Course Curriculum for Geological Modeling

Fall 2022 - 01:460:418/16:460:611

Instructors:

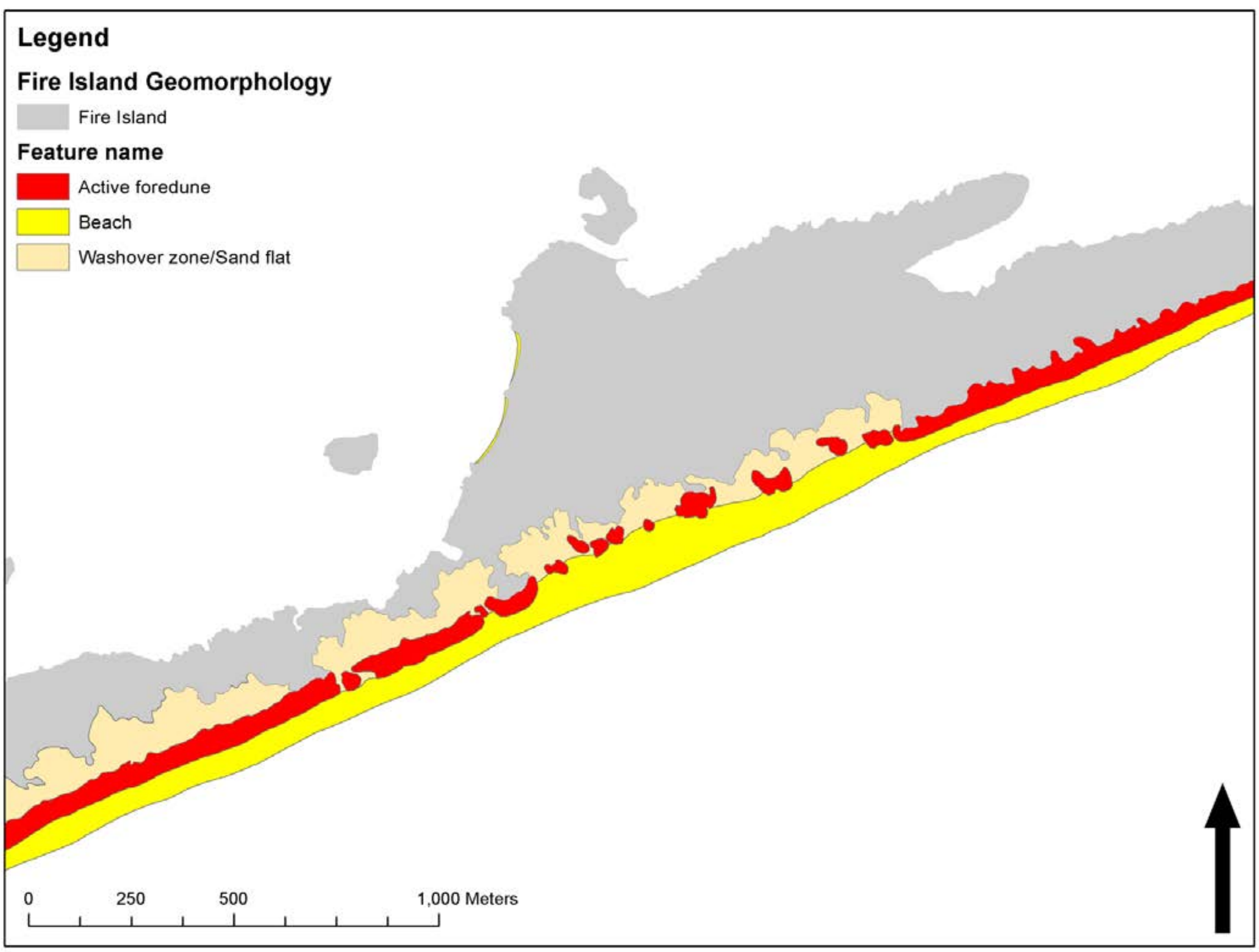
- **W. John Schmelz**: wjs107@eps.rutgers.edu
- **Professor Lauren Neitzke Adamo**: Lneitzke@eps.rutgers.edu

Department of Earth and Planetary Sciences
Rutgers, The State University of New Jersey

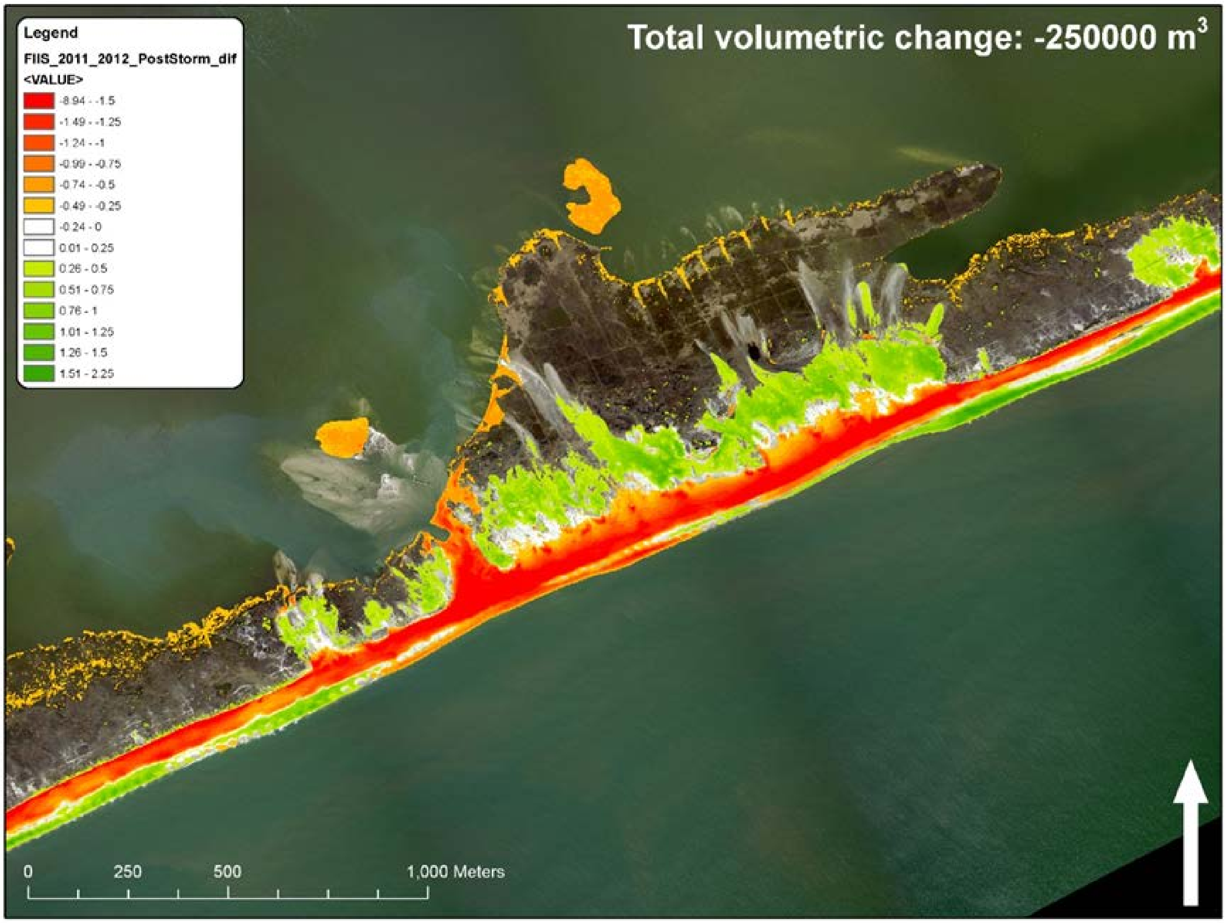
1. Introduction to ArcGIS and spatial analysis

Instructions for application here (https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Documents/SpatialAnalysis.pdf), data and code available on request (1.5 GB).

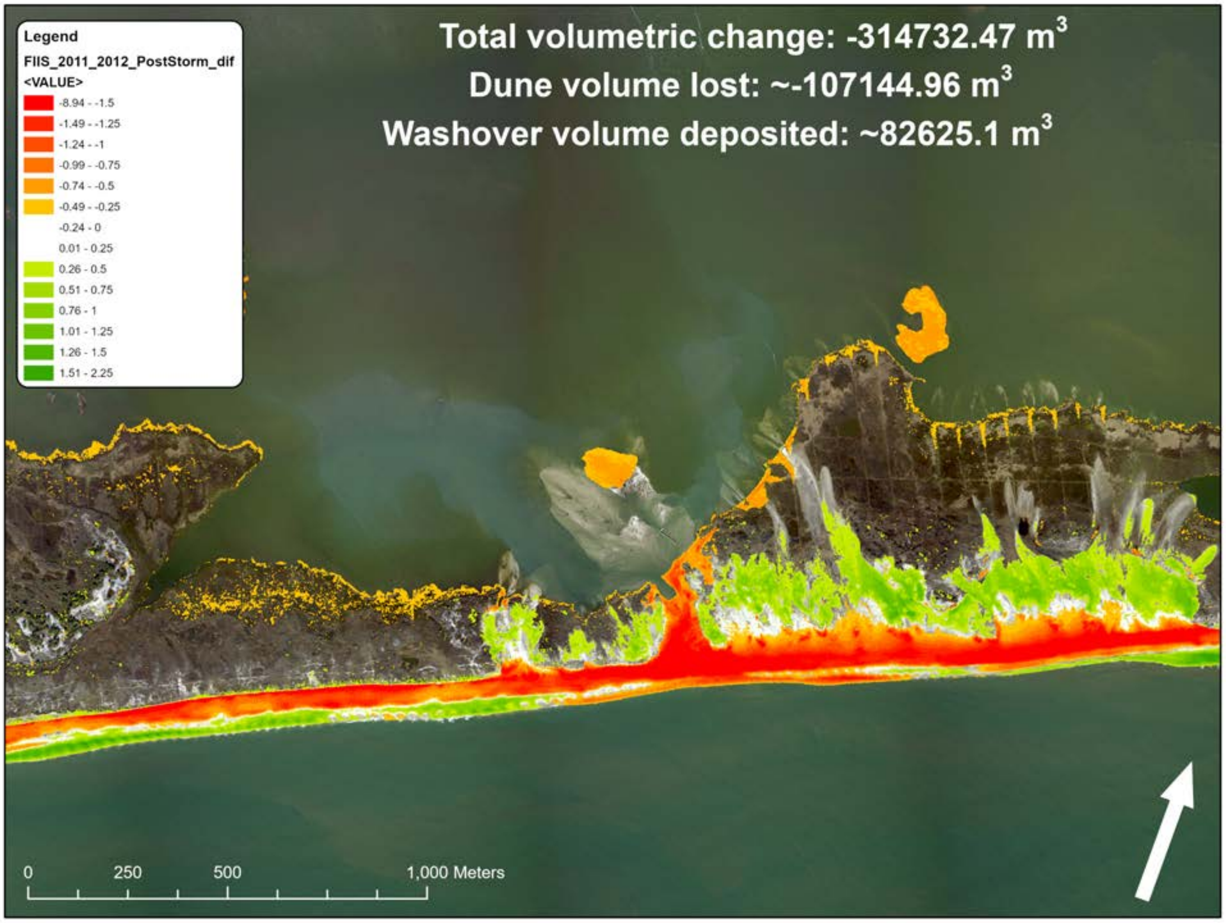
1.1 ArcGIS - Importing data, setting symbology, creating and editing a shapefile, and making a map



1.2 Spatial analysis in ArcGIS applied to geomorphological map of Fire Island, NY



1.3 Programmatically quantify geomorphological change occuring due to Superstorm Sandy at Fire Island using geomorpholgical maps, lidar, and ArcPy

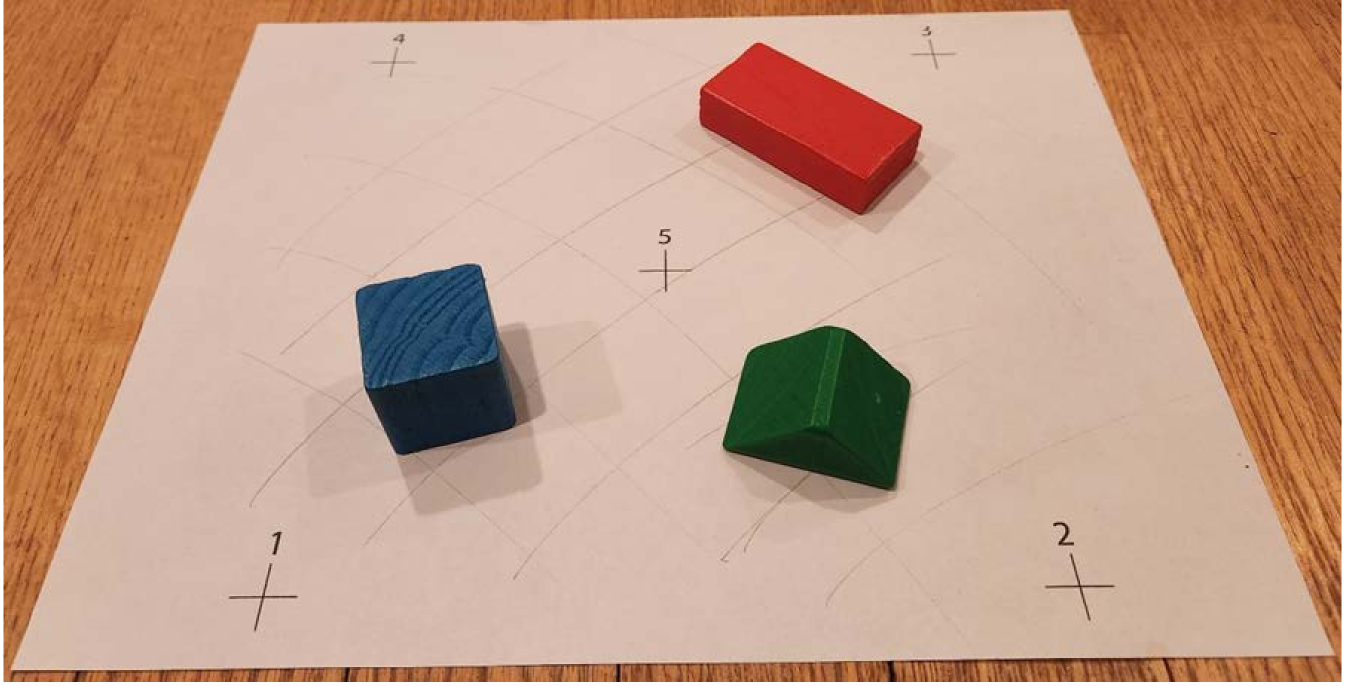


2. Photogrammetry with Pix4D

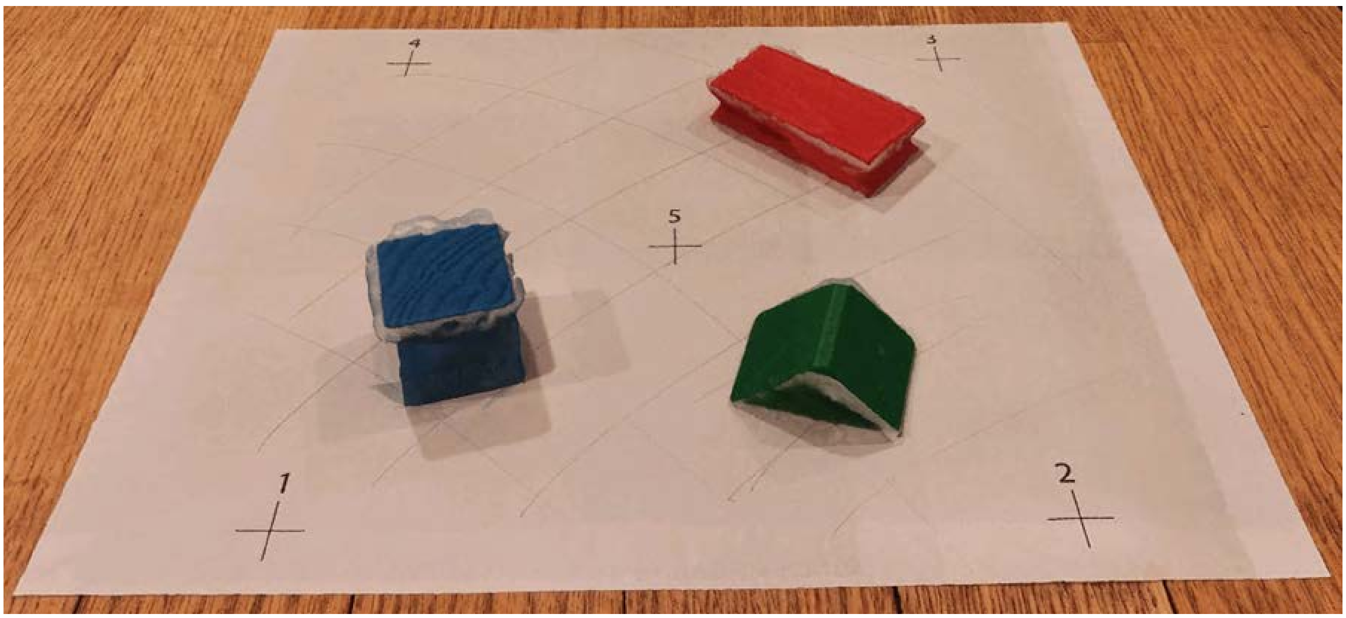
Instructions for application here (https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Documents/DronePhotogrammetry.pc
Data for part 2 available on request (121 MB).

2.1 Measurement of dimensions of toy blocks from photogrammetric model created using cell phone photos

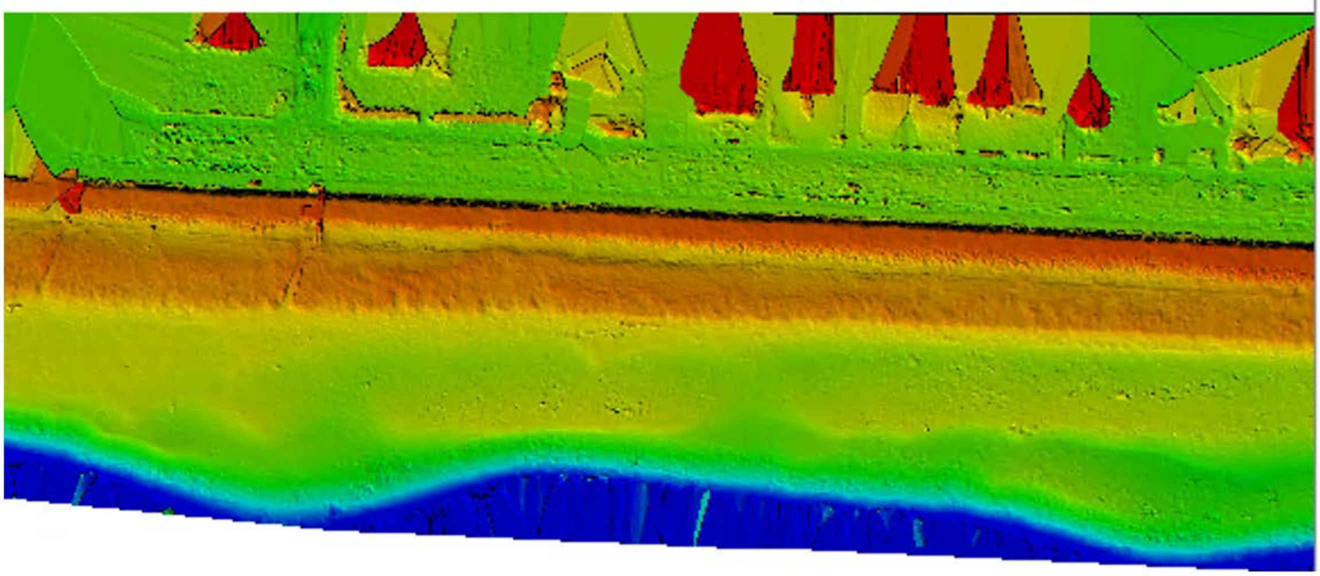
Real image:



3D model:



2.2 Photogrammetric model of beach and dune in Monmouth Beach, NJ



3. Python basics - data types, indexing, plotting

3.1 Introduction to Python

This is the first class we will learn to apply Python. In applying the variety of computational methods we will learn in this course, try to keep in mind that we are largely a group of earth scientists with expertise in that domain. I.e., we arent and dont have to be computer scientists, mathematicians, or statisticians. However, in familiarizing ourselves with data analysis methods, hopefully we can apply computational and statistical techniques in conjuction with our subject matter knowledge to help advance our research and better prepare us to evaluate and understand results produced within the earth science community considering the application of these computational methods is becoming more and more common.

In this class, we will use Python to make calculations and store/visualize the results. Python is a "higher-level" programming language. This simply means that the "lower-level" tasks, that concern directly communicating processes to the hardware we are working with, are integrated into the Python program. As a result, we can type natural language commands that initiate computations. For example, we can set a variable name, assign it a value, and print it using the lines of code in the cell below.

In [67]:
```
a = "YOUR_FIRST_NAME"
print (a)
```

```
YOUR_FIRST_NAME
```

There are certain rules associated with each command that instruct the Python interpreter to produce an output. For example, the "print" function allows us to create an output that catenates multiple input values.

In [68]:
```
b = "YOUR_LAST_NAME"
print (a,b)
```

```
YOUR_FIRST_NAME YOUR_LAST_NAME
```

The two variables, "a" and "b", are "objects" in Python. Objects are scalar or non-scalar, and each object is characterized by a specific data type. Variables "a" and "b" are comprised of a series of letters of text, and they are assigned a "string" data type. We can request the data type of any variable:

In [69]:
```
print(type(a))
print(type(b))
```

```
<class 'str'>
<class 'str'>
```

Strings are a non-scalar object. That simply means that they have an internal structure that can be indexed or segregated. Lists are a second type of non-scalar object. We can create a list, "c", from variables "a" and "b".

In [70]:
```
c = [a,b]
print(c)
```

```
['YOUR_FIRST_NAME', 'YOUR_LAST_NAME']
```

The list "c", and the strings that comprise it can be subdivided using an index that is specified within brackets. The first item in a nonscalar object, whether it is a list item or a letter in a string, is referenced using the index value "0". The nth item by integer n-1. A series of items can be called using a colon, e.g., c[0:n] calls the first n items in object "c."

In [71]:
```
print('index of "YOUR_FIRST_NAME":',c.index('YOUR_FIRST_NAME'))
print('item 1 in list "c":',c[0])
print('first letter in item 1 in list "c":',c[0][0])
print('index of "YOUR_LAST_NAME":',c.index('YOUR_LAST_NAME'))
print('item 2 in list "c":',c[1])
print('first four letters in item 2 in list "c":',c[1][0:4])
```

```
index of "YOUR_FIRST_NAME": 0
item 1 in list "c": YOUR_FIRST_NAME
first letter in item 1 in list "c": Y
index of "YOUR_LAST_NAME": 1
item 2 in list "c": YOUR_LAST_NAME
first four letters in item 2 in list "c": YOUR
```

So far, we have dealt exclusively with non-scalar objects. The scalar objects are indivisible, in the sense that the objects themselves cannont be subdivided. There are four types of scalar objects including two types of numbers, integers and floating point numbers. The other scalar data types are 'bool' and 'None'. Bolean values refer to "True" or "False", and 'none' is an item with no value at all, which is a value in itself and the only value in the 'none' data type.

Integers are represented by the 'int' data type. Floating point numbers, that comprise the data type 'float', are real numbers and are always represented with a decimal point in Python. Integers can be added, subtracted, multiplied, or divided by other integers and can maintain an integer data type, but will convert to a 'float' when an operation is applied with both integers and floating point numbers.

