**
Assignment 2 Instructor: Jay Lee
Due Date: 01/31/22 jeongwoo@ufl.edu

Instruction

Show your work and how you derive an answer step by step. Feel free to discuss your work with your classmates but do not copy solutions. Each student has to submit their own solutions in Canvas. Your scanned submission should be high-quality and professionally presented. Please remember that late submissions will not be accepted. The total score is 100 points. You will get 30 points just by submitting your solutions on time.

1 Bask*et* Robbins 31 (10 points)

Consider a variant of the extra point game we studied in class. Jay and Wonki are playing a Korean drinking game called "Bask*et* Robbins 31." They take turns counting the numbers in sequence from 1 to 31. Jay goes first, starting from 1. On each turn, each player may count either 1, 2, or 3 numbers, and they cannot double count a number or skip numbers. The player to count the last number, 31, loses the game.

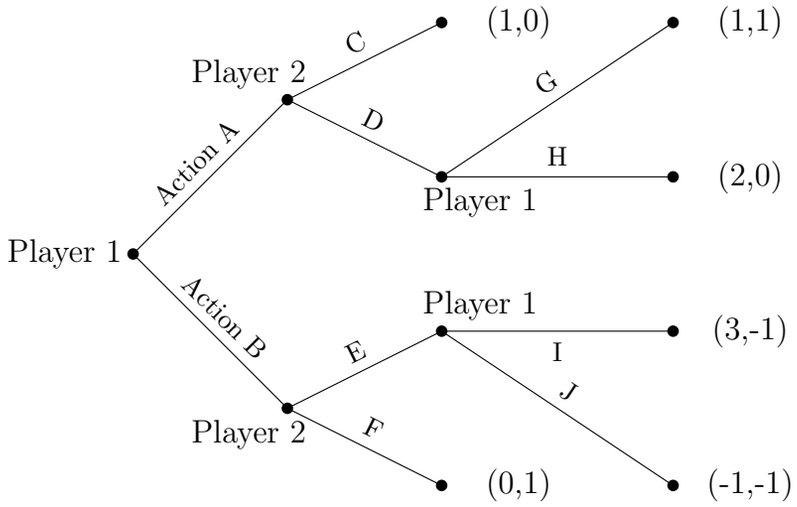
(a) If both players play optimally, who will win? How does the winning strategy look? (No need to draw a game tree. Just describe the winning strategy.)

(b) Consider another variant. Now, all the rules are the same but the player to count the last number, 31, WINS the game. If both players play optimally, who will win? How does the winning strategy look?

(c) Consider another variant. Again, the player to count the last number, 31, loses the game, but each player may count either 1, 2, 3 or 4 numbers. If both players play optimally, who will win? How does the winning strategy look?

(d) (Bonus Point: 5 points only if you provide a perfect answer. No partial credit.) Generalize the winning strategy for this game. How does the winning strategy look when the player to count the last number x loses the game and each player may count either 1, 2, 3, $\cdots$, N numbers? Is there any order advantage?

2 Sequential-Move Game (10 points)



(a) Find all the Subgame Perfect Nash equilibria in this game.

(b) Find all the Nash equilibria in this game. (Hint: write the game in strategic form.)

3 Corn Farmer (10 points)

Two farmers have a choice of growing 0, 40, or 60 tons of corn, are not able to observe each other's planting, and have the following payoff functions:

$$\text{Farmer 1 profit} = \begin{cases} 0 & \text{if} \quad q_1 = 0 \\ (120 - q_1 - q_2)q_1 - 1500 & \textit{otherwise.} \end{cases}$$

$$\text{Farmer 2 profit} = \begin{cases} 0 & \text{if} \quad q_2 = 0 \\ (120 - q_1 - q_2)q_2 - 1500 & \textit{otherwise.} \end{cases}$$

Where q_1 is the quantity of corn in tons produced by farmer 1, and q_2 is the quantity of corn in tons produced by farmer 2.

