An approximated method for calculation of shield's width for gamma cosine source

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2021

Abstract

A cosine source is common approximation for angular intensity distribution of various radiation source, e.g. an influx through wall penetrations in nuclear facilities. During designing a shielding assembly, not only thickness of the shield is important, but also width of the shield. Based on theory of build-up factors, an approximated equation for calculation of ratio between width and thickness of a shield has been developed. Sizes obtained by the equation have been verified using Monte Carlo simulation with good results, where maximal dose rates at the backside and at the frontside of the shield have been the same with accuracy mostly about 10%.

1 Introduction

Finding thickness of a shield for a requested attenuation factor and a particular radiation source, is one of the fundamental task of shielding design. Solution for one-dimensional problem are widely known and can be solved using tabularized data. However, due to radiation scattering and inhomogeneous source angular intensity, for many cases all three dimensions of the shield have to be assessed. It can be done by computational method like point kernel or Monte Carlo simulation, but such approach might be time-consuming and requires special software. There exists a need for development of more 'handy' methods for certain cases [1]. For particular geometries, the three-dimensional problem of shield design might be solved semi-analyticaly with sufficient approximation. One of those geometries is cosine source distribution.

Cosine source distribution is a good approximation for radiation emitting by wall penetration [2]. In fact, it can be applied to every influx of radiation through a hole in a shield. Depending on the hole length and radius or width, the distribution at the exit might be described by cosine function of higher degrees, like $\cos^2$, $\cos^3$, etc. Approximation that the angular intensity is proportional to cos is the most conservative approach.

If there is no possibility to add any shielding inside a penetration, external shielding might be necessary, mostly in shape of a block made of cement, iron or lead. According to conservative approach, the width of the shield shall be equal to the thickness of the shield, which leads to condition, that the shield shall be a cylinder or a rectangle with radius or side twice as big as the height. This is, intuitively, overesizing of the design.

It must be stressed, that every approximation of shield dimensions should be verified with more precise method, like Monte Carlo simulation. Any handy approximation might significantly reduce necessary amount of samples in trial-and-error approach, but cannot completely substitute computational methods.

2 Theory

Let us consider a group of mono-energetic collimated gamma radiation beams propagating in a homogeneous medium with linear attenuation factor μ. Intensity i of the beam after penetrating l distance is:

$$i = IBe^{-\mu l} \tag{1}$$

where I is the intensity at the beginning of the path and B is the build-up factor. Let us assume, that the beams came from the same source point and the original intensity varies depending on an azimuthal angle α between the beam path and a chosen axis:

$$I(\alpha) = I_0 f(\alpha) \tag{2}$$

where $f(\alpha)$ is a non-normalized[1] distribution function, where $f(0°) = 1$ and for every $\alpha > 90°$, $f(\alpha) = 0$. In this case equation (1) can be written as:

$$i(\alpha) = I(\alpha)B(\alpha)e^{-\mu l(\alpha)} = I_0 f(\alpha)B(\alpha)e^{-\mu l(\alpha)} \tag{3}$$

[1]It means that condition $\int_{\alpha_{min}}^{\alpha_{max}} f(\alpha)d\alpha = 1$ might not be fulfilled

Thus:

$$l(\alpha) = \frac{1}{\mu} \ln\left(\frac{I_0}{i(\alpha)} B(\alpha) f(\alpha)\right) = \frac{1}{\mu} \ln\left[D(\alpha) B(\alpha) f(\alpha)\right] \tag{4}$$

where $D(\alpha)$ is attenuation factor. If we mark $B(0°) = B_0$ and define relative build-up factor $b(\alpha)$:

$$b(\alpha) = \frac{B(\alpha)}{B_0} \tag{5}$$

equation (4) can be written as:

$$l(\alpha) = \frac{1}{\mu} \ln\left[D(\alpha) B_0 b(\alpha) f(\alpha)\right] = \frac{\ln[D(\alpha) B_0]}{\mu} + \frac{\ln[b(\alpha) f(\alpha)]}{\mu} \tag{6}$$

For $\alpha = 0°$, both $f(\alpha) = 1$ and $b(\alpha) = 1$:

$$l(0°) = l_0 = \frac{\ln[D(0°) B_0]}{\mu} + \frac{\ln(1)}{\mu} = \frac{1}{\mu} \ln[D(0°) B_0] \tag{7}$$