In [72]:
```
#data type of numbers
d = 3
e = 4
f = 4.

print(e,type(e))
print(f,type(f))
```

```
4 <class 'int'>
4.0 <class 'float'>
```

In [73]:
```
#data type of output of multiplication expressions
g = d*e
h = d*f

print(d,"*",e,"=",g,type(g))
print(d,"*",f,"=",h,type(h))
```

```
3 * 4 = 12 <class 'int'>
3 * 4.0 = 12.0 <class 'float'>
```

In [74]:
```
#data type of output of division expressions
i = g//d
j = g/d
k = e//d
l = e/d

print(g,"//",d,"=",i,type(i))
print(g,"/",d,"=",j,type(j))

print(e,"//",d,"=",k,type(k))
print(e,"/",d,"=",l,type(l))

#note the division of the integers either gives a float if using the '/' operator and th
#with no remainder using the "//" operator. use "%" to obtain the remainder of the calcu
m = e%d
print(e,"%",d,"=",m,type(m))
```

```
12 // 3 = 4 <class 'int'>
12 / 3 = 4.0 <class 'float'>
4 // 3 = 1 <class 'int'>
4 / 3 = 1.3333333333333333 <class 'float'>
4 % 3 = 1 <class 'int'>
```

In [75]:
```
#order of operations is maintained

n = d+f*d
o = (d+f)*d
print (n)
print (o)
```

```
15.0
21.0
```

We can make non-scalar objects, like a list, out of a series of scalar objects. Or a list of a mix of data types.

In [76]:
```
p = [a,c,d,f]
print (p)
print (type(p[0]))
print (type(p[1]))
print (type(p[2]))
print (type(p[3]))
```

```
['YOUR_FIRST_NAME', ['YOUR_FIRST_NAME', 'YOUR_LAST_NAME'], 3, 4.0]
<class 'str'>
<class 'list'>
<class 'int'>
<class 'float'>
```

We can test whether a statement is true using the boolean data type.

In [77]:
```
r = e==f
s = type(e)==type(f)

print(r)
print(s)
```

```
True
False
```

And a variable with no value carries the 'none' data type.

In [78]:
```
t = None
print (type(t))
```

```
<class 'NoneType'>
```

These concepts lead toward the formulation of simple programs or algorithms. An algorithm to derive the square root of a number can be derived. There are a couple of options for iteration in Python, and one is the "while" function. It tests whether a statement is true, i.e., returns a Boolean value of True and will continue to iterate as long as that is the case.

In [79]:
```
u = 20. #test value
v = 5. #initial value

while abs(u - v * v) > .01:
    v = (v + u/v)/2

print ("square root of",u,"is",v)
print (v**2.)
```

```
square root of 20.0 is 4.472222222222222
20.000771604938272
```

We could also use a for loop with a specific number of iterations

In [80]:
```
u = 20. #test value
v = 5. #initial value
i_2 = 10 #iterations

for index in range(0,i_2):
    v = (v + u/v)/2

print (v)
print (v**2.)
```

```
4.47213595499958
20.000000000000004
```

3.2 Working with data in Python - plotting, indexing, interpolation

3.2.1 Reading, writing, and indexing datafiles

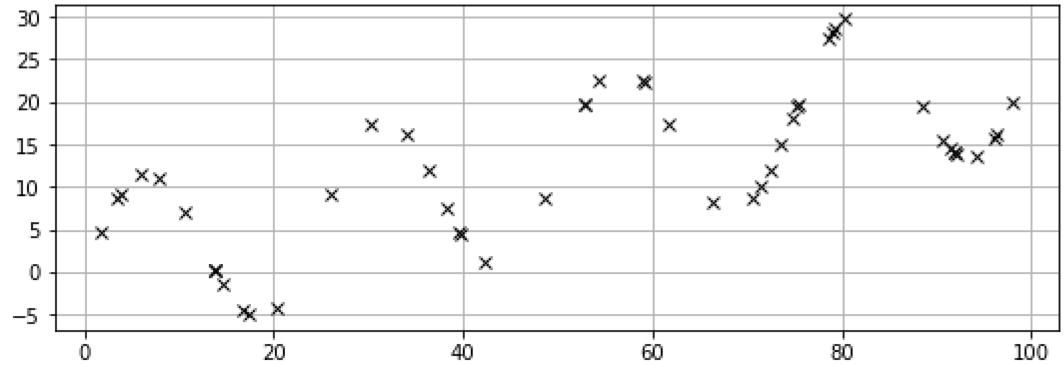
3.2.1.1 Create data and save as a text datafile

In [81]:
```
import numpy

#create data x and y

x = numpy.random.uniform(0,100,50)
y = 0.25 * x + 10*numpy.sin(2.*numpy.pi*x/25)

#create a 2D "array" to save data in
#see https://numpy.org/doc/stable/reference/generated/numpy.zeros.html

output = numpy.zeros((len(x),2))

#place x data into column 1 of the new matrix
output[:,0] = x

#place y data into column 2 of the new matrix
output[:,1] = y

#save as a text file
#fmt specifies that data values are float type and have 3 decimal places
#https://numpy.org/doc/stable/reference/generated/numpy.savetxt.html

filename = "sine_curve.txt"
numpy.savetxt(filename, output, fmt='%0.3f', delimiter=',')
```

3.3.1.2 Read text datafile and index data

This code uses a data file similar to the one created and saved as a text file in section 3.1. Here it is downloaded from Github so this code can run independently from other cells in this notebook. The script here can be applied to read any text file you have saved in your local directory.

In [82]:
```
import matplotlib.pyplot as plt
import numpy
import requests

#Download datafile and save to local directory as 'sine_curve.txt'
url = 'https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Data/Nonunifor
sine_curve = requests.get(url)

with open('sine_curve.txt', 'w') as f:
    f.write(sine_curve.text)

#read datafile, set comma as the delimiter
filename = "sine_curve.txt"
data = numpy.genfromtxt(filename,delimiter=',')

#make first data column the x values
x = data[:,0]

#make first data column the y values
y = data[:,1]

#create a figure, 9 inches by 9 inches
fig = plt.figure(1,figsize=(9,3))

#create a subplot and plot the x and y data
ax1 = plt.subplot(1,1,1)
ax1.plot(x,y,marker="x",linewidth=0,color="k")

ax1.grid()
plt.show()
```



3.3.1.3 Index data (cont.)

Determine the shape of an array. Count the number of elements in an array.

This code uses a data file similar to the one created and saved as a text file in section 3.1. It is downloaded from Github so this code can run independently from other cells in this notebook.

In [83]:
```
import matplotlib.pyplot as plt
import numpy
import requests

#Download datafile and save to local directory as 'sine_curve.txt'

url = 'https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Data/Nonunifor
sine_curve = requests.get(url)

with open('sine_curve.txt', 'w') as f:
    f.write(sine_curve.text)

#read datafile, set comma as the delimiter

filename = "sine_curve.txt"
data = numpy.genfromtxt(filename,delimiter=',')

#determine shape of data array
print("Shape of array:")
print (numpy.shape(data))

#determine length of a 1-dimensional array, i.e., column 1 of the data file
#this determine the number of values in a vector
print("Length of vector:")
print (len(data[:,0]))
```

```
Shape of array:
(50, 2)
Length of vector:
50
```

3.3.1.4 Index data (cont.)

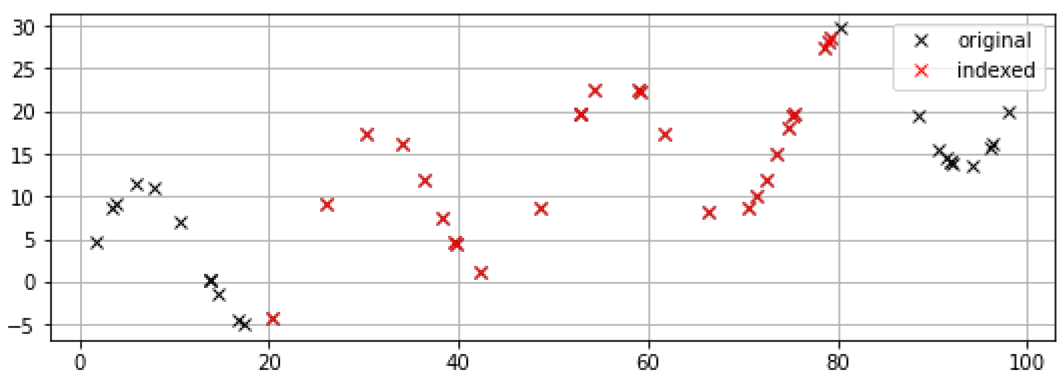
Find where dataset meets specified criteria. Here, where x values are **greater than 20 and less than 80**.

This code uses a data file similar to the one created and saved as a text file in section 3.1. It is downloaded from Github so this code can run independently from other cells in this notebook.

In [84]:
```
import matplotlib.pyplot as plt
import numpy
import requests
```

```
#Download datafile and save to local directory as 'sine_curve.txt'

url = 'https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Data/Nonunifor
sine_curve = requests.get(url)

with open('sine_curve.txt', 'w') as f:
    f.write(sine_curve.text)

#read datafile, set comma as the delimiter

filename = "sine_curve.txt"
data = numpy.genfromtxt(filename,delimiter=',')

#find where values in column 1 are greater than 20 and less than 80

w1 = numpy.where((data[:,0]>20)&(data[:,0]<80))[0]

# print the number of values where this condtion is true
print("Number of values greater than 20 and less than 80:")
print(len(w1))

#create a figure, 9 inches by 9 inches
fig = plt.figure(1,figsize=(9,3))

#create a subplot
ax1 = plt.subplot(1,1,1)

#plot the original x,y data
ax1.plot(data[:,0],data[:,1],marker="x",linewidth=0,color="k",label = "original")

#plot the indexed x,y data, with x values between 20 and 80
ax1.plot(data[w1,0],data[w1,1],marker="x",linewidth=0,color="r",label = "indexed")

ax1.legend()
ax1.grid()
plt.show()
```

```
Number of values greater than 20 and less than 80:
27
```



3.3.1.5 Index data (cont.)

Find where dataset meets specified criteria. Here, where x values are **less than 20 or greater than 80**.

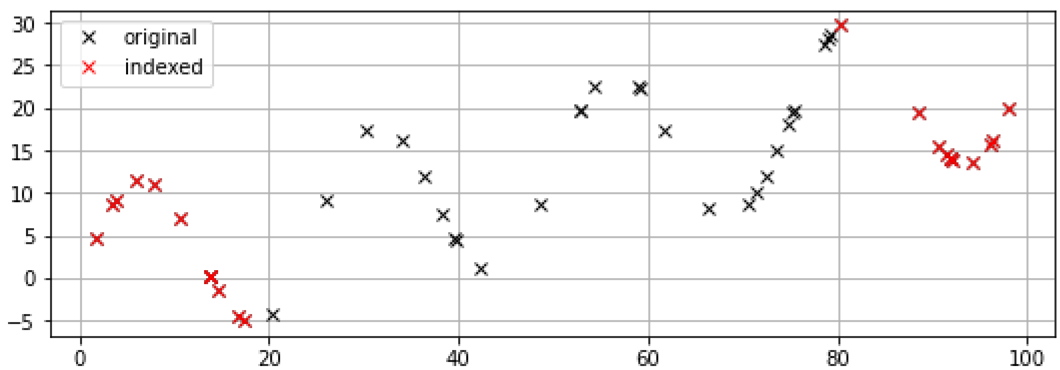
This code uses a data file similar to the one created and saved as a text file in the previous cell. It is downloaded from Github so this code can run independently from other cells in this notebook.

In [85]:

```
import matplotlib.pyplot as plt
import numpy
import requests
```

```
#Download datafile and save to local directory as 'sine_curve.txt'

url = 'https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Data/Nonunifor
sine_curve = requests.get(url)

with open('sine_curve.txt', 'w') as f:
    f.write(sine_curve.text)

#read datafile, set comma as the delimiter

filename = "sine_curve.txt"
data = numpy.genfromtxt(filename,delimiter=',')

#find where values in column 1 are greater than 20 and less than 80

w1 = numpy.where((data[:,0]<20) | (data[:,0]>80))[0]

# print the number of values where this condtion is true
print("Number of values less than 20 or greater than 80:")
print(len(w1))

#create a figure, 9 inches by 9 inches
fig = plt.figure(1,figsize=(9,3))

#create a subplot
ax1 = plt.subplot(1,1,1)

#plot the original x,y data
ax1.plot(data[:,0],data[:,1],marker="x",linewidth=0,color="k",label = "original")

#plot the indexed x,y data, with x values between 20 and 80
ax1.plot(data[w1,0],data[w1,1],marker="x",linewidth=0,color="r",label = "indexed")

ax1.legend()
ax1.grid()
plt.show()
```

```
Number of values less than 20 or greater than 80:
23
```

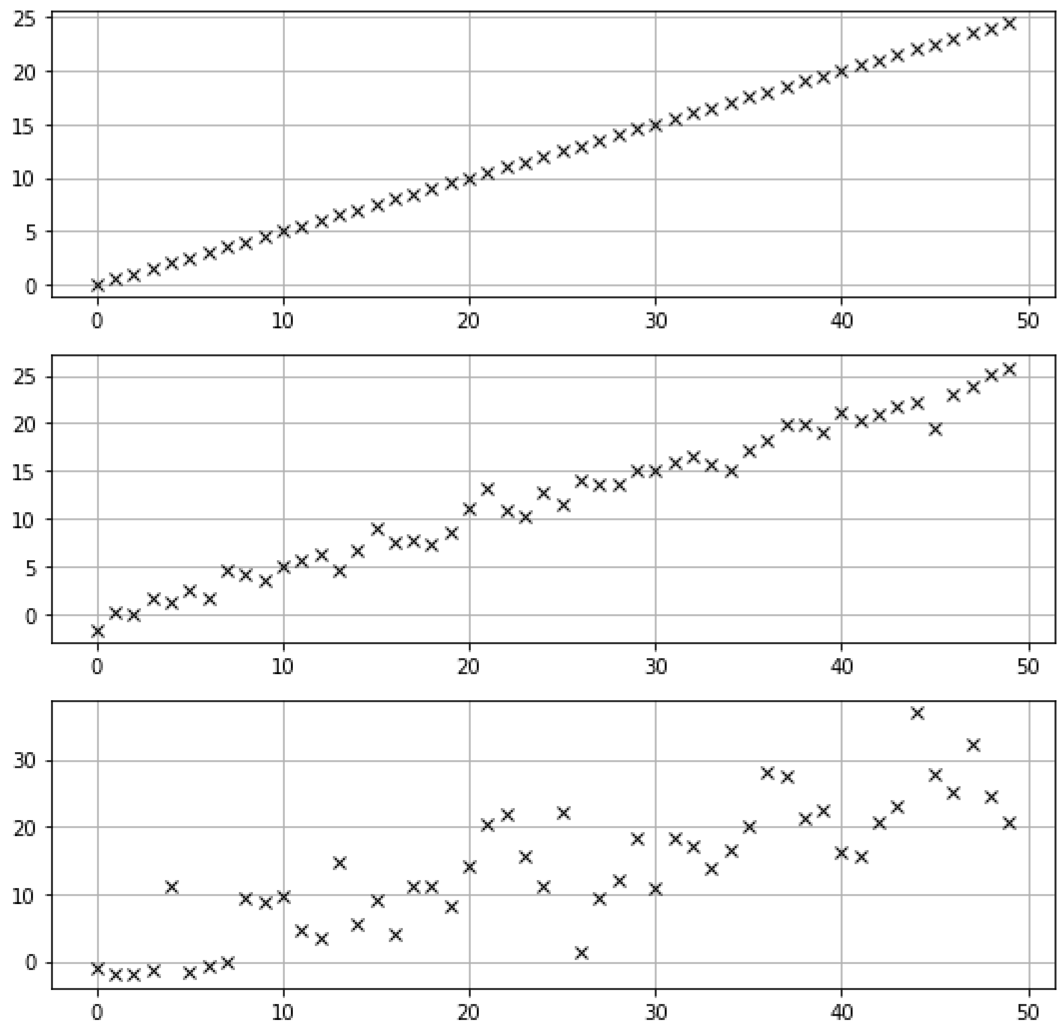


3.2.2 Plotting data

3.2.2.1 Create subplots

In [86]:
```
import matplotlib.pyplot as plt
import numpy

#create data x and y1, y2, and y3

x = numpy.arange(0,50,1)
y1 = (x * 0.5)
y2 = (x * 0.5) + numpy.random.normal(0,1,len(x))
```

```python
y3 = (x * 0.5) + numpy.random.normal(0,5,len(x))

#create a figure, 9 inches by 9 inches
fig = plt.figure(1,figsize=(9,9))

#create top subplot and plot y1
ax1 = plt.subplot(3,1,1)
ax1.plot(x,y1,marker="x",linewidth=0,color="k")
ax1.grid()

#create middle subplot and plot y2
ax2 = plt.subplot(3,1,2)
ax2.plot(x,y2,marker="x",linewidth=0,color="k")
ax2.grid()

#create lower subplot and plot y3
ax3 = plt.subplot(3,1,3)
ax3.plot(x,y3,marker="x",linewidth=0,color="k")
ax3.grid()

plt.show()
```



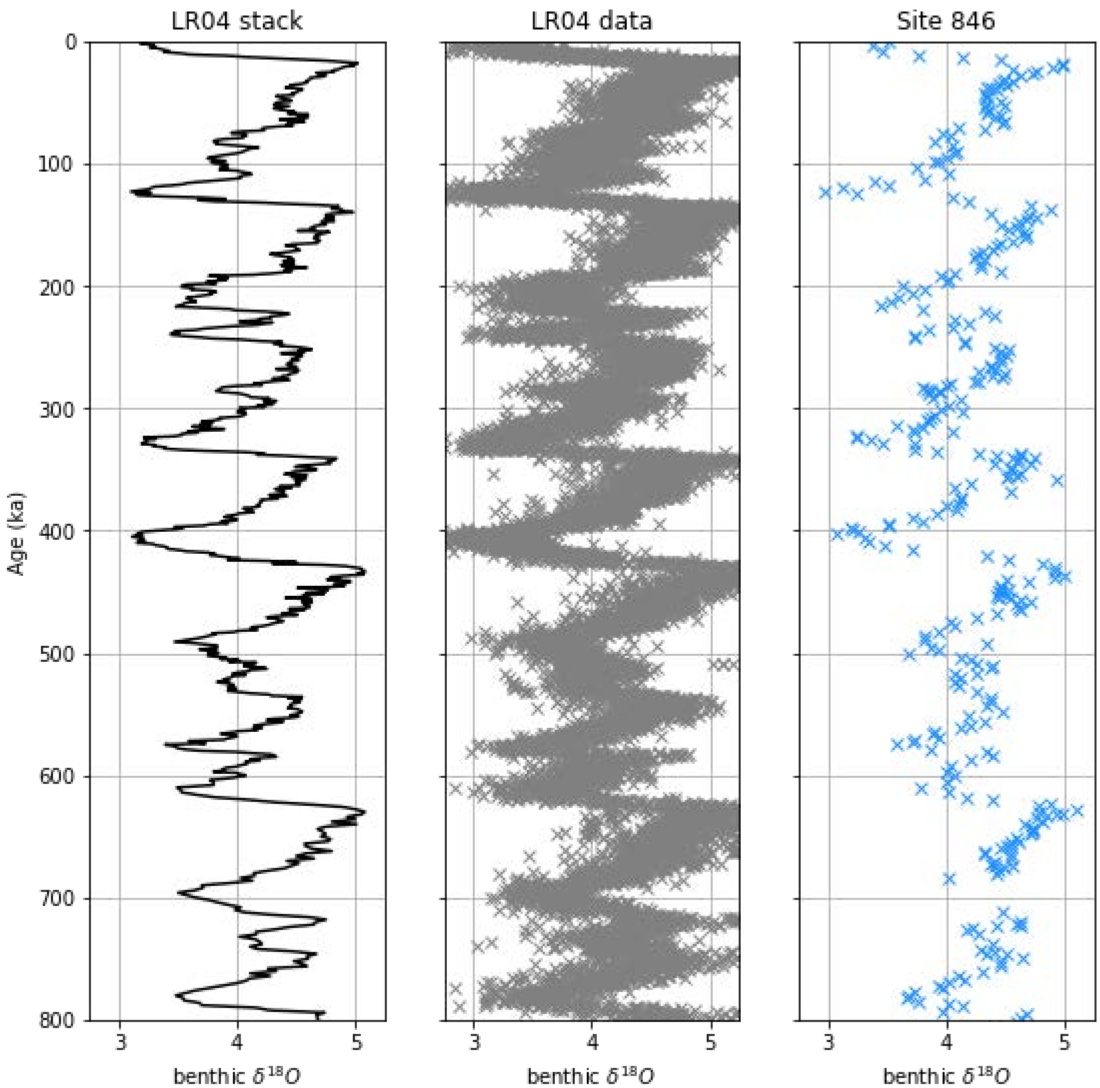
3.2.2.2 Plot data with mathematical expressions in labels

In [87]:

```python
import matplotlib.pyplot as plt
import numpy
import requests

#Download LR04 d18O stack datafile and save to local directory

url = 'https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Data/d18O/00_L
```

```
LR04 = requests.get(url)

with open('00_LR04.csv', 'w') as f:
    f.write(LR04.text)

#read datafile, set comma as the delimiter

filename = '00_LR04.csv'
LR04 = numpy.genfromtxt(filename,delimiter=',')

#Download datafile with all the LR04 d18O data and save to local directory

url = 'https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Data/d18O/01_L
LR_original_data = requests.get(url)

with open('01_LR_original_data.csv', 'w') as f:
    f.write(LR_original_data.text)

#read datafile, set comma as the delimiter

filename = "01_LR_original_data.csv"
LR04_orig = numpy.genfromtxt(filename,delimiter=',')

#Download datafile with the d18O data from ODP leg 138 site 846 and save to local direct

url = 'https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Data/d18O/02_O
ODP_138_846_d18O = requests.get(url)

with open('02_ODP_138_846_d18O.csv', 'w') as f:
    f.write(ODP_138_846_d18O.text)

#read datafile, set comma as the delimiter

filename = "02_ODP_138_846_d18O.csv"
ODP_138_846 = numpy.genfromtxt(filename,delimiter=',')

#plot data

fig = plt.figure(1,figsize=(9,9))

ax1 = plt.subplot(131)
ax1.plot(LR04[:,1],LR04[:,0],color="k")

ax1.set_ylim(800,0)
ax1.set_xlim(2.75,5.25)
ax1.grid()
ax1.set_title("LR04 stack")
ax1.set_ylabel("Age (ka)")
ax1.set_xlabel("benthic $\delta^{18}O$")

ax2 = plt.subplot(132)
ax2.plot(LR04_orig[:,1],LR04_orig[:,0],linewidth=0.0,marker='x',color="gray")

ax2.set_ylim(800,0)
ax2.set_xlim(2.75,5.25)
ax2.grid()
ax2.set_title("LR04 data")
ax2.set_yticklabels([])
ax2.set_xlabel("benthic $\delta^{18}O$")

ax3 = plt.subplot(133)
ax3.plot(ODP_138_846[:,1],ODP_138_846[:,0],linewidth=0.0,marker='x',color="dodgerblue")

ax3.set_ylim(800,0)
ax3.set_xlim(2.75,5.25)
ax3.grid()
```

```
ax3.set_title("Site 846")
ax3.set_xlabel("benthic $\delta^{18}O$")
ax3.set_yticklabels([])

plt.show()
```



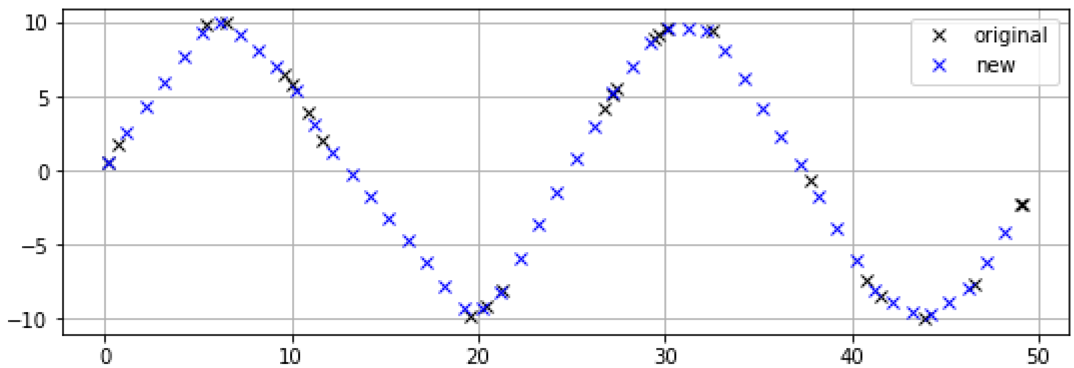
3.2.3 Interpolate data

3.2.3.1 Linear interpolation

In [88]:

```
import matplotlib.pyplot as plt
import numpy
import scipy
from scipy import interpolate

#create data x and y

x = numpy.random.uniform(0,50,25)
y = 10*numpy.sin(2.*numpy.pi*x/25)

#interpolate data, x and y

f = scipy.interpolate.interp1d(x, y)

#create new evenly spaced x data points between the minimum value
#and the maximum value in vector x

x_new = numpy.arange(numpy.min(x),numpy.max(x),1)
```

```
#create new interpolated y data points
#at the locations of the new evenly spaced x data points

y_new = f(x_new)

#create a figure, 9 inches by 9 inches
fig = plt.figure(1,figsize=(9,3))

#create a subplot. plot the original x and y data
#and the new x and y data
ax1 = plt.subplot(1,1,1)
ax1.plot(x,y,marker="x",linewidth=0,color="k",label="original")
ax1.plot(x_new,y_new,marker="x",linewidth=0,color="b",label="new")

ax1.legend()
ax1.grid()
plt.show()
```

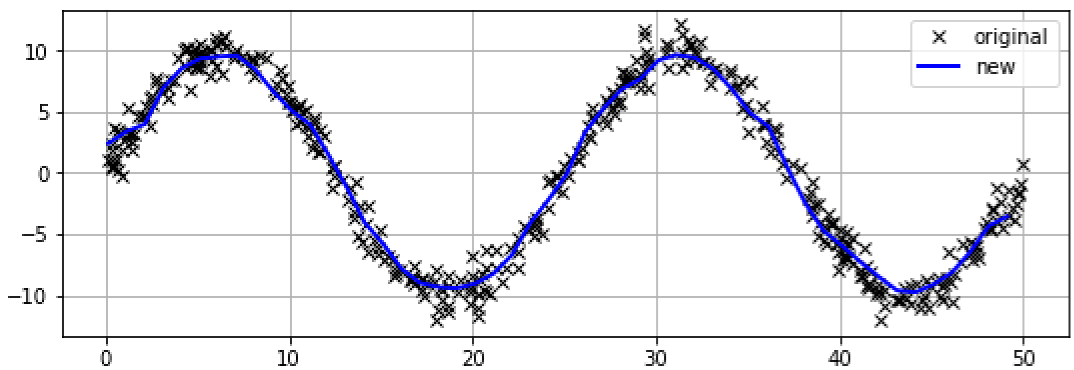


3.2.3.2 Moving average interpolation

The moving average function pulled for this script is available either on Github (https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Scripts/m_avg.py) or in the interpolation Jupyter Notebook we covered in week 4. Works best with a densely populated dataset.

