FITTING SYNTHETIC LIBRARIES TO PHOTOMETRIC DATA

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Posing the problem

Let's assume that we have an empirically calibrated template library T_i with $i = 1, n_T$ in a redshift grid z_j where $z_j = z_1+(j-1)dz$, $j = 1, n_z$. We also have a synthetic library S_k with $k = 1, n_S$, where, generally speaking, $n_S >> n_T$. The problem at hand is to calculate $p(S_k|\mathbf{C}$,$m_0, I)$,where $\mathbf{C} = \mathbf{m}-m_0$ are the colors of a galaxy and m_0 is the magnitude we use to define the redshift/magnitude prior. S_k can be very large and may contain many spectral types which are not realistic, in the sense of not being present in the galaxy population. It is therefore not practical to calculate directly these probability, using e.g. BPZ or other similar codes since the color/redshifts degenaracies will be much worse than for a compact, well-calibrated empirical library. We therefore try to solve the problem by forcing the solutions to belong to the color space defined by the empirical templates and the magnitude/redshift prior $p(z, T|m_0)$. And so:

$$p(S_k, z_i|\mathbf{C}, m_0) = \sum_{j=1}^{n_T} p(S_k, z_i, T_j|\, mathbfC, m_0) = \sum_{j=1}^{n_T} p(z_i, T_j|\mathbf{C}, m_0)p(S_k|z_i, T_j, \mathbf{C}, m_0) \tag{1}$$

Since the probability of having the S_k template is completely determined by its comparison with the T_j template, we can write:

$$p(S_k|z_i, T_j, \mathbf{C}, m_0) = p(S_k|z_i, T_j) \propto p_I(S_k)p(T_j|S_k, z_i) \tag{2}$$

The factor $p_I(S_k)$ is a prior for the synthetic template library. For instance when we produce the library we may want to assign equal weight to all metallicities, etc. It can be flat, or we can use it to modulate the parameter estimation. The factor $p(z_i, T_j|\mathbf{C}, m_0, I)$ is the posterior generated by BPZ. The factor $p(T_j|z_i, S_k)$ can be calculated as

$$p(T_j|z_i, S_k) \propto \exp(-\chi 2/2)$$

where

$$\chi^2(S_k; z_i, T_j) = \sum_\alpha [S_{k\alpha}(z_i) - T_{j\alpha}(z_i)]^2$$

where $S_{k\alpha}$ and $T_{j\alpha}$ are, respectively, the fluxes of the templates S_k and T_j, redshifted to z_i observed with the filter α, and the sum goes over all the observed n_F filters. We can calculate this expression using a formula similar to that in Benítez 2000. If we define

$$s_{S_k T_j} = \sum_\alpha S_{k\alpha} T_{j\alpha};\ s_{S_k} = \sum_\alpha S^2_{k\alpha};\ s_{T_j} = \sum_\alpha T^2_{j\alpha}$$

then we have

$$\chi^2(S_k; z_i, T_j) = s_{S_k}(z_i) - \frac{s^2_{S_k T_j}(z_i)}{s_{T_j}(z_i)}$$

The factor $p(S_k|z_i, T_j)$ is a data cube of $n_S \times n_z \times n_T$ size, but it has to be calculated only once. Moreover, we can greatly reduce the size of this array by culling those values of k for which the maximum of $p(S_k|z_i, T_j)$ is very small and shrinking the first axis of the data cube to $n'_S << n_S$. Calculating $p(S_k|z_i, T_j, \mathbf{C}, m_0)$ for each galaxy is thus reduced to multiplying the BPZ posterior, $p(z_i, T_j|\mathbf{C}, m_0)$, by the $p(S_k|z_i, T_j)$ array (and we can use python fancy indexing to operate only on those values for which $p(z_i, T_j|\mathbf{C}, m_0) > \epsilon$, further reducing the number of calculations involved) and then averaging over the T_j axis. That would yield the $n_S \times n_z$ array $p(S_k, z_i|\mathbf{C}, m_0)$.

Galaxy parameters

To calculate the probability of any parameter θ_S characterizing the spectra, where θ_S can be a scalar, like the luminosity L , the stellar mass $M_\star$, and the metallicity Z, or multivalued, as the SFH, we have:

$$p(\theta_S, z_i|\mathbf{C}, m_0) = \sum_{k=1}^{n_S} p(S_k, \theta_S, z_i|\mathbf{C}, m_0) = \sum_k p(S_k, z_i|\mathbf{C}, m_0) p(\theta_S|S_k, z_i, \mathbf{C}, m_0)] =$$

$$= \sum_k p(S_k, z_i|\mathbf{C}, m_0, I)\delta[\theta_S(S_k, z_i, m_0)]$$

where δ is the delta function, and we have taken into account that, generally speaking, θ will be fully determined by S_k, m_0 and z_i.

Thus, to calculate the galaxy parameters, one has first to find the analytical or numerical relationship

$\theta = \theta(S_k, z_i, m_0)$

In same cases, e.g. the metallicity, it is not necessary to use the redshift and magnitude. In others, e.g. the luminosity, SFR or the stellar mass that information will be relevant.

It is important here to take into account the prior $p_I(S_k)$ if the initial library S_k is not randomly distributed in the parameter space.

The resulting function will yield disjoint point estimates of $p(\theta_S)$, to calculate the final distribution of θ_S at each redshift, we have to bin in $\delta\theta$ intervals and integrate.