Taking a condition that $D(\alpha) = const$, equation (6) is basically an equation for path lengths, for which we obtain the same attenuation factor as for the beam alongside the axis of choice. Using (7) in (6), we receive:

$$l(\alpha) = l_0 + \frac{\ln[b(\alpha) f(\alpha)]}{\mu} = l_0 \left(1 + \frac{\ln[b(\alpha) f(\alpha)]}{\mu l_0}\right) \tag{8}$$

and also taking from (7) that $\mu l_0 = \ln(D B_0)$:

$$l(\alpha) = l_0 \left(1 + \frac{\ln[b(\alpha) f(\alpha)]}{\ln(D B_0)}\right) = l_0 \left(1 + \log_{D B_0}[b(\alpha) f(\alpha)]\right) \tag{9}$$

Now, consider projection of the path on the axis perpendicular to the axis of choice. The length of the projection $w(\alpha)$ is given by:

$$w(\alpha) = \sin(\alpha) l(\alpha) \tag{10}$$

$$w(\alpha) = \sin(\alpha) l_0 \left(1 + \log_{D B_0}[b(\alpha) f(\alpha)]\right) \tag{11}$$

And the ratio $k(\alpha)$ of length of the projection to the length of the path alongside the axis of choice is:

$$k(\alpha) = \frac{w(\alpha)}{l_0} = \sin(\alpha) \left(1 + \log_{D B_0}[b(\alpha) f(\alpha)]\right) \tag{12}$$

Finding maximum value of $k(\alpha)$ for particular attenuation factor (D) and material (B_0) will bring an answer, what should be minimum value of a shield width, if we want to ensure, that attenuation on backsides of the shield will be at least the same as at the main side (alongside the main stream of radiation) of the shield with thickness l_0.

To find $k_{max}(D, B_0)$, first, $b(\alpha)$ and $f(\alpha)$ must be considered. Let us assume a cosine source. Values of relative build-up factors depend on the shielding material properties, but conservatively could be approximated by linear function (fig. 1)

$$b(\alpha) \approx \frac{0.4\alpha}{90^\circ} + 1 \tag{13}$$

Eq. (13) leads to overestimated build-up factor values for heavy materials, like lead, but for purposes of finding the k_{max}, such overestimation is acceptable.

Solving eq. (12) analytically for $f(\alpha) = \cos(\alpha)$ leads to unwieldy solution. However, approximated solution can be obtained in more handy form with a numerical method. In the method, values in a range $0^\circ < \alpha < 90^\circ$ of functions (fig. 2) are calculated:

$$k_x(\alpha) = \sin(\alpha)\left(1 + \log_x[\left(\frac{0.4\alpha}{90^\circ} + 1\right)\cos(\alpha)]\right) \tag{14}$$

For every x, the greatest value of $k_x(\alpha)$ has been chosen as $k_{max}(x)$. It has occurred, that there is a good regression:

$$k_{max}(x) = 0.11\ln(\ln(x)) + 0.61 \tag{15}$$

Therefore, approximation for ratio between thickness and width of the shield for a cosine source is:

$$k(D, B_0) = 0.11\ln(\ln(DB_0)) + 0.61 \tag{16}$$

where D is requested attenuation factor and B_0 is the build-up factor in the main axis of the shield ($\alpha = 0^\circ$).

3 Validation

To verify corectness of the eq. (16), series of Monte Carlo simulation have been performed. Monte Carlo simulations have been run using OpenMC