In [89]:
```
import sys
import matplotlib.pyplot as plt
import numpy
import requests

##dowloaad moving average algorithm as a python script from Github this is the same scri
#in the Jupyter Notebook we went over in week 4

url = 'https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Scripts/m_avg.
m_avg_py = requests.get(url)

with open('m_avg_py.py', 'w') as f:
    f.write(m_avg_py.text)

import m_avg_py

#create data x and y

x = numpy.random.uniform(0,50,400)
y = 10*numpy.sin(2.*numpy.pi*x/25) + numpy.random.normal(0,1,len(x))

#create new evenly spaced x data points between the minimum value
#and the maximum value in vector x

x_new = numpy.arange(numpy.min(x),numpy.max(x),1)
```

```
#interpolate data and create new interpolated y data points
#at the locations of the new evenly spaced x data points
#using specified parameters m_avg(x_new,x_orig,y_orig,span)

#set new x values
x_new = x_new*1.0

#set x values to interpolate
x_orig = x*1.0

#set y values to interpolate
y_orig = y*1.0

#set window length
span = 2.

y_new = m_avg_py.m_avg(x_new,x_orig,y_orig,span)

#create a figure, 9 inches by 9 inches
fig = plt.figure(1,figsize=(9,3))

#create a subplot. plot the original x and y data
#and the new x and y data interpolated using moving average algorithm
ax1 = plt.subplot(1,1,1)
ax1.plot(x,y,marker="x",linewidth=0,color="k",label="original")
ax1.plot(x_new,y_new,linewidth=2,color="b",label="new")

ax1.legend()

ax1.grid()
plt.show()
```

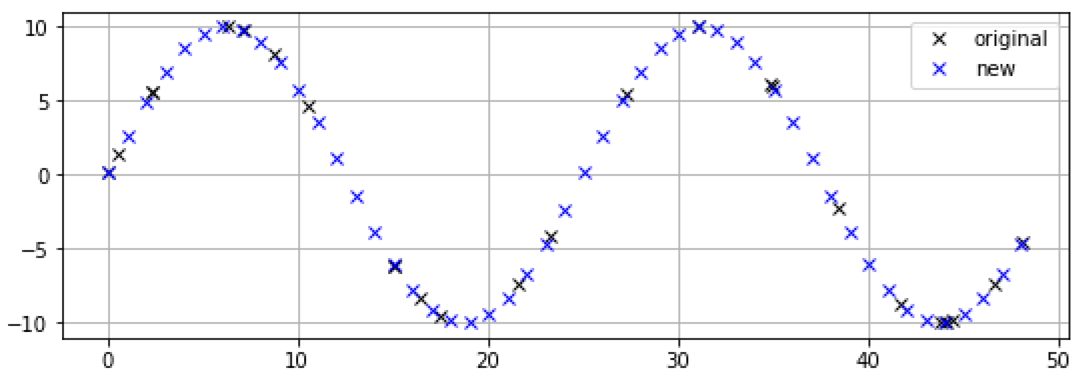


3.2.3.3 Spline interpolation

In [90]:

```
import sys
import matplotlib.pyplot as plt
import numpy
import scipy
from scipy import interpolate

#create data x and y

x = numpy.random.uniform(0,50,25)
y = 10*numpy.sin(2.*numpy.pi*x/25)

#create new evenly spaced x data points between the minimum value
#and the maximum value in vector x

x_new = numpy.arange(numpy.min(x),numpy.max(x),1)

#interpolate data and create new interpolated y data points
#at the locations of the new evenly spaced x data points
```

```
#using specified parameters m_avg(x_new,x_orig,y_orig,span)

#set new x values
x_new = x_new*1.0

#set x values to interpolate
x_orig = x*1.0

#set y values to interpolate
y_orig = y*1.0

f = scipy.interpolate.interp1d(x_orig,y_orig,'cubic')
y_new = f(x_new)

#create a figure, 9 inches by 9 inches
fig = plt.figure(1,figsize=(9,3))

#create a subplot. plot the original x and y data
#and the new x and y data interpolated using spline

ax1 = plt.subplot(1,1,1)
ax1.plot(x,y,marker="x",linewidth=0,color="k",label="original")
ax1.plot(x_new,y_new,marker="x",linewidth=0,color="b",label="new")

ax1.legend()

ax1.grid()
plt.show()
```

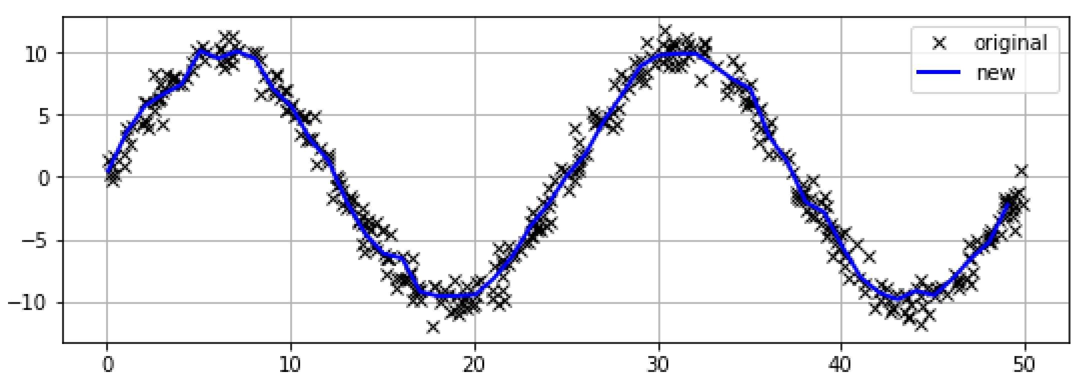


3.2.3.4 Inverse distance weighted (IDW) interpolation

The IDW function pulled for this script is available either on Github (https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Scripts/IDW.py) or in the interpolation Jupyter Notebook we covered in week 4. Works best with a densely populated dataset.

In [91]:
```
import sys
import matplotlib.pyplot as plt
import numpy
import requests

##dowloaad IDW algorithm as a python script from Github this is the same script
#in the Jupyter Notebook we went over in week 4

url = 'https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Scripts/IDW.py
IDW_py = requests.get(url)

with open('IDW_py.py', 'w') as f:
    f.write(IDW_py.text)

import IDW_py
```

```
#create data x and y

x = numpy.random.uniform(0,50,400)
y = 10*numpy.sin(2.*numpy.pi*x/25) + numpy.random.normal(0,1,len(x))

#create new evenly spaced x data points between the minimum value
#and the maximum value in vector x

x_new = numpy.arange(numpy.min(x),numpy.max(x),1)

#interpolate data and create new interpolated y data points
#at the locations of the new evenly spaced x data points
#using specified parameters IDW(x_new,x_orig,y_orig,span,factor)

#set new x values
x_new = x_new*1.0

#set x values to interpolate
x_orig = x*1.0

#set y values to interpolate
y_orig = y*1.0

#set window length
span = 2.5

#set factor
factor = 2.

y_new = IDW_py.IDW(x_new,x_orig,y_orig,span,factor)

#create a figure, 9 inches by 9 inches
fig = plt.figure(1,figsize=(9,3))

#create a subplot. plot the original x and y data
#and the new x and y data interpolated using moving average algorithm
ax1 = plt.subplot(1,1,1)
ax1.plot(x,y,marker="x",linewidth=0,color="k",label="original")
ax1.plot(x_new,y_new,linewidth=2,color="b",label="new")

ax1.legend()

ax1.grid()
plt.show()
```



3.2.3.5 LOESS interpolation

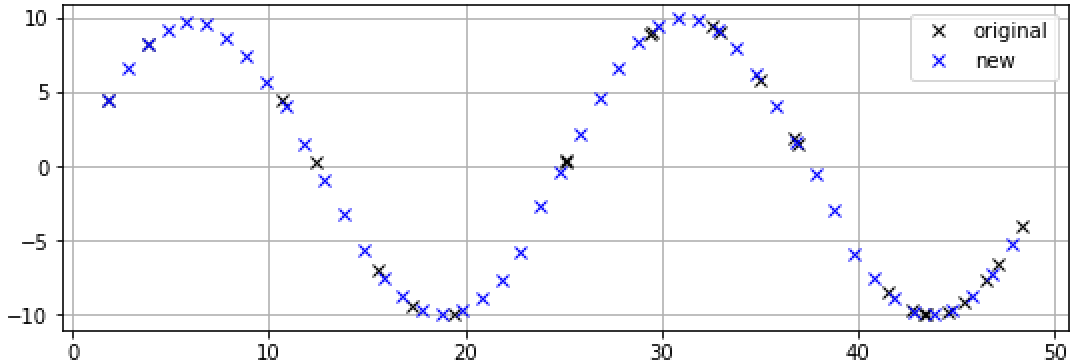
3.2.3.5.1 LOESS interpolation of sample dataset

The loess function stored on Github and pulled for this script is available either on Github (https://github.com/wschmelz/GeologicalModeling/blob/main/Scripts/loess.py) or in the interpolation

Jupyter Notebook we covered in week 4.

In [92]:
```
import sys
import matplotlib.pyplot as plt
import numpy
import requests

##dowloaad LOESS algorithm as a python script from Github this is the same script
#in the Jupyter Notebook we went over in week 4

url = 'https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Scripts/loess.
loess_py = requests.get(url)

with open('loess_py.py', 'w') as f:
    f.write(loess_py.text)

import loess_py

#create data x and y

x = numpy.random.uniform(0,50,25)
y = 10*numpy.sin(2.*numpy.pi*x/25)

#create new evenly spaced x data points between the minimum value
#and the maximum value in vector x

x_new = numpy.arange(numpy.min(x),numpy.max(x),1)

#interpolate data and create new interpolated y data points
#at the locations of the new evenly spaced x data points
#using specified parameters loess(x_new,x,y,pt_min_r,dist_r,factor)

#set new x values
x_new = x_new*1.0

#set x values to interpolate
x = x*1.0

#set y values to interpolate
y = y*1.0

#set minimum points to consider (must be integer)
pt_min_r = 5

#set window length
dist_r = 5.

#set factor of 1 (linear) or 2 (polynomial)
factor = 2

y_new = loess_py.loess(x_new,x,y,pt_min_r,dist_r,factor)

#create a figure, 9 inches by 3 inches
fig = plt.figure(1,figsize=(9,3))

#create a subplot. plot the original x and y data
#and the new x and y data interpolated using loess algorithm
ax1 = plt.subplot(1,1,1)
ax1.plot(x,y,marker="x",linewidth=0,color="k",label="original")
ax1.plot(x_new,y_new,marker="x",linewidth=0,color="b",label="new")

ax1.legend()

ax1.grid()
plt.show()
```

```
LOESS Regression complete
```



3.2.3.5.2 LOESS interpolation of d18O data

The loess function stored on Github and pulled for this script is available either on Github (https://github.com/wschmelz/GeologicalModeling/blob/main/Scripts/loess.py) or in the interpolation Jupyter Notebook we covered in week 4.

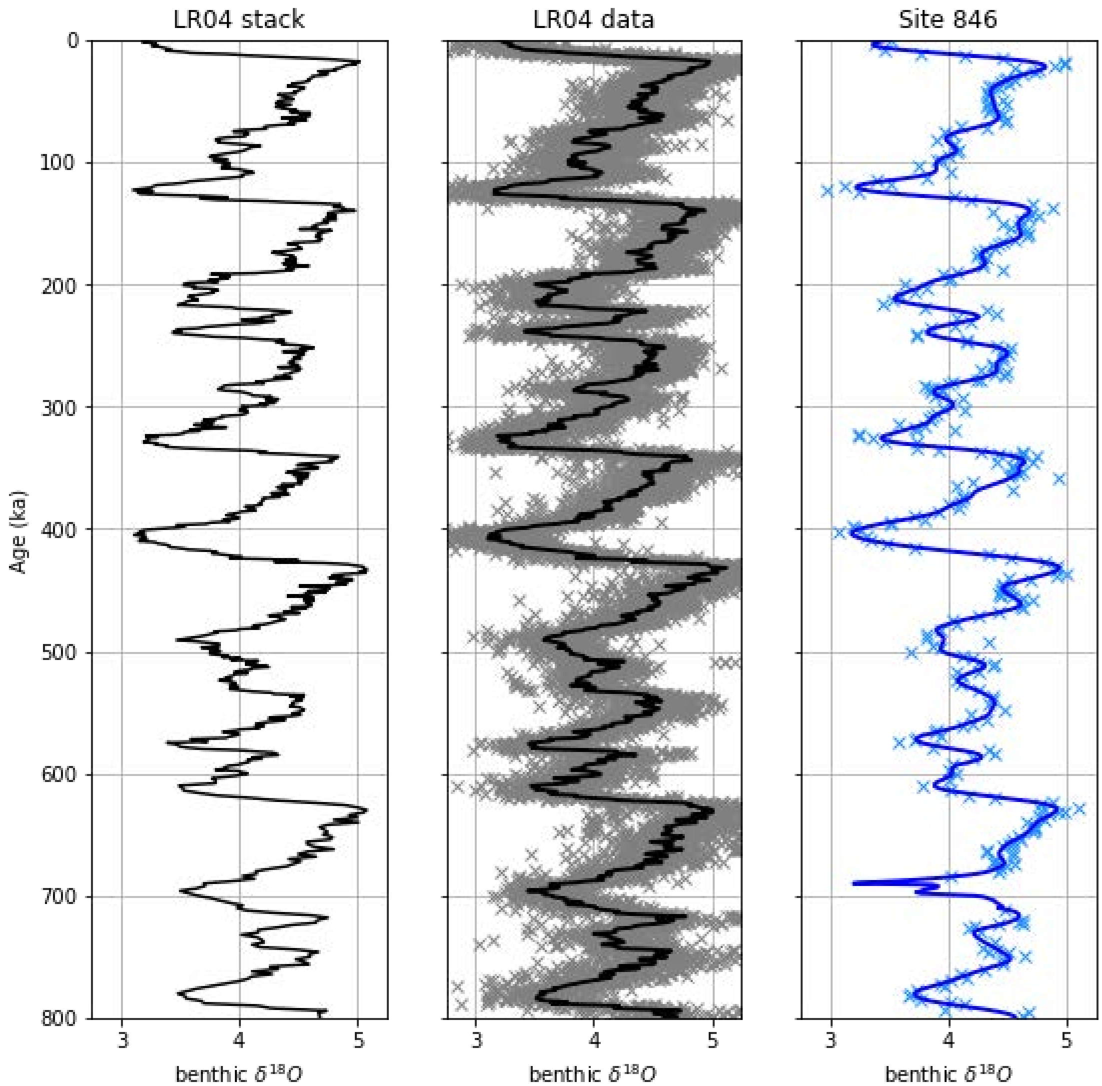
In [93]:

```
import matplotlib.pyplot as plt
import numpy
import requests

#Download LR04 d18O stack datafile and save to local directory

url = 'https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Data/d18O/00_L
LR04 = requests.get(url)

with open('00_LR04.csv', 'w') as f:
    f.write(LR04.text)

#read datafile, set comma as the delimiter

filename = '00_LR04.csv'
LR04 = numpy.genfromtxt(filename,delimiter=',')

#Download datafile with all the LR04 d18O data and save to local directory

url = 'https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Data/d18O/01_L
LR_original_data = requests.get(url)

with open('01_LR_original_data.csv', 'w') as f:
    f.write(LR_original_data.text)

#read datafile, set comma as the delimiter

filename = "01_LR_original_data.csv"
LR04_orig = numpy.genfromtxt(filename,delimiter=',')

#Download datafile with the d18O data from ODP leg 138 site 846 and save to local direct

url = 'https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Data/d18O/02_O
ODP_138_846_d18O = requests.get(url)

with open('02_ODP_138_846_d18O.csv', 'w') as f:
    f.write(ODP_138_846_d18O.text)

#read datafile, set comma as the delimiter

filename = "02_ODP_138_846_d18O.csv"
ODP_138_846 = numpy.genfromtxt(filename,delimiter=',')
```

```
##dowloaad LOESS algorithm as a python script from Github this is the same script
#in the Jupyter Notebook we went over in week 4

url = 'https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Scripts/loess.
loess_py = requests.get(url)

with open('loess_py.py', 'w') as f:
    f.write(loess_py.text)

import loess_py

#create interpolation values
dt = 1.
time_start = 800.
t_vals = numpy.arange(0,time_start+dt,dt)

#set window length
dist_r1 = 2.5
dist_r2 = 20.
#set factor
pt_min_r = 10

#apply function

LOESS_LR04_orig = loess_py.loess(t_vals,LR04_orig[:,0],LR04_orig[:,1],pt_min_r,dist_r1,2
LOESS_Site846 = loess_py.loess(t_vals,ODP_138_846[:,0],ODP_138_846[:,1],pt_min_r,dist_r2

#replot

fig = plt.figure(2,figsize=(9,9))

ax1 = plt.subplot(131)
ax1.plot(LR04[:,1],LR04[:,0],color="k")

ax1.set_ylim(800,0)
ax1.set_xlim(2.75,5.25)
ax1.grid()
ax1.set_title("LR04 stack")
ax1.set_ylabel("Age (ka)")
ax1.set_xlabel("benthic $\delta^{18}O$")

ax2 = plt.subplot(132)
ax2.plot(LR04_orig[:,1],LR04_orig[:,0],linewidth=0.0,marker='x',color="gray")
ax2.plot(LOESS_LR04_orig,t_vals,linewidth=2.0,color="k")

ax2.set_ylim(800,0)
ax2.set_xlim(2.75,5.25)
ax2.grid()
ax2.set_title("LR04 data")
ax2.set_yticklabels([])
ax2.set_xlabel("benthic $\delta^{18}O$")

ax3 = plt.subplot(133)
ax3.plot(ODP_138_846[:,1],ODP_138_846[:,0],linewidth=0.0,marker='x',color="dodgerblue")
ax3.plot(LOESS_Site846,t_vals,linewidth=2.0,color="blue")

ax3.set_ylim(800,0)
ax3.set_xlim(2.75,5.25)
ax3.grid()
ax3.set_title("Site 846")
ax3.set_xlabel("benthic $\delta^{18}O$")
ax3.set_yticklabels([])

plt.show()
```

```
LOESS Regression complete
```

```
LOESS Regression complete
```



3.2.3.5.3 2D LOESS interpolation of drone topography

The loess function stored on Github and pulled for this script is available either on Github (https://github.com/wschmelz/GeologicalModeling/blob/main/Scripts/loess.py) or in the interpolation Jupyter Notebook we covered in week 4.