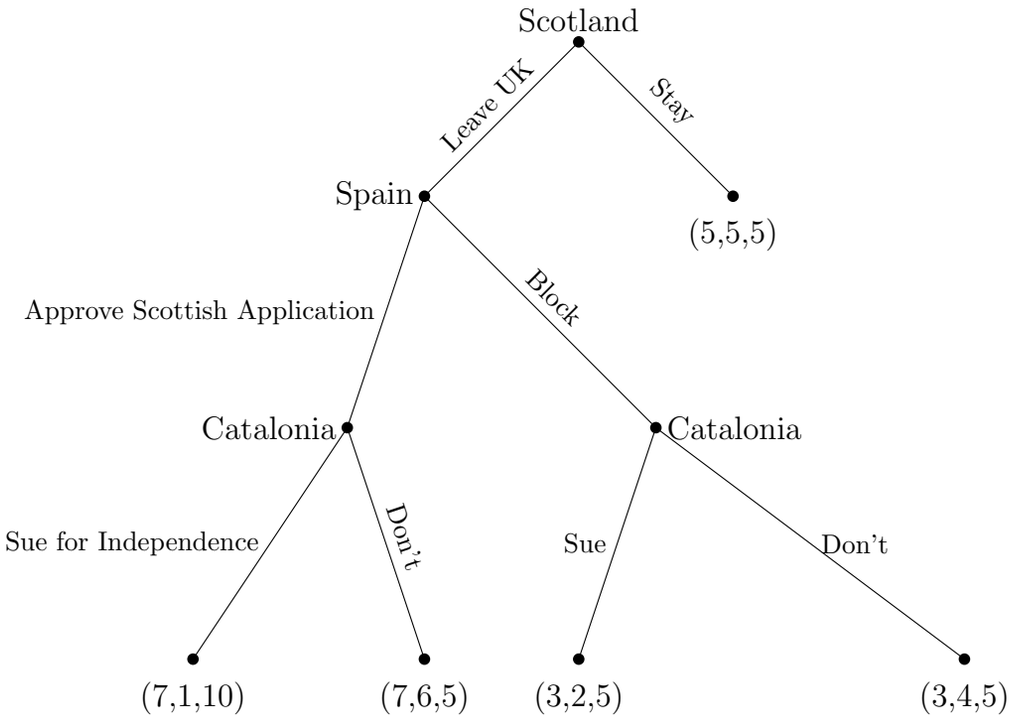
(a) Construct the strategic form for this scenario, and find any (pure strategy) NE.

(b) Construct the extensive form for this game. Be careful to label which player makes the decision at each node, and remember to indicate information sets between nodes where necessary.

(c) Is it possible for one player to achieve his best possible outcome by credibly declaring a strategic move in the pregame? If not, why not? And if so, what sort of strategic move would the player use, and how might he phrase his declaration?

4 Brexit (10 points)

Scotland is considering 2nd Brexit, whether to leave the UK and apply for EU membership. Spain is the deciding vote on Scotland's potential application, which will bolster the legal cause for Catalan independence if accepted. In each payoff, the first number represents the payoff to Scotland, the second number Spain, and the third number Catalonia.

