

# Developing fully transparent, site-level, measurement-based inventories using continuous monitoring data

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**COLORADO SCHOOL OF MINES**



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**Measurement-based** = only use measurement data to build the inventory

**Site-level** = only measurements from the specific site used to build the inventory

**Fully transparent** = all of the methods are open source!

# site-level & measurement-based

Site-level = only measurements from the specific site used to build the inventory

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## Advantages

- No assumptions about similar sites following similar distributions
- No potentially for underestimation to leak through from the inventory

# site-level & measurement-based

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**Measurement-based** = only use measurement data to build the inventory

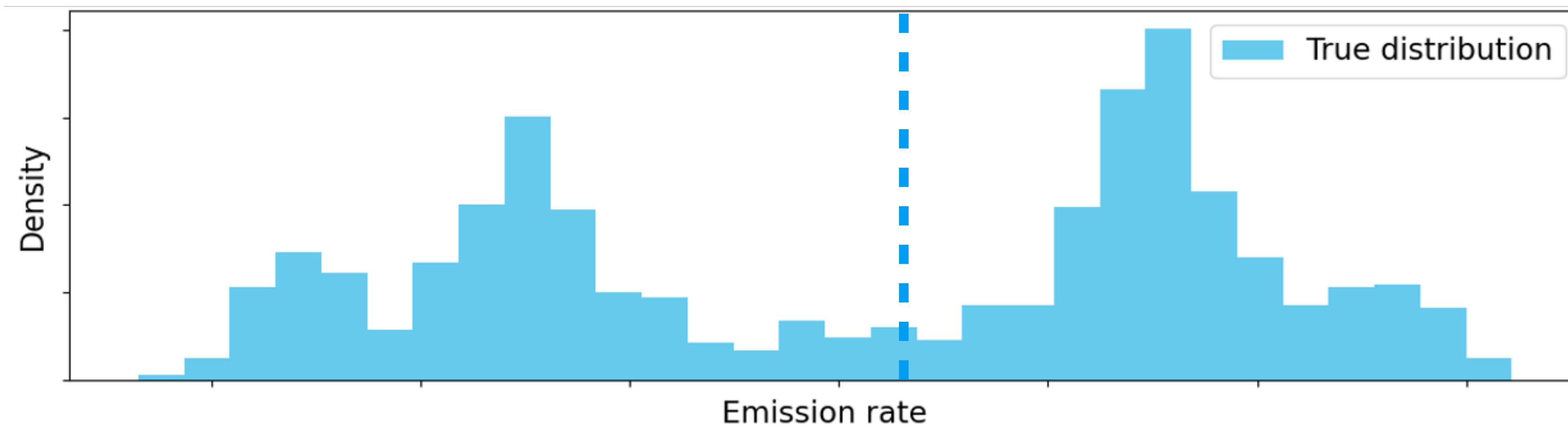
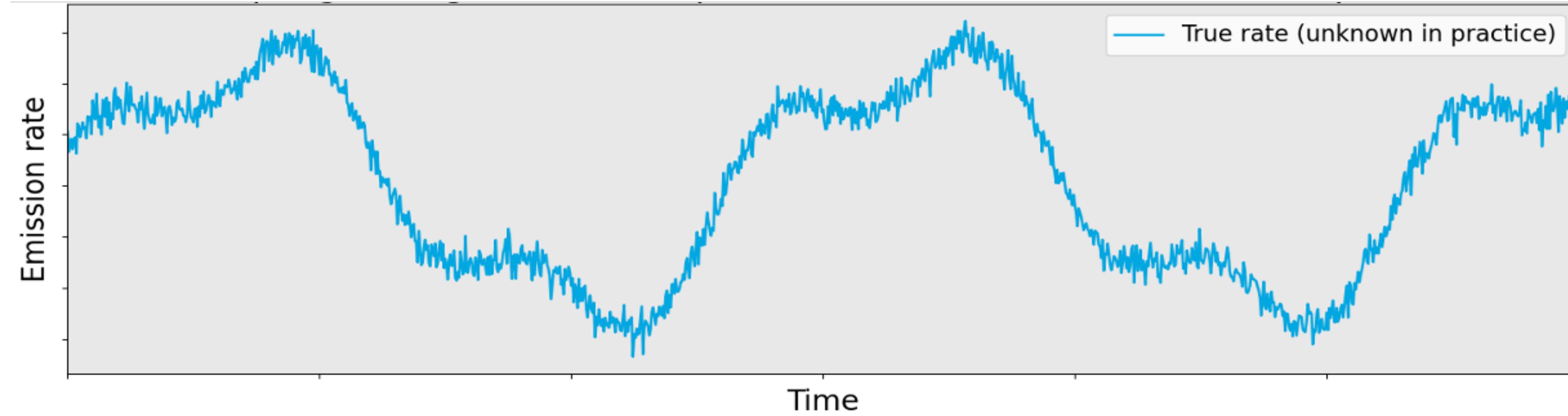
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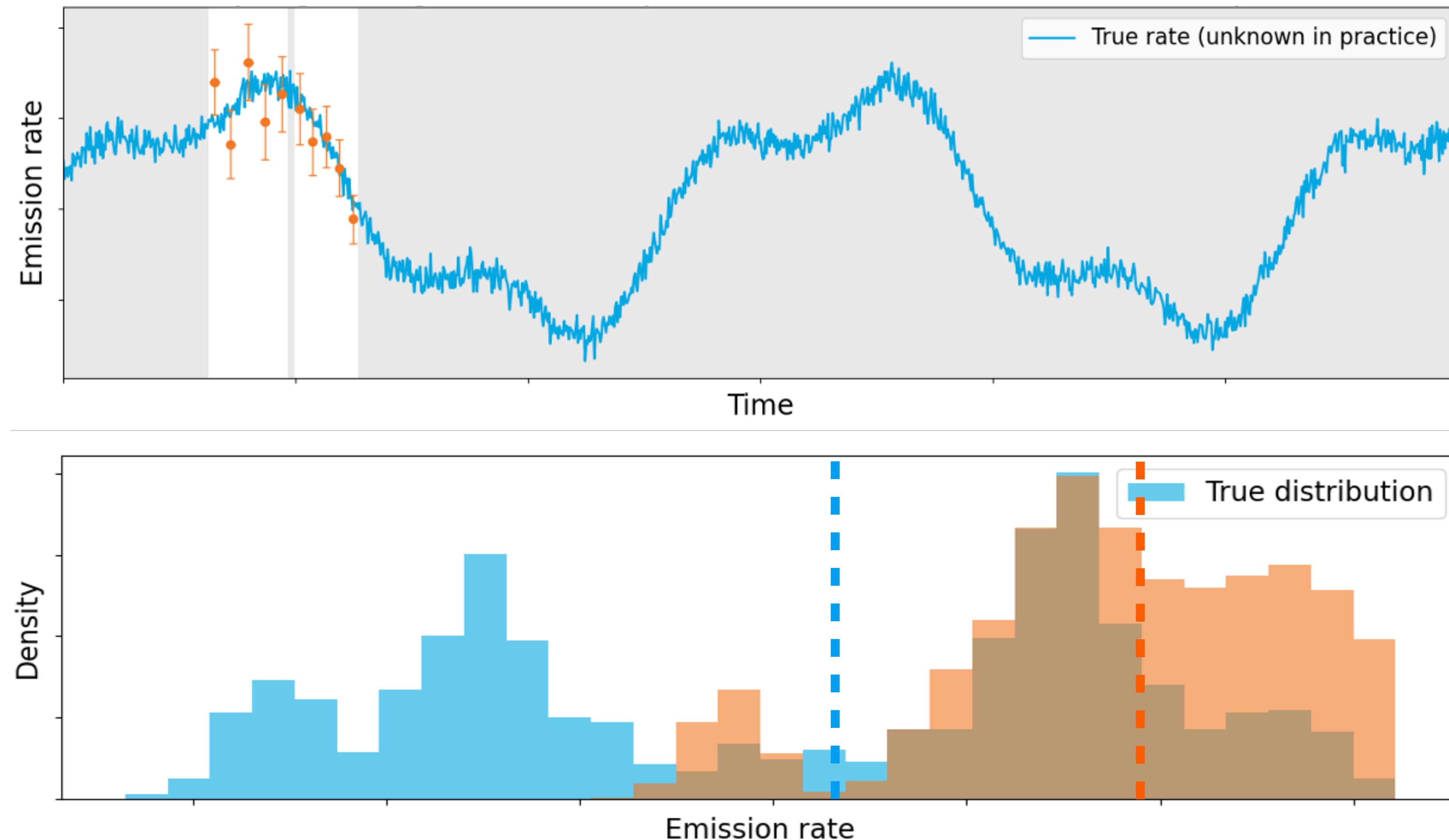
## Challenges

- Source estimate does not necessarily equal root cause
- Need a lot of measurements on each site
- Scaling up requires lots of measurements on lots of sites

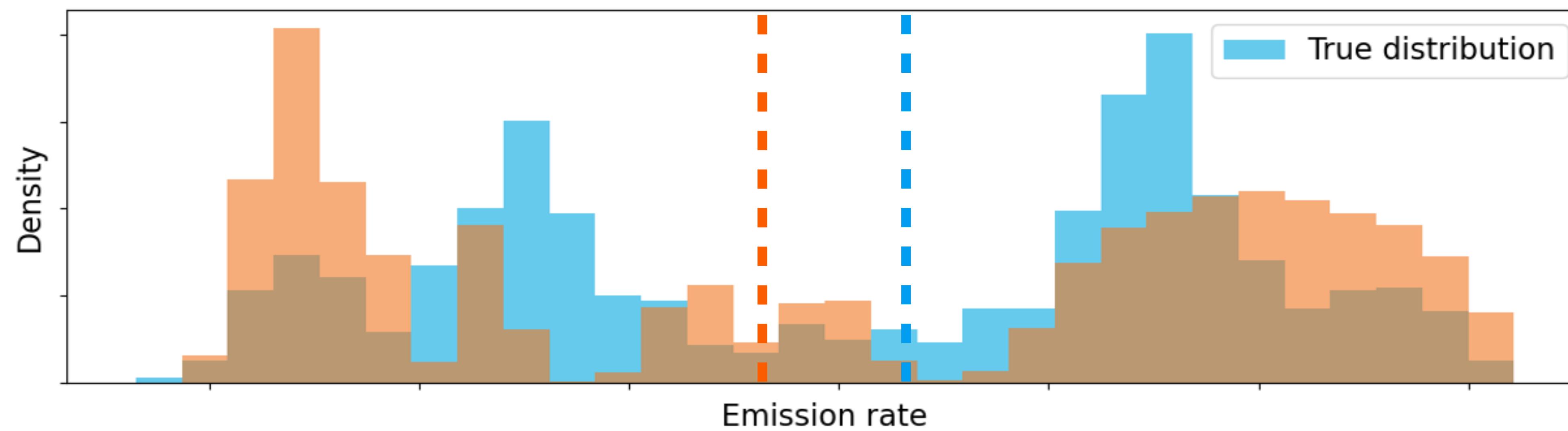
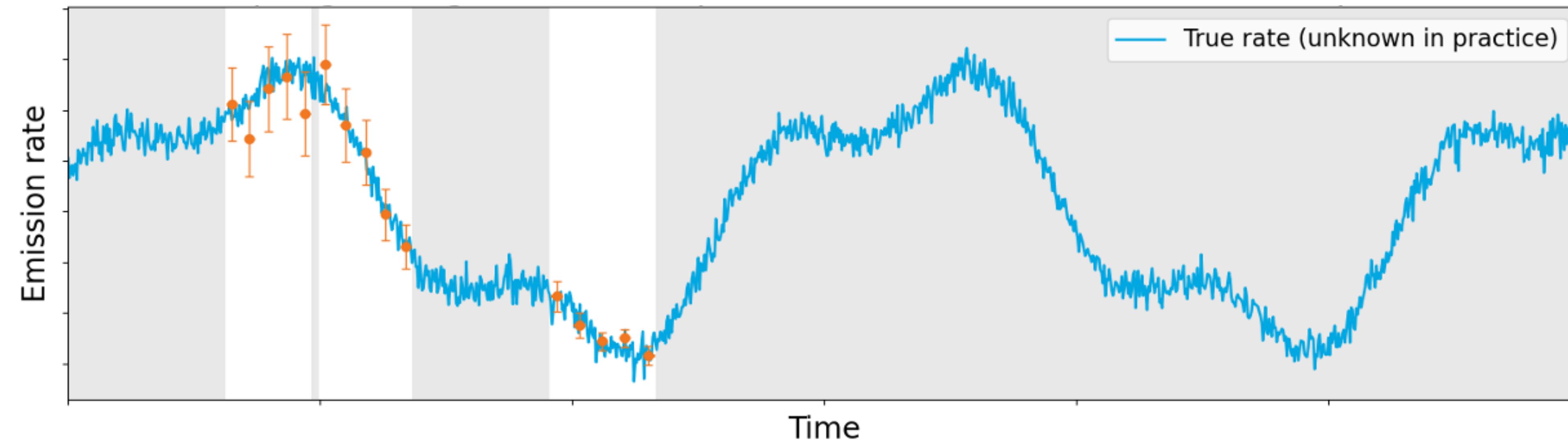
# Why do you need so many measurements?



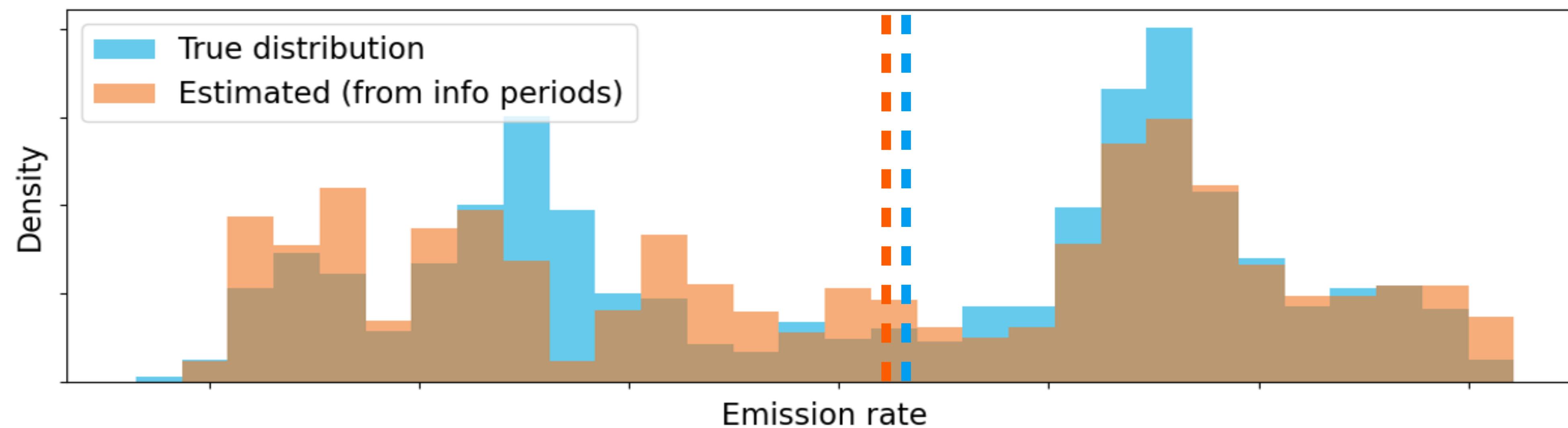
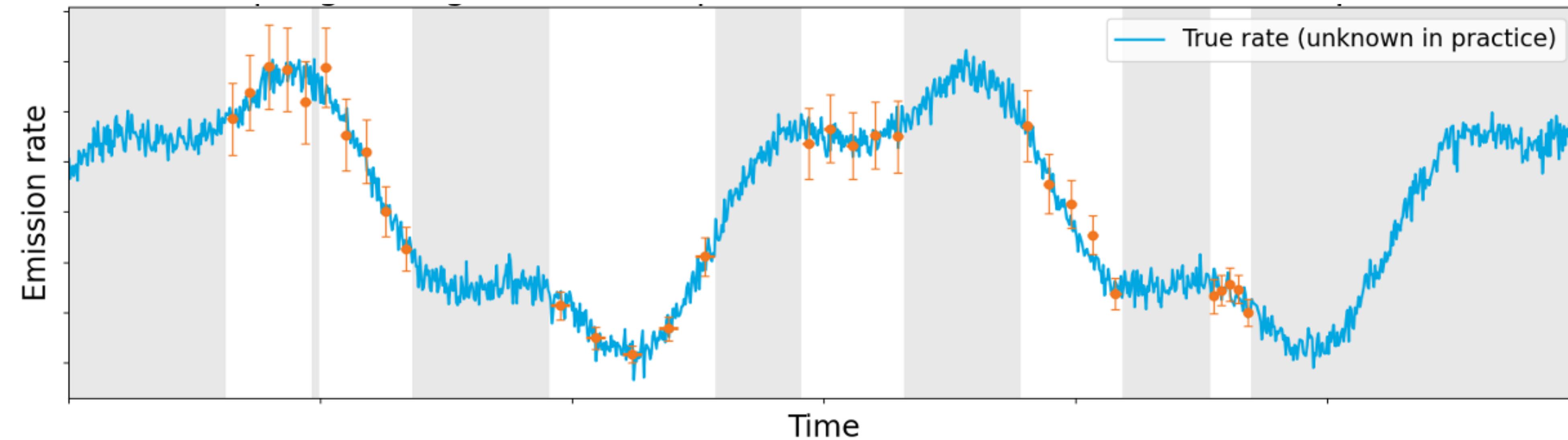
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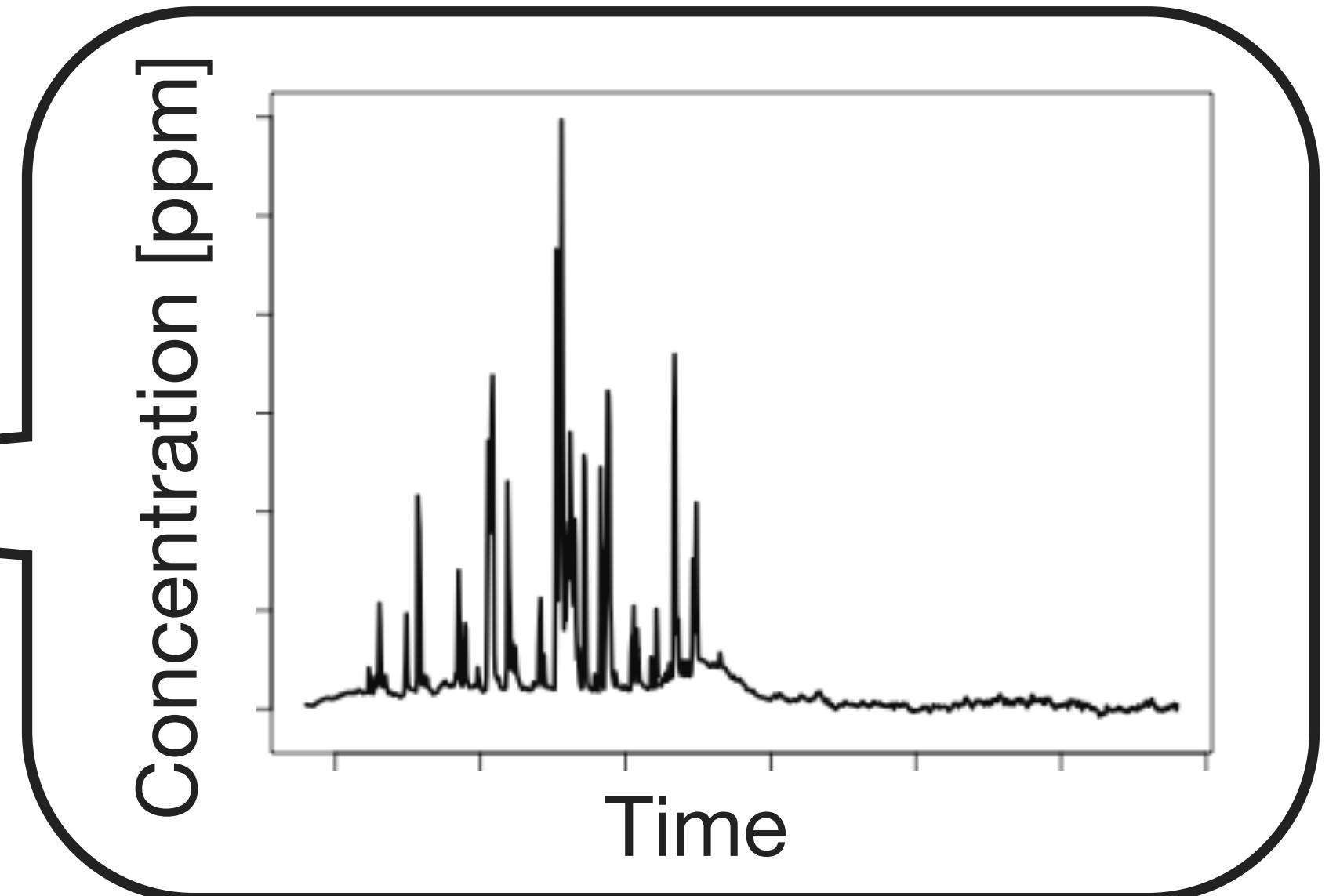
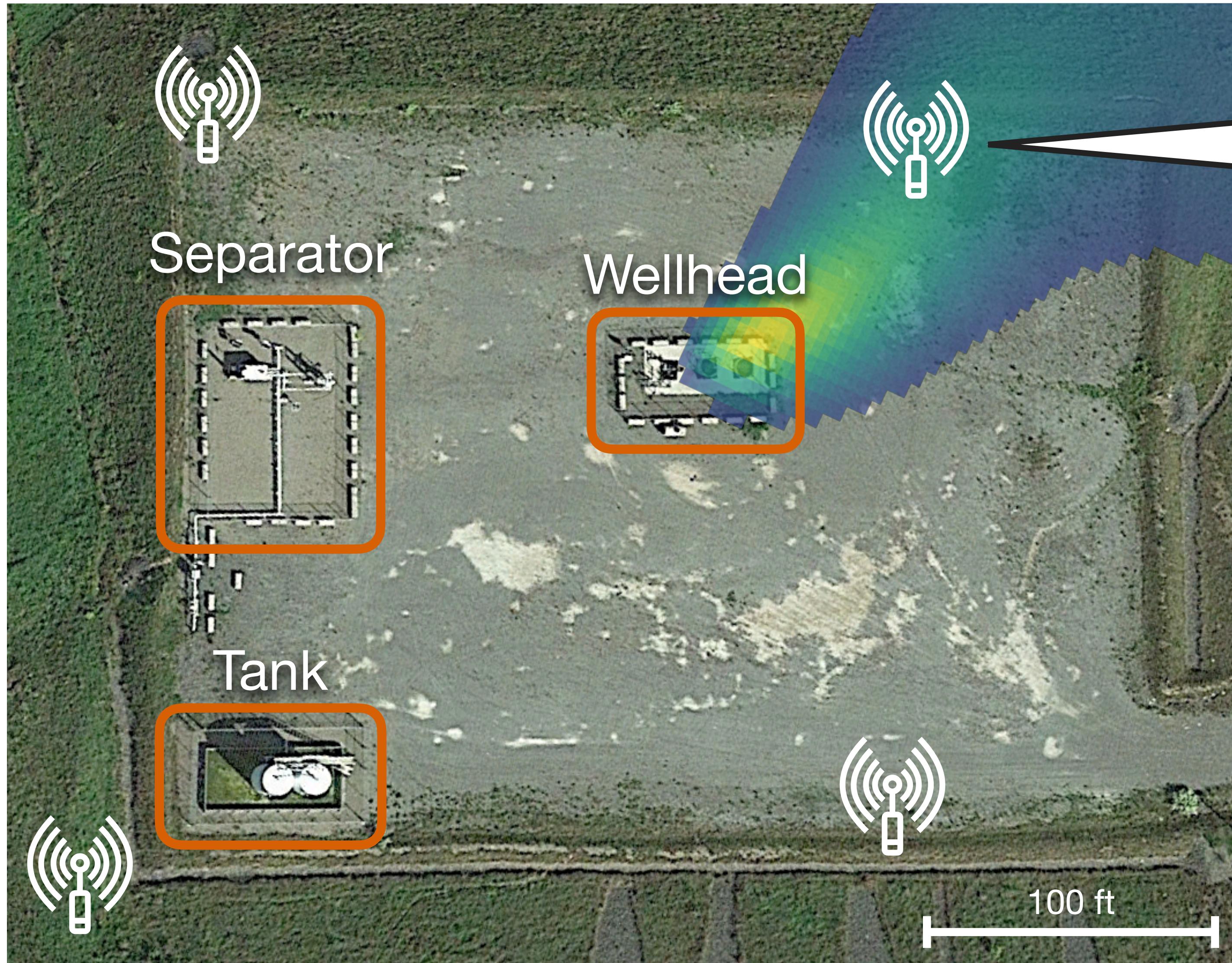
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# Why do you need so many measurements?