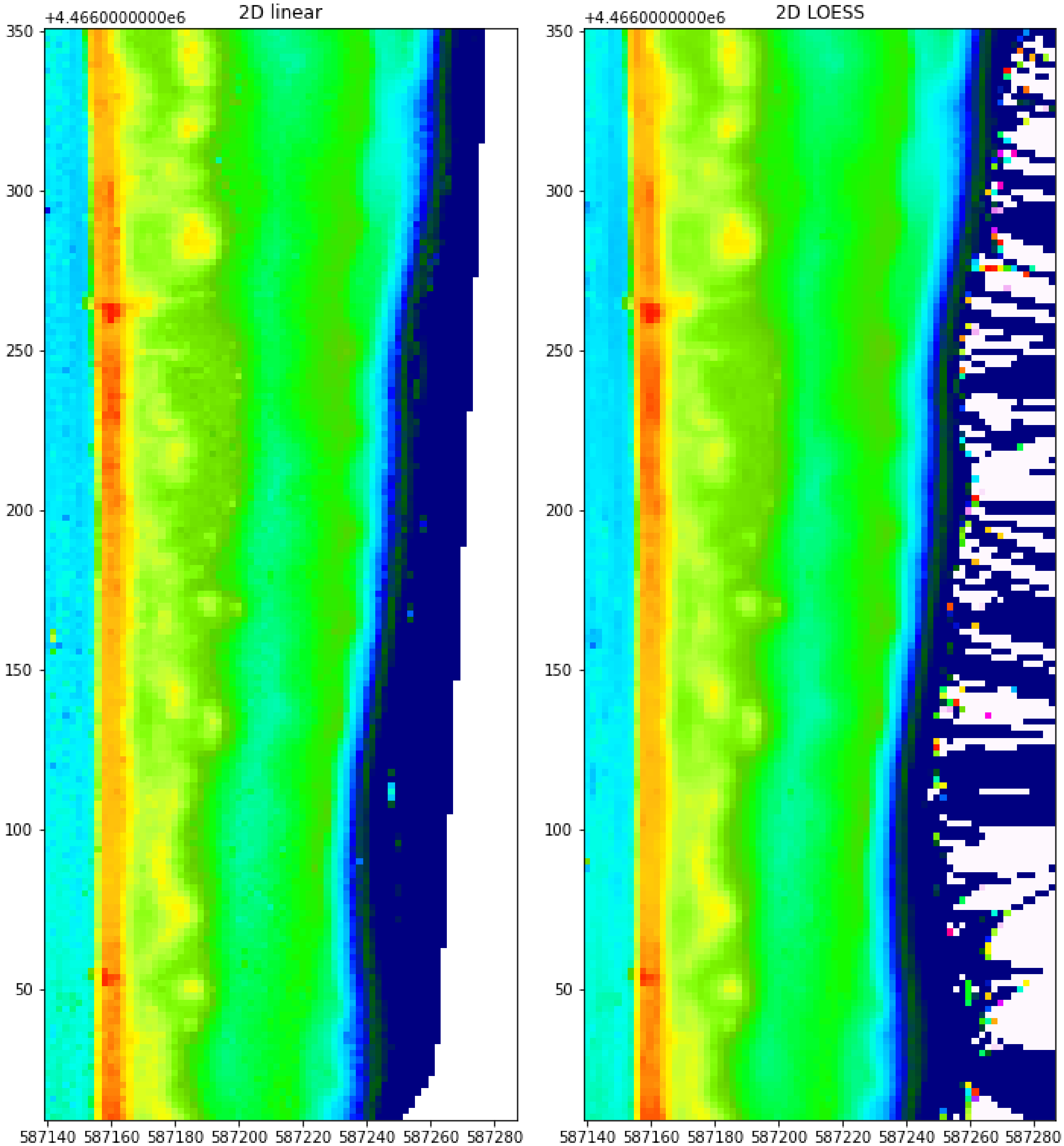
In [94]:

```
import matplotlib.pyplot as plt
import numpy
import requests

#Download drone topography datafile and save to local directory

url = 'https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Data/DroneTopo
MB_20220522_rpd = requests.get(url)

with open('MB_20220522_rpd.csv', 'w') as f:
    f.write(MB_20220522_rpd.text)

##dowloaad LOESS algorithm as a python script from Github this is the same script
#in the Jupyter Notebook we went over in week 4

url = 'https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Scripts/loess.
loess_py = requests.get(url)

with open('loess_py.py', 'w') as f:
    f.write(loess_py.text)
```

```
import loess_py

drone_datafile1 =  "MB_20220522_rpd.csv"
drone_data1 = numpy.genfromtxt(drone_datafile1, delimiter = ",")

#make new grid

x_max = 587285.
x_min = 587140.
y_max = 4466350.
y_min = 4466010.

cellsize = 2.
new_x = numpy.arange(x_min,x_max+cellsize,cellsize)
new_y = numpy.arange(y_min,y_max+cellsize,cellsize)

X, Y = numpy.meshgrid(new_x, new_y)

print (numpy.shape(X))

print ("Number of nterpolated data points: ", len(numpy.ndarray.flatten(X)))

#interpolate data and create new interpolated y data points
#at the locations of the new evenly spaced x data points
#using specified parameters loess2D(x_new,y_new,x,y,z,pt_min_r,dist_r,factor)

#set new x values
x_new = numpy.ndarray.flatten(X)*1.0

#set new y values
y_new = numpy.ndarray.flatten(Y)*1.0

#set x values to interpolate
x = drone_data1[::2,0]*1.0

#set y values to interpolate
y = drone_data1[::2,1]*1.0

#set z values to interpolate
z = drone_data1[::2,2]*1.0

#set minimum points to consider (must be integer)
pt_min_r = 10

#set window length
dist_r = 3.

#set factor of 1 (linear) or 2 (polynomial)
factor = 2

Z1_loess = loess.loess_2D(x_new,y_new,x,y,z,pt_min_r,dist_r,factor)
Z1_loess = numpy.reshape(Z1_loess,numpy.shape(X))

#linear interpolation for comparison

Z1 = interpolate.griddata(drone_data1[:,0:2], drone_data1[:,2], (X, Y), method='linear')

#plot results

fig = plt.figure(figsize=(10,12))

ax1 = plt.subplot(121)

ax1.pcolor(X,Y,Z1,cmap="gist_ncar",vmin=-32.5,vmax=-24.5,shading="auto")
ax1.set_aspect("equal")
ax1.set_title("2D linear")
```

```
ax2 = plt.subplot(122)

ax2.pcolor(X,Y,Z1_loess,cmap="gist_ncar",vmin=-32.5,vmax=-24.5,shading="auto")
ax2.set_aspect("equal")
ax2.set_title("2D LOESS")
plt.tight_layout()
plt.show()
```

```
(74,)
(171,)
(171, 74)
(171, 74)
Number of nterpolated data points:  12654
LOESS Regression complete
```



4. Summary statistics, regression, and correlation

4. Summary statistics

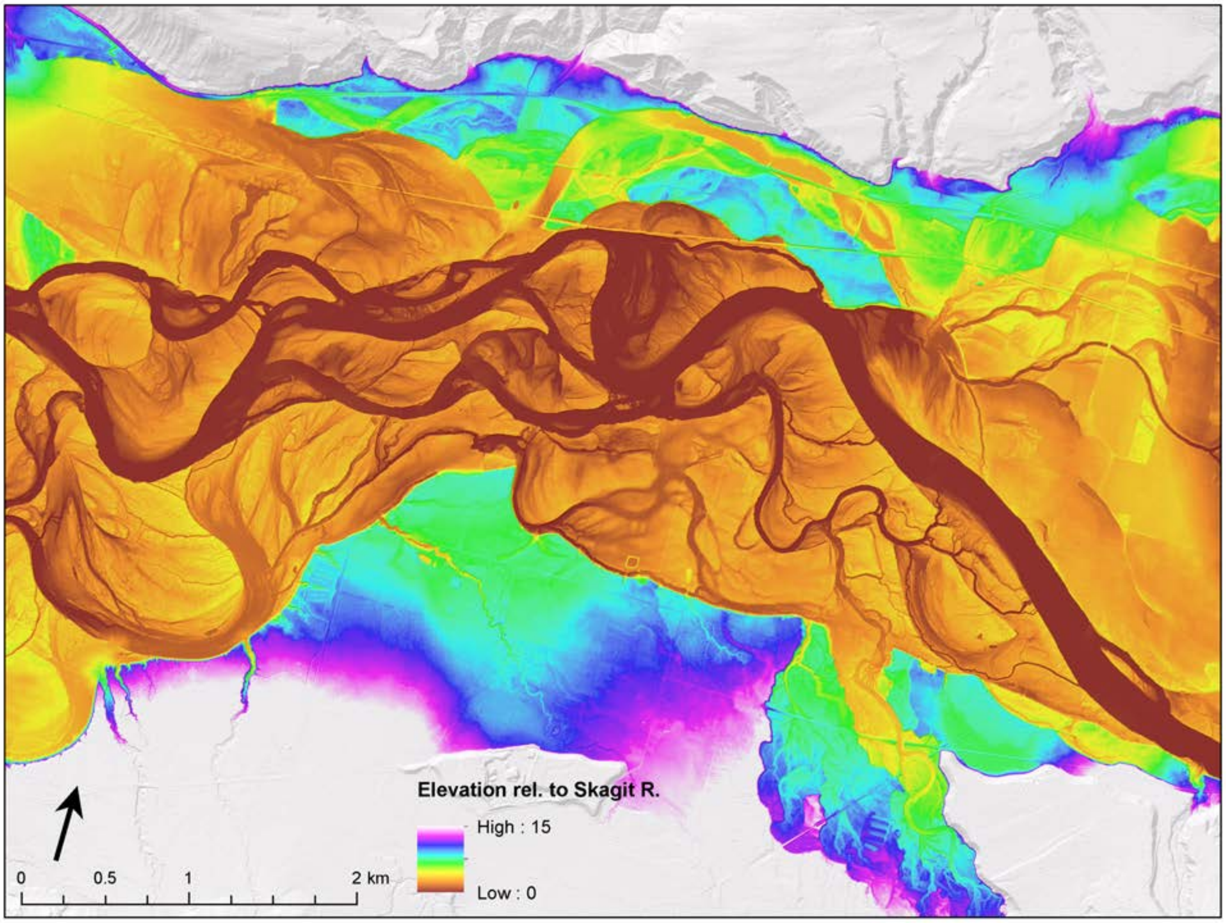
In [1]:
```
import numpy

#take 100 samples from a normal distribution with a mean of 5 and stdev of 2
# numpy.random.normal(mean,stdev,samples)

x = numpy.random.normal(5,2,100)

#calculate the number of data values in x (we set this to 100)

N = len(x)
print("N:",N)

#calculate the mean

mean_x = numpy.mean(x)
print("Sample mean:",round(mean_x,3))

#calculate the standard deviation

std_x = numpy.std(x)
print("Standard deviation:",round(std_x,3))

#calculate the standard error on the estimate of the mean

se_x = numpy.std(x)/numpy.sqrt(N)
print("Standard error:",round(se_x,3))

#95% Confidence intervals on the estimate of the mean

high_975 = mean_x + (1.96*se_x)
low_025 = mean_x - (1.96*se_x)

print("The mean of x is", round(mean_x,3),"; 95% CI ranges from", round(low_025,3), "to"
```

```
N: 100
Sample mean: 4.927
Standard deviation: 1.991
Standard error: 0.199
The mean of x is 4.927 ; 95% CI ranges from 4.537 to 5.317
```

5. Mapping earth surface processes

5.1 Relative elevation model of the Skagit River

Instructions for application here (), data and code available on request (1.5 GB).



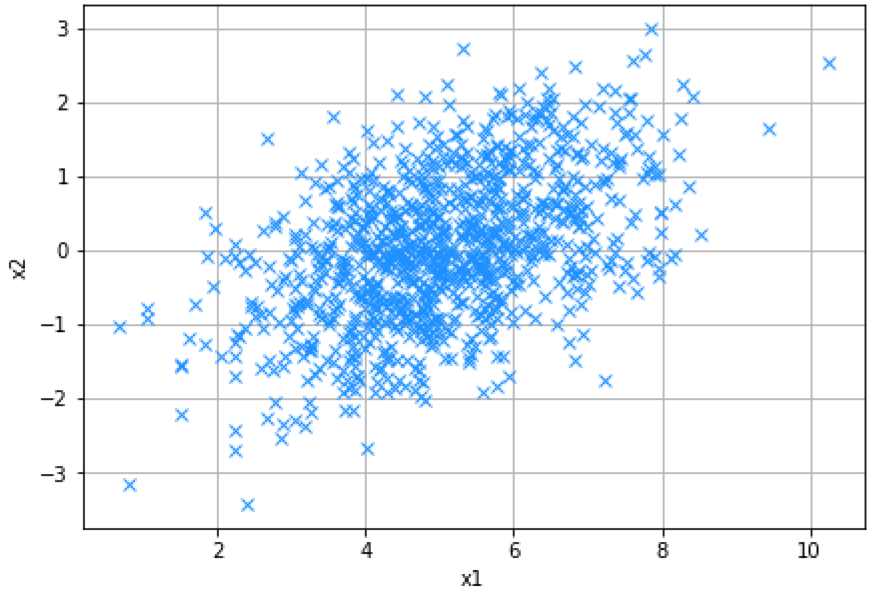
6. Correlation and regression

6.1 Covariance and correlation

In [96]:
```
import matplotlib.pyplot as plt
import numpy

#define functions for covariance and correlation coefficient

def cov(x1,x2):
    N = len(x1)
    mean_x1 = numpy.mean(x1)
    mean_x2 = numpy.mean(x2)
    cov_tmp = (1./(N-1)) * numpy.sum((x1-mean_x1)*(x2-mean_x2))
    return cov_tmp

def corr_coeff(x1,x2):
    N = len(x1)
    mean_x1 = numpy.mean(x1)
    mean_x2 = numpy.mean(x2)
    stdev_x1 = numpy.std(x1)
    stdev_x2 = numpy.std(x2)
    cov_tmp = (1./(N-1)) * numpy.sum((x1-mean_x1)*(x2-mean_x2))
    r_tmp = cov_tmp/(stdev_x1*stdev_x2)
    return r_tmp

#Create a correlated distribution of data with a covariance set to 0.75

var_x1 = 2.
```

```python
var_x2 = 1.
cov_x1_x2 = 0.75
mean_x1 = 5.0
mean_x2 = 0.0

u = numpy.array([mean_x1,mean_x2])
k = numpy.array([[var_x1, cov_x1_x2], [cov_x1_x2, var_x2]])
dist = numpy.random.multivariate_normal(u, k, size=1000)
x1, x2 = dist[:,0],dist[:,1]

#calculate covariance and correlation coefficient ("r") using defined functions

covariance = cov(x1,x2)
r = corr_coeff(x1,x2)
cov_mat = numpy.cov(x1,x2)

print(r)
print("Sample covariance:", round(covariance,3))
print("Correlation coefficient (r):", round(r,3))
print("Sample covariance matrix:\n",cov_mat)

#estimate variance in sample of x2 explained by sample of x1

r2 = r**2.
print("r^2:",r2)

#create a figure, 7 inches by 7 inches
fig = plt.figure(1,figsize=(7,7))

#create a subplot
ax1 = plt.subplot(1,1,1)

#plot the original x,y data
ax1.plot(x1,x2,marker="x",linewidth=0,color="dodgerblue")
ax1.set_xlabel("x1")
ax1.set_ylabel("x2")
ax1.set_aspect("equal")
ax1.grid()
plt.show()
```

```
0.4812431381081024
Sample covariance: 0.649
Correlation coefficient (r): 0.481
Sample covariance matrix:
 [[1.87505769 0.64863935]
 [0.64863935 0.9708047 ]]
r^2: 0.2315949579761341
```



6.2 Regression

y_i is the data with a some component of error, ε, and $y\hat{}_i$ is the regression model

b_1 is the slope, and b_0 is the intercept

$$y_i = \widehat{y}_i + \varepsilon_i$$

$$\widehat{y}_i = b_0 + b_1 x_i$$

6.2.1 Calculate regression model parameters, and estimate error

In [97]:
```
import numpy
import scipy
from scipy import stats

#Create variables that are linearly related, with normally distributed noise

x = numpy.random.uniform(0,50,100)
y = (x * 0.25) + numpy.random.normal(0,2.5,100)

#run regression through scipy
res = scipy.stats.linregress(x, y)

#slope and intercept values
b1 = res.slope
b0 = res.intercept

#standard error estimates for slope and intercept
b1_se = res.stderr
b0_se = res.intercept_stderr

#correlation coefficient (r) and estimate of residual variance (r^2)

r = res.rvalue
r2 = r**2.

print("slope:",round(b1,3))
print("intercept:",round(b0,3))
print("slope se:",round(b1_se,3))
print("intercept se:",round(b0_se,3))
```

```
print("correlation coefficient (r):",round(r,3))
print("r^2:",round(r2,3))
```

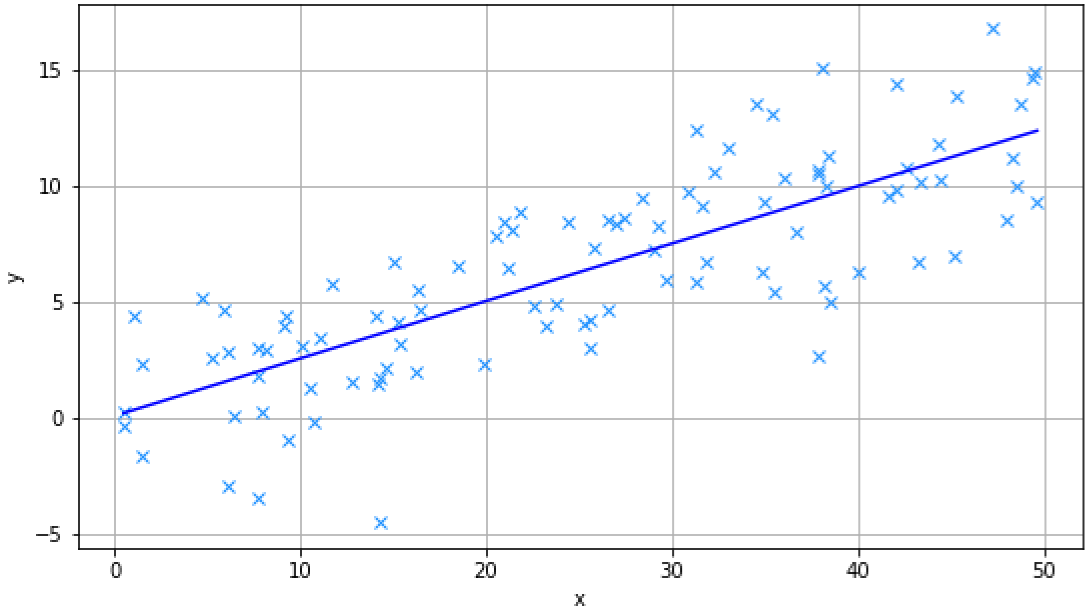
```
slope: 0.255
intercept: -0.519
slope se: 0.018
intercept se: 0.506
correlation coefficient (r): 0.825
r^2: 0.681
```

6.2.2 Plot the regression model

In [98]:

```
import numpy
import scipy
from scipy import stats
import matplotlib.pyplot as plt

#Create variables that are linearly related, with normally distributed noise

x = numpy.random.uniform(0,50,100)
y = (x * 0.25) + numpy.random.normal(0,2.5,100)

#run regression through scipy
res = scipy.stats.linregress(x, y)

#slope and intercept values
b1 = res.slope
b0 = res.intercept

#standard error estimates for slope and intercept
b1_se = res.stderr
b0_se = res.intercept_stderr

#correlation coefficient (r) and estimate of residual variance (r^2)

r = res.rvalue
r2 = r**2.

print("slope:",round(b1,3))
print("intercept:",round(b0,3))
print("slope se:",round(b1_se,3))

print("correlation coefficient (r):",round(r,3))
print("r^2:",round(r2,3))

#calculate estimate of y from regression

x_new = numpy.linspace(numpy.min(x),numpy.max(x),2)
y_est = b1*x_new + b0

#create a figure
fig = plt.figure(1,figsize=(9,5))

#create a subplot
ax1 = plt.subplot(1,1,1)

#plot the original x,y data
ax1.plot(x,y,marker="x",linewidth=0,color="dodgerblue")
#plot the regression line
ax1.plot(x_new,y_est,linewidth=1.5,color="blue")
ax1.set_xlabel("x")
ax1.set_ylabel("y")
ax1.grid()
plt.show()
```

```
slope: 0.247
intercept: 0.089
slope se: 0.019
correlation coefficient (r): 0.797
r^2: 0.635
```



6.3 Multivariate regression

y is the data, which is a function of several coefficients (b_1,...,b_n), several independent variables (x_1,...,x_n), and some amount of noise, ε.

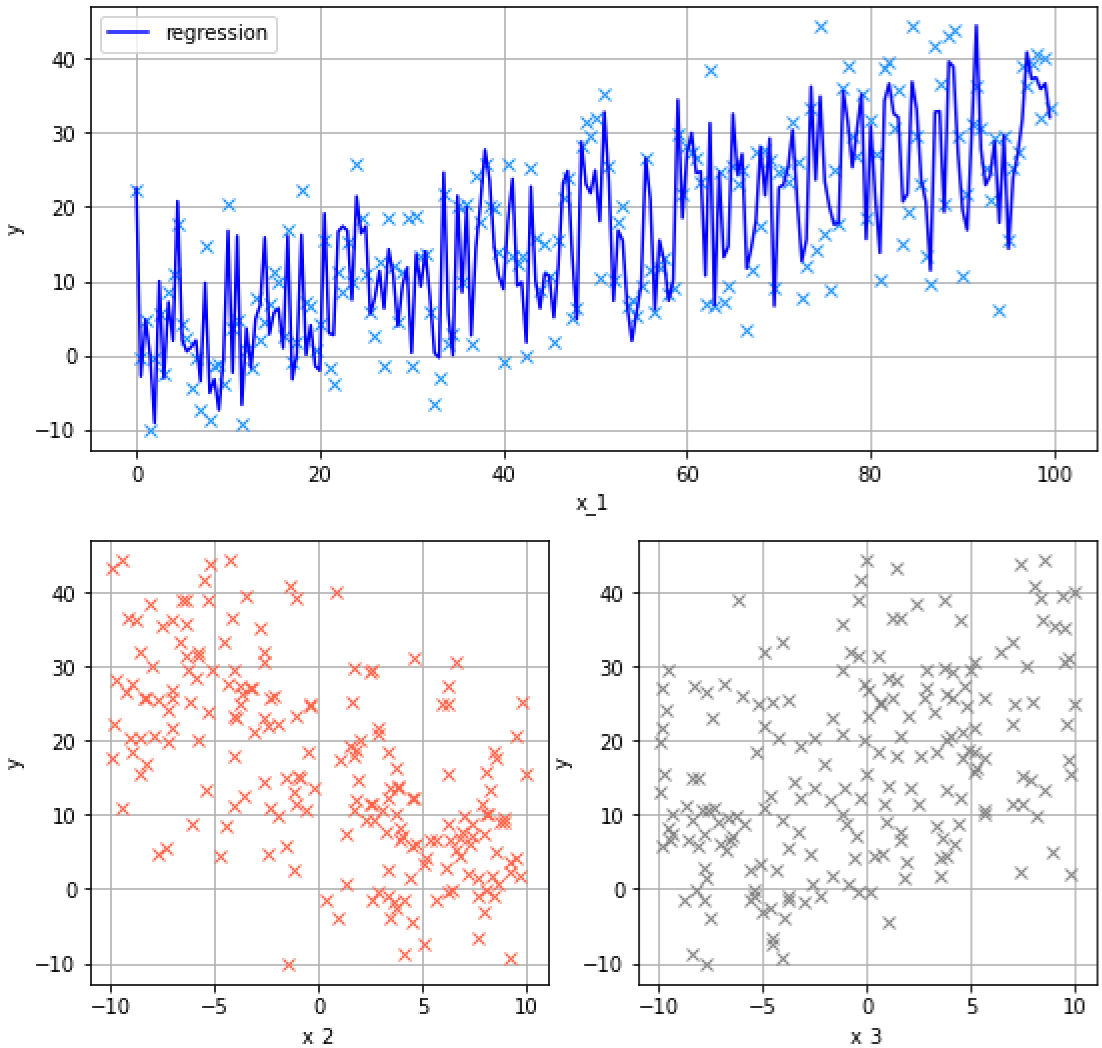
$$y = b_0 + b_1 x_1 + b_2 x_2 + \ldots + b_N x_N + \varepsilon$$

In [99]:
```
import numpy
import scipy
from scipy import stats
import matplotlib.pyplot as plt

#Create independent variables, regularly sampled at times in X_1

x_1 = numpy.arange(0.,100.,0.5)
x_2 = numpy.random.uniform(-10.,10.,200)
x_3 = numpy.random.uniform(-10.,10.,200)

#create dependent variable that is linearly related to the independent variables
#add noise to observations of the process

y_true = 5.0 + (x_1 * 0.25) + (x_2 * -1.25) + (x_3 * 0.75)
y_obs = y_true + numpy.random.normal(0,5.,200)

#number of independent variables

N = 3

#Create data matrix
y = numpy.reshape(y_obs,(-1,1))
A = numpy.ones((len(x_1),N+1))

#Place observations of independent variables into the data matrix
#leave first column as a vector of ones

A[:,0] = A[:,0] * 1.0
```

```
A[:,1] = x_1
A[:,2] = x_2
A[:,3] = x_3

#Run least squares

b = numpy.linalg.lstsq(A, y,rcond=None)[0]
b_0 = float(b[0])
b_1 = float(b[1])
b_2 = float(b[2])
b_3 = float(b[3])

#print parameter values
print("Parameter values")
print ("B_0:",numpy.round(b_0,3))
print ("B_1:",numpy.round(b_1,3))
print ("B_2:",numpy.round(b_2,3))
print ("B_3:",numpy.round(b_3,3))
print("")

#Calculate estimate of y from observations x_1, x_2, and x_3

y_new = b_0 + (A[:,1]*b_1) + (A[:,2]*b_2) + (A[:,3]*b_3)

#calculate r2 (Thomson and Emery (1998) - CH. 3)

SST = numpy.sum((y_obs-numpy.mean(y_obs))**2.)
SSR = numpy.sum((y_new-numpy.mean(y_obs))**2.)
r2 = SSR/SST

print ("r^2:",round(r2,3))

#create a figure
fig = plt.figure(1,figsize=(9,9))

#create a subplot for the observations of y and
#the estimate of y from regression as a function of x_1
ax1 = plt.subplot(2,1,1)

#plot the original x,y data
ax1.plot(x_1,y_obs,marker="x",linewidth=0,color="dodgerblue")
#plot the regression line
ax1.plot(x_1,y_new,linewidth=1.5,color="blue",label="regression")
ax1.set_xlabel("x_1")
ax1.set_ylabel("y")
ax1.legend()
ax1.grid()

#create a subplot for the observations of y
#cross-plotted with observations of x_2
ax2 = plt.subplot(2,2,3)

#plot x_2 and y
ax2.plot(x_2,y_obs,marker="x",linewidth=0,color="tomato")
ax2.set_xlabel("x_2")
ax2.set_ylabel("y")

ax2.grid()

#create a subplot for the observations of y
#cross-plotted with observations of x_3
ax3 = plt.subplot(2,2,4)

#plot x_3 and y
ax3.plot(x_3,y_obs,marker="x",linewidth=0,color="gray")
ax3.set_xlabel("x_3")
```

```
ax3.set_ylabel("y")

ax3.grid()

plt.show()
```

```
Parameter values
B_0: 3.911
B_1: 0.263
B_2: -1.137
B_3: 0.769

r^2: 0.859
```



6.3 Monte Carlo simulation to estimate regression parameter uncertainty

This code uses a data file that contains a dataset similar to the one created in section 5.3. It is downloaded from Github so this code can run independently from other cells in this notebook.