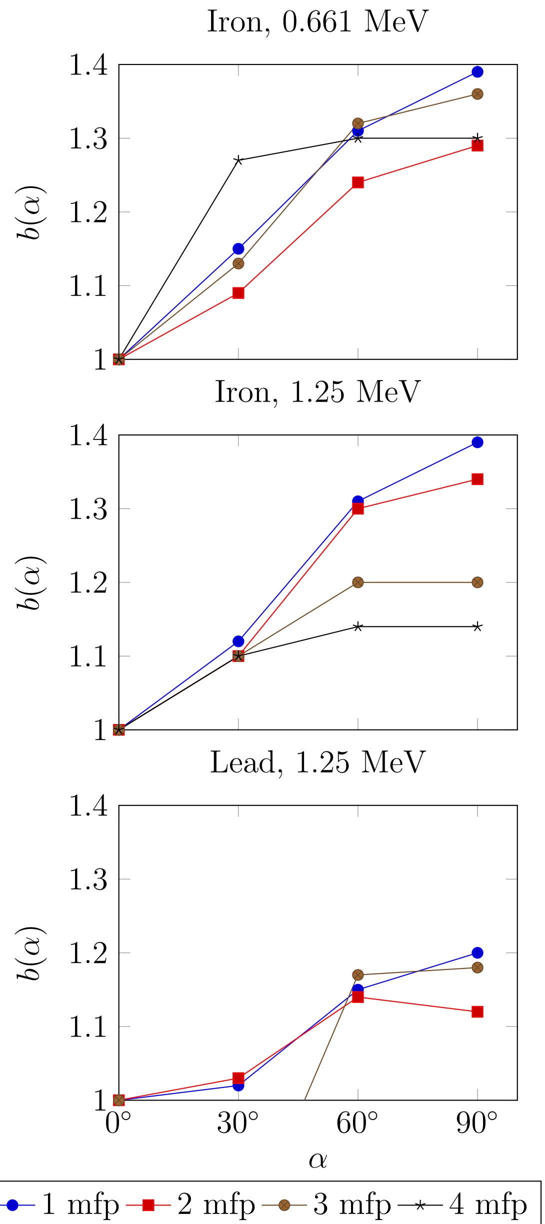


Figure 1: Calculated the relative build-up factors for the mono-energetic gamma cosine source in a shield depending on thickness of the shield and azimuthal angle; based on [3]

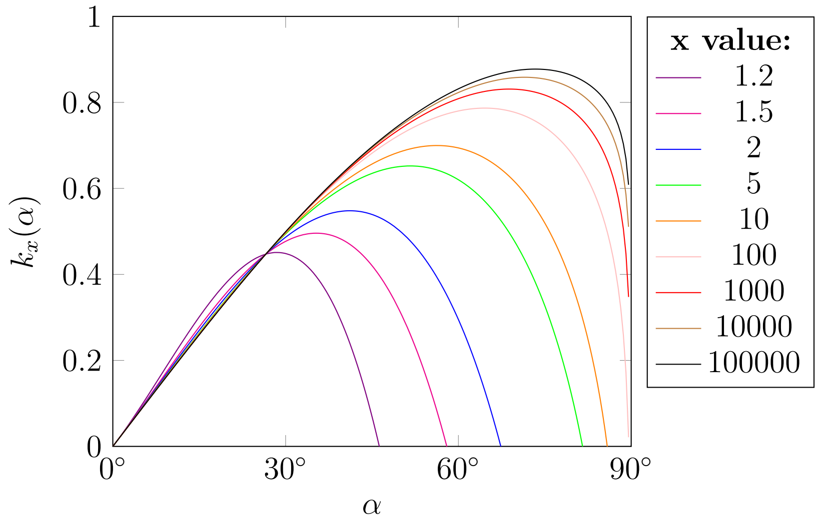


Figure 2: Plots of function $k_x(\alpha)$ for different x values

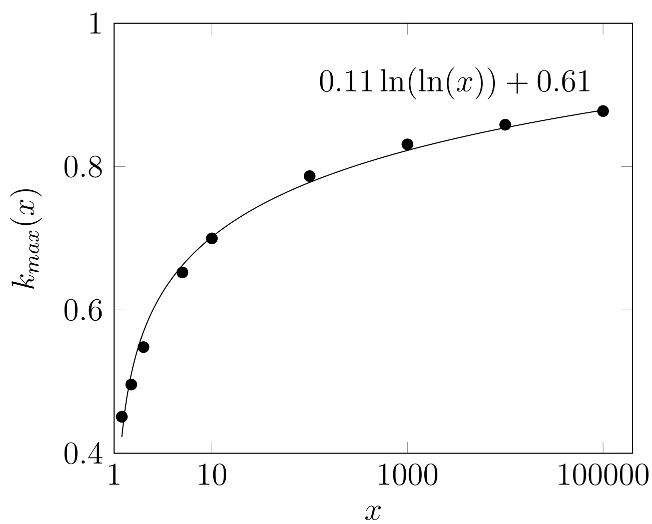


Figure 3: Regression of $k_{max}(x)$ function ($R^2 = 0.9916$)

software, version 0.12.1 [4], running on Linux Mint 19.1 with ENDF/B-VII.1 cross check library. Material composition has been taken from [5]. Python API code for the simulations are available on GitHub [6].