# Continuous monitoring 101

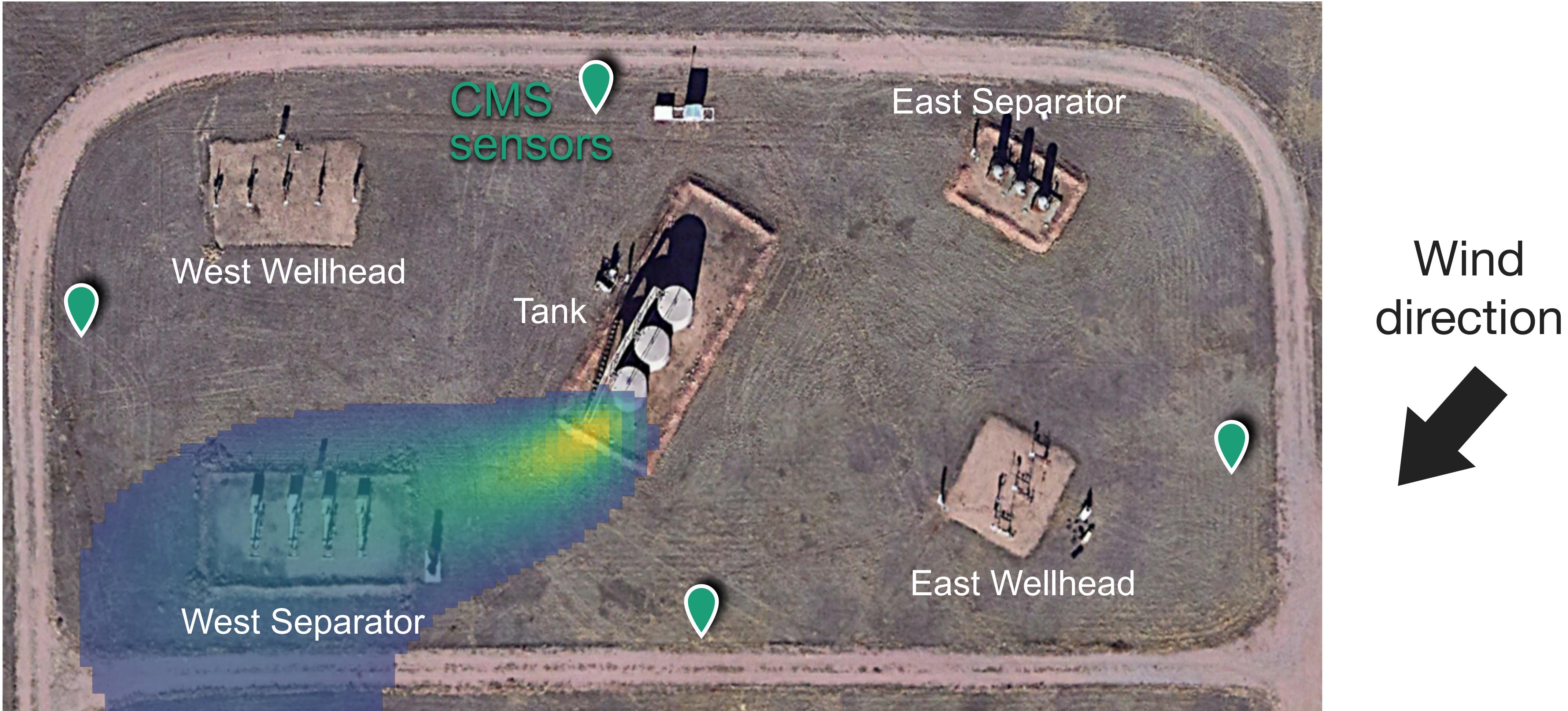


Aerial measurement technology

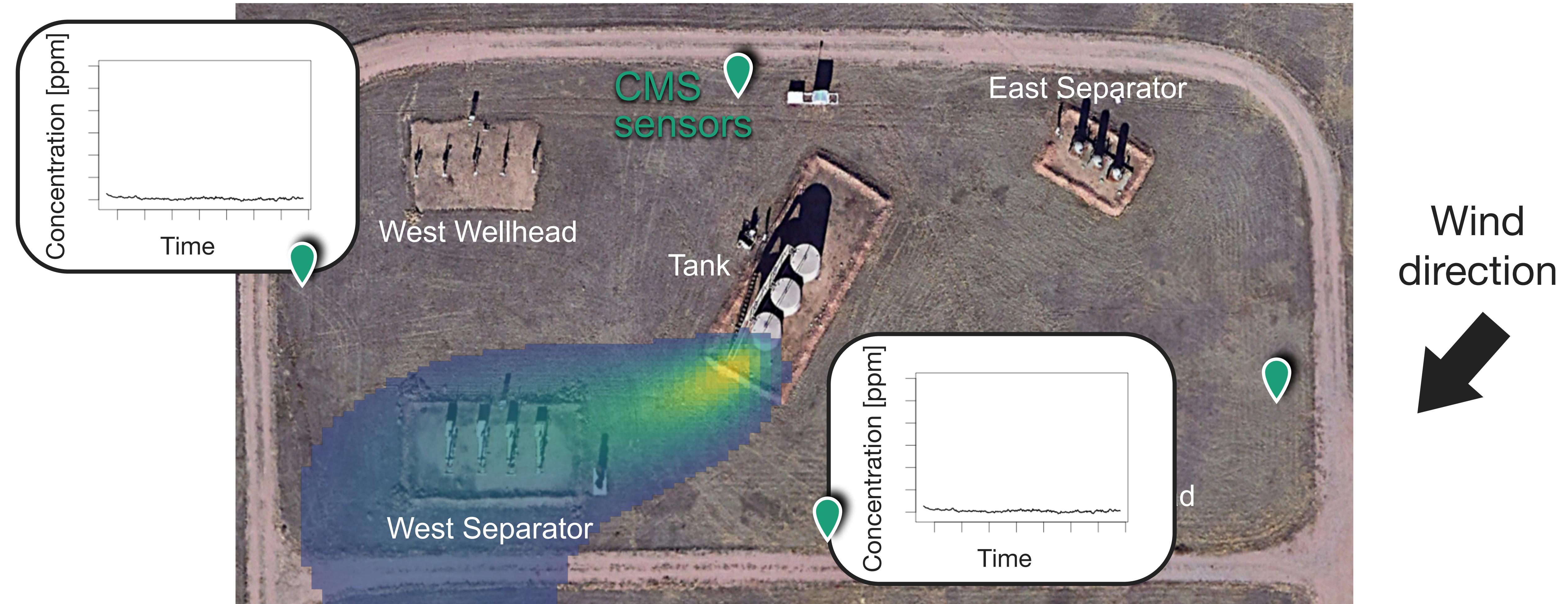
# “Continuous” is rarely truly continuous



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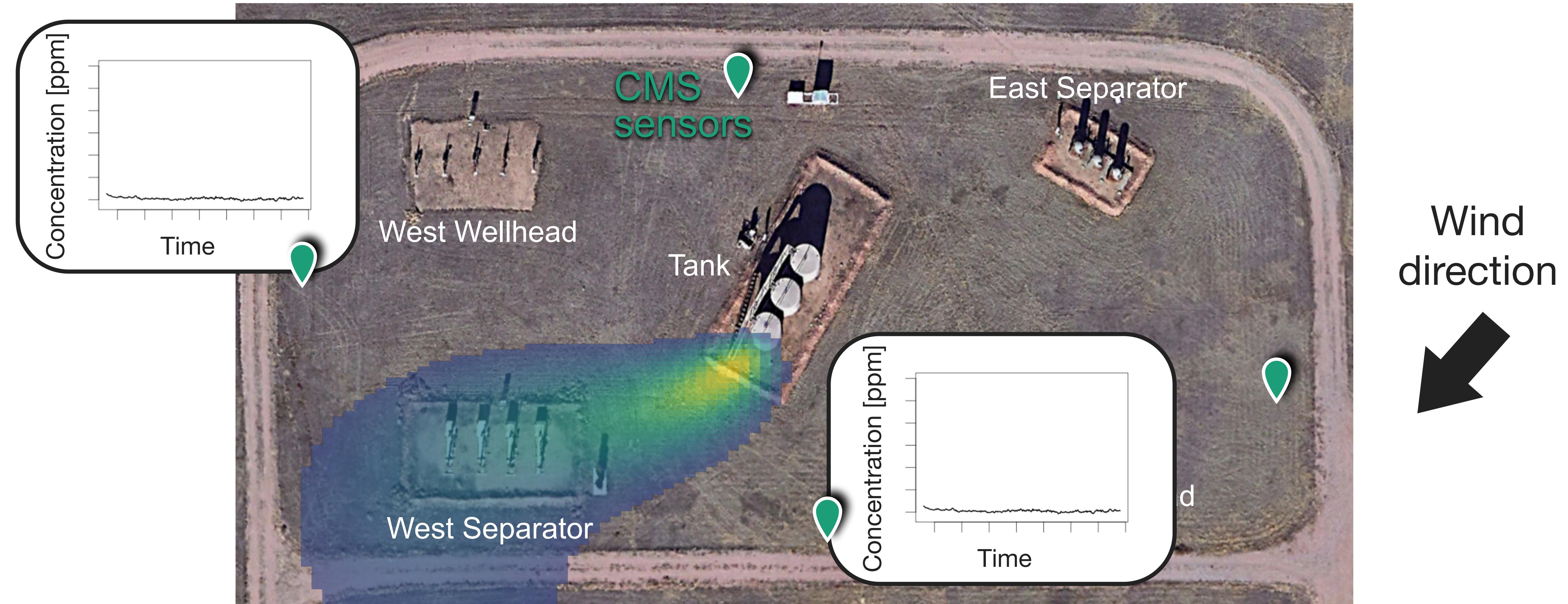


# “Continuous” is rarely truly continuous



No emissions information when the wind blows between sensors

# “Continuous” is rarely truly continuous



This cannot be interpreted as 0 kg/hr!

# However... we can estimate when this happens!



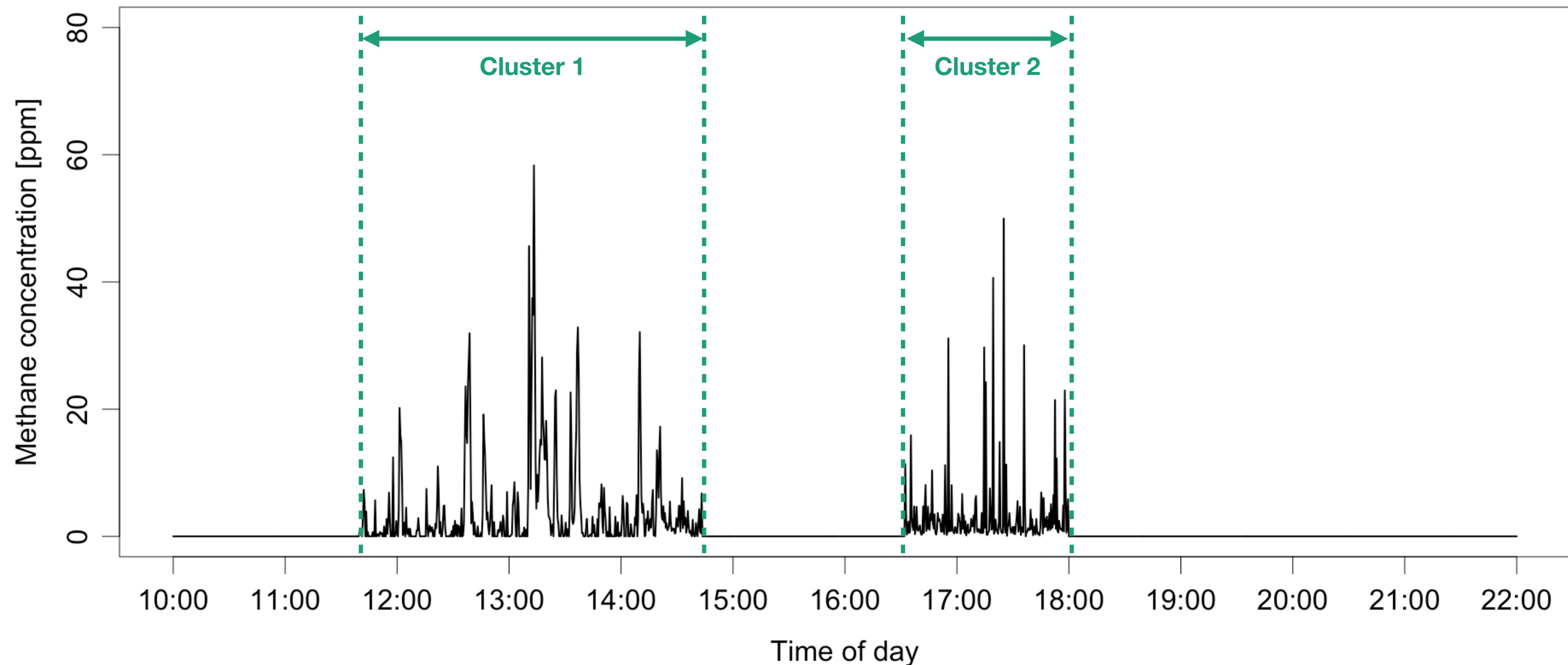
Downwind region **does not** overlap with CMS sensors  
= period of “**no information**”

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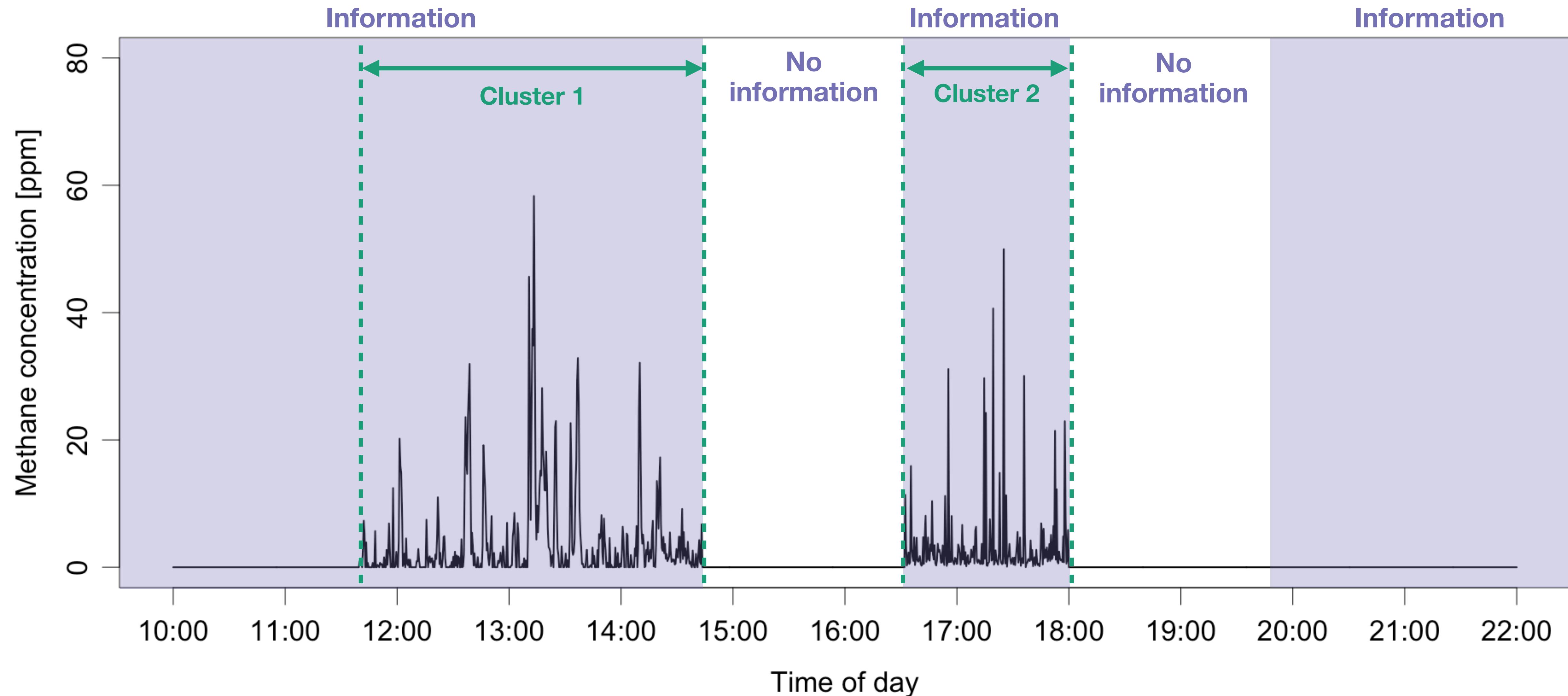


Downwind region **does** overlap with CMS sensors  
= period of “**information**”

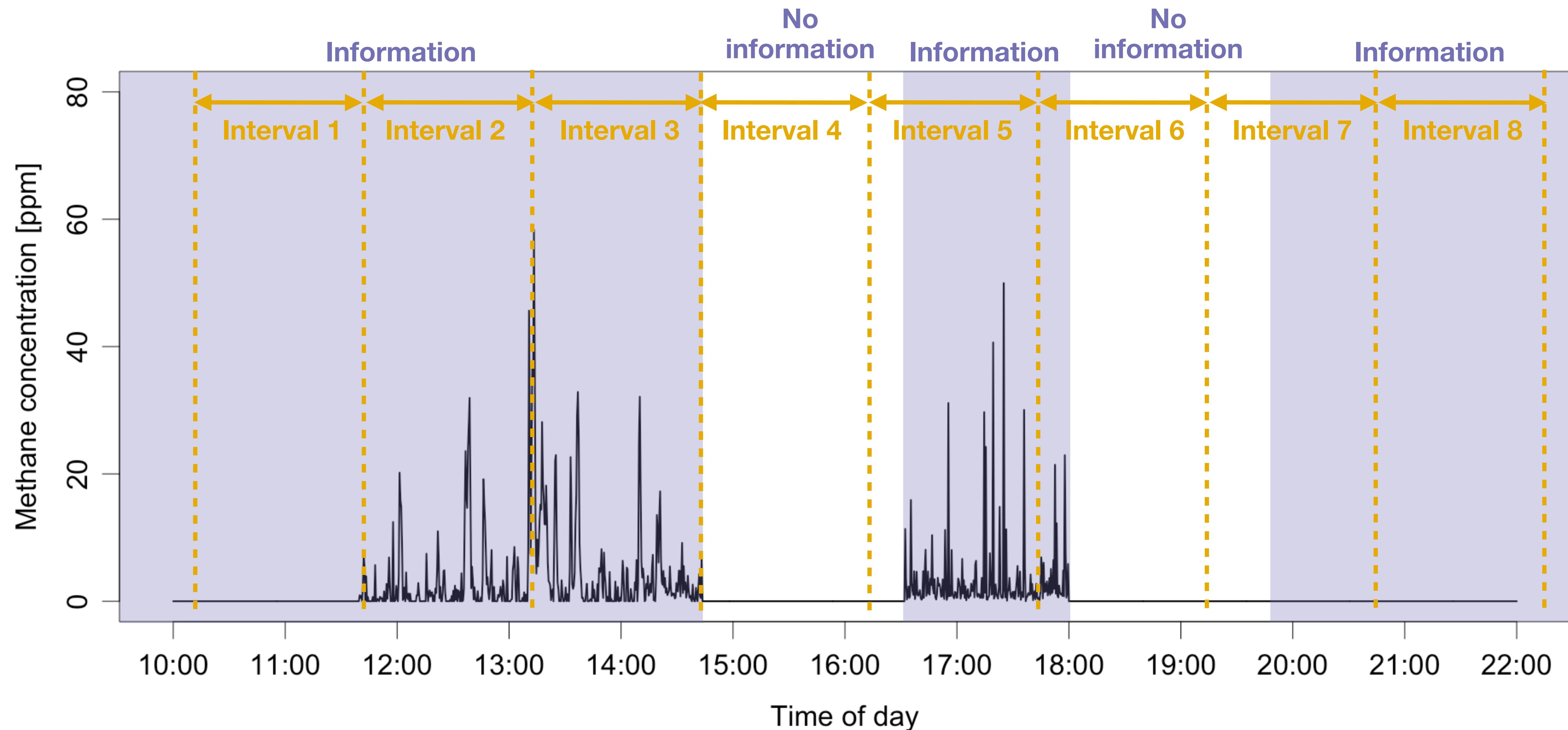
# How do periods of information and no information present themselves in the data?