In [100…

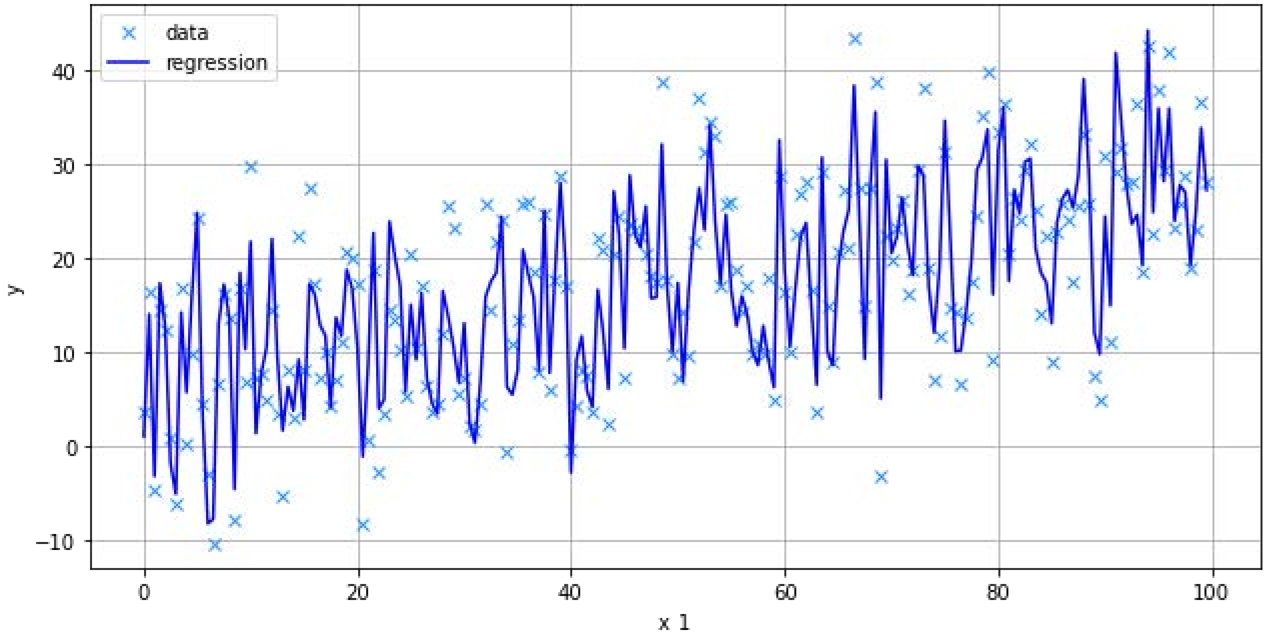
```
import matplotlib.pyplot as plt
import numpy
import requests

# Download datafile and save to local directory as 'Multiple_reg.txt'

url = 'https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Data/Multiple_
Multiple_reg = requests.get(url)

with open('Multiple_reg.txt', 'w') as f:
    f.write(Multiple_reg.text)
```

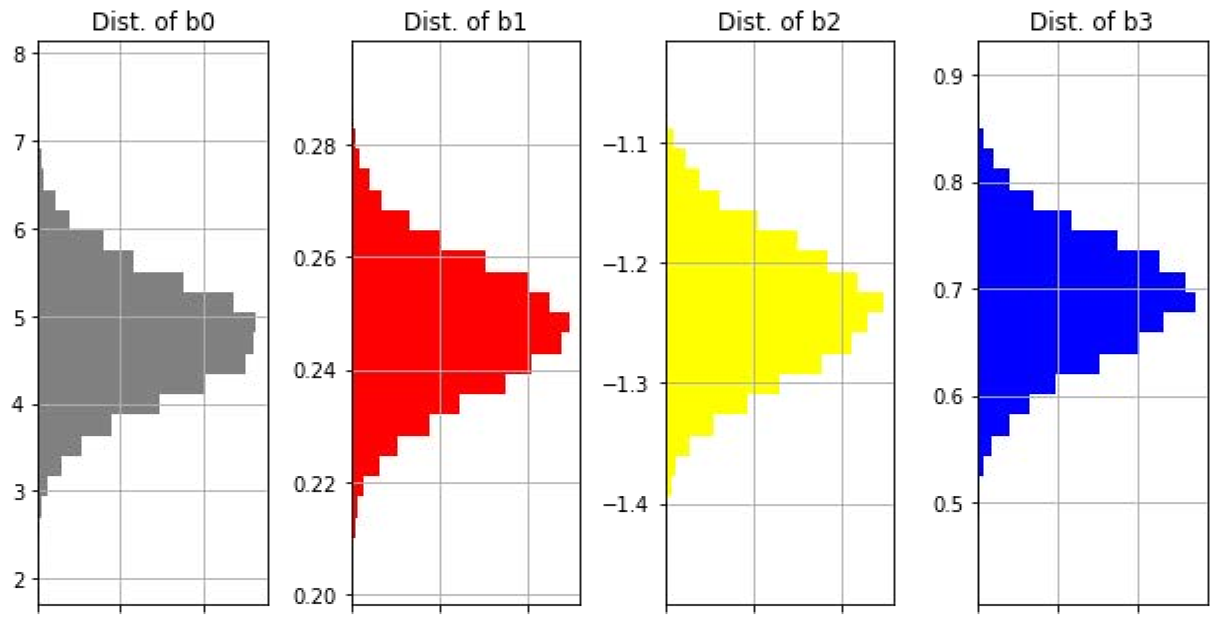
```
## read datafile, set comma as the delimiter

filename = "Multiple_reg.txt"
data = numpy.genfromtxt(filename,delimiter=',')

## define variables

x_1 = data[:,0]
x_2 = data[:,1]
x_3 = data[:,2]
y_obs = data[:,3]

# Run initial Regression

## number of independent variables

N = 3

## Create data matrix
y = numpy.reshape(y_obs,(-1,1))
A = numpy.ones((len(x_1),N+1))

##Place observations of independent variables into the data matrix
##leave first column as a vector of ones

A[:,0] = A[:,0] * 1.0
A[:,1] = x_1
A[:,2] = x_2
A[:,3] = x_3

##Run least squares

b = numpy.linalg.lstsq(A, y,rcond=None)[0]
b_0 = float(b[0])
b_1 = float(b[1])
b_2 = float(b[2])
b_3 = float(b[3])

##print parameter values
print("True parameter values")
print ("B_0:",5.0)
print ("B_1:",0.25)
print ("B_2:",-1.25)
print ("B_3:",0.75)
print("")

print("Parameter values from regession of sample with noise")
print ("B_0:",numpy.round(b_0,3))
print ("B_1:",numpy.round(b_1,3))
print ("B_2:",numpy.round(b_2,3))
print ("B_3:",numpy.round(b_3,3))
print("")

##Calculate estimate of y from observations x_1, x_2, and x_3

y_new = b_0 + (A[:,1]*b_1) + (A[:,2]*b_2) + (A[:,3]*b_3)

##Calculate residuals of regression analysis
##and the standard deviation of the distribution

residuals = y_obs - y_new
residuals_stdev = numpy.std(residuals)

#Run monte carlo using distribution of residuals to assess
#regression estimate and parameter uncertainties
```

```
##set number of iterations

iters = 10000

##create an output matrix with as many rows as iterations,
##and as many columns as model coefficients

output_coeff = numpy.zeros((iters,N+1))

for n in range(0,iters):
    #add noise to initial regression model output that mirrors variance of
    #residuals from that regression
    y_mc = y_new + numpy.random.normal(0,residuals_stdev,len(y_new))
    y_mc = numpy.reshape(y_mc,(-1,1))
    b_mc = numpy.linalg.lstsq(A, y_mc,rcond=None)[0]

    #record new coefficients
    output_coeff[n,0] = b_mc[0]
    output_coeff[n,1] = b_mc[1]
    output_coeff[n,2] = b_mc[2]
    output_coeff[n,3] = b_mc[3]

#calculate uncertainty of regression coefficients

b_0_mean = numpy.mean(output_coeff[:,0])
b_0_stdev = numpy.std(output_coeff[:,0])

b_1_mean = numpy.mean(output_coeff[:,1])
b_1_stdev = numpy.std(output_coeff[:,1])

b_2_mean = numpy.mean(output_coeff[:,2])
b_2_stdev = numpy.std(output_coeff[:,2])

b_3_mean = numpy.mean(output_coeff[:,3])
b_3_stdev = numpy.std(output_coeff[:,3])

print("Parameter values and uncertainty (95% CI) from Monte Carlo simulation")
print ("b0 is",round(b_0_mean,3),"+/-",round(1.96*b_0_stdev,3))
print ("b1 is",round(b_1_mean,3),"+/-",round(1.96*b_1_stdev,3))
print ("b2 is",round(b_2_mean,3),"+/-",round(1.96*b_2_stdev,3))
print ("b3 is",round(b_3_mean,3),"+/-",round(1.96*b_3_stdev,3))

# create a figure

#create a figure
fig = plt.figure(1,figsize=(9,9))

#create a subplot for the observations of y;
#the estimate of y from multiple regression as a function of x_1;
#and the uncertainty of that estimate derived from bootstrapping
ax1 = plt.subplot(2,1,1)

#plot the original x,y data
ax1.plot(x_1,y_obs,marker="x",linewidth=0,color="dodgerblue",label="data")
#plot the regression line
ax1.plot(x_1,y_new,linewidth=1.5,color="blue",label="regression")

ax1.set_xlabel("x_1")
ax1.set_ylabel("y")
ax1.legend()
ax1.grid()

#create four subplots that will show histograms of coefficient values in the bootstrap a

#b0
```

```python
ax2 = plt.subplot(2,4,5)

#plot the original x,y data
ax2.hist(output_coeff[:,0],bins=25,color="gray", orientation='horizontal')
ax2.set_xticklabels([])
ax2.set_title("Dist. of b0")
ax2.grid()

#b1

ax3 = plt.subplot(2,4,6)

#plot the original x,y data
ax3.hist(output_coeff[:,1],bins=25,color="red", orientation='horizontal')
ax3.set_xticklabels([])
ax3.set_title("Dist. of b1")
ax3.grid()

#b2

ax4 = plt.subplot(2,4,7)

#plot the original x,y data
ax4.hist(output_coeff[:,2],bins=25,color="yellow", orientation='horizontal')
ax4.set_xticklabels([])
ax4.set_title("Dist. of b2")
ax4.grid()

#b3

ax5 = plt.subplot(2,4,8)

#plot the original x,y data
ax5.hist(output_coeff[:,3],bins=25,color="blue", orientation='horizontal')
ax5.set_xticklabels([])
ax5.set_title("Dist. of b3")
ax5.grid()
plt.tight_layout()
plt.show()
```

```
True parameter values
B_0: 5.0
B_1: 0.25
B_2: -1.25
B_3: 0.75

Parameter values from regession of sample with noise
B_0: 4.744
B_1: 0.247
B_2: -1.239
B_3: 0.69

Parameter values and uncertainty (95% CI) from Monte Carlo simultation
b0 is 4.751 +/- 1.336
b1 is 0.247 +/- 0.023
b2 is -1.238 +/- 0.109
b3 is 0.69 +/- 0.113
```





7. Autocorrelation and crosscorrelation

7.1 Normalized autocovariance

$$C_{yy}(\tau) \equiv E[\{y(t)-\mu\}\{y(t+\tau)-\mu\}]$$

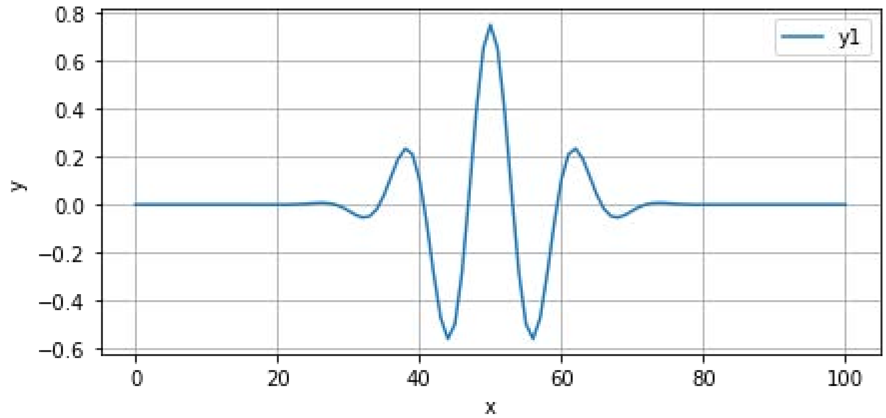
$$= \frac{1}{N-k}\sum_{i=1}^{N-k}[y_i-\overline{y}]\,[y_{i+k}-\overline{y}]$$

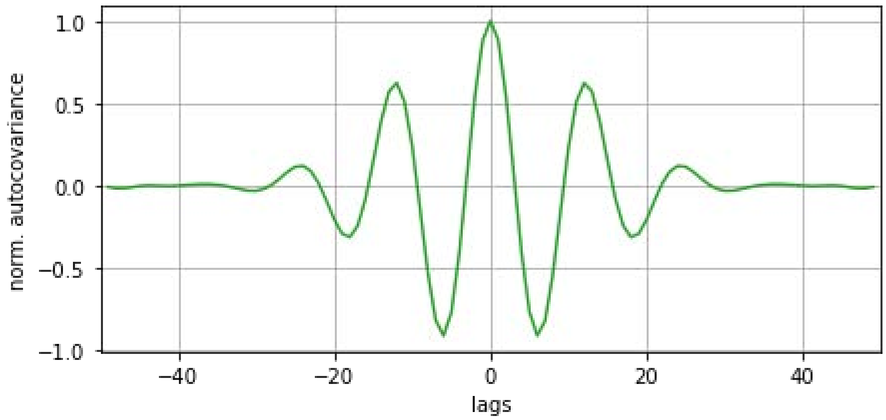
$$\rho_{yy}(\tau) = \frac{C_{yy}(\tau)}{\sigma^2}$$

In [101…

```
import matplotlib.pyplot as plt
import numpy
import scipy
from scipy import signal

#create data
x = numpy.arange(0.,100.+1.,1.)
```

```
#y1 is a morlet wavelet
y1 = numpy.real(scipy.signal.morlet(len(x), w=4., s=1.0))

#create a function to calculate normalized autocovariance, as defined by Thomson and Eme
#input dataseries y and the maximum number of lags to evaluate

def autocovariance_normalized(y,nmax):
    a_corr = numpy.zeros(nmax)
    lags = numpy.arange(0,nmax,1)
    for n in range(0,len(a_corr)):
        if n ==0:
            a_corr[n] = ((1./len(y))*numpy.sum((y-numpy.mean(y))**2.))/numpy.var(y)
        if n >0:
            a_corr[n] = ((1./(len(y)-float(n)))*numpy.sum((y[0:-n]-numpy.mean(y[0:-n]))*

    lags_neg = numpy.arange(1,len(a_corr),1)*-1.
    a_corr_neg = numpy.zeros(nmax-1)
    for n in range(1,len(a_corr)):
        a_corr_neg[n-1] = ((1./(len(y)-float(n)))*numpy.sum((y[n:]-numpy.mean(y[n:]))*(y

    lags = numpy.append(numpy.flipud(lags_neg),lags)
    a_corr = numpy.append(numpy.flipud(a_corr_neg),a_corr)

    return lags,a_corr

#calculate normalized crosscovariance of y1 and y2

lags,a_corr = autocovariance_normalized(y1,50)

#plot the autocovariance as a function of lags

#create a figure
fig = plt.figure(2,figsize=(7,7))

#create a subplot that shows y1 plotted as a function of x
ax1 = plt.subplot(211)
ax1.plot(x,y1,color='C0',label = "y1")
ax1.set_ylabel("y")
ax1.set_xlabel("x")
ax1.legend()
ax1.grid()

#create a subplot that shows autocovariance of y1 plotted as a function of lags
ax2 = plt.subplot(212)
ax2.plot(lags,a_corr,color='C2')
ax2.set_ylabel("norm. autocovariance")
ax2.set_xlabel("lags")
ax2.set_xlim(-50,50)
ax2.grid()
plt.show()
```





7.2 Normalized cross-covariance

$$C_{xy}(\tau) \equiv E\left[\left\{y(t) - \mu_y\right\}\left\{x(t+\tau) - \mu_x\right\}\right]$$

$$= \frac{1}{N-k}\sum_{i=1}^{N-k}[y_i - \overline{y}][x_{i+k} - \overline{x}]$$

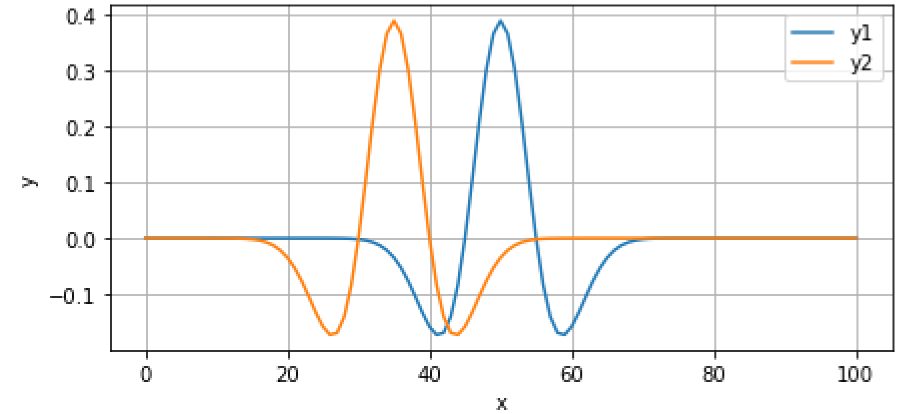
$$\rho_{xy} \equiv \frac{C_{xy}(\tau)}{\sigma_x \sigma_y}$$

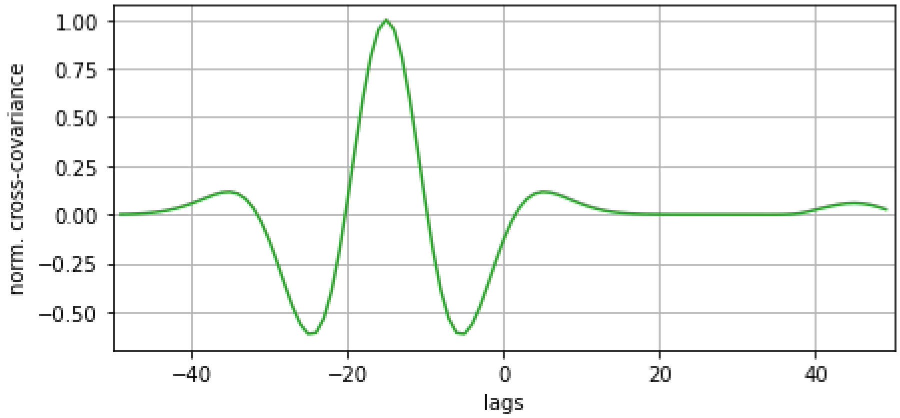
In [102…

```
import matplotlib.pyplot as plt
import numpy
import scipy
from scipy import signal

#create data
x = numpy.arange(0.,100.+1.,1.)
#y1 is a ricker wavelet
y1 = scipy.signal.ricker(len(x), 5)
#y1 is anidentical ricker wavelet, 15 time steps behind y1
y2 = y1[15:]
y2 = numpy.append(y2,numpy.zeros(len(y1)-len(y2)))

#create a function to calculate normalized crosscovariance, as defined by Thomson and Em
#input dataseries y1, dataseries y2, and the maximum number of lags to evaluate
#y1 and y2 must be 1 dimencional vectors of the same length
def crosscovariance_normalized(y1,y2,nmax):

    c_corr = numpy.zeros(nmax)
    lags = numpy.arange(0,nmax,1)

    for n in range(0,len(c_corr)):
```

```
        if n ==0:
            c_corr[n] = ((1./(len(y1)-float(n)))*numpy.sum((y1-numpy.mean(y1))*(y2-numpy
        if n >0:
            c_corr[n] = ((1./(len(y1)-float(n)))*numpy.sum((y1[0:-n]-numpy.mean(y1[0:-n]

    lags_neg = numpy.arange(1,nmax,1)*-1.
    c_corr_neg = numpy.zeros(nmax-1)
    for n in range(1,len(c_corr)):
        c_corr_neg[n-1] = ((1./(len(y1)-float(n)))*numpy.sum((y1[n:]-numpy.mean(y1[n:]))

    lags = numpy.append(numpy.flipud(lags_neg),lags)
    c_corr = numpy.append(numpy.flipud(c_corr_neg),c_corr)

    return lags,c_corr

#calculate normalized crosscovariance of y1 and y2

lags,c_corr = crosscovariance_normalized(y1,y2,50)

#identify the lag value with the maximum normalized cross-covariance

w_lag = numpy.argmax(c_corr)

#print lead of y1 relative to y2

print("Lags relating y1 to y2",lags[w_lag])

if lags[w_lag] >=0:

    print("y1 is ",(lags[w_lag]), "lags ahead of y1")

if lags[w_lag] <0:

    print("y2 is ",(lags[w_lag]), "lags behind y2")

#plot the crosscovariance as a function of lags

#create a figure
fig = plt.figure(2,figsize=(7,7))

#create a subplot that shows y1 and y2 plotted as a function of x
ax1 = plt.subplot(211)
ax1.plot(x,y1,color='C0',label = "y1")
ax1.plot(x,y2,color='C1',label = "y2")
ax1.set_ylabel("y")
ax1.set_xlabel("x")
ax1.legend()
ax1.grid()

#create a subplot that shows cross-covariance of y1 and y2 plotted as a function of lags
ax2 = plt.subplot(212)
ax2.plot(lags,c_corr,color='C2')
ax2.set_ylabel("norm. cross-covariance")
ax2.set_xlabel("lags")
ax2.set_xlim(-50,50)
ax2.grid()
plt.show()
```

```
Lags relating y1 to y2 -15.0
y2 is  -15.0 lags behind y2
```





8. Spectral analysis

8.1 Fourier series

Equations from Thomson and Emery (2014) - Ch. 5.

$$y(t) = \frac{1}{2}A_0 + \sum_{p=1}^{\infty}\left[A_p \cos(\omega_p t) + B_p \sin(\omega_p t)\right] \tag{5.246}$$

in which

$$\omega_p = 2\pi f_p = 2\pi p f_1 = 2\pi p / T; \quad p = 1, 2, \ldots \tag{5.247}$$

$$A_p = \frac{2}{T}\int_0^T y(t)\cos(\omega_p t)\mathrm{d}t, \quad p = 0, 1, 2, \ldots \tag{5.248a}$$