A cylindrical shield with radius r and thickness h made of concrete, iron or lead has been put on a point source emitting gamma radiation with one energy line – 661 keV (representing ^{137}Cs source), or with two energy lines – 1173 keV and 1332 keV with ratio 1:1 (representing ^{60}Co). Subsequently, the dose rates at the surface on the shield have been measured as a function of a gamma radiation flux multiplied by energy coefficients according to ICRP Publication 116 [7] for the isotropic irradiation geometry. The dose rates have been measured on mesh net with size 1 mm $\times$ 1 mm $\times$ 10 mm, where 10 mm is the dimension in direction perpendicular to the cross section surface. The obtained dose rates have been normalised to the value of the maximum dose rate at the frontside of the shield as a dimensionless ratio ρ. Results below 1 are indicating on overestimation of the shield's radius in regards to the shield's thickness (radiation at the top of the shield will be greater than on the backsides) and results above 1 are indicating underestimation (radiation at the backsides of the shield will be greater than on the top). The radius of the shield has been found using eq. (16):

$$r = hk(D, B_0) = h(0.11 \ln(\ln(DB_0)) + 0.61) \tag{17}$$

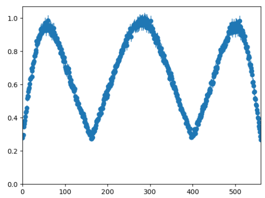
where D is requested attenuation factor and h is the thickness of the shield. Obtained results have been rounded up with accuracy to 1 mm.

The thickness has been found using Gamma Emitter Rad Pro Calculator [8], a popular simple engineering tool. The build-up factor B_0 might be found from various tables. However, in this case the build-up factors has been found also from Rad Pro Calculator as a ratio of dose rates after shielding for the same shield thickness and initial dose rate, with and without considering the build-up factor. For example, if for an iron shield with thickness 6.6 cm, the dose rate from ^{137}Cs will decrease from 100 mSv/h to 10 mSv/h (calculated with build factor), and for the same shield thickness and source, dose rate will decrease from 100 mSv/h to 2.2 mSv/h (calculated without build-up factor), it means that build-up factor for this case (source and thickness) is $10/2.2 = 4.55$.

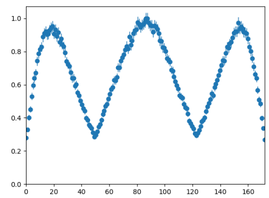
Results are presented in tab. 1 and on fig. 4. The normalized dose rates are mainly in a range 0.9-1.1, which shows acceptable overestimation or underestimation of the design, using the developed equation. Minimum (0.87 $\pm$

Source, Material	D	h [cm]	B_0	$k(D, B_0)$	r [cm]	ρ
^{137}Cs, Lead	5	1.6	1.43	0.68	1.1	1.07 ± 0.04 1.08 ± 0.04
^{137}Cs, Iron	5	4.9	3.33	0.72	3.6	0.95 ± 0.03 0.97 ± 0.03
^{137}Cs, Concrete	5	16.1	4.00	0.73	11.8	0.99 ± 0.03 0.98 ± 0.03
^{137}Cs, Lead	100	4.2	2.00	0.79	3.4	0.98 ± 0.06 0.88 ± 0.06
^{137}Cs, Iron	100	11.8	9.52	0.82	9.7	1.0 ± 0.1 0.9 ± 0.1
^{137}Cs, Concrete	100	40.2	17.86	0.83	33.5	0.87 ± 0.08 0.89 ± 0.08
^{60}Co, Lead	5	3.3	1.74	0.69	2.3	1.14 ± 0.08 1.07 ± 0.08
^{60}Co, Iron	5	6.6	3.26	0.72	4.8	1.06 ± 0.05 1.02 ± 0.05
^{60}Co, Concrete	5	21.0	3.85	0.73	15.4	0.94 ± 0.02 0.94 ± 0.02
^{60}Co, Lead	100	8.6	2.86	0.80	6.9	0.93 ± 0.04 0.97 ± 0.04
^{60}Co, Iron	100	16.4	10.00	0.82	13.5	0.97 ± 0.08 0.92 ± 0.08
^{60}Co, Concrete	100	51.7	11.36	0.82	42.7	0.9 ± 0.1 0.9 ± 0.1