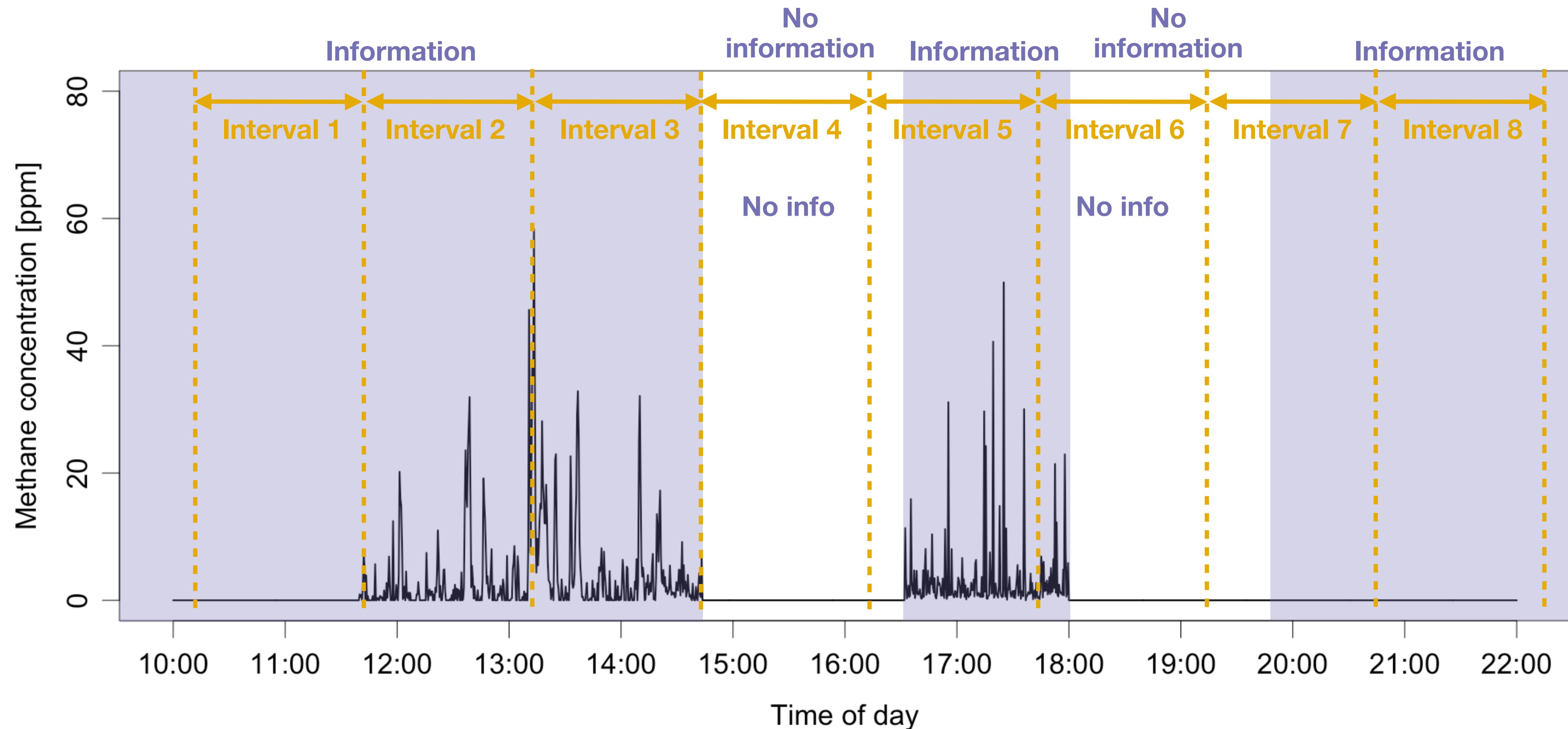
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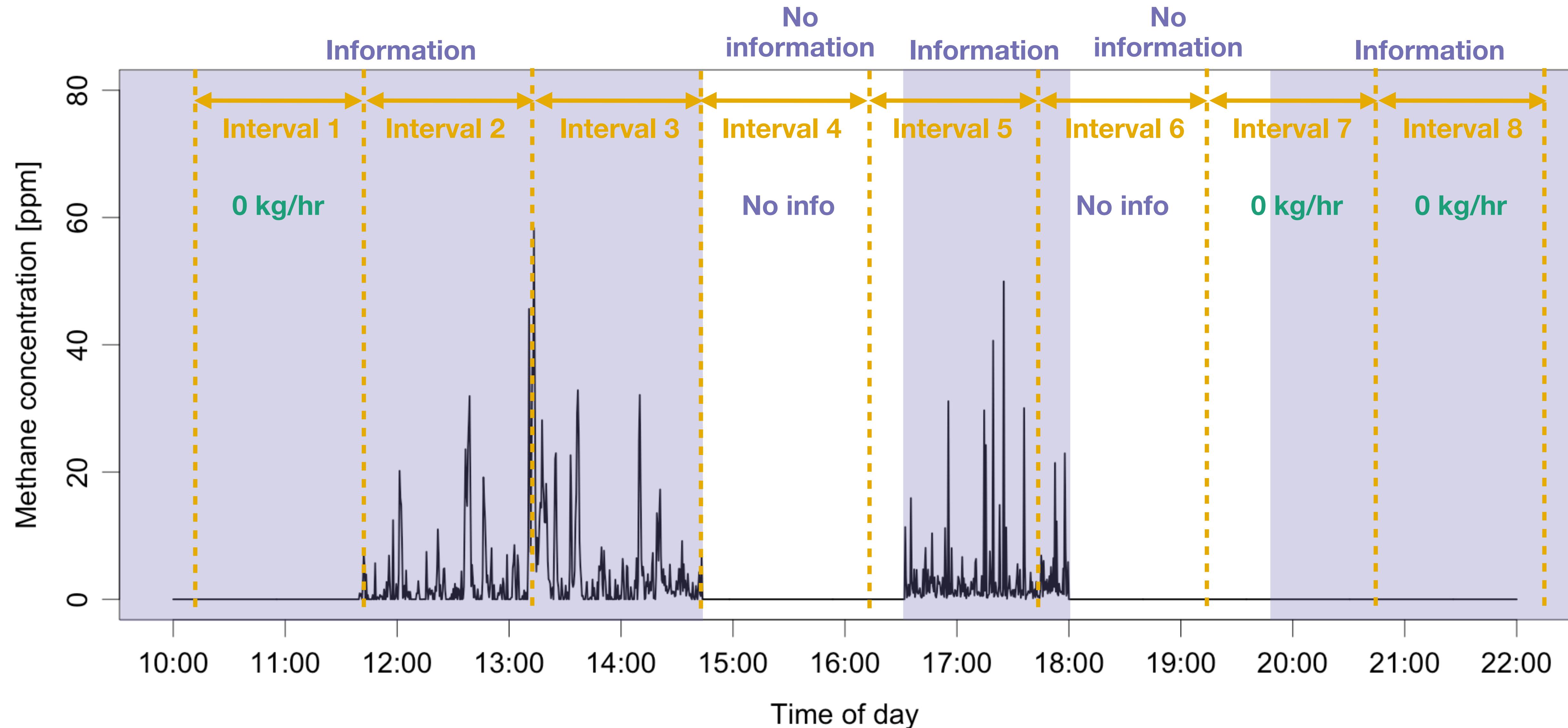
In practice, we estimate emissions on fixed **fixed intervals**



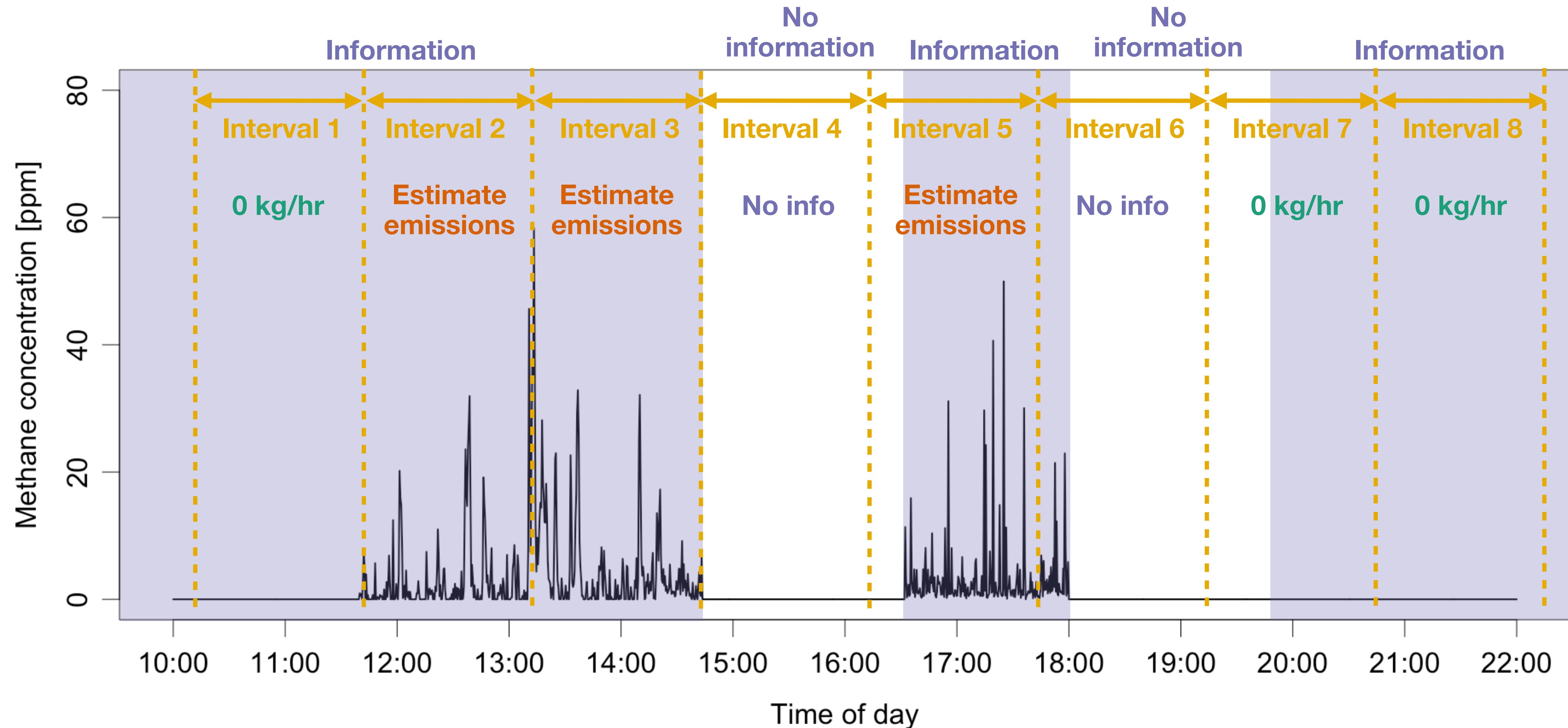
Before building an inventory, we need to identify when an interval has **no information**,



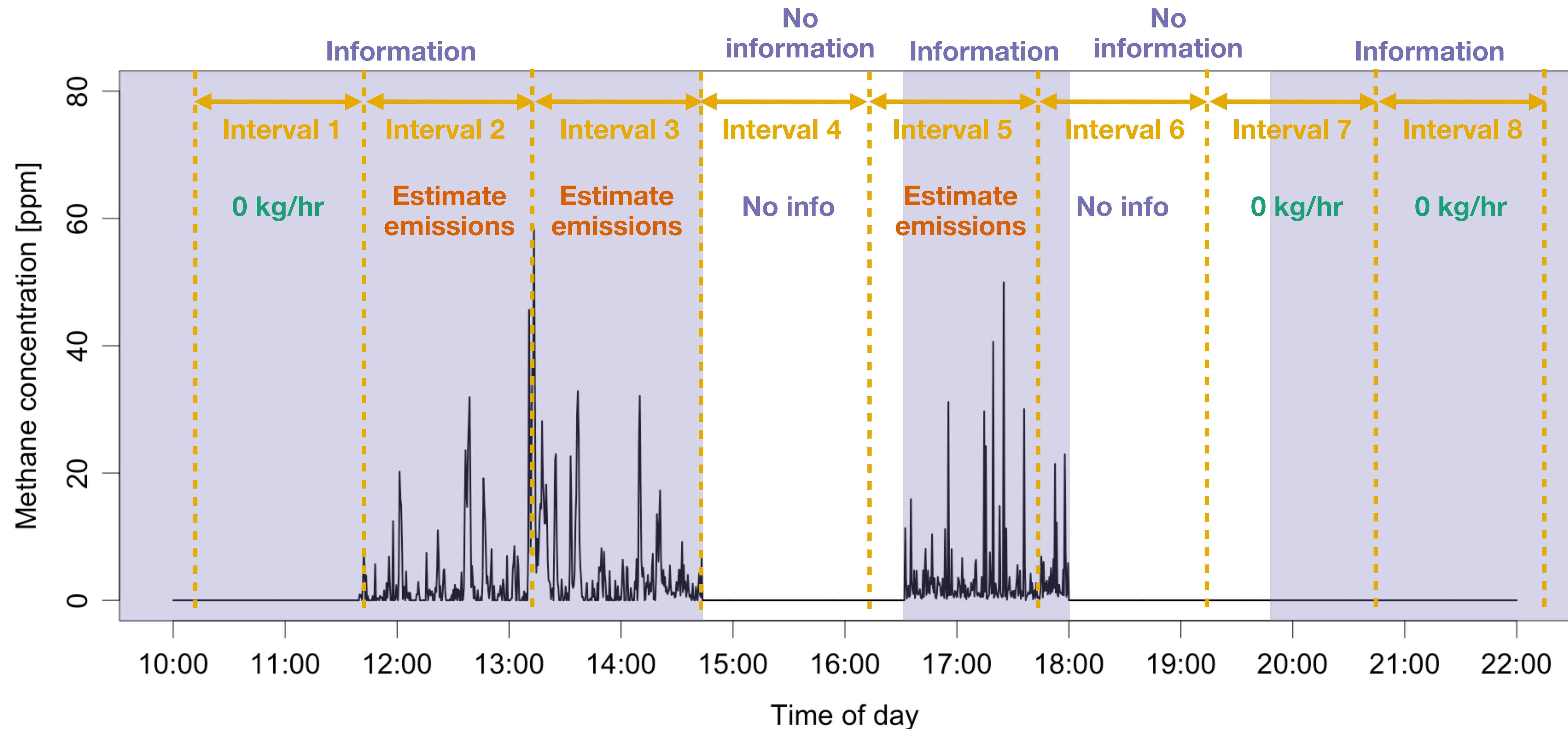
Before building an inventory, we need to identify when an interval has **no information, no emissions**,

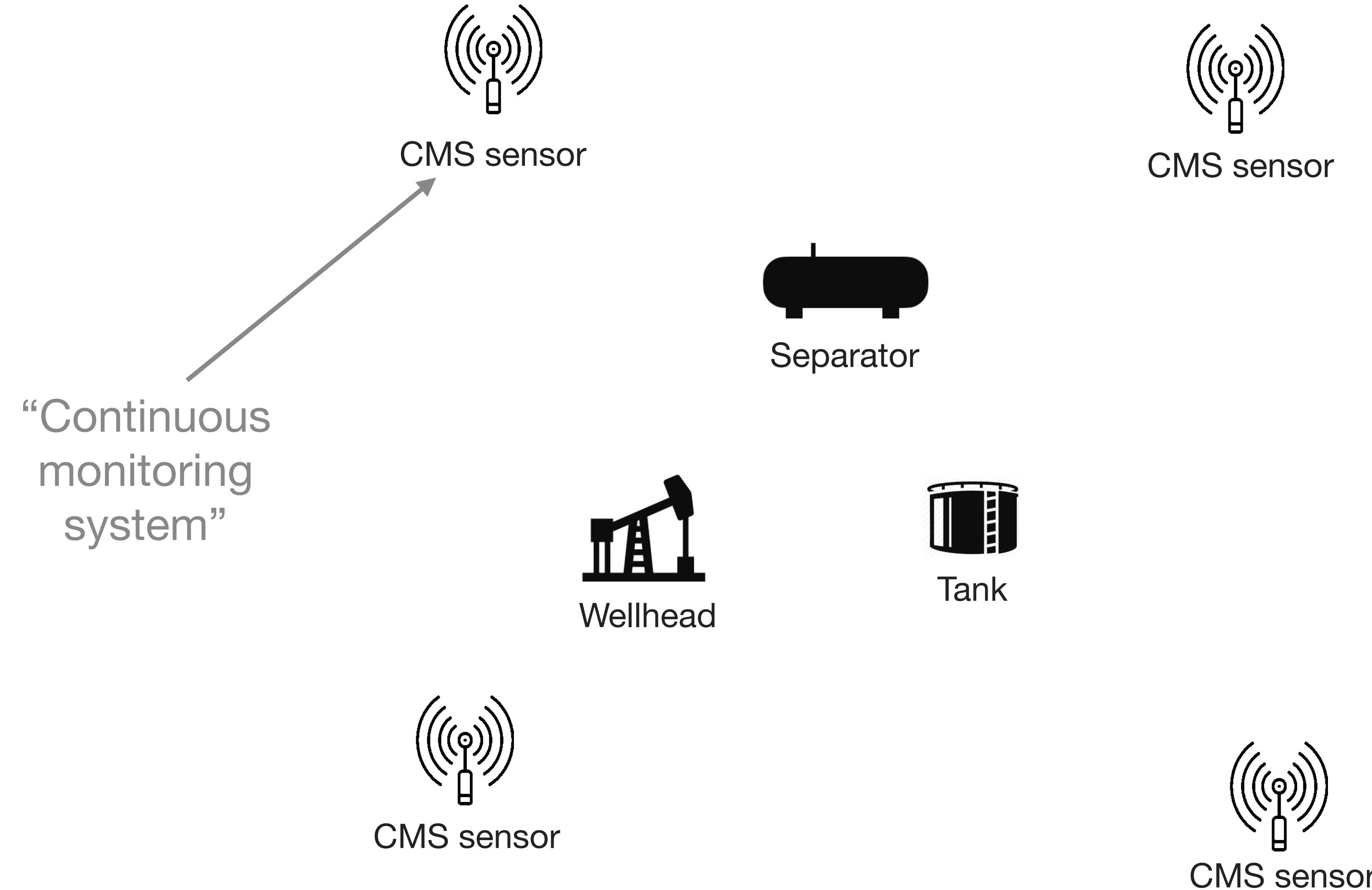


Before building an inventory, we need to identify when an interval has **no information**, **no emissions**, or a **non-zero emission**