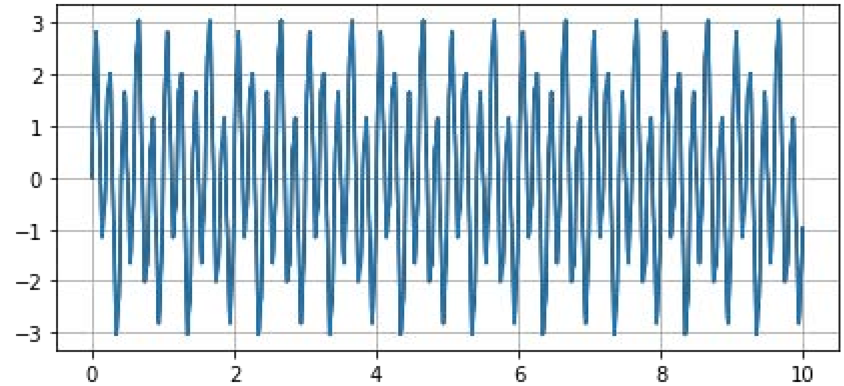
$$B_p = \frac{2}{T}\int_0^T y(t)\sin(\omega_p t)\mathrm{d}t, \quad p = 1, 2, \ldots \tag{5.248b}$$

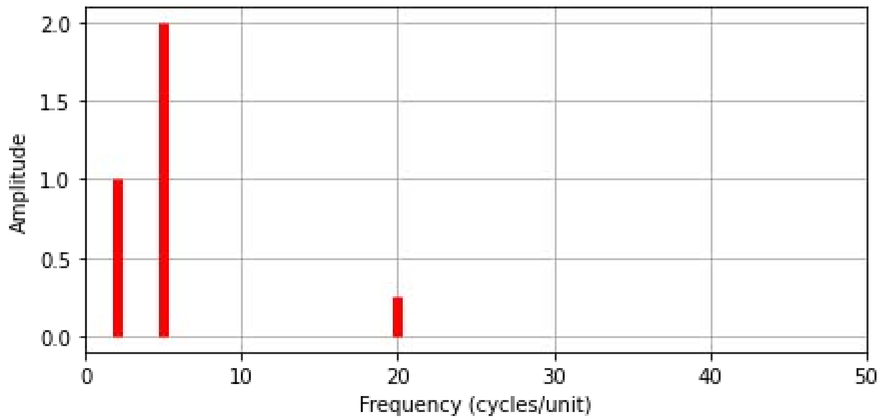
$$C_p = \left(A_p^2 + B_p^2\right)^{1/2}, \quad p = 0, 1, 2, \ldots \tag{5.250}$$

In [103…

```
import numpy
import matplotlib.pyplot as plt
from scipy import signal

#generate a vector, 1000 values from 0 to T
T = 10.0
x = numpy.linspace(0.0, T, 1000, endpoint=False)
dt = x[1] - x[0] #calculate dt
```

```
#make a square wave that has a frequency of 2 cycles per unit (or 2pi * 2 cycles per 2pi
sine_wave = numpy.sin(x*2.*numpy.pi*2.) + 2.*numpy.sin(x*2.*numpy.pi*5.) + 0.25*numpy.si

#make length of record a variable (N), as well as number of lags well use (M)
N = len(sine_wave)
M = 500

#create a figure with two subplots

fig = plt.figure(1,figsize=(7,7))
ax1= plt.subplot(211)
ax2= plt.subplot(212)

ax1.plot(x,sine_wave,'k') #plot sine wave
sum1 = 0.
#create loop to calculate coefficient for fourier series using M frequencies
for p in range(0,M+1):

    #calculate C0 coefficient
    C0 = 2. * numpy.mean(sine_wave)

    if p == 0:

        f_x = numpy.zeros(len(sine_wave)) + (1./2. * C0)

    #calculate the Ap, Bp, and Cp coefficients. Phase is a combination of Ap and Bp.

    if p >0:
        Ap = (2. / T) *  numpy.sum(sine_wave * numpy.cos((2. * numpy.pi * p * x)/T)* dt)
        Bp = (2. / T) *  numpy.sum(sine_wave * numpy.sin((2. * numpy.pi * p * x)/T)* dt)

        f_x = f_x + (Ap * numpy.cos((2. * numpy.pi * p * x)/T)) + (Bp * numpy.sin((2. *

        Cp = numpy.sqrt((Ap**2.) + (Bp**2.))
        ax2.plot([(p)/T,(p)/T],[0,Cp],'red',linewidth=5.0,solid_capstyle='butt')
        sum1 += (((Cp)))
ax1.plot(x,f_x)

ax2.set_xlabel("Frequency (cycles/unit)")
ax2.set_ylabel("Amplitude")
ax2.set_xlim(0,50)

ax1.grid()
ax2.grid()

plt.show()
```





8.2 Discrete Fourier transform

8.2.1 FFT to calculate spectral density

The Discrete Fourier transform is calculated as follows in the script below.

$$
\begin{aligned}
Y_k &= \Delta t \sum_{n=1}^{N} y_n e^{-i2\pi f_k n \Delta t} \\
&= \Delta t \sum_{n=1}^{N} y_n e^{-i2\pi k n / N}; \\
f_k &= k/N\Delta t, \; k = 0, \ldots, N \qquad (5.27)
\end{aligned}
$$

We check that our calculation adheres to Parseval's theorem, which states that signal energy in the time domain corresponds to energy in the frequency domain.

$$\Delta t \sum_{n=1}^{N} |y_n|^2 = \Delta f \sum_{k=0}^{N-1} |Y_k|^2$$

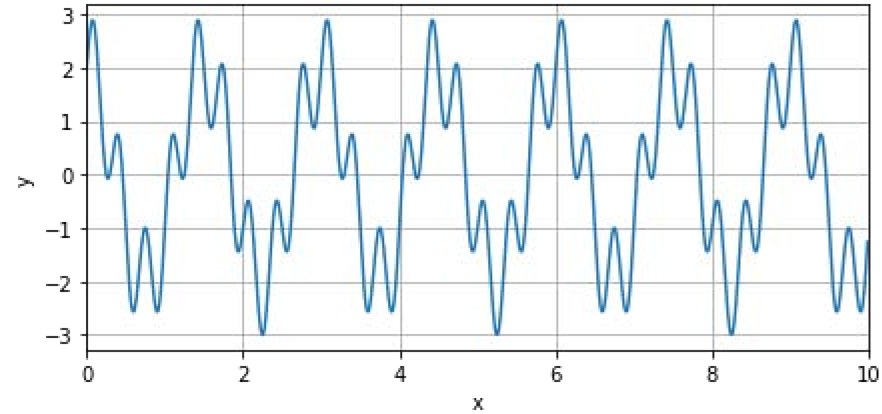
Equations from Thomson and Emery (2014) - Ch. 5.

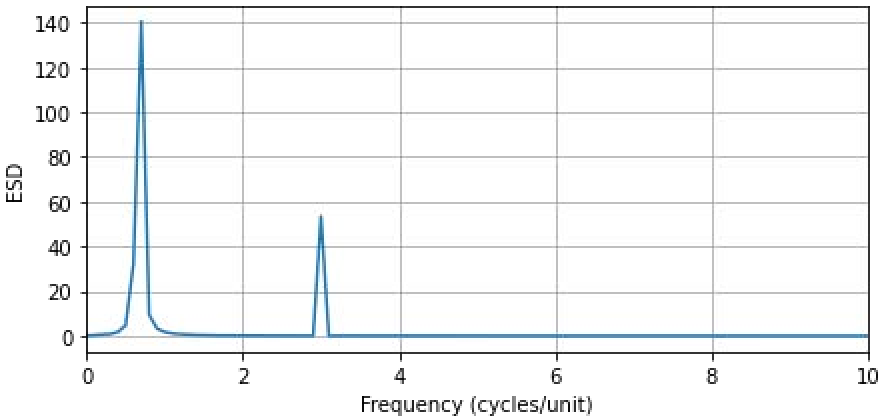
In [104…

```
import numpy
import matplotlib.pyplot as plt

#generate a vector, 1000 values from 0 to T, where T = 10

T = 10.0
N = 1000
x = numpy.linspace(0.0, T, N, endpoint=False)

#calculate dt

dt = x[1] - x[0]
```

```
#create a signal that is a combination of sine waves with wwavelengths of 1/3 amd 2.

a = numpy.sin(x*2.*numpy.pi*3.) + (2.*numpy.cos((x*2.*numpy.pi)/1.5))

#calculate fft multiplied by dt

fft_sinewave = dt * numpy.fft.fft(a)[:int(N/2)]

freq_sw = numpy.fft.fftfreq(len(a),d=dt)[:int(N/2)] #fft frequencies

df_fft = freq_sw[1] - freq_sw[0]

#double and square the result to obtain ESD as defined by Thomson and Emery (2014)

G_k = numpy.zeros(len(fft_sinewave))

G_k = 2.*(numpy.abs(fft_sinewave)**2.)
G_k[0] = numpy.abs(fft_sinewave[0])**2.

#create a figure

fig = plt.figure(2,figsize=(7,7))

#plot original data series

ax1 = plt.subplot(211)
ax1.plot(x,a)
ax1.set_xlim(0,numpy.max(10))
ax1.set_xlabel("x")
ax1.set_ylabel("y")
ax1.grid()

#plot ESD

ax2 = plt.subplot(212)

ax2.plot(freq_sw,G_k)
ax2.set_xlim(0,10)
ax2.set_xlabel("Frequency (cycles/unit)")
ax2.set_ylabel("ESD")
ax2.grid()

#Calculate signal energy in the time and frequency domain

print("Energy in the time domain:",round(numpy.sum(a**2.)*dt,3))
print("Energy in the freq. domain:",round(sum(G_k)*df_fft,3))

plt.show()
```

```
Energy in the time domain: 25.556
Energy in the freq. domain: 25.556
```





8.2.2 Bandpass filter

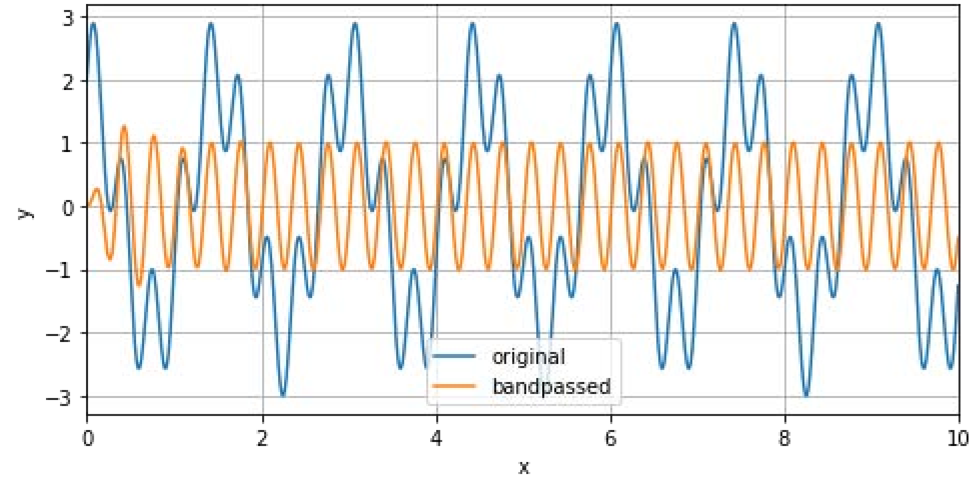
Application of a butterworth bandpass filter to extract energy from a specific bandwidth within the data series.

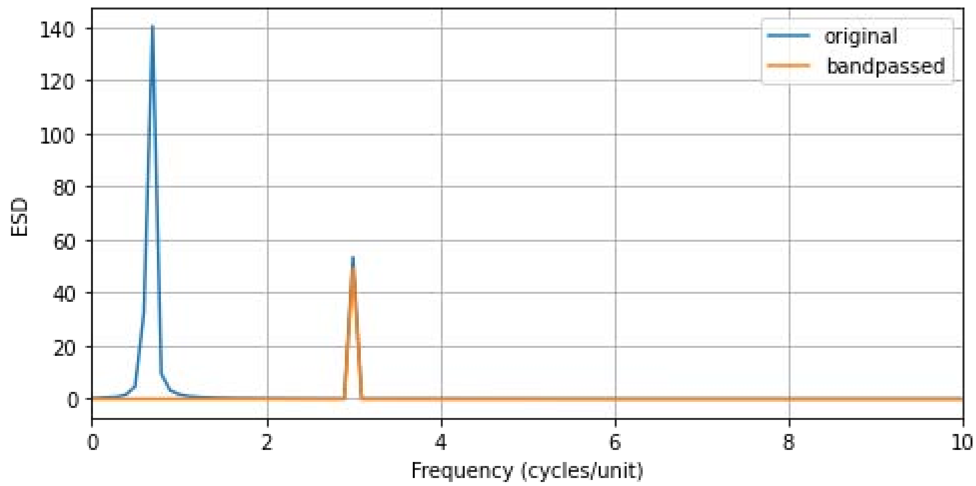
In [105…

```
import numpy
import matplotlib.pyplot as plt
import scipy
from scipy import signal
from scipy.signal import butter, lfilter

#generate a vector, 1000 values from 0 to T, where T = 10

T = 10.0
N = 1000
x = numpy.linspace(0.0, T, N, endpoint=False)

#calculate dt

dt = x[1] - x[0]

#create a signal that is a combination of sine waves with wwavelengths of 1/3 amd 2.

a = numpy.sin(x*2.*numpy.pi*3.) + (2.*numpy.cos((x*2.*numpy.pi)/1.5))

#calculate fft multiplied by dt

fft_sinewave = dt * numpy.fft.fft(a)[:int(N/2)]

freq_sw = numpy.fft.fftfreq(len(a),d=dt)[:int(N/2)] #fft frequencies

df_fft = freq_sw[1] - freq_sw[0]

#double and square the result to obtain ESD as defined by Thomson and Emery (2014)

G_k = numpy.zeros(len(fft_sinewave))
```

```
G_k = 2.*(numpy.abs(fft_sinewave)**2.)
G_k[0] = numpy.abs(fft_sinewave[0])**2.

#Calculate signal energy in the time and frequency domain

print("Energy in the time domain:",round(numpy.sum(a**2.)*dt,3))
print("Energy in the freq. domain:",round(sum(G_k)*df_fft,3))

#Apply butterworth bandpass filter, cutoff frequencies are set to 2 and 4

sos = signal.butter(3, [2.,4.], 'bp', fs=1/dt, output='sos')
filtered = signal.sosfilt(sos, a)

#calculate ESD of bandpassed signal

fft_bandpass = dt * numpy.fft.fft(filtered)[:int(N/2)]
freq_bandpass = numpy.fft.fftfreq(len(filtered),d=dt)[:int(N/2)] #fft frequencies
df_fft_bandpass = freq_bandpass[1] - freq_bandpass[0]

G_k_bandpass = numpy.zeros(len(fft_bandpass))

G_k_bandpass = 2.*(numpy.abs(fft_bandpass)**2.)
G_k_bandpass[0] = numpy.abs(fft_bandpass[0])**2.

#create a figure

fig = plt.figure(2,figsize=(7,7))

#create suplot for orignal and bandpassed data series

ax1 = plt.subplot(211)

#plot original data series

ax1.plot(x,a,label="original")

#plot bandpassed data series

ax1.plot(x,filtered,label="bandpassed")
ax1.set_xlim(0,numpy.max(10))
ax1.legend()
ax1.set_xlabel("x")
ax1.set_ylabel("y")
ax1.grid()

#plot ESD

ax2 = plt.subplot(212)

ax2.plot(freq_sw,G_k,label="original")
ax2.plot(freq_bandpass,G_k_bandpass,label="bandpassed")
ax2.legend()
ax2.set_xlim(0,10)
ax2.set_xlabel("Frequency (cycles/unit)")
ax2.set_ylabel("ESD")
ax2.grid()
print("Energy in the freq. domain for the bandpassed signal:",round(sum(G_k_bandpass)*df

plt.tight_layout()
plt.show()
```

```
Energy in the time domain: 25.556
Energy in the freq. domain: 25.556
Energy in the freq. domain for the bandpassed signal: 5.006
```





8.2.3 Red-noise spectrum

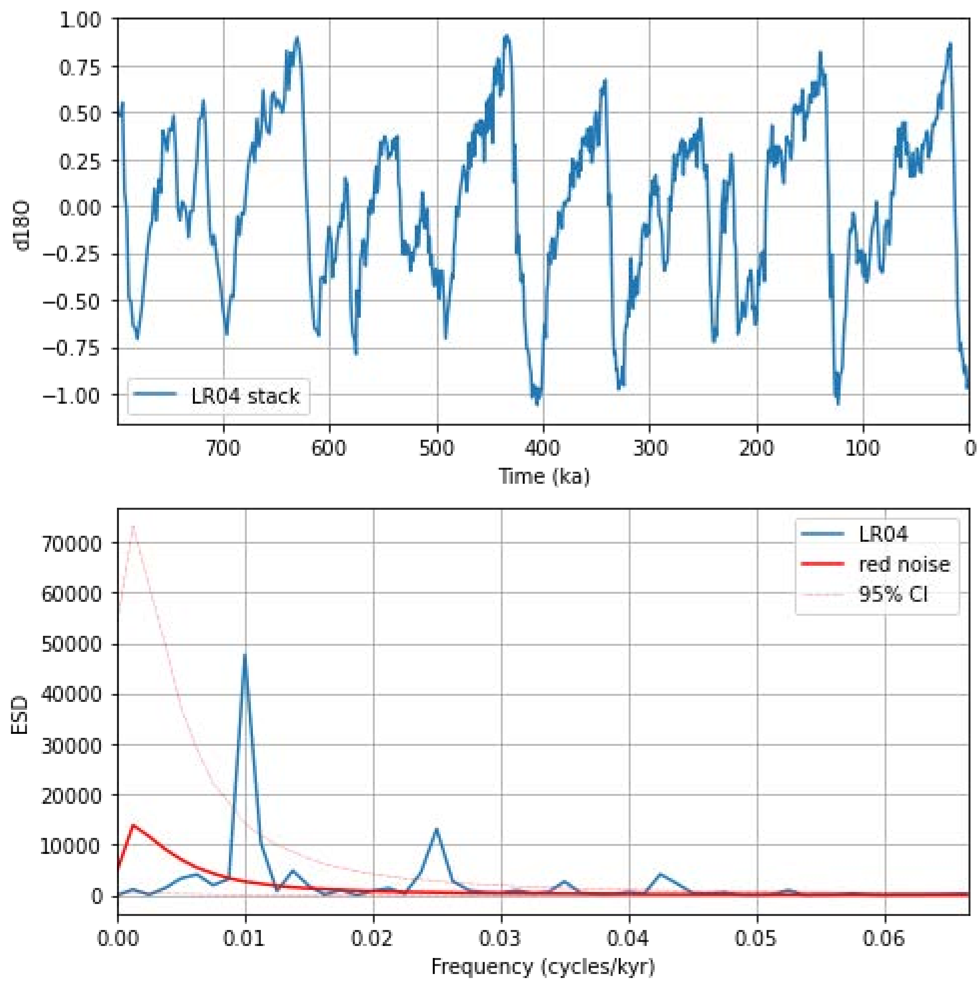
Test if the signal can be replicated by autocorrelated noise

In [106…

```
import matplotlib.pyplot as plt
import scipy
from scipy import signal
import numpy
import requests

##dowloaad red noise algorithm as a python script from Github

url = 'https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Scripts/red_no
red_noise = requests.get(url)

with open('red_noise.py', 'w') as f:
    f.write(red_noise.text)

import red_noise

##dowloaad LR04 benthic foraminiferal stack as a python script from Github

url = 'https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Data/LR04_d18O
LR04_d18O_stack = requests.get(url)

with open('LR04_d18O_stack.txt', 'w') as f:
    f.write(LR04_d18O_stack.text)

## read datafile, set comma as the delimiter

filename = "LR04_d18O_stack.txt"
data = numpy.genfromtxt(filename,delimiter=',')
```

```
#isolate data from last 800 kyr
w1 = numpy.where(data[:,0]<=800)[0]

#reinterpolate to constant sampling rate
T = numpy.max(data[w1,0])
dt = 1.
x = numpy.arange(0,T,dt)

N = len(x)

f = scipy.interpolate.interp1d(data[w1,0],data[w1,1])
a = scipy.signal.detrend(f(x),type="linear")

#calculate fft of LR04 stack, multiplied by dt

fft_LR04 = dt * numpy.fft.fft(a)[:int(N/2)]

freq_LR04 = numpy.fft.fftfreq(len(a),d=dt)[:int(N/2)] #fft frequencies

df_fft = freq_LR04[1] - freq_LR04[0]

#double and square the result to obtain ESD as defined by Thomson and Emery (2014)

G_k = numpy.zeros(len(fft_LR04))

G_k = 2.*(numpy.abs(fft_LR04)**2.)
G_k[0] = numpy.abs(fft_LR04[0])**2.

#calculate a red noise spectrum to test if spectral estimates can be explained by correl

monte_carlo_i = 10000

output = numpy.zeros((monte_carlo_i,len(freq_LR04)))


for n in range(0,monte_carlo_i):
    a2 = red_noise.red_noise(a)
    fft_rn = dt * numpy.fft.fft(a2)[:int(N/2)]
    freq_rn = numpy.fft.fftfreq(len(a2),d=dt)[:int(N/2)]
    df_rn = freq_rn[1] - freq_rn[0]
    output[n,:] = 2.*(numpy.abs(fft_rn)**2.)
    output[n,0] = numpy.abs(fft_rn[0])**2.

G_k_rn = numpy.quantile(output,0.5,axis=0)
G_k_rn_l = numpy.quantile(output,0.025,axis=0)
G_k_rn_h = numpy.quantile(output,0.975,axis=0)

#create a figure

fig = plt.figure(2,figsize=(7,7))

#create suplot for orignal and bandpassed data series

ax1 = plt.subplot(211)

#plot original data series

ax1.plot(x,a,label="LR04 stack")
ax1.set_xlim(numpy.max(x),0)

ax1.legend()
ax1.set_xlabel("Time (ka)")
ax1.set_ylabel("d18O")
ax1.grid()
```

```
#plot ESD

ax2 = plt.subplot(212)

ax2.plot(freq_LR04,G_k,label="LR04")
ax2.plot(freq_rn,G_k_rn,color='r',label="red noise")
ax2.plot(freq_rn,G_k_rn_h,linewidth=0.5,linestyle=":",color='r',label="95% CI")
ax2.plot(freq_rn,G_k_rn_l,linewidth=0.5,linestyle=":",color='r')
ax2.legend()
ax2.set_xlim(0,1/15)
ax2.set_xlabel("Frequency (cycles/kyr)")
ax2.set_ylabel("ESD")
ax2.grid()

plt.tight_layout()
plt.show()
```



