Introduction to Numerical Analysis and Applications

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Homework Bonus (**DUE**: June 11th)

This homework includes the materials from the past weeks, including *Numerical Integration,* and *Numerical Differentiation, and Ordinary Differential Equation.*

1. (10%) Derive the following centered finite-difference formula for 1st derivative with high-accuracy $O(h^4)$:

$$f'(x_i) = \frac{-f(x_{i+2}) + 8f(x_{i+1}) - 8f(x_{i-1}) + f(x_{i-2})}{12h}$$

2. Use the following data to find the velocity and acceleration at $t = 10$ seconds.

t (s)	0	2	4	6	8	10	12	14	16
x (m)	0	0.7	1.8	3.4	5.1	6.3	7.3	8.0	8.4

Apply the second-order correct for the following methods.

a. (10%) Forward finite-difference method.

b. (10%) Centered finite-difference method.

3. (30%) Find the approximation of the following integrals using Romberg Integration with a stopping criterion of 10^{-6}. Show how your Romberg Integration converge. Approximations are provided for your reference (*your answer should not be too different from the reference values!).*

a. $\int_0^1 x^x dx$ ($\simeq \mathbf{0.7834}$)

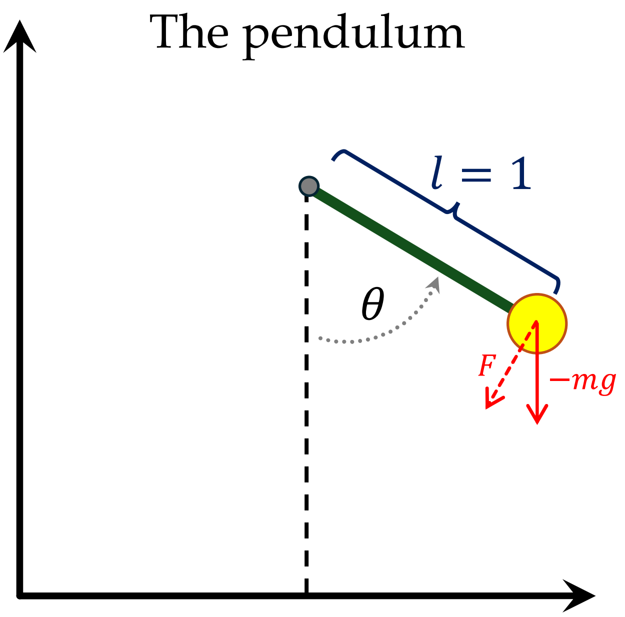
b. $\int_0^{\pi/2} \ln(\cos x + \sin x)\, dx$ ($\simeq \mathbf{0.3716}$)

Recall the Romberg algorithm iteration (slide #44) for numerical integration:

$$I_{j,k} = \frac{4^{k-1} \times I_{j,k-1} - 1 \times I_{j,k-1}}{4^{k-1} - 1}$$

where j is the index to the segments ($N = 2^j$) used for the integration, and k represents the order of the approximation.

4. (40%) **Simple harmonic motion**. Newton's second law of motion, $F = ma$, can be used to resolve the motion of a pendulum. A pendulum is bonded to a rod of length l, and therefore moves along a circle of radius l. Let θ be the angle (in radian) of the pendulum with respect to the vertical axis, then the tangential component of the acceleration is $l\theta''$. The force along the direction of motion is $-mg\sin\theta$. The differential equation is therefore $ml\theta'' = F = -mg\sin\theta$.



Apply the Euler's method to simulate the motion of our pendulum in terms angle (θ) and the angular velocity ($\theta' = d\theta/dt$) with the initial conditions of $\theta(0) = \frac{\pi}{2}$ and $\theta'(0) = 0$. The gravitational acceleration $g = 9.81$ m/sec^2 and the length of the rod $l = 1$. To start, you can set $y_1 = \theta$ and $y_2 = \theta'$ and solve the system of equations. Use (a) a step size of 0.01 second and (b) a step size of 0.001 second to simulate the motion for over 12 seconds. Show $\theta(t)$ and $\theta'(t)$ in a <u>Figure</u> and compare your results. *You can see how errors accumulate along with the calculation.*

(**BONUS** 15%) Redo the simulation with RK4.