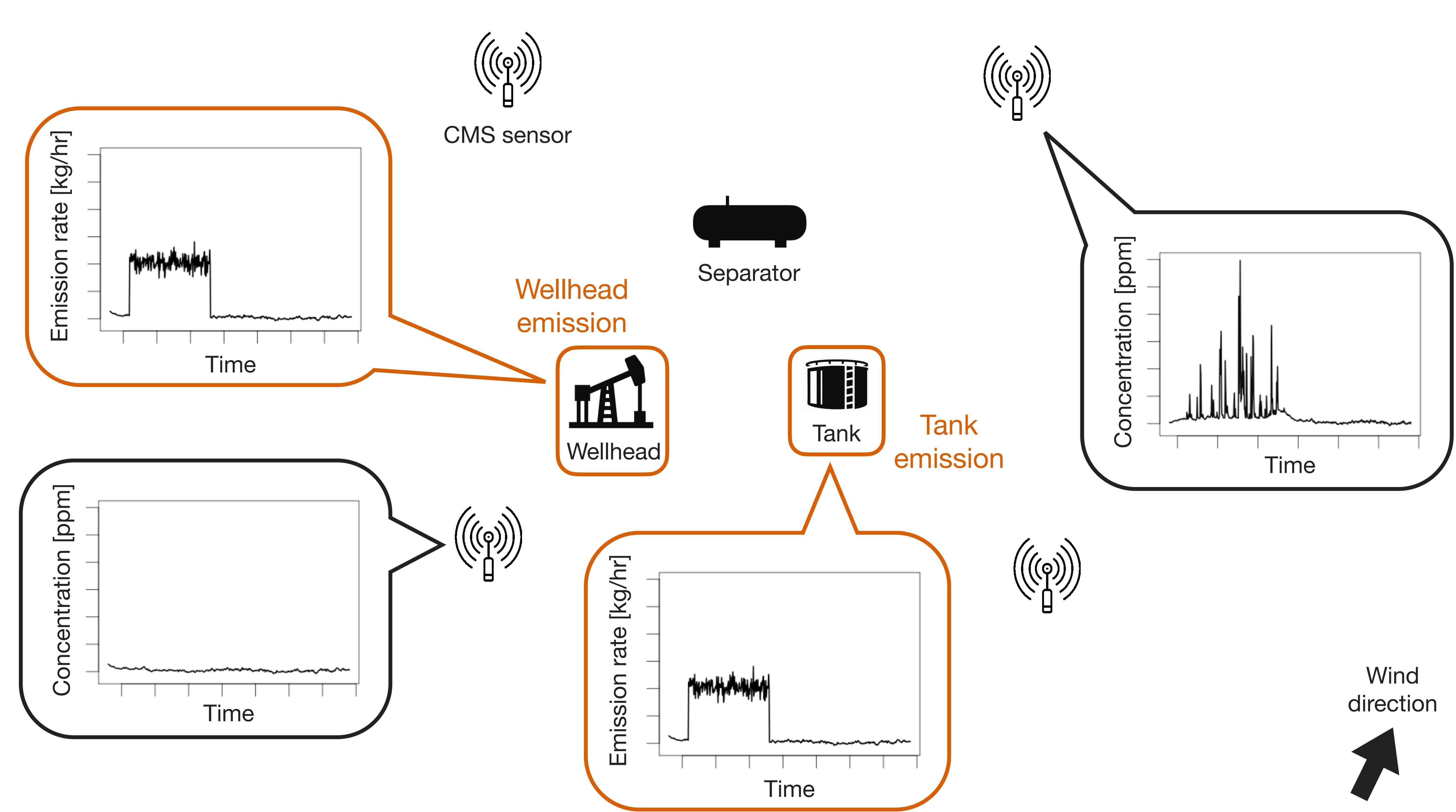


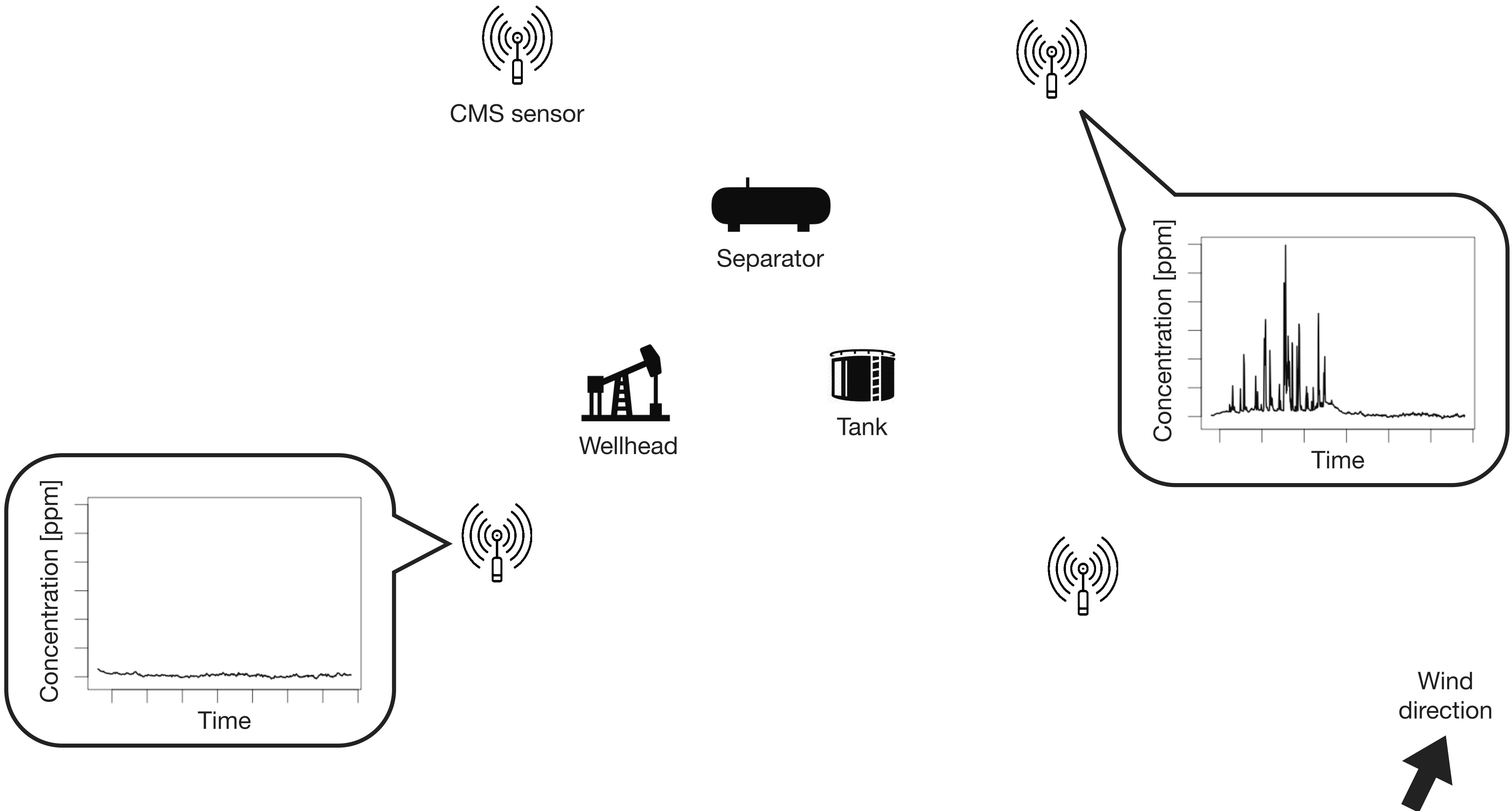
# How do you get the inventory? We're almost there...





## Estimating multi-source emissions with continuous monitors





# Multisource detection, localization, and quantification (MDLQ) model

Assume a multiple linear regression model at the data level

$n$  = number of observations

$p$  = number of potential sources

$$y = X\beta + \epsilon$$

$$y \equiv \{y_1, \dots, y_n\}, \beta \equiv \{\beta_1, \dots, \beta_p\}, X \in \mathbb{R}^{n \times p}$$

Concentration  
observations  
from CMS sensors

Emission rates for  
each source

Simulated concentrations  
from forward model, with  
each column assuming a  
different source

# Gaussian puff model: mathematical definition

Set up coordinate system so that source is at (0,0,H) and positive x-axis aligns with downwind vector

$$c_p(x, y, z, t, Q) = \frac{Q}{(2\pi)^{3/2} \sigma_y^2 \sigma_z} \exp\left(-\frac{(x - ut)^2 + y^2}{2\sigma_y^2}\right) \left[ \exp\left(-\frac{(z - H)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z + H)^2}{2\sigma_z^2}\right) \right]$$

Total volume of methane contained in puff  $p$

Predicted methane concentration at sensor location  $(x, y, z)$  and time  $t$  from puff  $p$

Exponential decay in concentration in horizontal plane  $(x, y)$

Exponential decay in concentration in vertical dimension  $(z)$

# Gaussian puff model: mathematical definition

Set up coordinate system so that source is at (0,0,H) and positive x-axis aligns with downwind vector

$$c(x, y, z, t, Q) = \sum_{p=1}^P c_p(x, y, z, t, Q)$$

Total concentration at  $(x, y, z, t)$

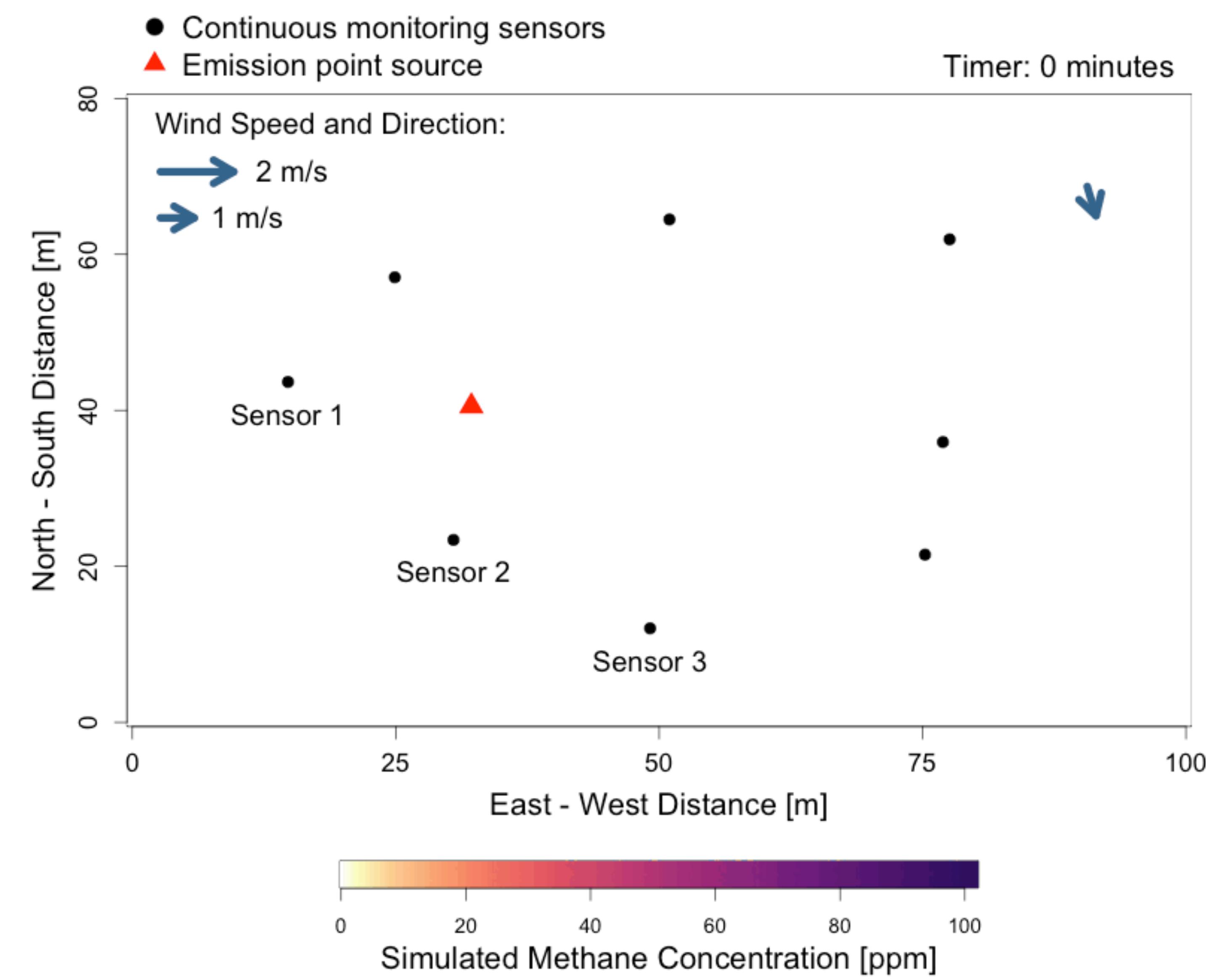
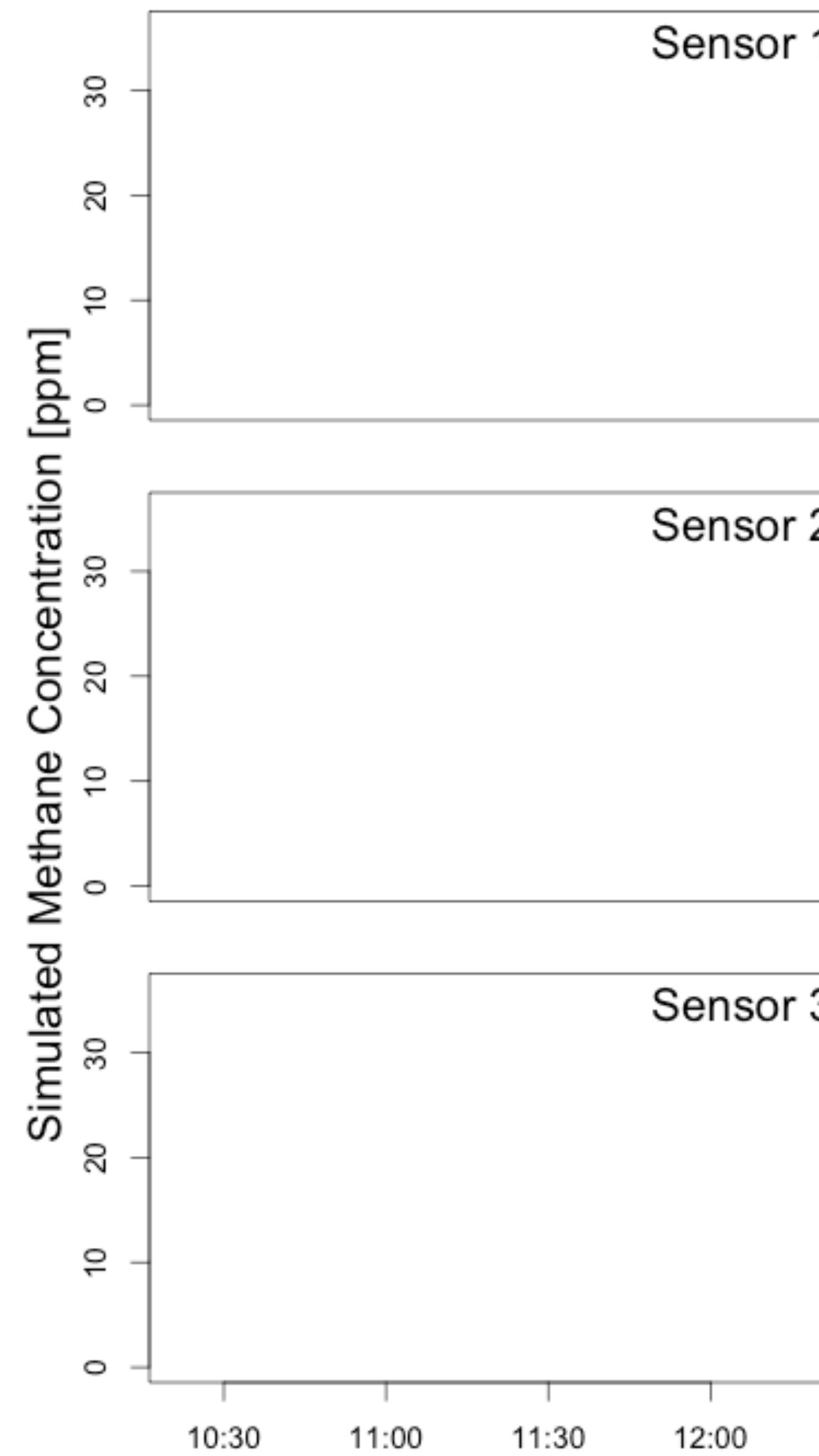
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Predicted methane concentration at sensor location  $(x, y, z)$  and time  $t$  from puff  $p$

Exponential decay in concentration in horizontal plane  $(x, y)$

Exponential decay in concentration in vertical dimension  $(z)$





Repeat this for all other potential sources!

# Multisource detection, localization, and quantification (MDLQ) model

Assume a multiple linear regression model at the data level

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Assume that the errors  $\epsilon \equiv \{\epsilon_1, \dots, \epsilon_n\}$  are identically distributed, Gaussian, and autocorrelated such that

$$\epsilon \sim N(0, \sigma^2 R)$$

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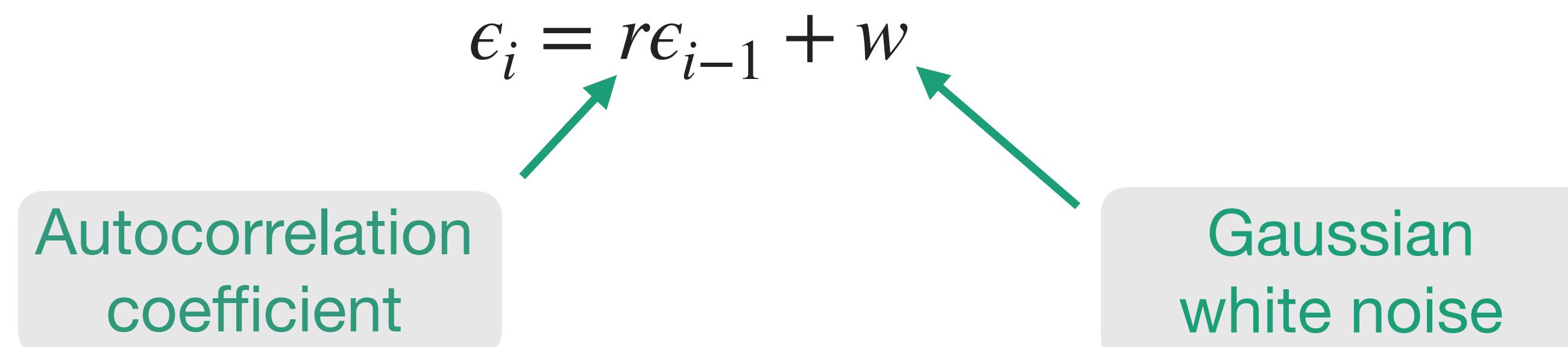
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Let the errors follow an AR(1) process such that

$$\epsilon_i = r\epsilon_{i-1} + w$$


Autocorrelation coefficient

Gaussian white noise

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This gives us:  $y \sim N(X\beta, \sigma^2 R)$

# Multisource detection, localization, and quantification (MDLQ) model

Data-level:  $y = X\beta + \epsilon$

$$\epsilon \sim N(0, \sigma^2 R)$$

$n$  = number of observations

$p$  = number of potential sources

The remainder of the hierarchy takes the following form

Spike-and-slab prior allows samples to be identically zero

Proportion of samples where  $z_i = 1$  gives posterior probability that source  $i$  is emitting

$$\beta_i \sim \begin{cases} 0, \\ \text{Exp}(\tau_i^2 \sigma^2), \end{cases} \quad z_i = 0$$

$$z_i = 1$$

“Slab” component is non-negative

$$z_i \sim \text{Bernoulli}(\theta_i)$$

$$\theta_i \sim \text{Beta}(a_i, b_i)$$

$a_i, b_i, c_i, d_i$  can contain operator insight

$$\tau_i^2 \sim \text{Inv-Gamma}(c_i, d_i)$$

$$\sigma^2 \sim \text{Inv-Gamma}(\nu/2, \nu/2)$$

$$\nu \sim \text{Inv-Gamma}(\alpha_1, \alpha_2)$$

$$r \sim \text{Uniform}(0, 1)$$

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$$\beta_i \sim \begin{cases} 0, & z_i = 0 \\ \text{Exp}(\tau_i^2 \sigma^2), & z_i = 1 \end{cases}$$

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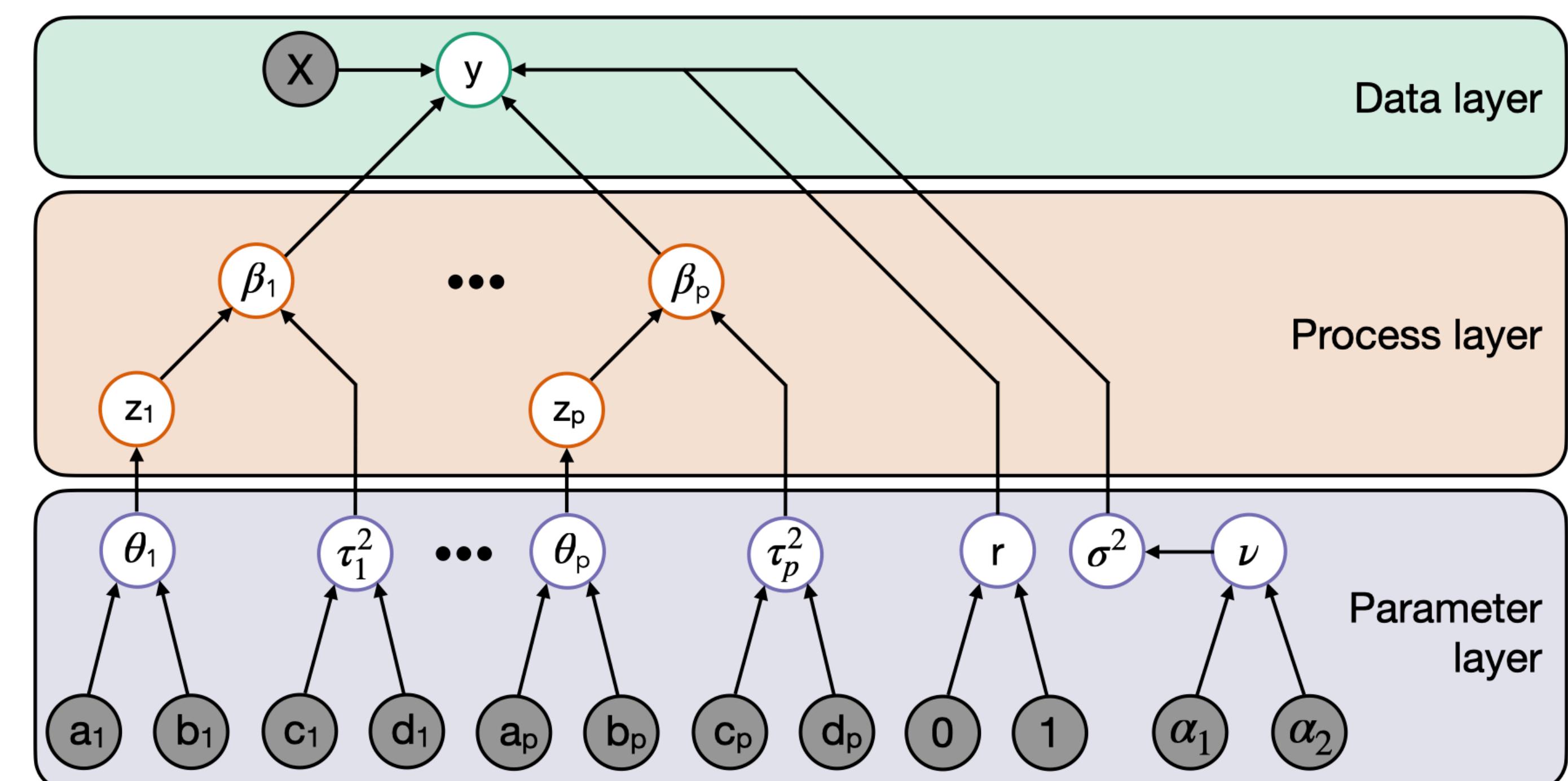
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$$\nu \sim \text{Inv-Gamma}(\alpha_1, \alpha_2)$$

$$r \sim \text{Uniform}(0, 1)$$



# Sampling from the posterior

We can derive Gibbs updates for all parameters except  $\nu$ .