9. Maximum likelihood estimation and Gaussian Processes

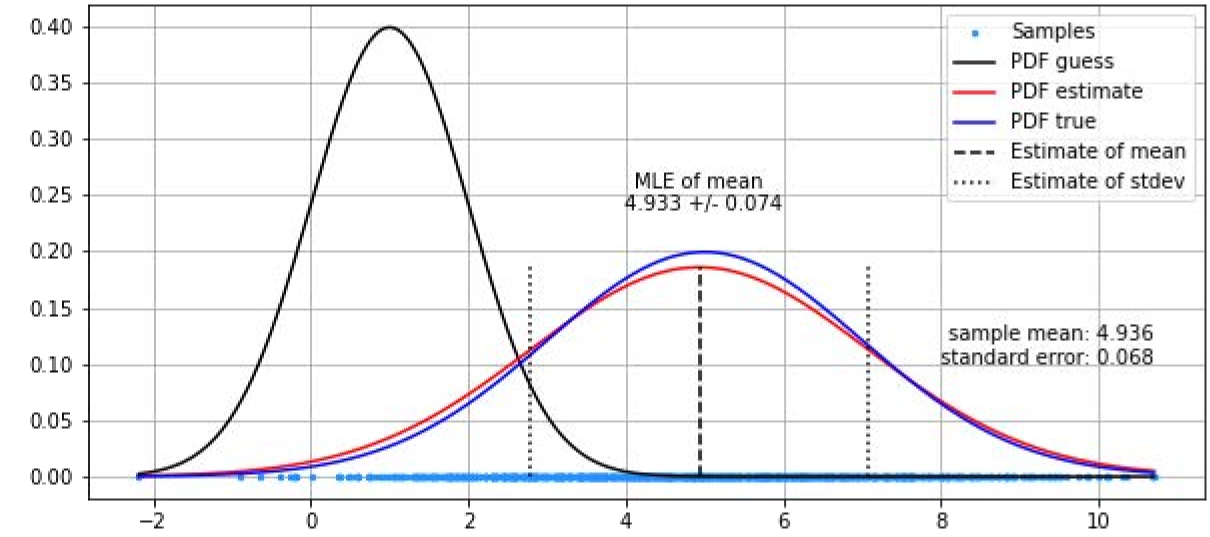
9.1 Maximimum likelihood estimation

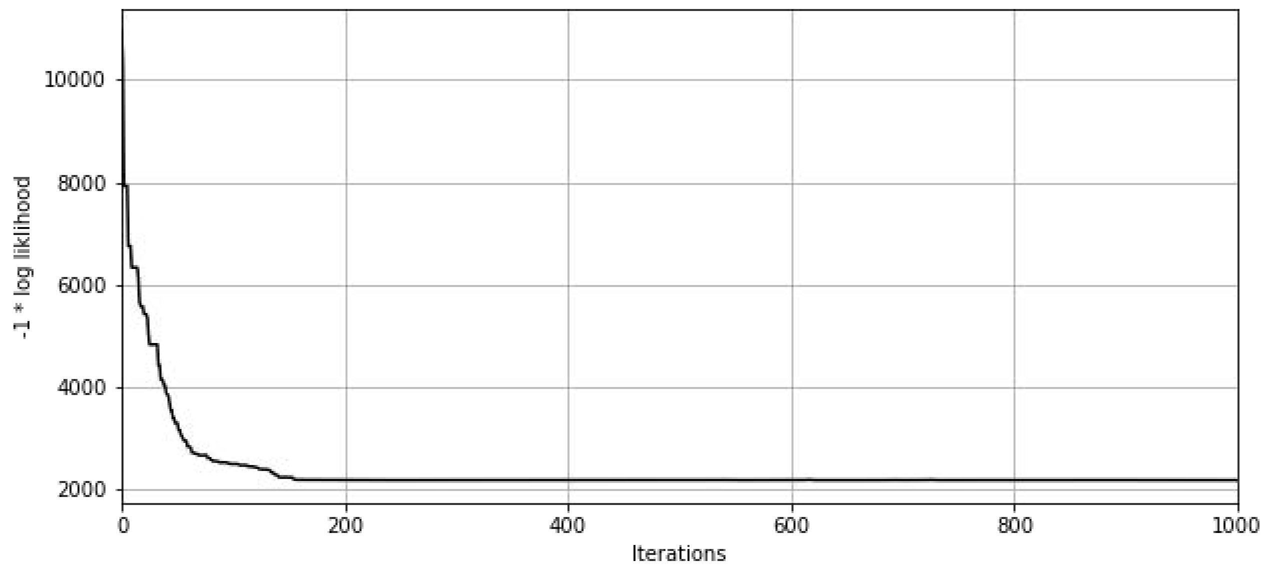
In [107…

```
import os
import sys
import numpy
import scipy
import matplotlib.pyplot as plt

mean_stdev = numpy.array([5.,2.])

def log_lik(y,model,stdev):
```

```
    n_in = len(y)

    loglik = - ((n_in/2.)*numpy.log(stdev**2)) -  ((n_in/2.) * numpy.log(2. * numpy.pi))

    return -1. * loglik

def function(theta):
    return theta[0]

def MCMC(function,data,iterations,theta_guess,stepsize,burn_in):
    output_matrix_A = numpy.zeros((iterations,len(theta_guess)))
    loglik_output = numpy.zeros((iterations,1))
    accept_output = numpy.zeros((iterations,1))
    iter_vec= numpy.arange(0,iterations,1)
    x_new = numpy.linspace(numpy.min(data),numpy.max(data),1000)
    for n in range(MCMC_iters):

        if n==0:
            model_out = function(theta_guess)
            old_loglik = log_lik(data,model_out,theta_guess[1])
            new_theta  = theta_guess * 1.0
        if n > 0:
            old_theta  = output_matrix_A[n-1,:]
            old_loglik = loglik_output[n-1,0]
            new_theta = numpy.ndarray.flatten((numpy.random.normal(loc = old_theta, scal

        model_out2 = function(new_theta)
        new_loglik = log_lik(data,model_out2,new_theta[1])

        if numpy.isnan(new_loglik) == False:
            if (new_loglik < old_loglik):
                output_matrix_A[n,:] = new_theta
                loglik_output[n,0] = new_loglik
                accept_output[n,0] = 1.0

            else:
                u = numpy.random.uniform(0.0,1.0)

                if (u < numpy.exp(old_loglik - new_loglik)):
                    output_matrix_A[n,:] = new_theta
                    loglik_output[n,0] = new_loglik
                    accept_output[n,0] = 1.0

                else:
                    output_matrix_A[n,:] = old_theta
                    loglik_output[n,0] = old_loglik
                    accept_output[n,0] = 0.0

        else:
            output_matrix_A[n,:] = old_theta
            loglik_output[n,0] = old_loglik
            accept_output[n,0] = 0.0

    return output_matrix_A,loglik_output

MCMC_iters = 1000
burn_in = 350

y_1 = numpy.random.normal(mean_stdev[0],mean_stdev[1],1000)

fig = plt.figure(1,figsize=(10,10))

theta_guess = numpy.array([1.,1.])
stepsize = numpy.array([1./10.,1./10.])
output_matrix_A,loglik_output = MCMC(function,y_1,MCMC_iters,theta_guess,stepsize,burn_i
```

```
mean_est = numpy.mean(output_matrix_A[burn_in:,0])
mean_est_error = numpy.std(output_matrix_A[burn_in:,0])
std_est = numpy.mean(output_matrix_A[burn_in:,1])
std_est_error = numpy.std(output_matrix_A[burn_in:,1])

ax1 = plt.subplot(211)
ax1.plot(y_1,y_1*0.0,color="dodgerblue",linewidth=0.0,marker="o",markersize=2.5,label="S
x_new = numpy.linspace(numpy.min(y_1),numpy.max(y_1),1000)
PDF_normal_original = (1./(output_matrix_A[0,1]*numpy.sqrt(2.*numpy.pi)))*numpy.exp(-1.*
PDF_normal_current = (1./(std_est*numpy.sqrt(2.*numpy.pi)))*numpy.exp(-1.*(1./(2.*std_es
PDF_true = (1./(mean_stdev[1]*numpy.sqrt(2.*numpy.pi)))*numpy.exp(-1.*(1./(2.*mean_stdev
ax1.plot(x_new,PDF_normal_original,color='k',label="PDF guess")
ax1.plot(x_new,PDF_normal_current,color='r',label="PDF estimate")
ax1.plot(x_new,PDF_true,color='blue',label="PDF true")


ax1.plot([mean_est,mean_est],[0,numpy.max(PDF_normal_current)],color='k',linestyle="--",
ax1.plot([mean_est+std_est,mean_est+std_est],[0,numpy.max(PDF_normal_current)],color='k'
ax1.plot([mean_est-std_est,mean_est-std_est],[0,numpy.max(PDF_normal_current)],color='k'

label1 = "MLE of mean\n " + str(round(mean_est,3)) + " +/- " + str(round(mean_est_error,
ax1.text(mean_est,numpy.max(PDF_normal_current)+0.05,label1,color='k',horizontalalignmen

label2 = "sample mean: " + str(round(numpy.mean(y_1),3)) + "\n standard error: " + str(r
ax1.text(numpy.max(y_1),.1,label2,color='k',horizontalalignment="right")

ax1.grid()
ax1.legend()
ax2 = plt.subplot(212)
iter_vec= numpy.arange(0,MCMC_iters,1)
ax2.plot(iter_vec,loglik_output,color='k')
ax2.set_ylabel("-1 * log liklihood")
ax2.set_xlabel("Iterations")
ax2.set_xlim(0,MCMC_iters)
ax2.grid()
plt.show()
```





9.2 Gaussian process regression

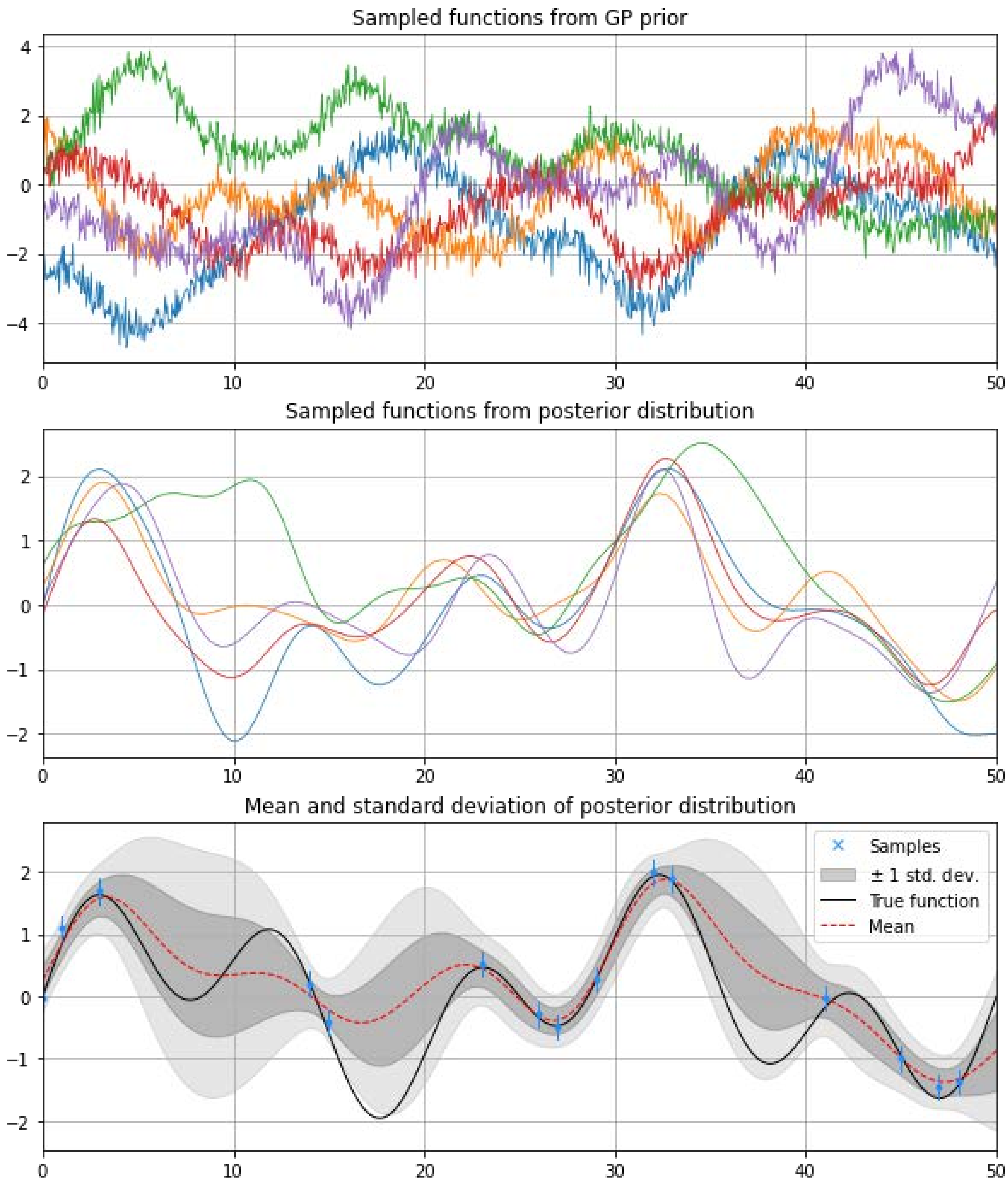
In [108…

```
#import modules
import numpy
import sklearn
import sklearn.gaussian_process
from sklearn.gaussian_process import GaussianProcessRegressor
from sklearn.gaussian_process.kernels import ConstantKernel, DotProduct, Matern, RBF,Exp

import matplotlib
import matplotlib.pyplot as plt

x_data_orig = numpy.arange(0,50+1,1)
numpy.random.shuffle(x_data_orig)
x_data = x_data_orig[0:15]
x_data_orig = numpy.linspace(0,50,1000)

sine_curve = numpy.sin(2.*numpy.pi*x_data/25.) + numpy.sin(2.*numpy.pi*x_data/10.) + num
sine_curve_orig = numpy.sin(2.*numpy.pi*x_data_orig/25.) + numpy.sin(2.*numpy.pi*x_data_

kernel1 = 1.5**2. * RBF(length_scale=3.)
kernel2 = WhiteKernel(noise_level=0.1)

kernel = kernel1 + kernel2

X_train = numpy.reshape(x_data,(-1,1))
```

```python
y_train = sine_curve * 1.0

gpr = GaussianProcessRegressor(kernel=kernel,alpha=0.1,n_restarts_optimizer = 10,random_

fig = plt.figure(1,figsize=(10,12))
n_samples = 5

ax1 = plt.subplot(311)
x = numpy.linspace(0,50+1,1000)
X = numpy.reshape(x,(-1,1))

y_samples = gpr.sample_y(X, n_samples)

for n in range(0,n_samples):

    ax1.plot(x,y_samples[:,n],linestyle="-",linewidth=0.75,alpha=1.0)

ax1.set_xlim(0,50)

ax1.grid()
ax1.set_title("Sampled functions from GP prior")

gpr.fit(X_train, y_train)
n_samples = 5
ax1 = plt.subplot(312)

x = numpy.linspace(0,50+1,1000)
X = numpy.reshape(x,(-1,1))
y_samples = gpr.sample_y(X, n_samples)

for n in range(0,n_samples):

    ax1.plot(x,y_samples[:,n],linestyle="-",linewidth=0.75,alpha=1.0)

print(gpr.log_marginal_likelihood())
print(gpr.kernel_)

ax1.set_xlim(0,50)

ax1.grid()
ax1.set_title("Sampled functions from posterior distribution")
ax2 = plt.subplot(313)

y_mean, y_std = gpr.predict(X, return_std=True)
ax2.plot([0,0],[numpy.nan,numpy.nan],marker="x",linewidth=0,color="dodgerblue",label="Sa
ax2.fill_between(x,y_mean - 2.*y_std,y_mean + 2.*y_std,alpha=0.1,color="black")
ax2.fill_between(x,y_mean - y_std,y_mean + y_std,alpha=0.2,color="black",label=r"$\pm$ 1

ax2.plot(x_data_orig, sine_curve_orig, color="black", linewidth=1.0, label="True functio
ax2.plot(x, y_mean, color="red", linewidth=1.0,linestyle="--", label="Mean")


for n in range(0,len(x_data)):

    ax2.plot([x_data[n],x_data[n]],[sine_curve[n]-0.2,sine_curve[n]+0.2],linewidth=1.0,c

ax2.plot(x_data,sine_curve,marker=".",linewidth=0,color="dodgerblue")

ax2.set_title("Mean and standard deviation of posterior distribution")
ax2.set_xlim(0,50)
ax2.legend()
ax2.grid()
plt.show()
```

```
C:\Users\racecar\anaconda3\lib\site-packages\sklearn\gaussian_process\kernels.py:420: Co
```

```
nvergenceWarning: The optimal value found for dimension 0 of parameter k2__noise_level i
s close to the specified lower bound 1e-05. Decreasing the bound and calling fit again m
ay find a better value.
  warnings.warn(
-16.19936773633101
1.02**2 * RBF(length_scale=2.83) + WhiteKernel(noise_level=1e-05)
```



10. Logistic regression, random forest, and neural network algorithms

10.1 Logistic regression

In [109…

```
import matplotlib.pyplot as plt
import numpy
import sklearn
from sklearn.linear_model import LogisticRegression
from sklearn import metrics

#Create a correlated distribution of data with a covariance set to 0.75, N = 1000
```

```
N = 1000

var_x1 = 1.
var_x2 = 1.
cov_x1_x2 = 0.75
mean_x1 = 0.0
mean_x2 = 0.0

u = numpy.array([mean_x1,mean_x2])
k = numpy.array([[var_x1, cov_x1_x2], [cov_x1_x2, var_x2]])
dist = numpy.random.multivariate_normal(u, k, size=N)
x1, x2 = dist[:,0],dist[:,1]

#make variable y equal to 1 if x2>=0 and equal to 0 if x2<0

y = x2*0.0

w1 = numpy.where(x2>=0)[0]

y[w1] = 1.0

#start logistic regression

logisticRegr = LogisticRegression()

#fit the model on first 500 points in

x_train = numpy.reshape(x1[0:int(N/2)],(-1,1))
y_train = numpy.reshape(y[0:int(N/2)],(-1,1))

logisticRegr.fit(x_train, y_train)

#make predictions and validate model using test data

x_test = numpy.reshape(x1[int(N/2):],(-1,1))
y_test = numpy.reshape(y[int(N/2):],(-1,1))

predictions = logisticRegr.predict(x_test)

w1 = numpy.where(predictions==y_test[:,0])[0]

perc_correct = len(w1)/len(x_test)

print(perc_correct)

#print the 'score' of the model

score = logisticRegr.score(x_test, y_test)
print(score)

cm = metrics.confusion_matrix(y_test, predictions)
print(cm)

#create a figure
fig = plt.figure(1,figsize=(14,7))

#plot the original distribution

ax1 = plt.subplot(121)

ax1.plot(x1, x2,color="dodgerblue",marker=".",linewidth=0.0,alpha=0.5,label="data points
ax1.grid()

#plot the model
```

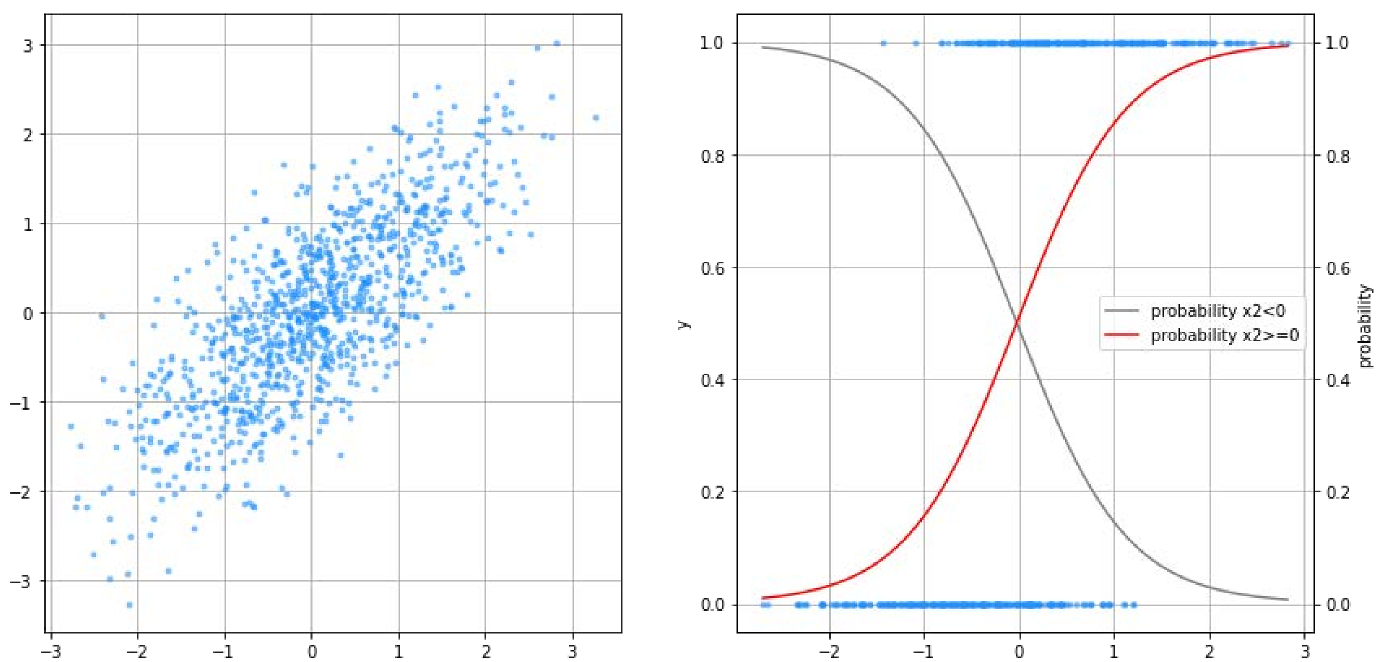
```
ax2 = plt.subplot(122)

ax2.plot(x_test,y_test,color="dodgerblue",marker=".",linewidth=0.0,alpha=0.5,label="data
ax2.grid()
ax2.set_ylabel("y")
ax2.set_ylim(-0.05,1.05)
x_vector = numpy.reshape(numpy.linspace(numpy.min(x_test),numpy.max(x_test),250),(-1,1))
prob_x1_0 = logisticRegr.predict_proba(x_vector)[:,0]
prob_x1_1 = logisticRegr.predict_proba(x_vector)[:,1]

ax3 = ax2.twinx()
ax3.plot(x_vector,prob_x1_0,color="gray",label="probability x2<0")
ax3.plot(x_vector,prob_x1_1,color="r",label="probability x2>=0")
ax3.legend()
ax3.set_ylim(-0.05,1.05)
ax3.set_ylabel("probability")
plt.show
```

```
0.766
0.766
[[187  68]
 [ 49 196]]
```

```
C:\Users\racecar\anaconda3\lib\site-packages\sklearn\utils\validation.py:993: DataConver
sionWarning: A column-vector y was passed when a 1d array was expected. Please change th
e shape of y to (n_samples, ), for example using ravel().
  y = column_or_1d(y, warn=True)
```

Out[109]:

```
<function matplotlib.pyplot.show(close=None, block=None)>
```



10.2 Logistic regression 2

Example with 1 independent variable and three classifications (0,1,2)

In [110…

```
import matplotlib.pyplot as plt
import numpy
import sklearn
from sklearn.linear_model import LogisticRegression
from sklearn import metrics

#Create a correlated distribution of data with a covariance set to 0.75, N = 1000

N = 1000
```

```
var_x1 = 1.
var_x2 = 1.
cov_x1_x2 = 0.75
mean_x1 = 0.0
mean_x2 = 0.0

u = numpy.array([mean_x1,mean_x2])
k = numpy.array([[var_x1, cov_x1_x2], [cov_x1_x2, var_x2]])
dist = numpy.random.multivariate_normal(u, k, size=N)
x1, x2 = dist[:,0],dist[:,1]

#make variable y equal to: 1) 0 if x2<-0.5; 2) 1 if x2>=-0.75 and x2<=0.5; 3) or 2 if x2

y = x2*0.0

w1 = numpy.where((x2>=-.5)&(x2<.5))[0]

y[w1] = 1.0

w2 = numpy.where((x2>=.5))[0]

y[w2] = 2.0

#start logistic regression

logisticRegr = LogisticRegression()

#fit the model on first 500 points in

x_train = numpy.reshape(x1[0:int(N/2)],(-1,1))
y_train = numpy.reshape(y[0:int(N/2)],(-1,1))

logisticRegr.fit(x_train, y_train)

#make predictions and validate model using test data

x_test = numpy.reshape(x1[int(N/2):],(-1,1))
y_test = numpy.reshape(y[int(N/2):],(-1,1))

predictions = logisticRegr.predict(x_test)

w1 = numpy.where(predictions==y_test[:,0])[0]

perc_correct = len(w1)/len(x_test)

print(perc_correct)

#print the 'score' of the model

score = logisticRegr.score(x_test, y_test)
print(score)

cm = metrics.confusion_matrix(y_test, predictions)
print(cm)

#create a figure
fig = plt.figure(1,figsize=(14,7))

#plot the original distribution

ax1 = plt.subplot(121)

ax1.plot(x1, x2,color="dodgerblue",marker=".",linewidth=0.0,alpha=0.5,label="data points
ax1.grid()

ax1.set_xlabel("x1")
```