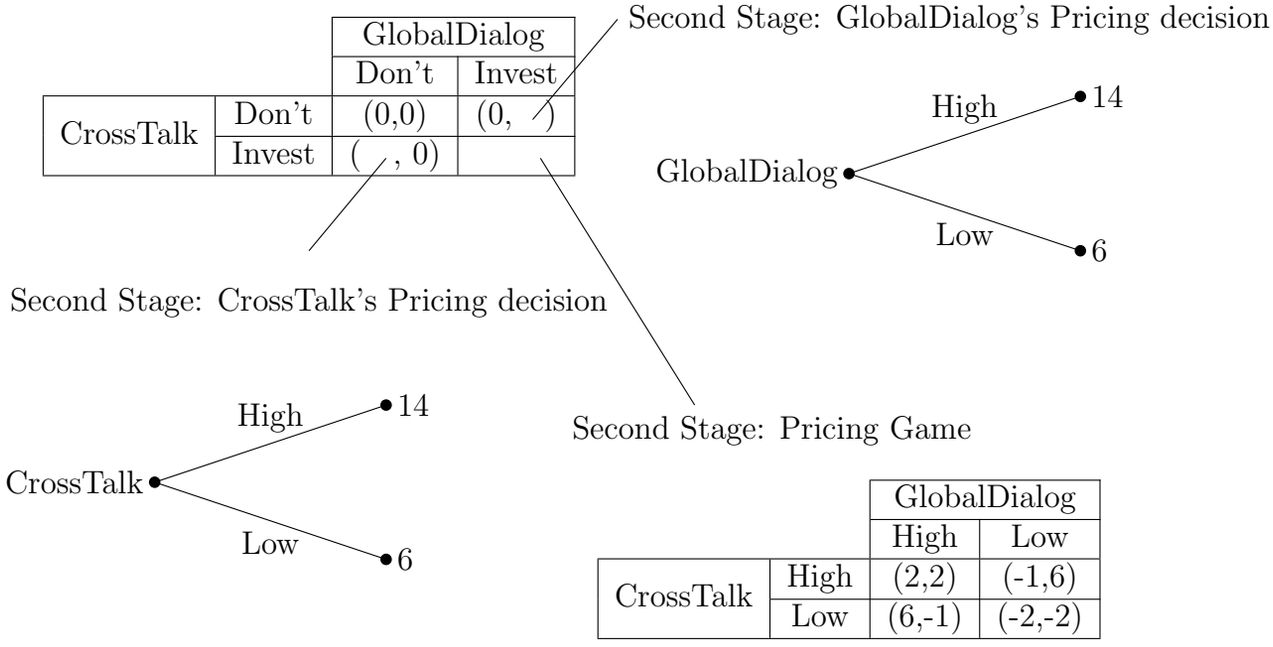


(a) Find all (pure strategy) SPNE.

(b) Is it possible for Spain to achieve its best possible outcome (payoff 6) by credibly declaring a strategic move in the pregame? If not, why not? And if so, what sort of strategic move would Spain use, and how might it phrase its declaration? (If needed, you can assume that any strategic moves are credible.)

5 Sequential-Move Game (10 points)

Consider a variant of the game between CrossTalk and GlobalDialog we studied in class. Specifically, the payoffs in the pricing game (second stage) are slightly different from the original game.



		GlobalDialog	
		High	Low
CrossTalk	High	(2,2)	(-1,6)
	Low	(6,-1)	(-2,-2)

(a) Find all (pure strategy) SPNE.

(b) Represent the entire two-stage game with a single game tree. Be careful to label which player makes the decision at each node, and remember to indicate information sets between nodes where necessary.

6 Trade (10 points)

Two identical Chinese firms are currently exporting to the US. The inverse demand function for the market is $P = 80 - Q$, where Q is the sum of the quantities chosen by firms 1 and 2. For example, if two firms produce 40 units in total, the market price will be $80 - 40 = 40$. Each firm has a marginal cost of 20. Each firm's payoff is its profit, which is defined as

$$\text{Profit} = (\text{Price} - \text{Marginal Cost}) \times \text{Quantity produced by the firm} - \text{Fixed cost}$$

Currently, there is a fixed cost of 100 to each exporting firm to establish a joint venture with a US firm. The US consumers receive (70−Price) utility per unit of a good imported from China. For example, if 30 goods are sold in the US at the price \$50, then the US receives $(70 - 50) \times 30 = 600$.