$$\theta_i | \xi \sim \text{Beta}(z_i + a_i, 1 - z_i + b_i)$$

$$\sigma^2 | \xi \sim \text{Inv-Gamma} \left( \frac{\nu}{2} + \frac{n}{2}, \frac{\nu}{2} + \frac{1}{2}(y - X\beta)^T R^{-1}(y - X\beta) \right)$$

$$r | \xi \sim \begin{cases} \mathcal{N}(X\beta, \sigma^2 R) & 0 < r < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\tau_i^2 | \xi \sim \text{Inv-Gamma} \left( z_i + c_i, \frac{\beta_i}{\sigma^2} + d_i \right)$$

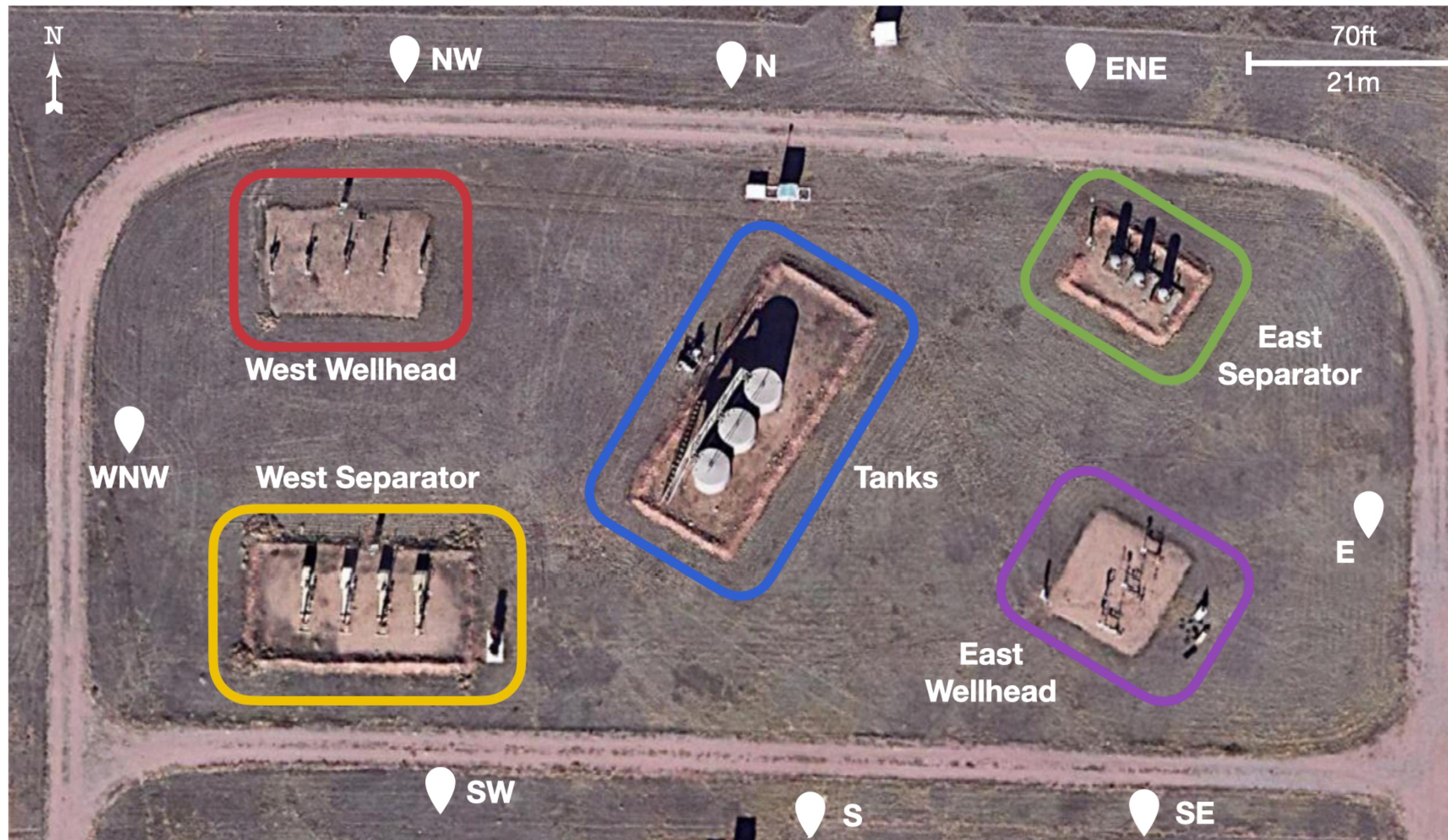
$$\beta_i | \xi \sim \begin{cases} 0 & z_i = 0 \\ \mathcal{N} \left( \left( \frac{X^T R^{-1} X}{\sigma^2} \right)^{-1} \left( \frac{X^T R^{-1} y}{\sigma^2} - \frac{e_i}{\tau_i^2 \sigma^2} \right), \left( \frac{X^T R^{-1} X}{\sigma^2} \right)^{-1} \right) & z_i = 1 \end{cases}$$

$$z_i | \xi \sim \text{Bernoulli} \left( 1 - \frac{1 - \theta_i}{(1 - \theta_i) + \theta_i \left( \frac{1}{\tau_i^2 \sigma^2} \right) \exp \left( \frac{\left( \sum_{j=1}^n (w_j X_{j,i}^* + w_j^* X_{j,i}) - \frac{2}{\tau_i^2} \right)^2}{4\sigma^2 \sum_{j=1}^n X_{j,i} X_{j,i}^*} \right) \left( \frac{2\sigma^2 \pi}{\sum_{j=1}^n X_{j,i} X_{j,i}^*} \right)^{1/2} \left( \frac{1}{2} \right)} \right)$$

$\nu | \xi \sim ?$  (Use a Metropolis–Hastings step)

Iterative samples from each full conditional gives you samples from the joint posterior!

# Model evaluation on multi-source controlled release data



337 controlled releases:

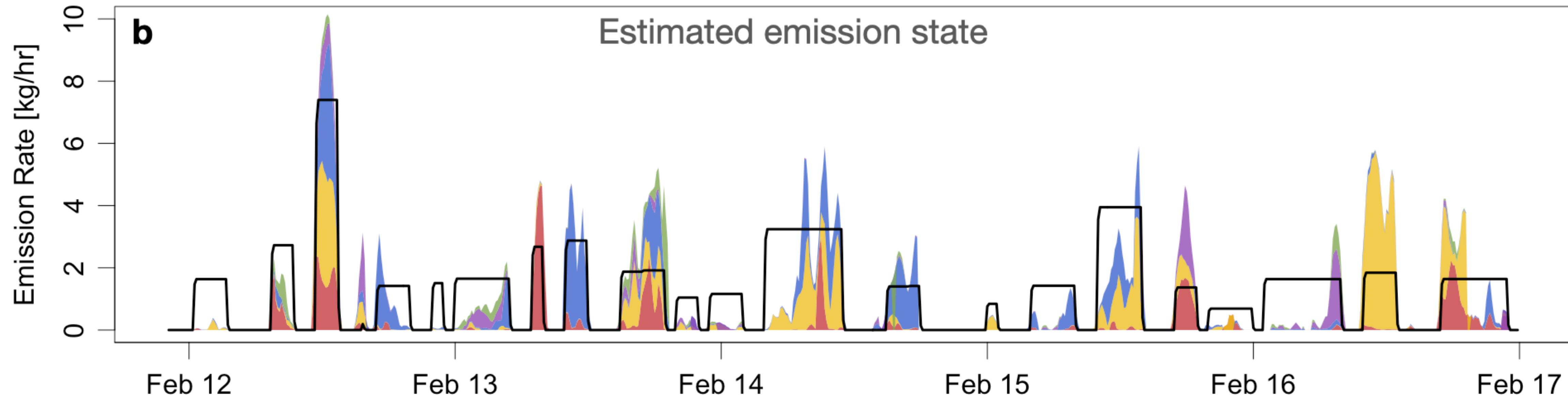
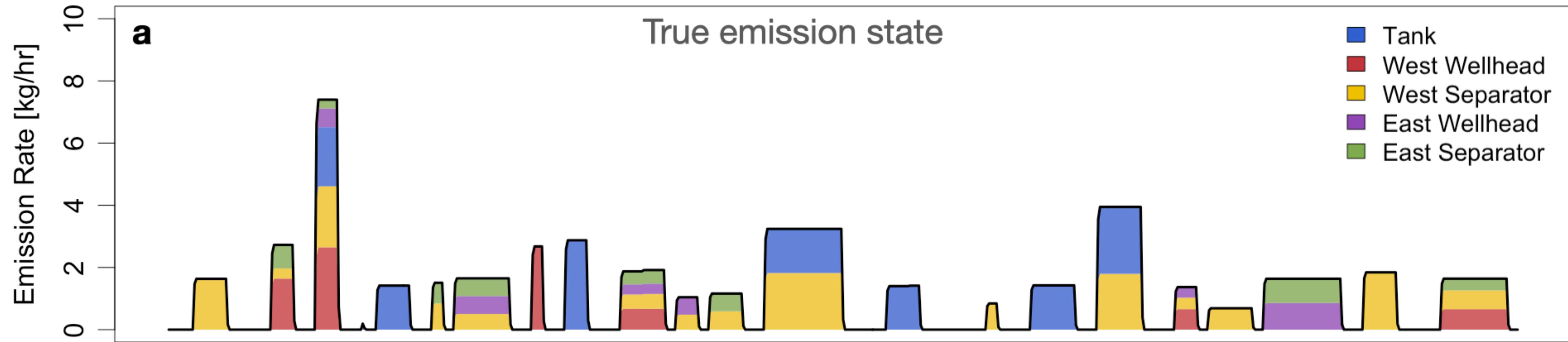
- 99 (29%) single-source
- 238 (71%) multi-source

Emission rates range from 0.08 to 7.2 kg/hr

Emission durations range from 0.5 to 8 hours

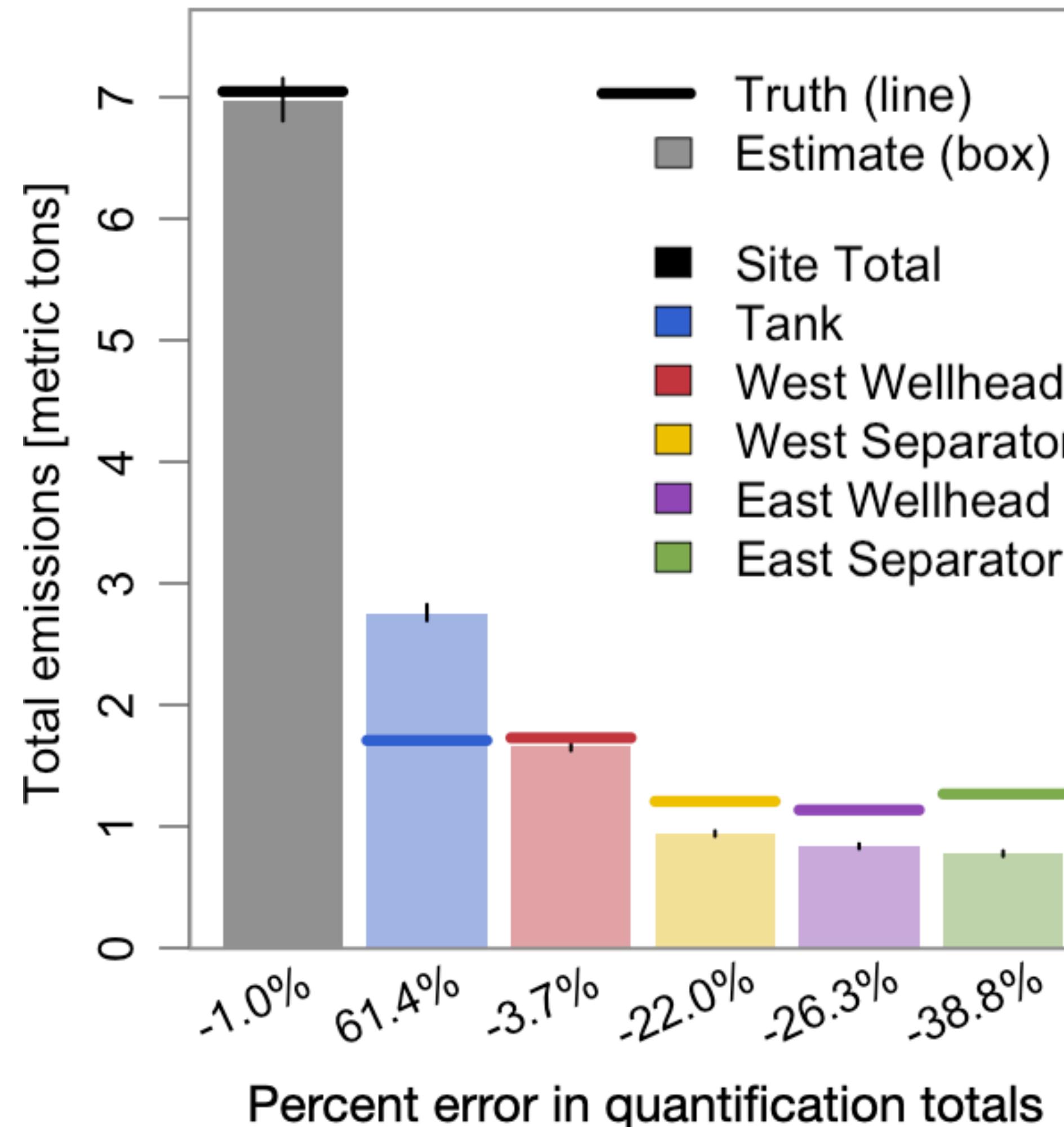
Methane Emissions Technology Evaluation Center (METEC)

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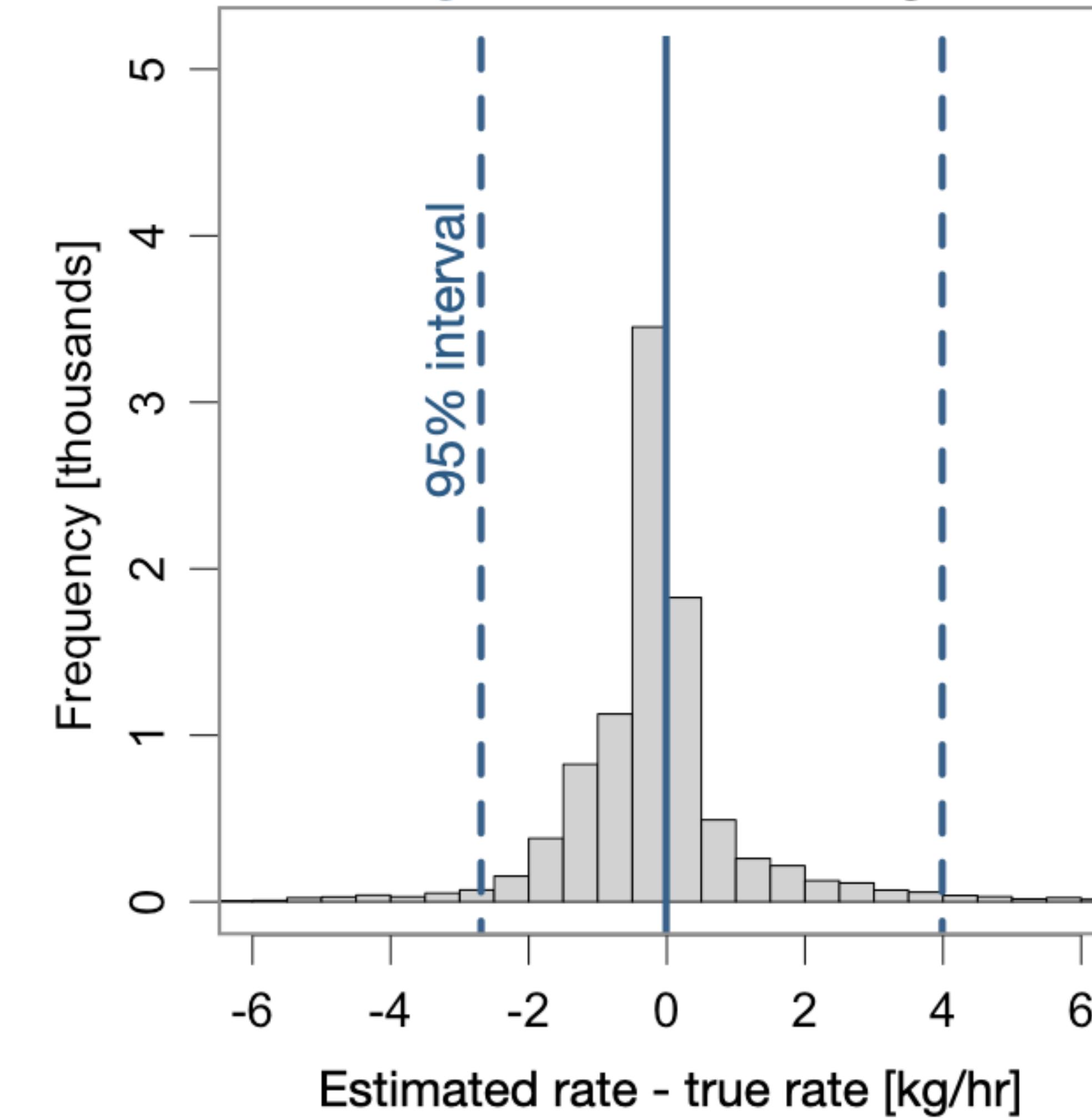


# Model evaluation on multi-source controlled release data

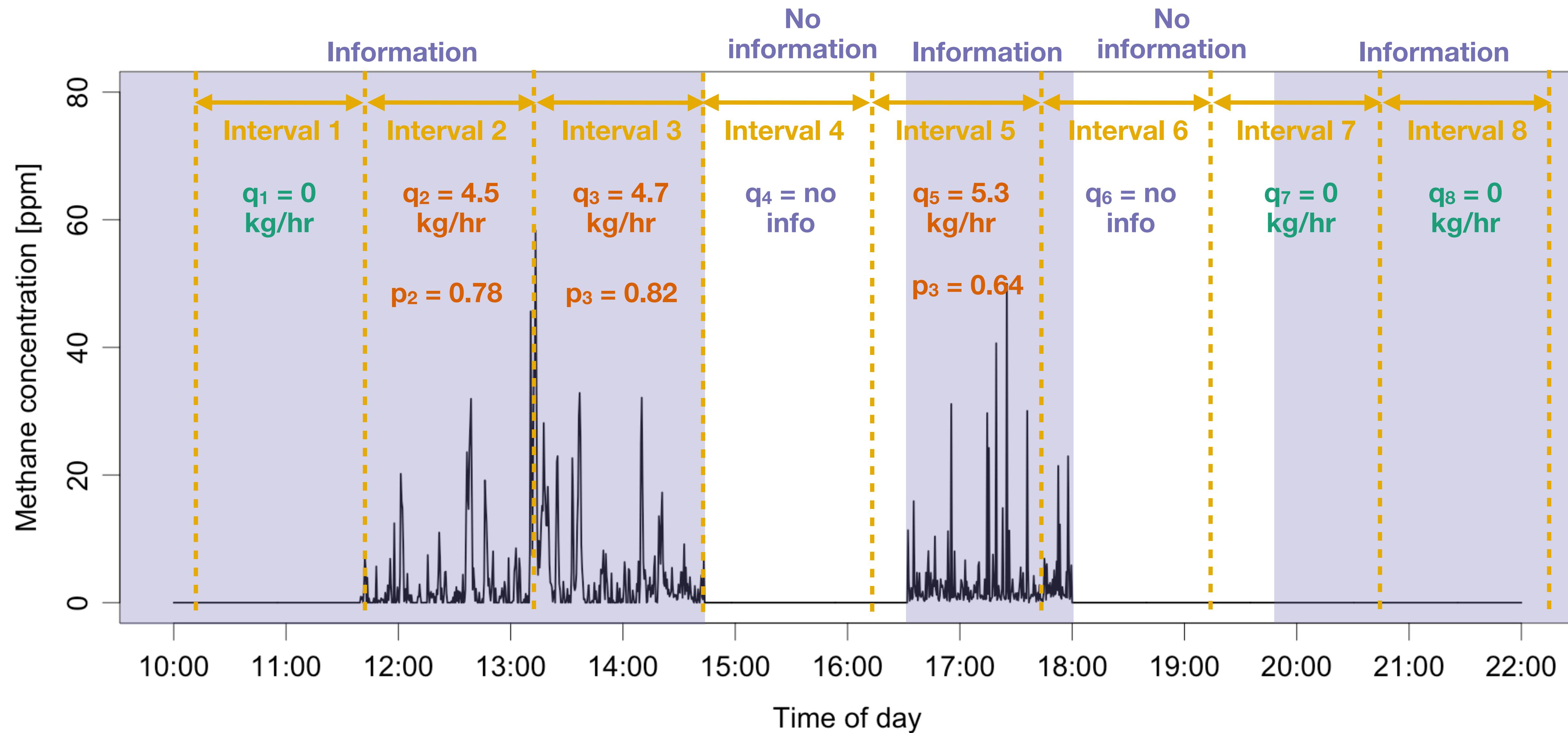
**a** Site-level and source-level emission inventories