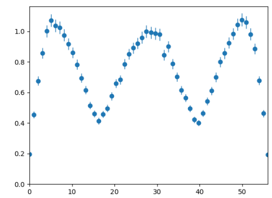
Table 1: Results of validation using Monte Carlo. Double results of ρ are for respectively left and right side of the shield



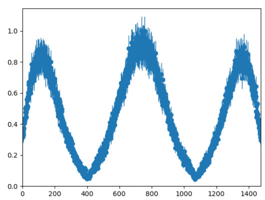
(a) ^{137}Cs, con., $D = 5$



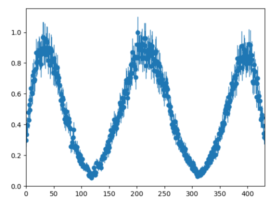
(e) ^{137}Cs, iron, $D = 5$



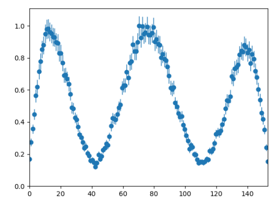
(i) ^{137}Cs, lead, $D = 5$



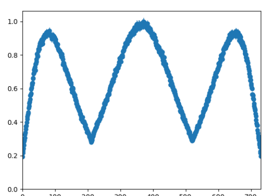
(b) ^{137}Cs, con., $D = 100$



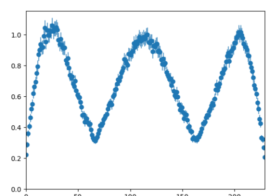
(f) ^{137}Cs, iron, $D = 100$



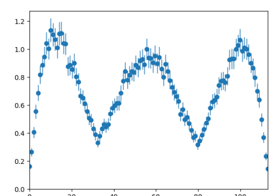
(j) ^{137}Cs, lead, $D = 100$



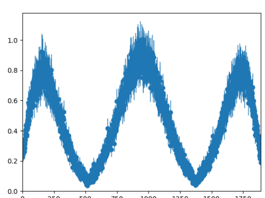
(c) ^{60}Co, con., $D = 5$



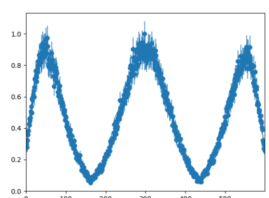
(g) ^{60}Co, iron, $D = 5$



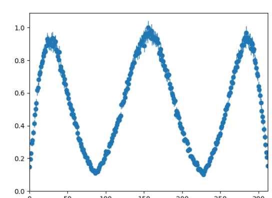
(k) ^{60}Co, lead, $D = 5$



(d) ^{60}Co, con., $D = 100$



(h) ^{60}Co, iron, $D = 100$



(l) ^{60}Co, lead, $D = 100$

Figure 4: Normalized dose rates at the surface of the shield. Visible three maxima represent maximum at the left backside, at the top, and at the right backside of the shield

0.08) have been obtained for left backside of a shield made of concrete with ^{137}Cs source. Maximum (1.14 ± 0.08) have been obtained for left backside of a shield made of lead with ^{60}Co source. The data trend shows, that the equation might lead to slightly overestimated width for light materials and sources with lower energies and to slightly underestimated width for heavier materials and sources with higher energies. However, all results within investigated range are considered as good.

4 Conclusion

Approximated handy equation for calculation ratio of width to length for a shield for cosine source has been found, and it is

$$k(D, B_0) = 0.11(\ln(\ln(DB_0)) + 0.61$$

where D is requested attenuation factor for the shield and B_0 is the build-up factor through the centre of the shield. The equation has been verified by series of Monte Carlo simulation for shields made of concrete, lead, and iron; for sources containing ^{137}Cs or ^{60}Co; and for attenuation factors $D = 5$ and $D = 100$. According to the simulations, for mentioned conditions the difference between maximum dose rates at the top of the shield and at the backsides of the shield were ca. 10%, which is considered as sufficient approximation for engineering design purposes.

References

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[6] Code on GitHub: `https://github.com/wsaczawa/MCcos/blob/main/width.py`

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