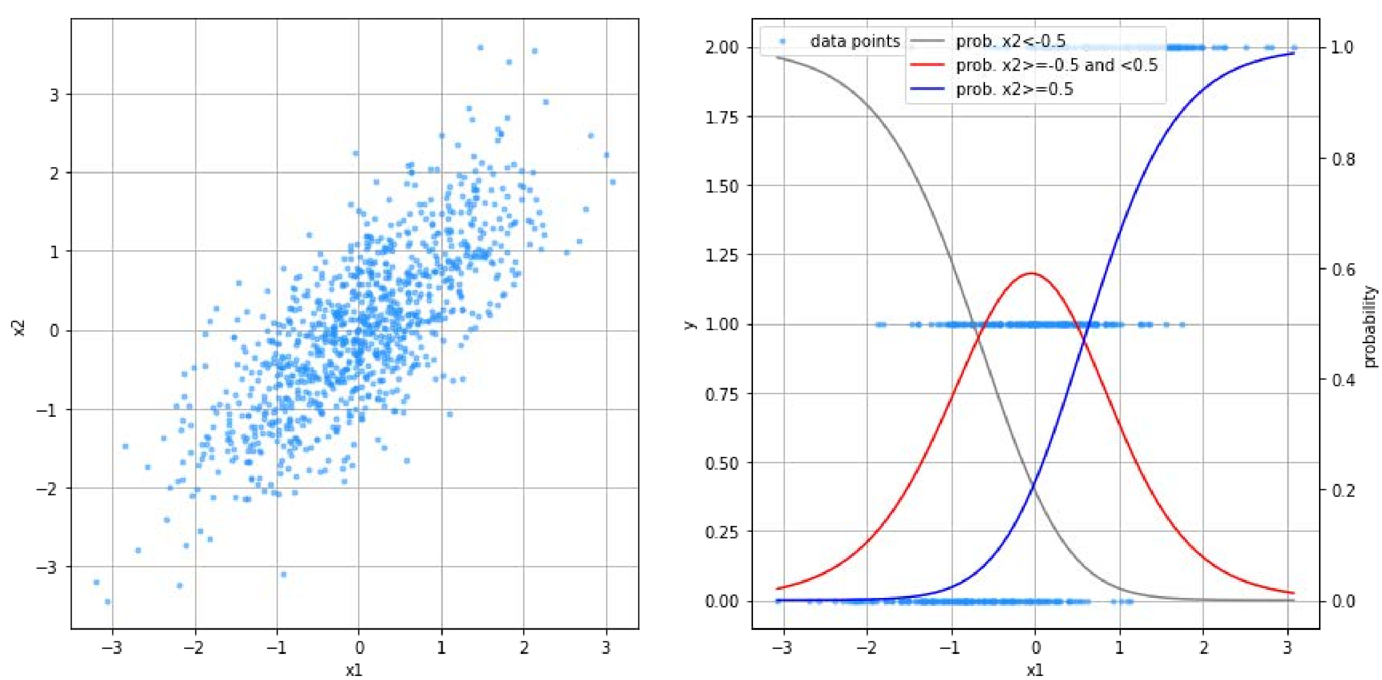
```
ax1.set_ylabel("x2")

#plot the model

ax2 = plt.subplot(122)

ax2.plot(x_test,y_test,color="dodgerblue",marker=".",linewidth=0.0,alpha=0.5,label="data
ax2.grid()
ax2.set_ylabel("y")
ax2.set_xlabel("x1")
ax2.set_ylim(-0.1,2.1)
ax2.legend()
x_vector = numpy.reshape(numpy.linspace(numpy.min(x_test),numpy.max(x_test),250),(-1,1))
prob_x1_0 = logisticRegr.predict_proba(x_vector)[:,0]
prob_x1_1 = logisticRegr.predict_proba(x_vector)[:,1]
prob_x1_2 = logisticRegr.predict_proba(x_vector)[:,2]

ax3 = ax2.twinx()
ax3.plot(x_vector,prob_x1_0,color="gray",label="prob. x2<-0.5")
ax3.plot(x_vector,prob_x1_1,color="r",label="prob. x2>=-0.5 and <0.5")
ax3.plot(x_vector,prob_x1_2,color="blue",label="prob. x2>=0.5")
ax3.legend()
ax3.set_ylim(-0.05,1.05)
ax3.set_ylabel("probability")
plt.show
```

```
0.624
0.624
[[ 90  66   5]
 [ 32 118  33]
 [  3  49 104]]
```

```
C:\Users\racecar\anaconda3\lib\site-packages\sklearn\utils\validation.py:993: DataConver
sionWarning: A column-vector y was passed when a 1d array was expected. Please change th
e shape of y to (n_samples, ), for example using ravel().
  y = column_or_1d(y, warn=True)
```

Out[110]:

```
<function matplotlib.pyplot.show(close=None, block=None)>
```

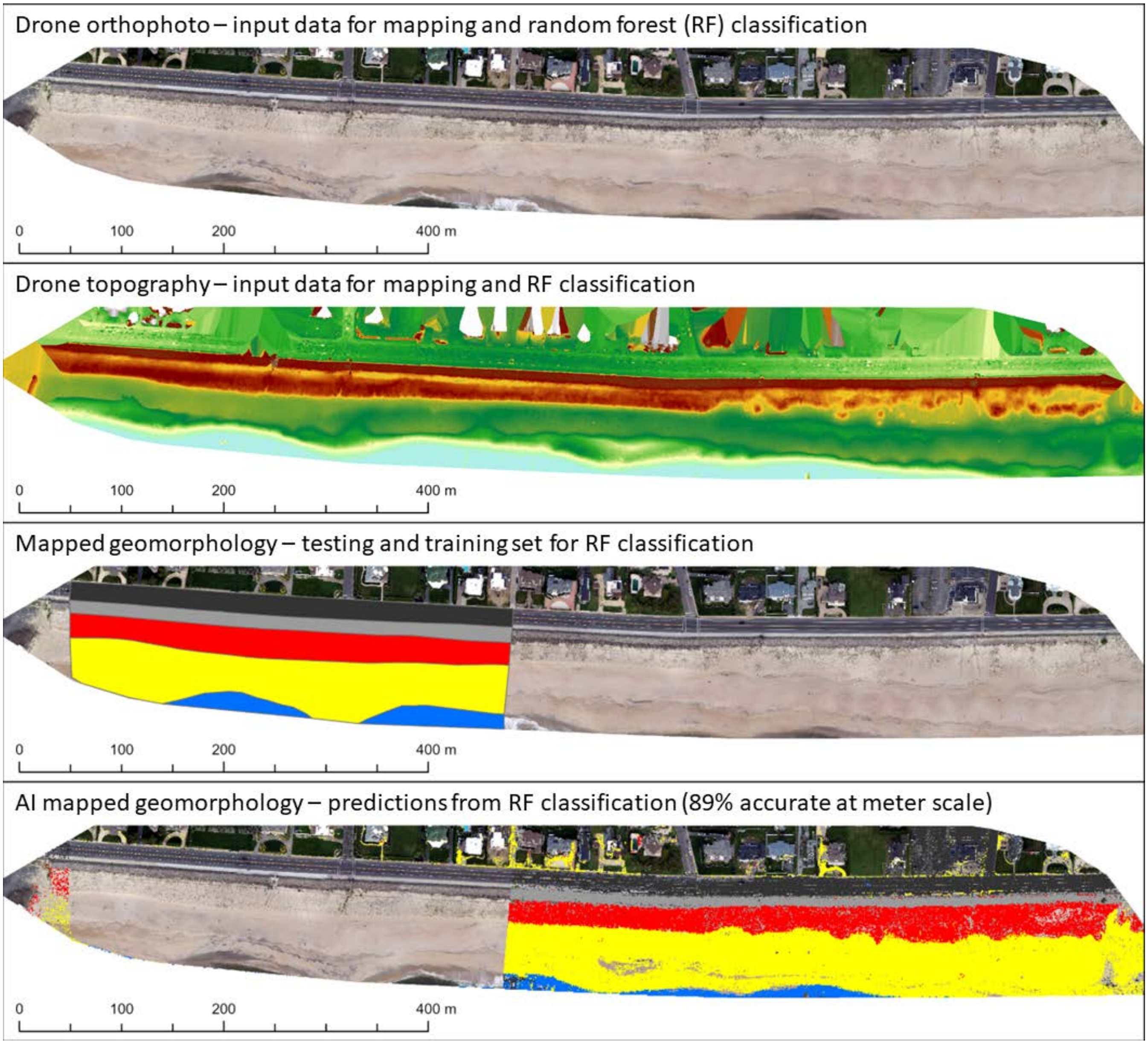


10. Logistic regression, random forest, and neural network algorithms

10.3.1 Random forest algorithm - geomorph map

1) Only using elevation and RGB bands make predictions about geomorphology. 2) Find estimator importance. 3) Plot predictions.

Instructions and data can be found here (https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Documents/RandomForestClassificatic and here (https://github.com/wschmelz/GeologicalModeling/raw/main/Data/RandomForest/Pix4d_1m_files.zip), respectively.



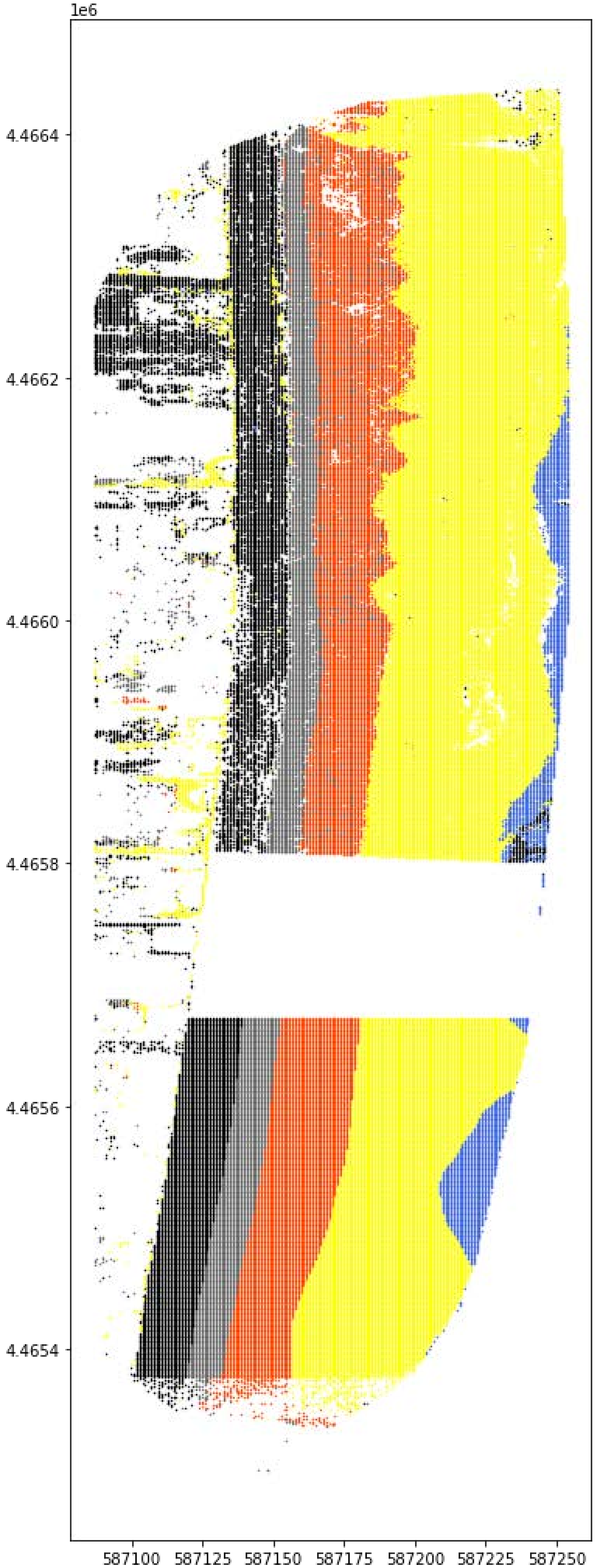
In [111…

```
import os
import sys
import matplotlib.pyplot as plt
import numpy
import sklearn
from sklearn.ensemble import RandomForestClassifier
from sklearn import metrics
from sklearn import tree
import requests

#Download map dataset and save to local directory as 'RF_pts_zRGB_m.csv'
url = 'https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Data/RF_pts_zR
RF_pts_zRGB_m = requests.get(url)
```

```
with open('RF_pts_zRGB_m.csv', 'w') as f:
    f.write(RF_pts_zRGB_m.text)

backslash = '\\'

wkspc = str(os.getcwd()).replace(backslash,"/") + "/"

#import map dataset - make sure this line matches your filename

filename = wkspc + "RF_pts_zRGB_m.csv"

map_data = numpy.genfromtxt(filename,delimiter=",")

#sort training, testing, and prediction data components
#pls ask if this portion of the code does not make sense

#remove invalid data points - where X,Y,Z,R,G, or B,
#i.e., columns 0-5, are equal to -9999

w1 = numpy.where(map_data[:,2:6] == -9999.)[0]
map_data = numpy.delete(map_data,w1,axis=0)

#prediction only set

w2 = numpy.where(map_data[:,6]==-9999.)[0]
X_predict = map_data[w2,2:6]
X_predict_xy = map_data[w2,0:2]

#training set

w3 = numpy.where(map_data[:,6]!=-9999.)[0]

N = len(w3)

index = numpy.arange(0,N,1)
numpy.random.shuffle(index)

X_train = map_data[index[0:int(N/2)],2:6]
y_train = numpy.reshape(map_data[index[0:int(N/2)],6],(-1,1))
X_train_xy = map_data[index[0:int(N/2)],0:2]

#testing set

X_test = map_data[index[int(N/2):],2:6]
y_test = numpy.reshape(map_data[index[int(N/2):],6],(-1,1))
X_test_xy = map_data[index[int(N/2):],0:2]

#make an sklearn random forest classier

clf = RandomForestClassifier(n_estimators=50)

#fit the model

clf.fit(X_train, y_train)

#make predictions using the test set

y_pred = clf.predict(X_test)

#evaluate model accuracy

score = clf.score(X_test, y_test)
print(score)

importance = clf.feature_importances_
```

```
print(importance)

#make predictions using prediction set

y_pred2 = clf.predict(X_predict)

#create figure

fig = plt.figure(1,figsize=(6,18))

ax1 = plt.subplot(111)

classes = [1,2,3,4,5]
class_names = ["Dune","Seawall","Pavement","Beach","Ocean"]
class_colors = ["orangered","dimgrey","black","yellow","royalblue"]

for n in range(0,len(classes)):
    w1 = numpy.where(y_pred2==classes[n])[0]
    ax1.plot(X_predict_xy[w1,0],X_predict_xy[w1,1],color=class_colors[n],marker=".",alph
    w2 = numpy.where(y_train[:,0]==classes[n])[0]
    ax1.plot(X_train_xy[w2,0],X_train_xy[w2,1],color=class_colors[n],marker=".",alpha=1,
    w3 = numpy.where(y_test[:,0]==classes[n])[0]
    ax1.plot(X_test_xy[w3,0],X_test_xy[w3,1],color=class_colors[n],marker=".",alpha=1,ma

output_matrix = numpy.zeros((len(y_pred2),3))
output_matrix[:,0:2] = X_predict_xy
output_matrix[:,2] = y_pred2

filename = "Map_predictions.csv"
numpy.savetxt(filename, output_matrix, fmt=['%0.3f','%0.3f','%i'], delimiter=',')

plt.show()
```

```
C:\Users\racecar\AppData\Local\Temp\ipykernel_39664\62386417.py:68: DataConversionWarnin
g: A column-vector y was passed when a 1d array was expected. Please change the shape of
y to (n_samples,), for example using ravel().
  clf.fit(X_train, y_train)
0.87113422845853
[0.44372072 0.22987089 0.16231606 0.16409232]
```



1e6
4.4664
4.4662
4.4660
4.4658
4.4656
4.4654
587100
587125
587150
587175
587200
587225
587250

10.3.2 Random forest algorithm - geomorph map

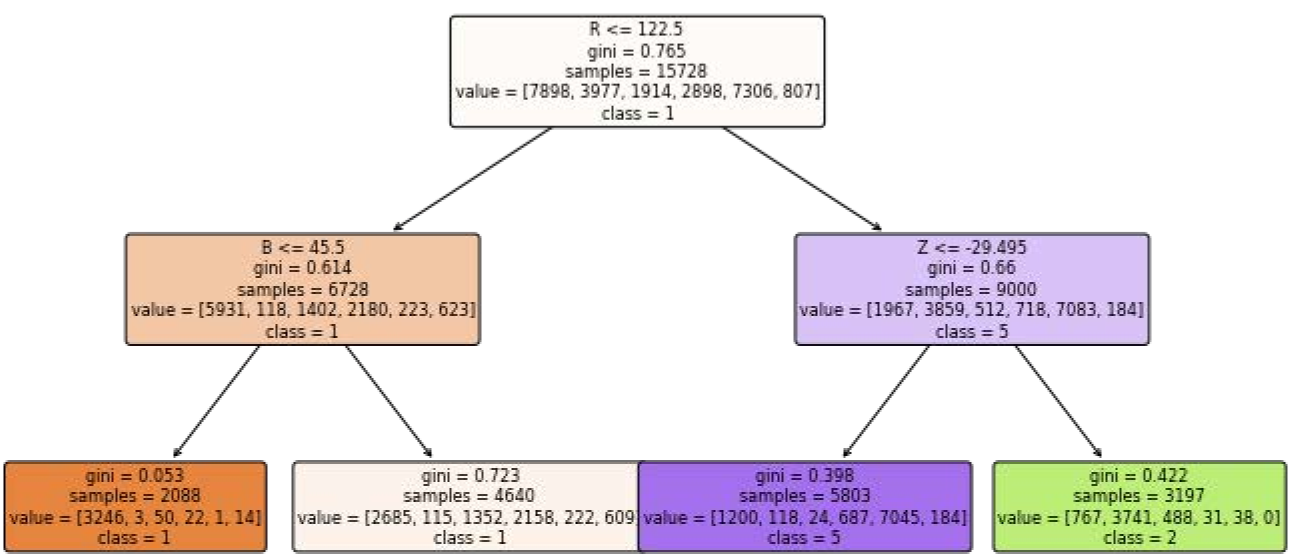
1) Only using elevation and RGB bands make predictions about geomorphology. 2) View decision tree map with fewer estimators and lesser prediction accuracy.

In [112…

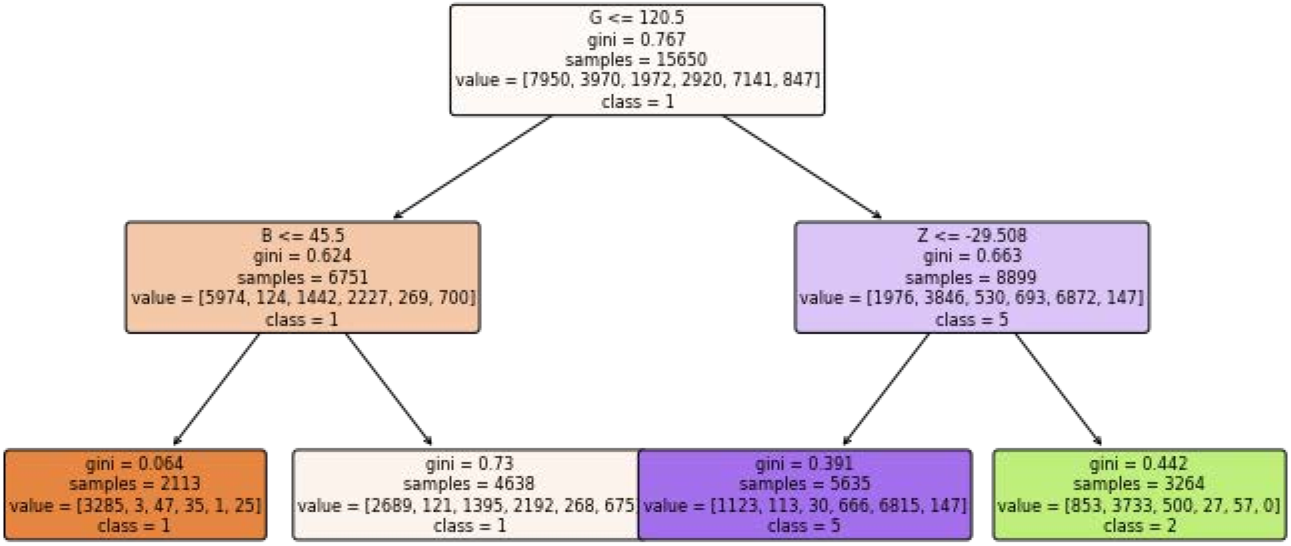
```
import os
import sys
import matplotlib.pyplot as plt
import numpy
import sklearn
from sklearn.ensemble import RandomForestClassifier
from sklearn import metrics
from sklearn import tree

#Download map dataset and save to local directory as 'RF_pts_zRGB_m.csv'
url = 'https://raw.githubusercontent.com/wschmelz/GeologicalModeling/main/Data/RF_pts_zR
RF_pts_zRGB_m = requests.get(url)

with open('RF_pts_zRGB_m.csv', 'w') as f:
    f.write(RF_pts_zRGB_m.text)

backslash = '\\'

wkspc = str(os.getcwd()).replace(backslash,"/") + "/"

#import map dataset - make sure this line matches your filename

filename = wkspc + "RF_pts_zRGB_m.csv"

map_data = numpy.genfromtxt(filename,delimiter=",")

#sort training, testing, and prediction data components
#pls ask if this portion of the code does not make sense

#remove invalid data points - where X,Y,Z,R,G, or B,
#i.e., columns 0-5, are equal to -9999

w1 = numpy.where(map_data[:,2:6] == -9999.)[0]
map_data = numpy.delete(map_data,w1,axis=0)

#prediction only set

w2 = numpy.where(map_data[:,6]==-9999.)[0]

X_predict = map_data[w2,2:6]

#training set

w3 = numpy.where(map_data[:,6]!=-9999.)[0]

N = len(w3)

index = numpy.arange(0,N,1)
numpy.random.shuffle(index)

X_train = map_data[index[0:int(N/2)],2:6]
y_train = numpy.reshape(map_data[index[0:int(N/2)],6],(-1,1))

#testing set

X_test = map_data[index[int(N/2):],2:6]
y_test = numpy.reshape(map_data[index[int(N/2):],6],(-1,1))

#make an sklearn random forest classier
```

```python
clf = RandomForestClassifier(n_estimators=3,max_depth=2,random_state=42)

#fit the model

clf.fit(X_train, y_train)

#make predictions using the test set

y_pred = clf.predict(X_test)

#evaluate model accuracy

score = clf.score(X_test, y_test)
print(score)

importance = clf.feature_importances_

print(importance)

features = ["Z","R","G","B"]
classes = ["Dune","Seawall","Pavement","Beach","Ocean"]
classes = ["1","2","3","4","5"]
for estimator in clf.estimators_:
    print(estimator)
    plt.figure(figsize=(12,6))
    tree.plot_tree(estimator,
                   feature_names=features,
                   class_names=classes,
                   fontsize=8,
                   filled=True,
                   rounded=True)
    plt.show()
```

```
C:\Users\racecar\AppData\Local\Temp\ipykernel_39664\1865531839.py:65: DataConversionWarning: A column-vector y was passed when a 1d array was expected. Please change the shape of y to (n_samples,), for example using ravel().
  clf.fit(X_train, y_train)
0.6663037780734647
[0.30905941 0.13185747 0.3284244  0.23065872]
DecisionTreeClassifier(max_depth=2, max_features='auto',
                       random_state=1608637542)
```




```
DecisionTreeClassifier(max_depth=2, max_features='auto',
                       random_state=1273642419)
```




```
DecisionTreeClassifier(max_depth=2, max_features='auto',
                       random_state=1935803228)
```