**b** Site-level quantification errors  
Average error = -0.01 kg/hr



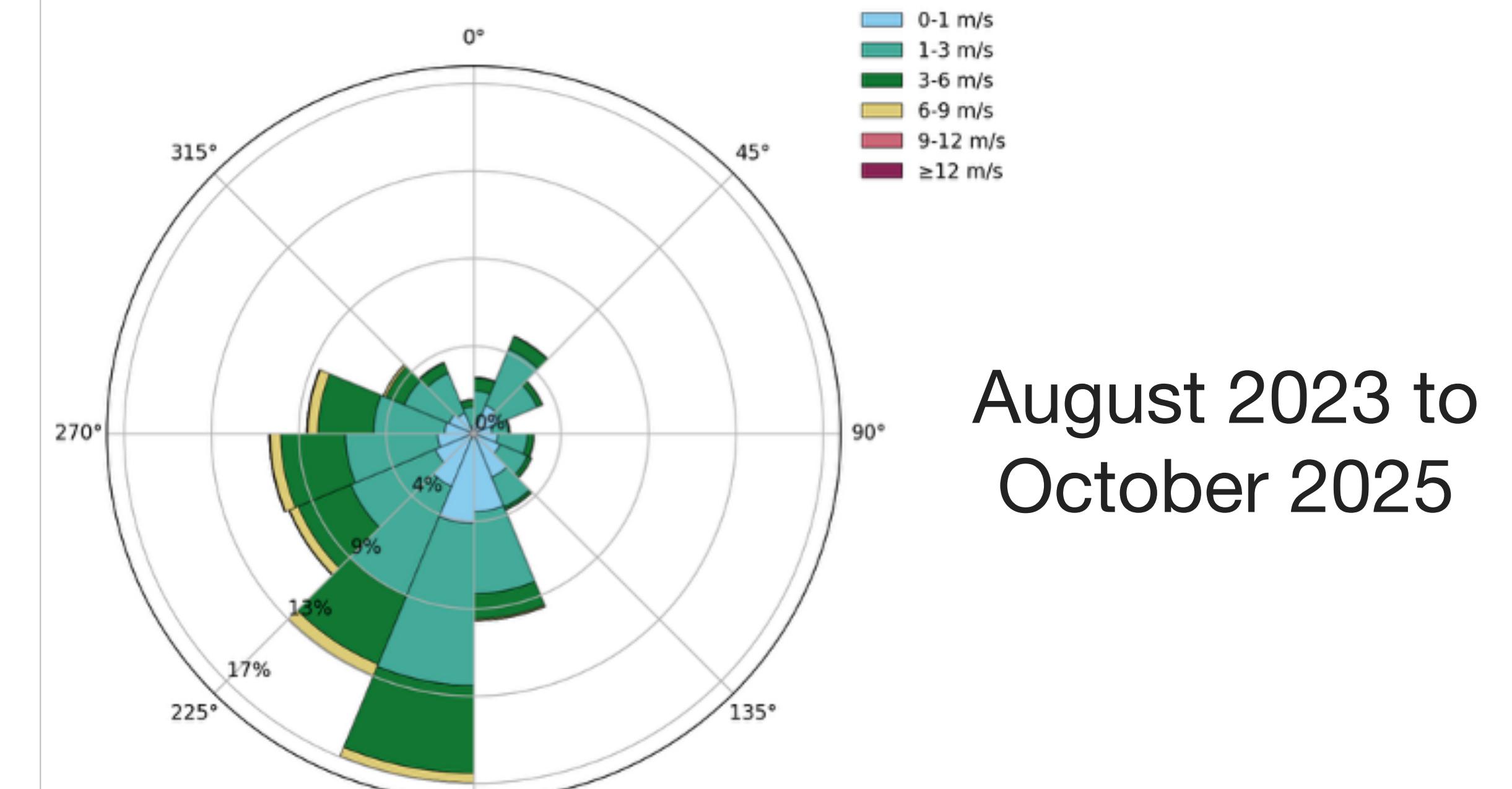
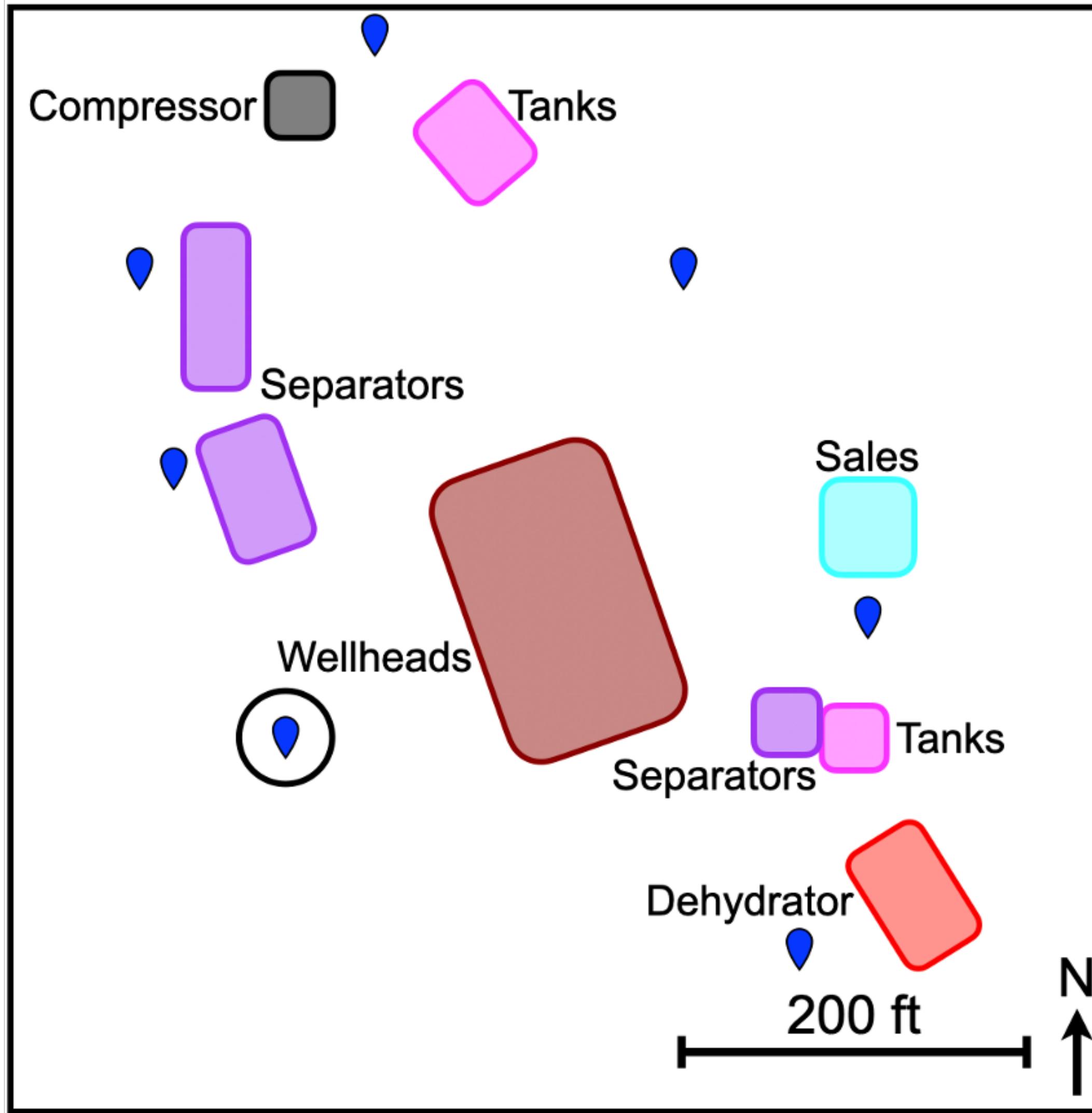
Need to identify when an interval is **no information**, **no emissions**, or a **non-zero emission**



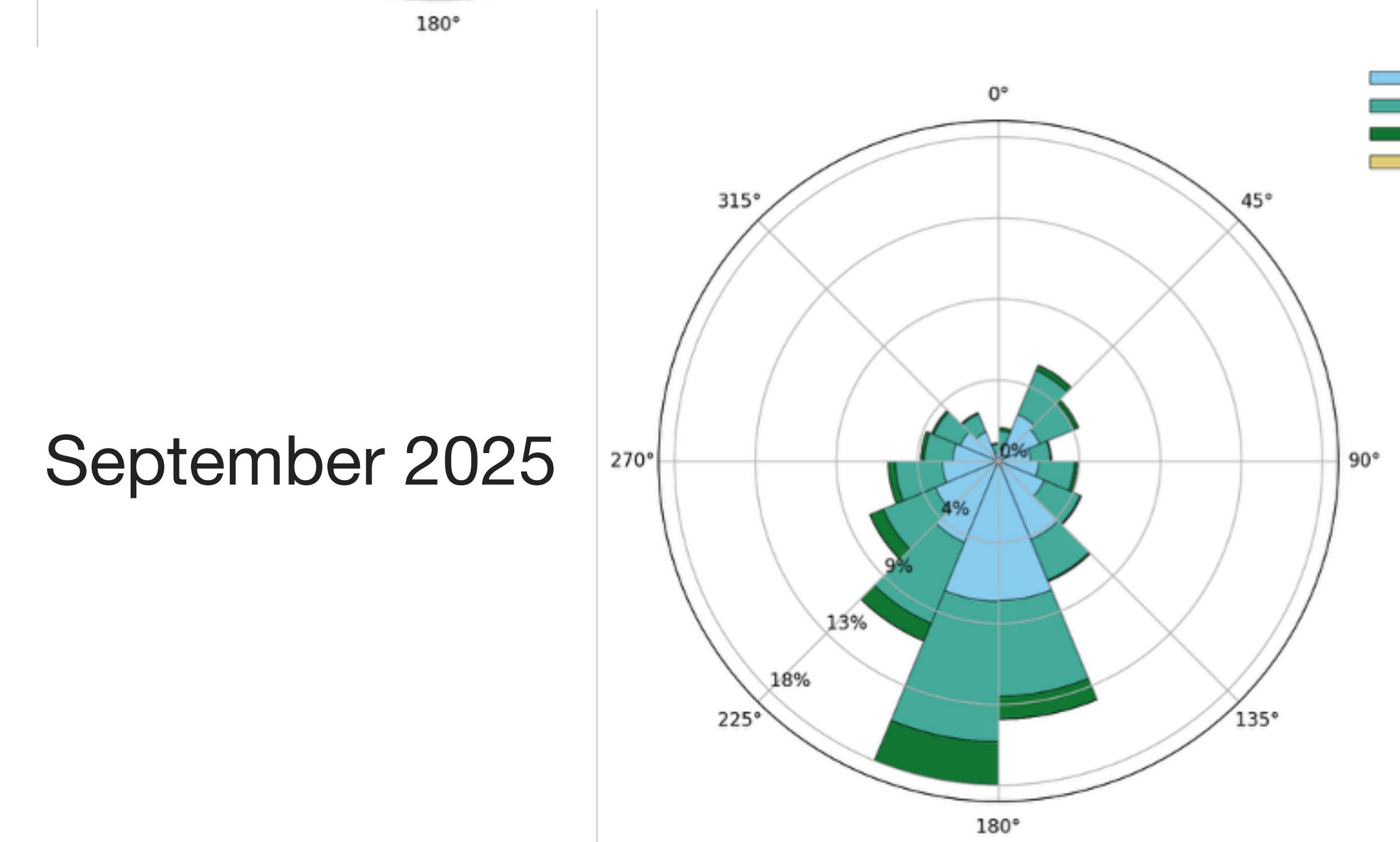
# Measurement-based inventory results for the Appalachian Methane Initiative (AMI)

- **We have data from 26 production sites**
  - All are equipped with high-end continuous monitoring point sensors
  - Number of sensors per site varies from 3 to 7
- **57.82 total years of data**
  - Average of 2.22 years per site

# Example: Site 25



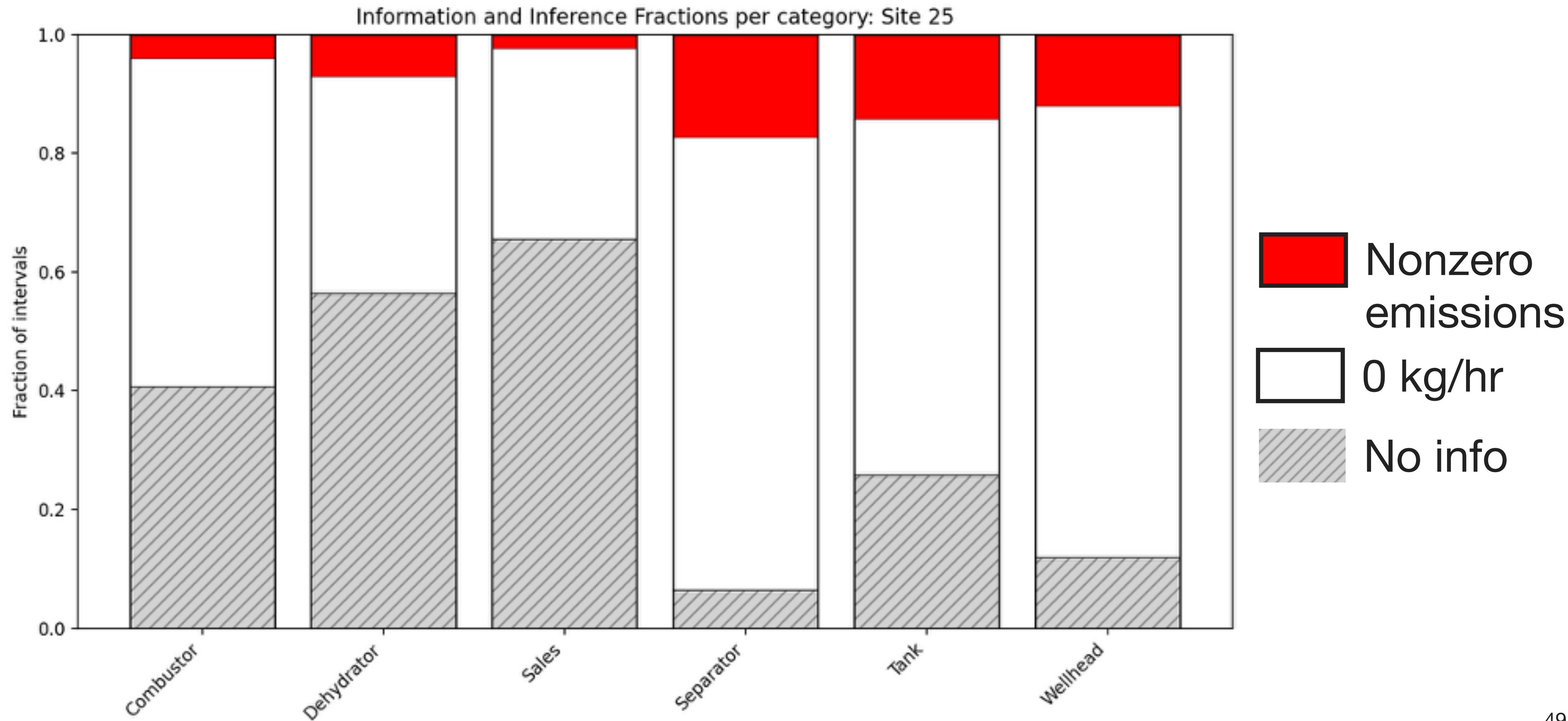
August 2023 to  
October 2025



September 2025

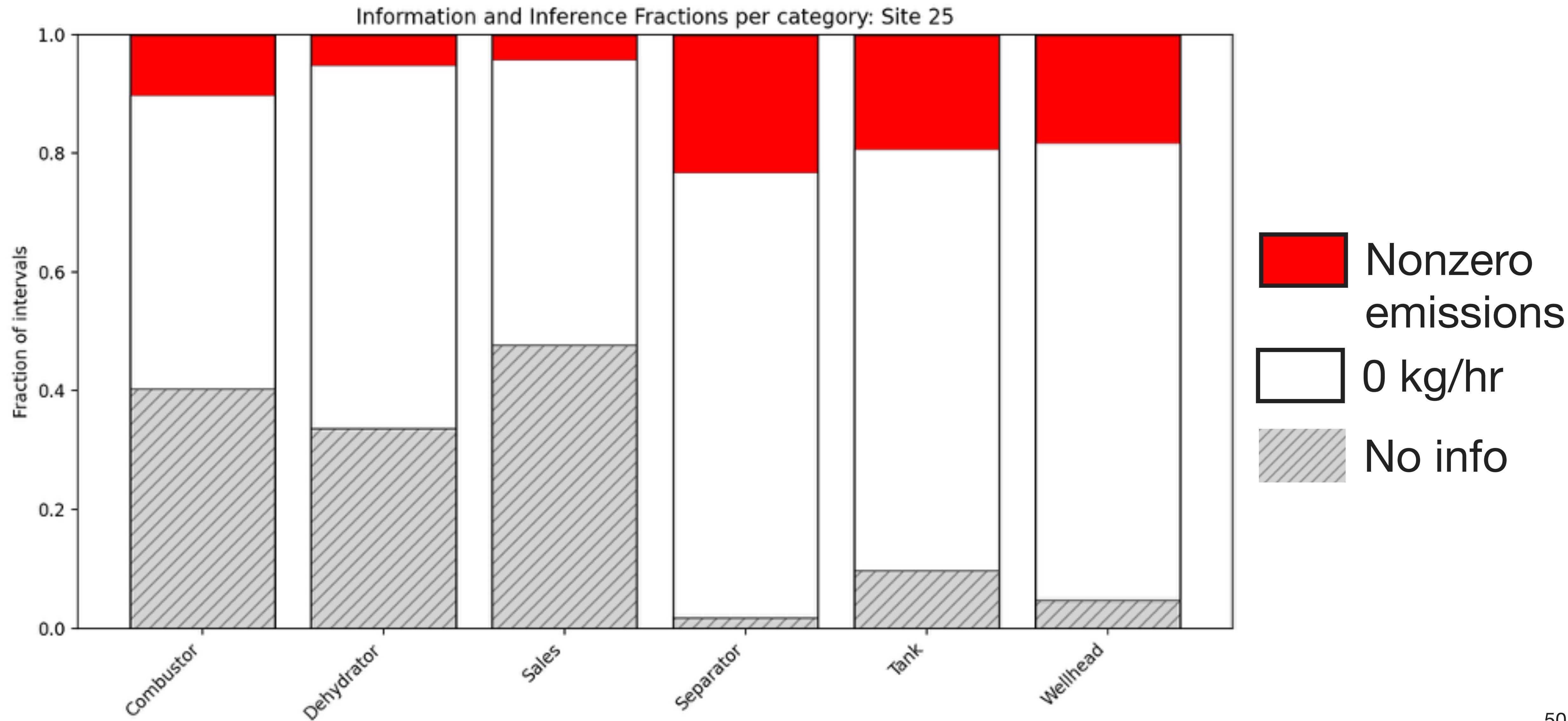
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# Ratio of no information to information



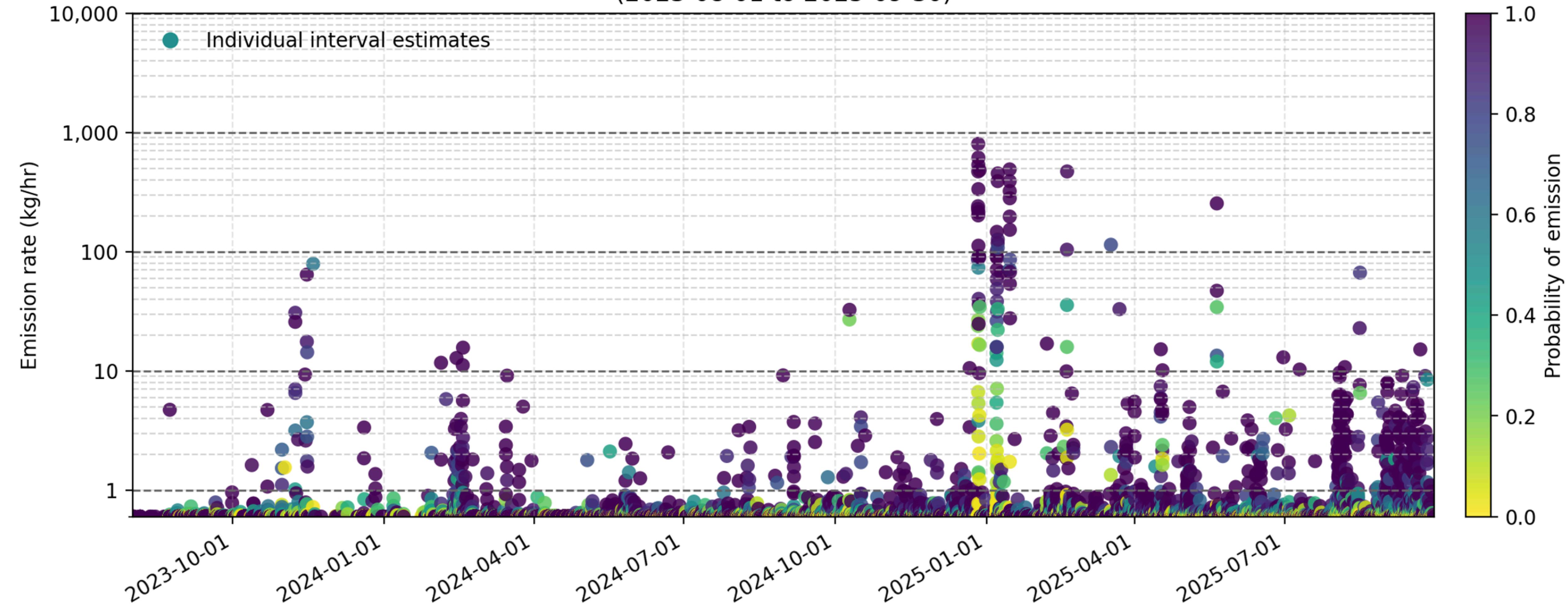
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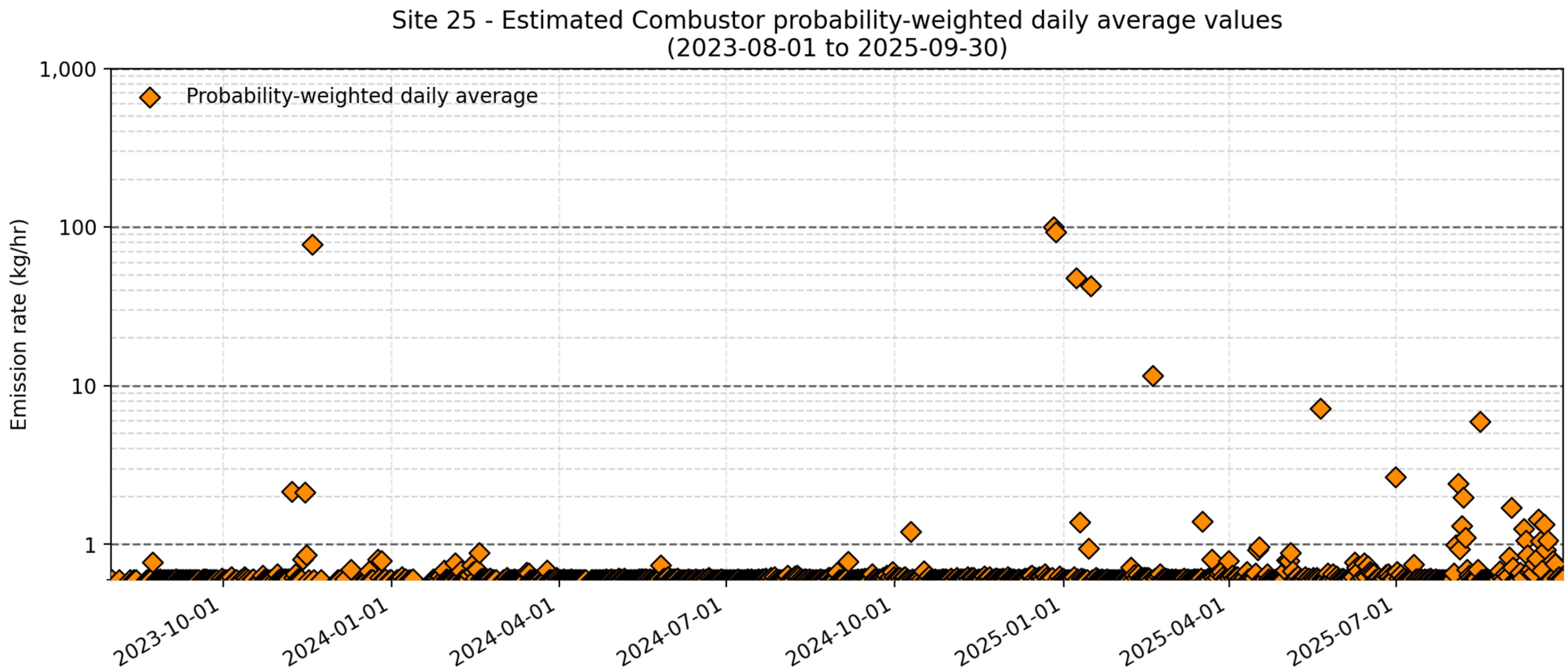