The US is considering whether to increase tariffs against China, following which the two Chinese firms simultaneously consider whether to exit the US market. If trade barriers are enacted, there is an additional fixed cost of 100 to each exporting firm to clear regulatory hurdles. Assume that the tariff revenue goes to the government but it is not re-distributed to the US consumers. The US administration only cares the US consumer utility.

(a) Each firm has the choice to not enter the US market and receive a payoff of 0, or produce either 20 or 30 units for export. Construct the payoffs of the two firms for each choice combination.

		Firm 2		
		No Export	20	30
Firm 1	No Export	(,)	(,)	(,)
	20	(,)	(,)	(,)
	30	(,)	(,)	(,)

(b) The US consumers receive (70 − Price) utility for each individual good imported from China. Calculate the payoffs for the US when the total number of consumed goods is 0, 20, 30, 40, 50 and 60, respectively.

(c) The US moves first and decides whether to impose tariff or not in the first stage. In the second stage, two firms simultaneously choose their production decisions: No export, 20, or 30. Fill out the payoffs in the following game tree. The payoff for the US should be the first entry, then Firm 1, Firm 2 in order.

US

No

		Firm 2		
		No Export	20	30
Firm 1	No Export	(, ,)	(, ,)	(, ,)
	20	(, ,)	(, ,)	(, ,)
	30	(, ,)	(, ,)	(, ,)

Tariff

		Firm 2		
		No Export	20	30
Firm 1	No Export	(, ,)	(, ,)	(, ,)
	20	(, ,)	(, ,)	(, ,)
	30	(, ,)	(, ,)	(, ,)

(d) Find all (pure strategy) Subgame Perfect Nash Equilibria.

(e) Recently, the US administration tried to ban TikTok and WeChat from the US App stores. This is not very surprising given the current ongoing trade war between the US and China. Since 2018, the US has imposed tariffs on hundreds of Chinese goods, and China has responded by imposing tariffs on US goods. Discuss briefly the current US trade policy against China relating this model. You can also discuss any flaws in this model that can change the result. Any answer will be given full credit.

7 Prisoner's Dilemma (10 points)

Let us elaborate on the prisoner's dilemma. Two criminals, Wonki and Bomi, have been caught doing a job for Hyesoo, resulting in the following game:

		Bomi	
		Confess	Don't
Wonki	Confess	(-10,-10)	(0,-15)
	Don't	(-15,0)	(-1,-1)

Negative utilities represent years spent in prison. Hyesoo also goes to jail for 10 years if one player confesses, and 15 years if both confess. However, Hyesoo gets 5 years worth of utility per confessor if she orders any confessor "taken care of". (Hyesoo has two choices: to give the order for any confessor or not give the order.) For example, if both Wonki and Bomi confess, then Hyesoo's utility from "Give order" will be $(-15+5+5)$. If one of them confesses, then Hyesoo's utility from "Give order" will be $(-10+5)$. The confessor takes a penalty of 50 years to his or her utility due to their untimely death if the order is given. Additionally, Hyesoo gets 1 year worth of utility if the order is given but not needed for having their bases covered, and otherwise has a utility of 0. (In other words, Hyesoo's payoff will be 1 if she gives an order, and 0 if she gives no order when neither of Wonki and Bomi confesses.)

(a) The following represents the above game with Hyesoo as the panel player. Fill out the payoffs in the matrix.

		Bomi	
		Confess	Don't
Wonki	Confess	(, ,)	(, ,)
	Don't	(, ,)	(, ,)

Hyesoo: No Order

		Bomi	
		Confess	Don't
Wonki	Confess	(, ,)	(, ,)
	Don't	(, ,)	(, ,)

Hyesoo: Order

(b) Find any (pure strategy) NE. If you use the IESDS, give the order in which the eliminations occur. If you use the best response analysis, you do not need to give the order, but mark the circles clearly.

(c) Are the prisoners made worse off by the new outcome(s), or do they actually prefer the threat of death from Hyesoo over the unmodified prisoner's dilemma?