# Combustor emission rate estimates over time

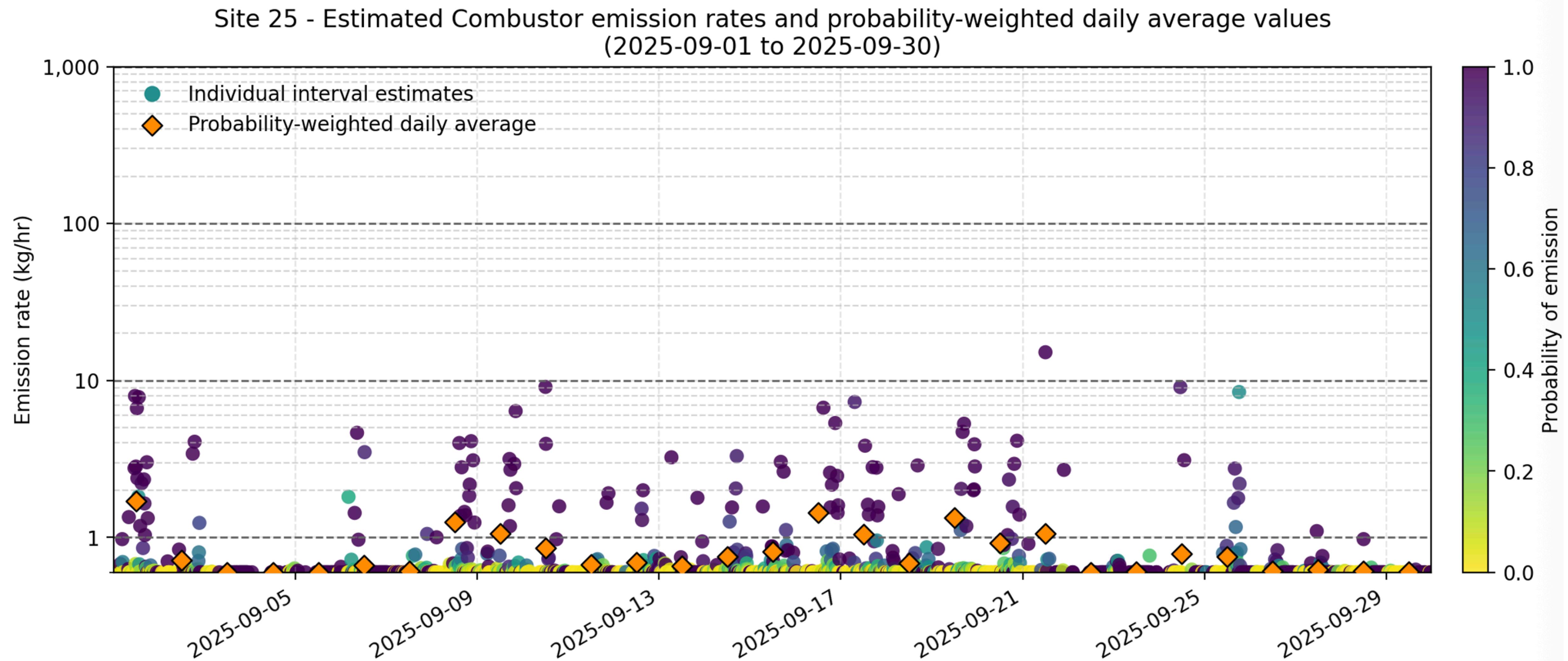
Site 25 - Estimated Combustor emission rates  
(2023-08-01 to 2025-09-30)



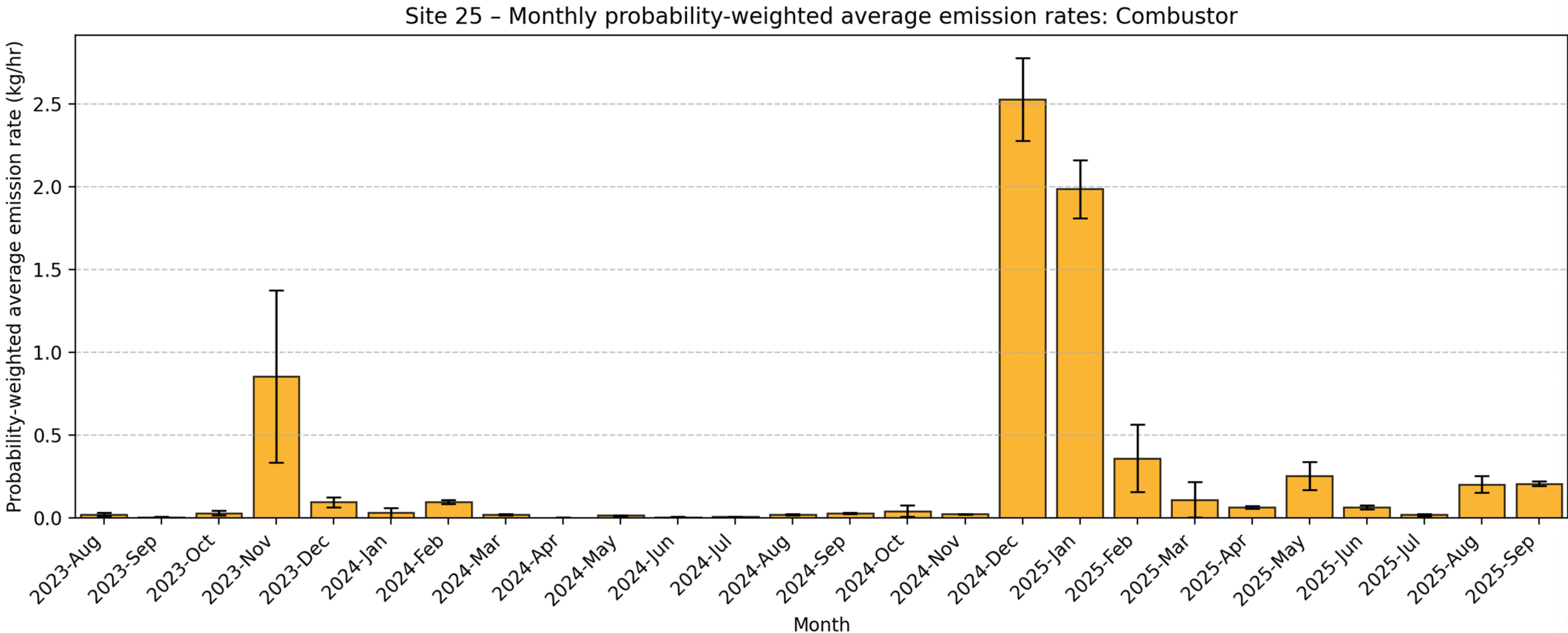
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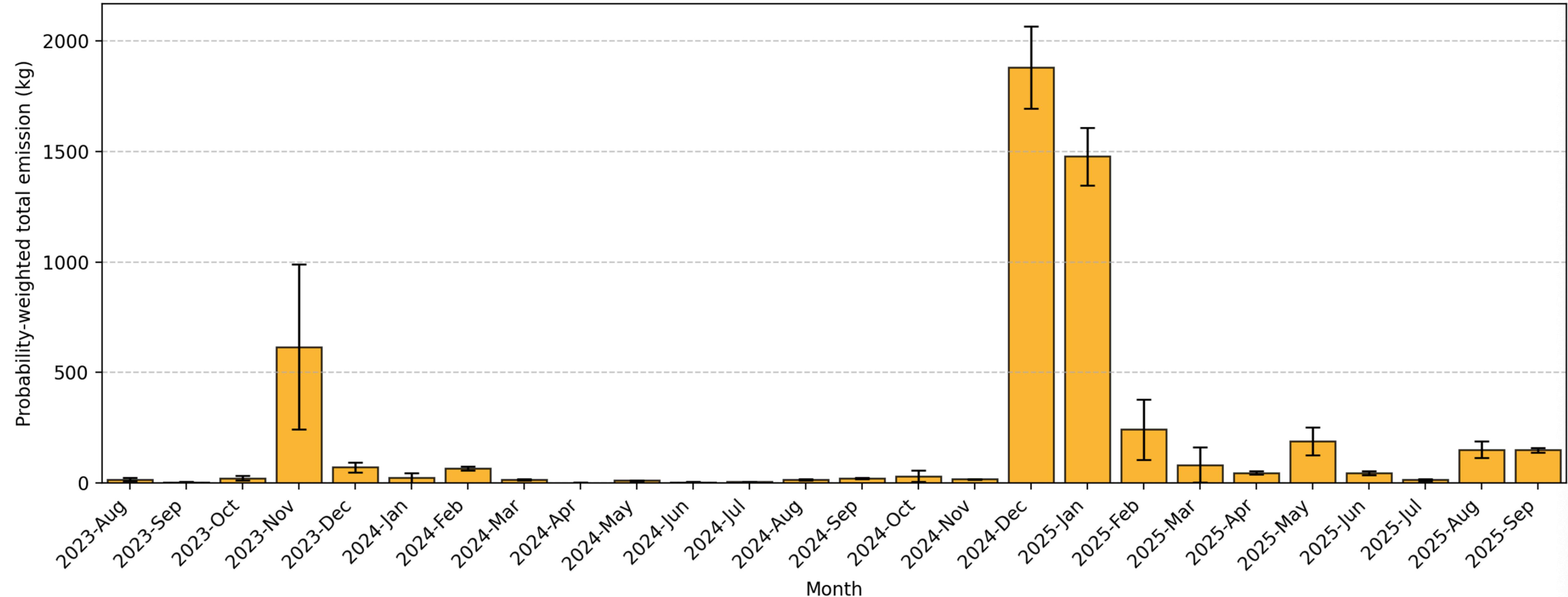


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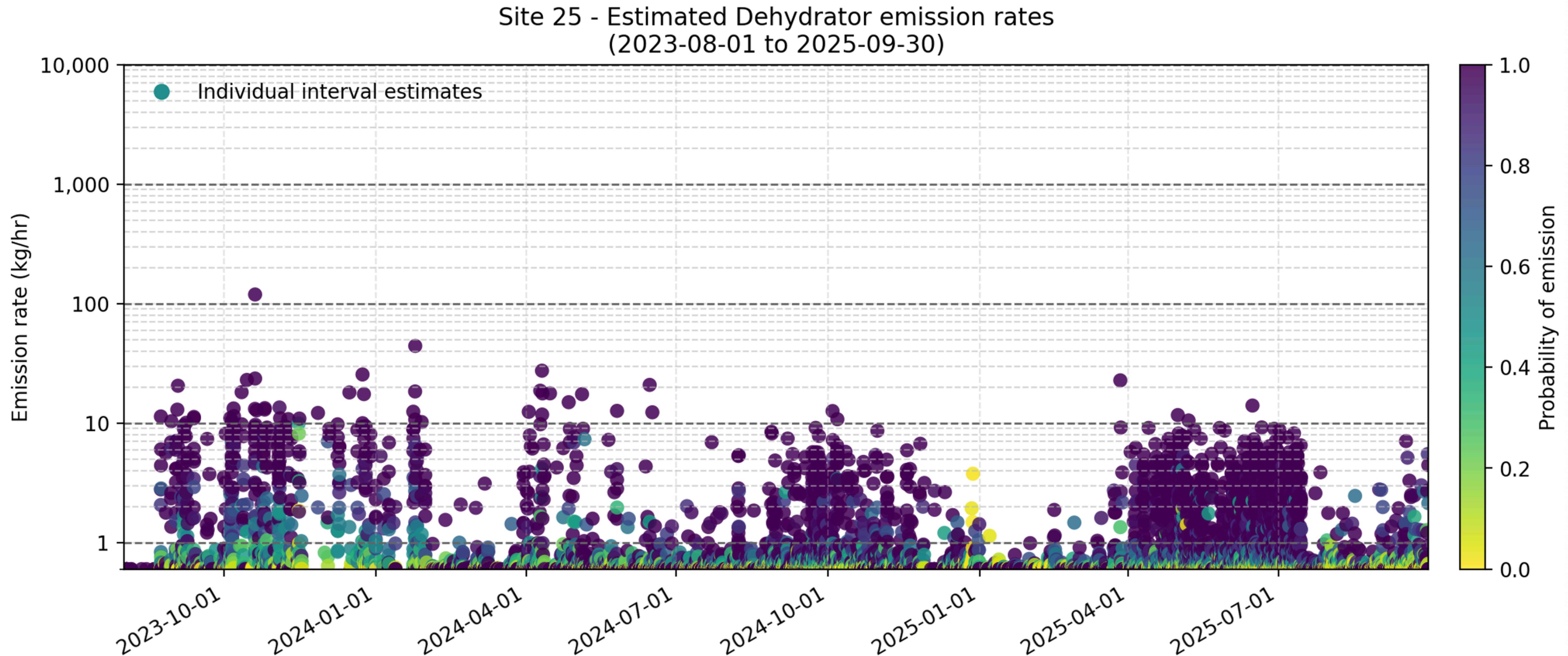


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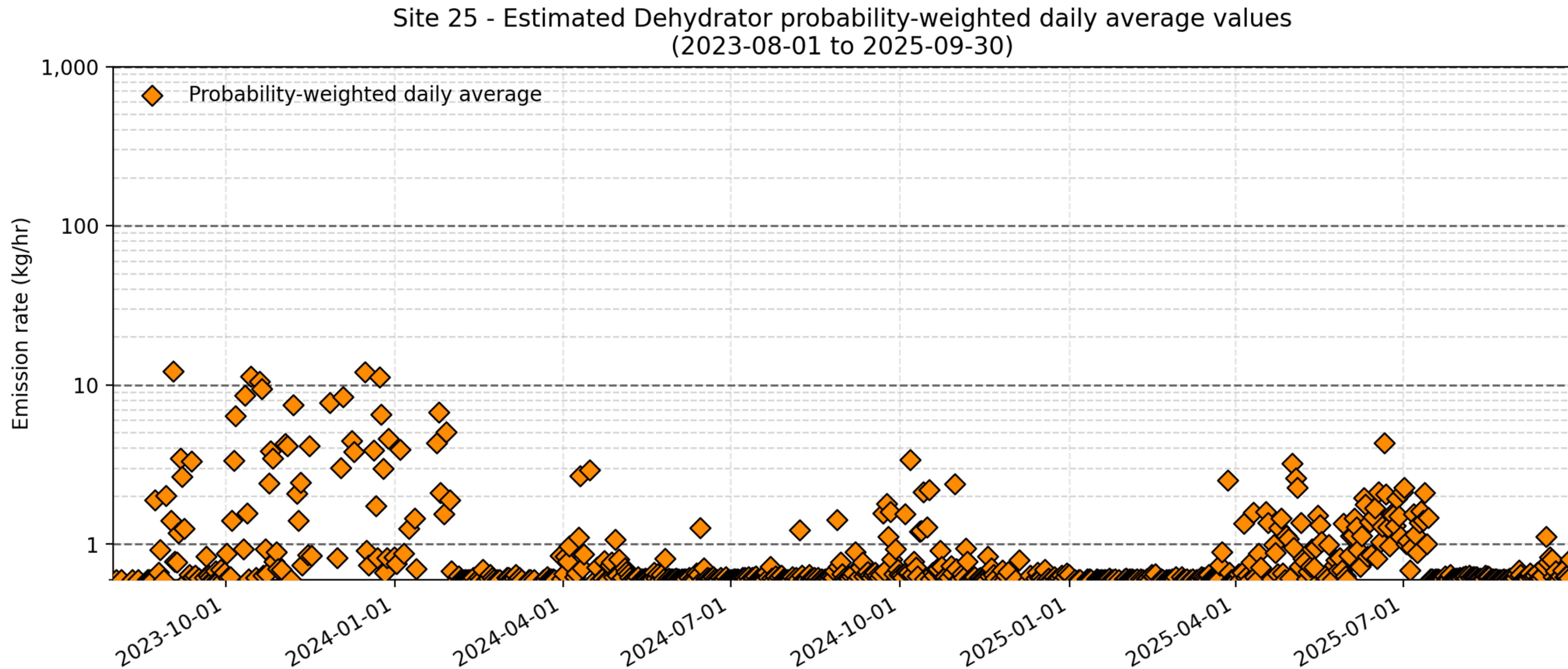
Site 25 – Monthly probability-weighted emission totals: Combustor



# Dehydrator emission rate estimates over time

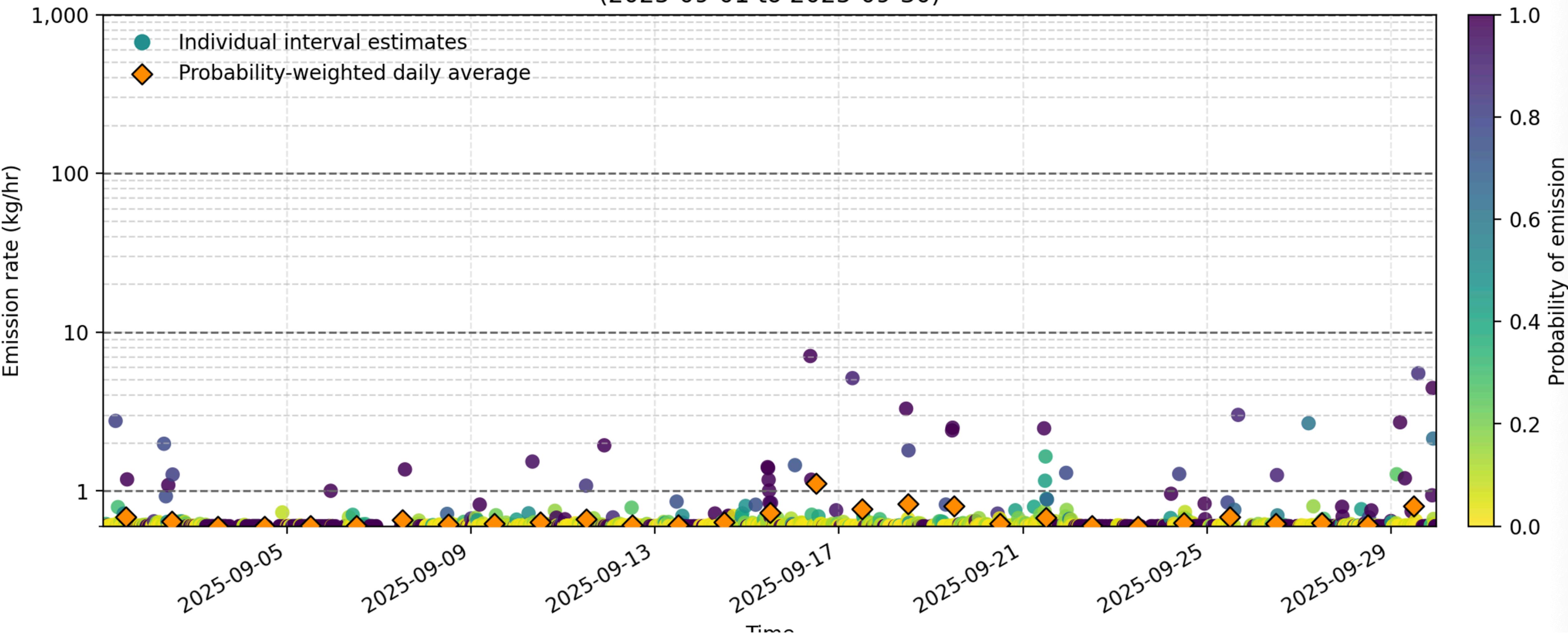


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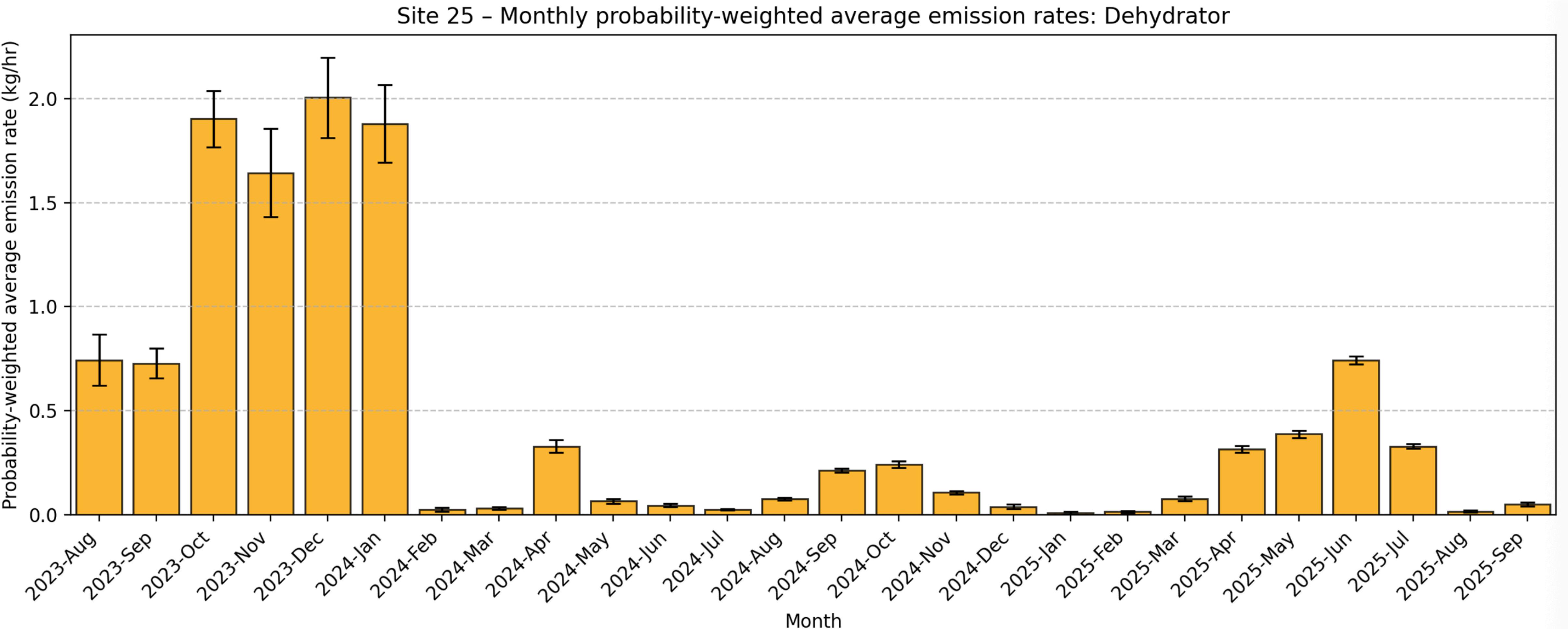


# Dehydrator emission rate estimates over time

Site 25 - Estimated Dehydrator emission rates and probability-weighted daily average values  
(2025-09-01 to 2025-09-30)

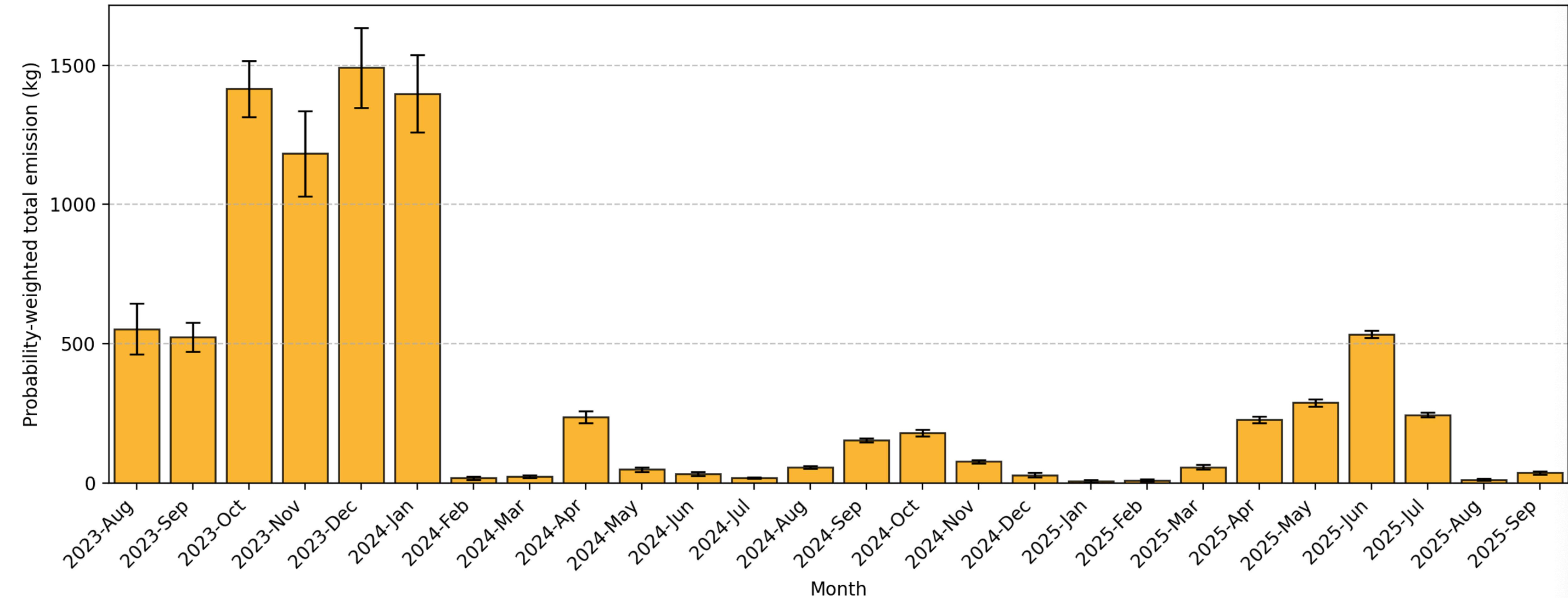


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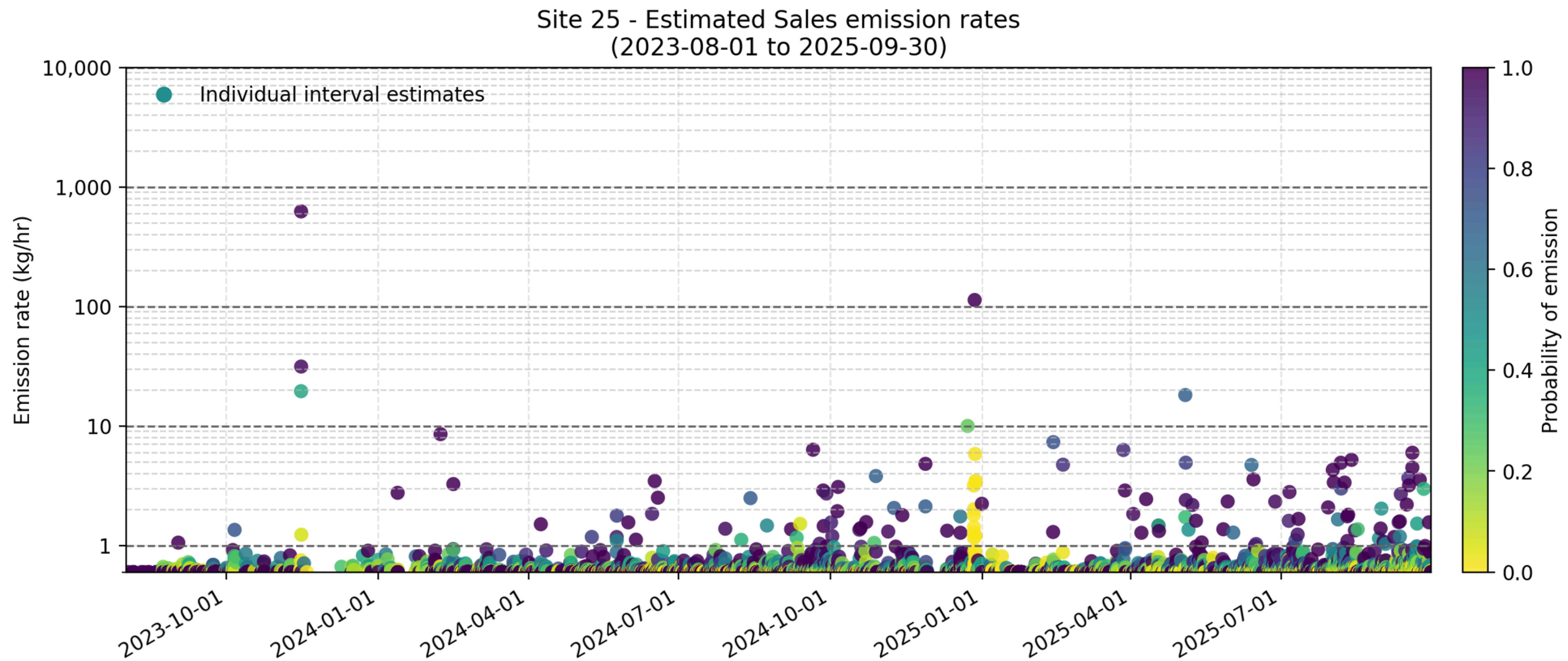


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Site 25 - Monthly probability-weighted emission totals: Dehydrator

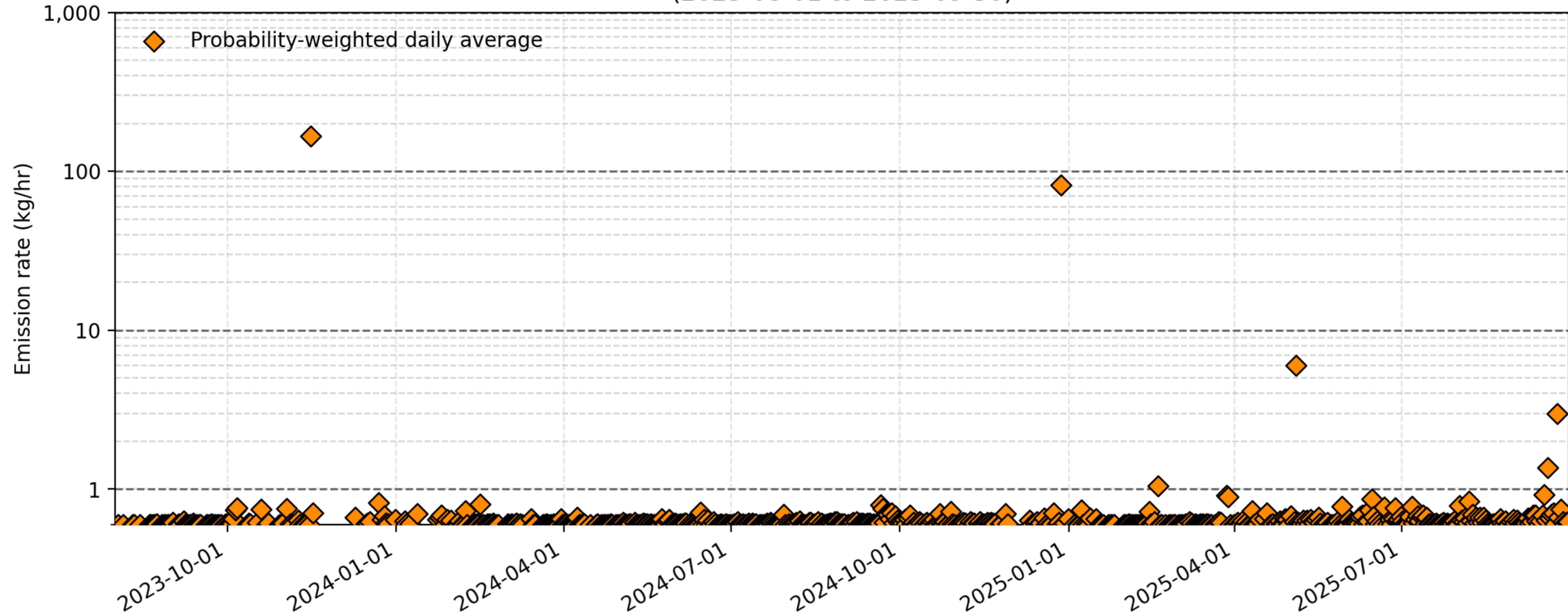


# Sales line emission rate estimates over time

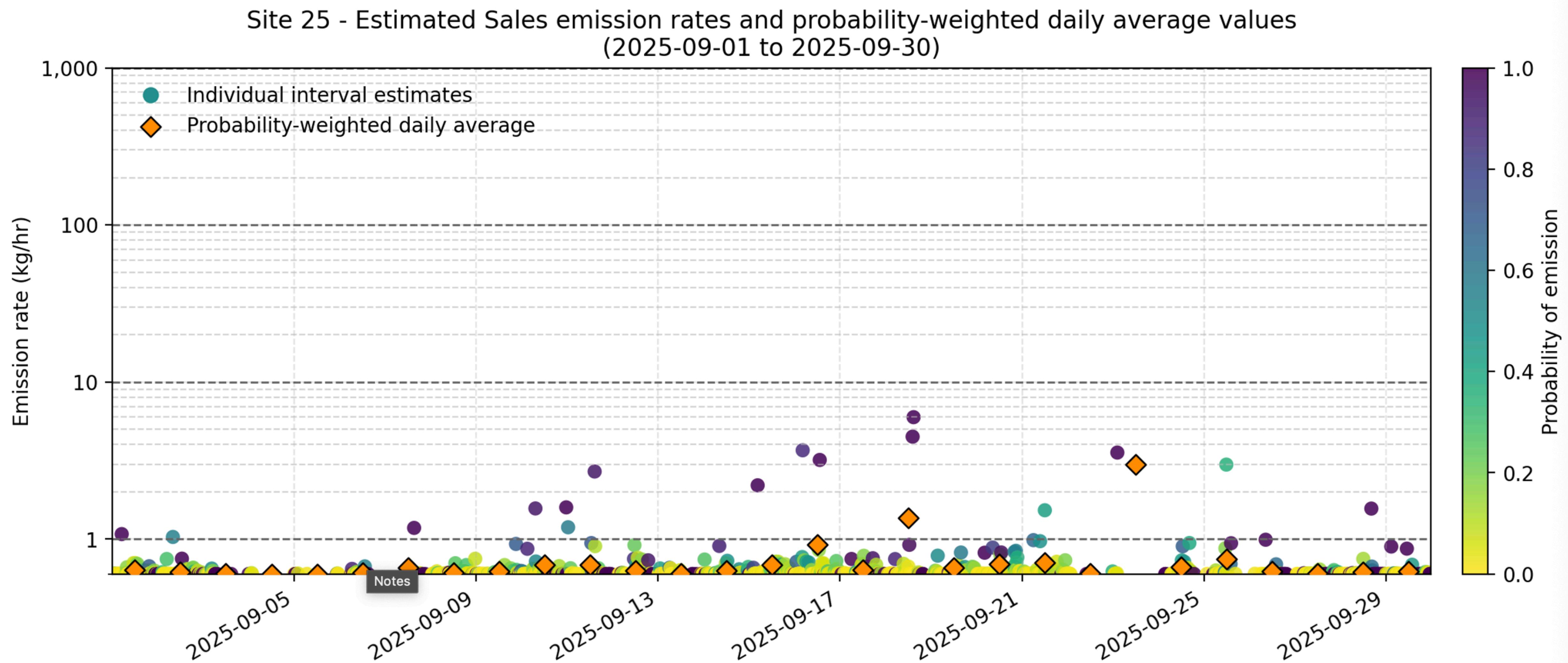


# Sales line emission rate estimates over time

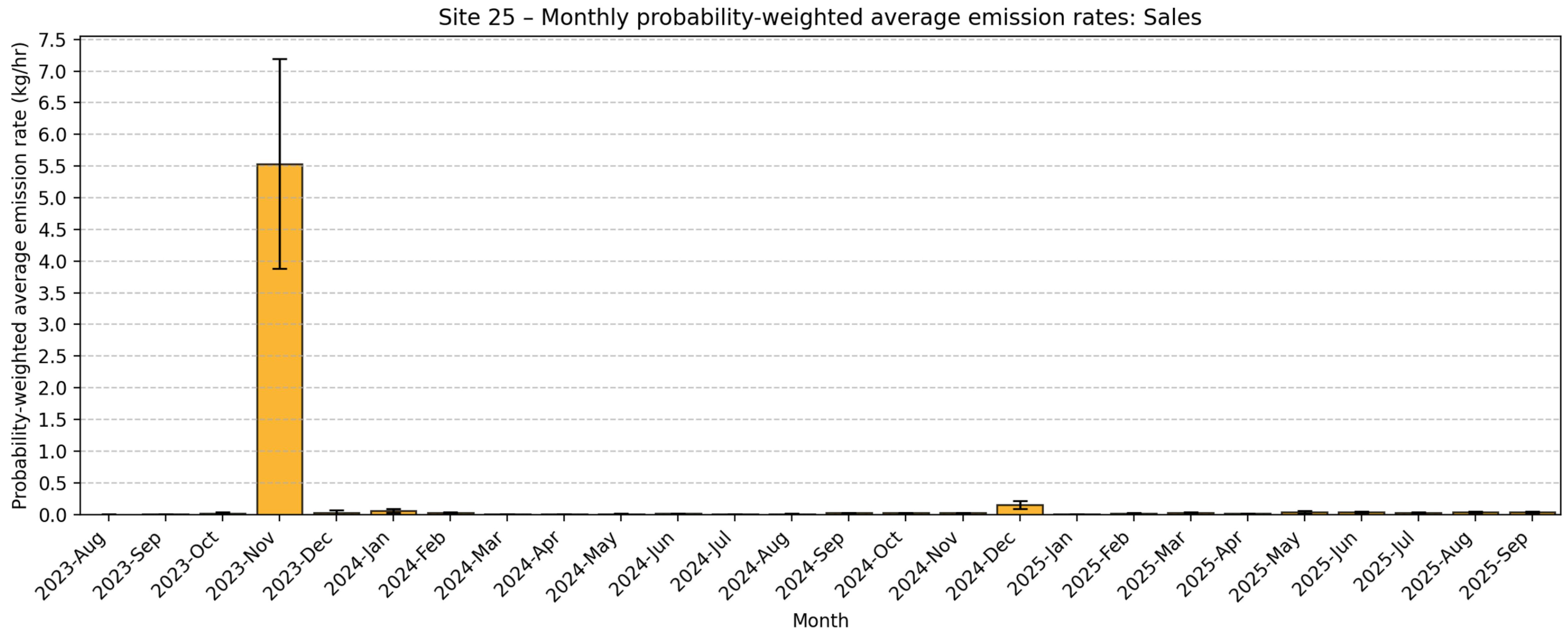
Site 25 - Estimated Sales probability-weighted daily average values  
(2023-08-01 to 2025-09-30)



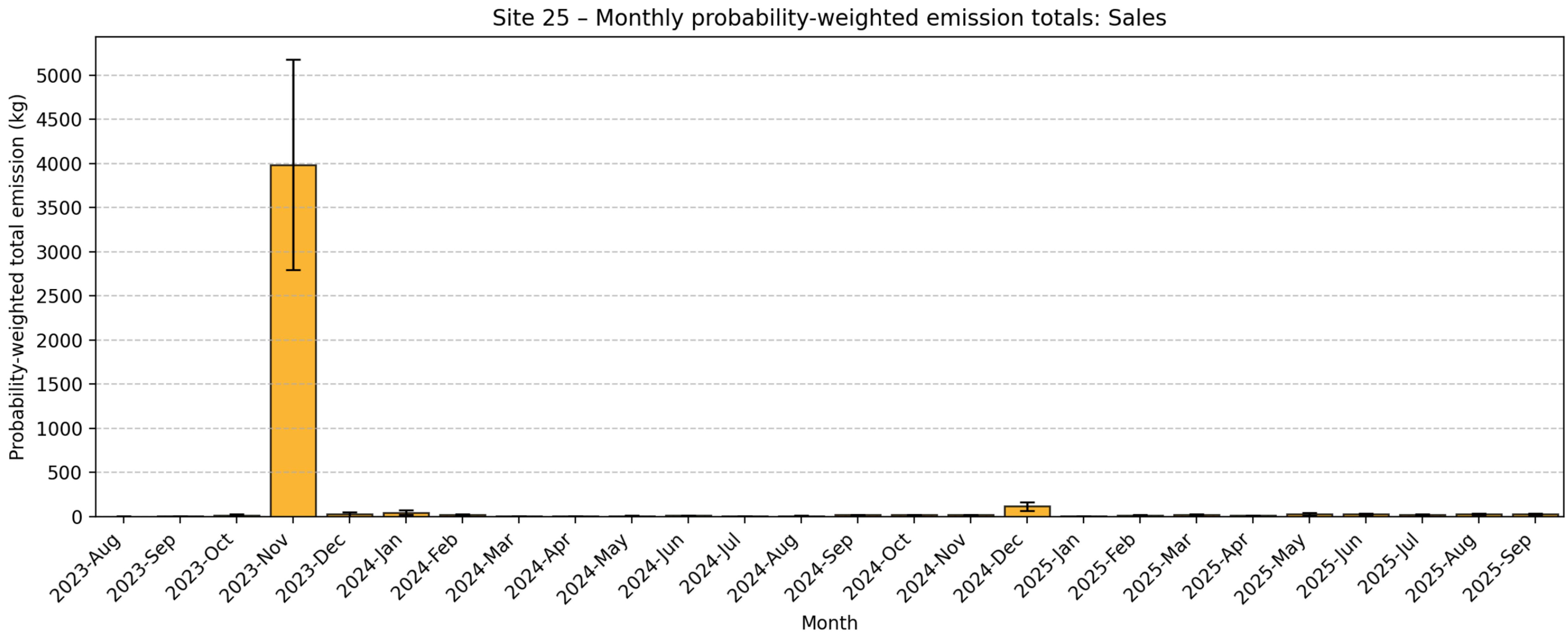
# Sales line emission rate estimates over time



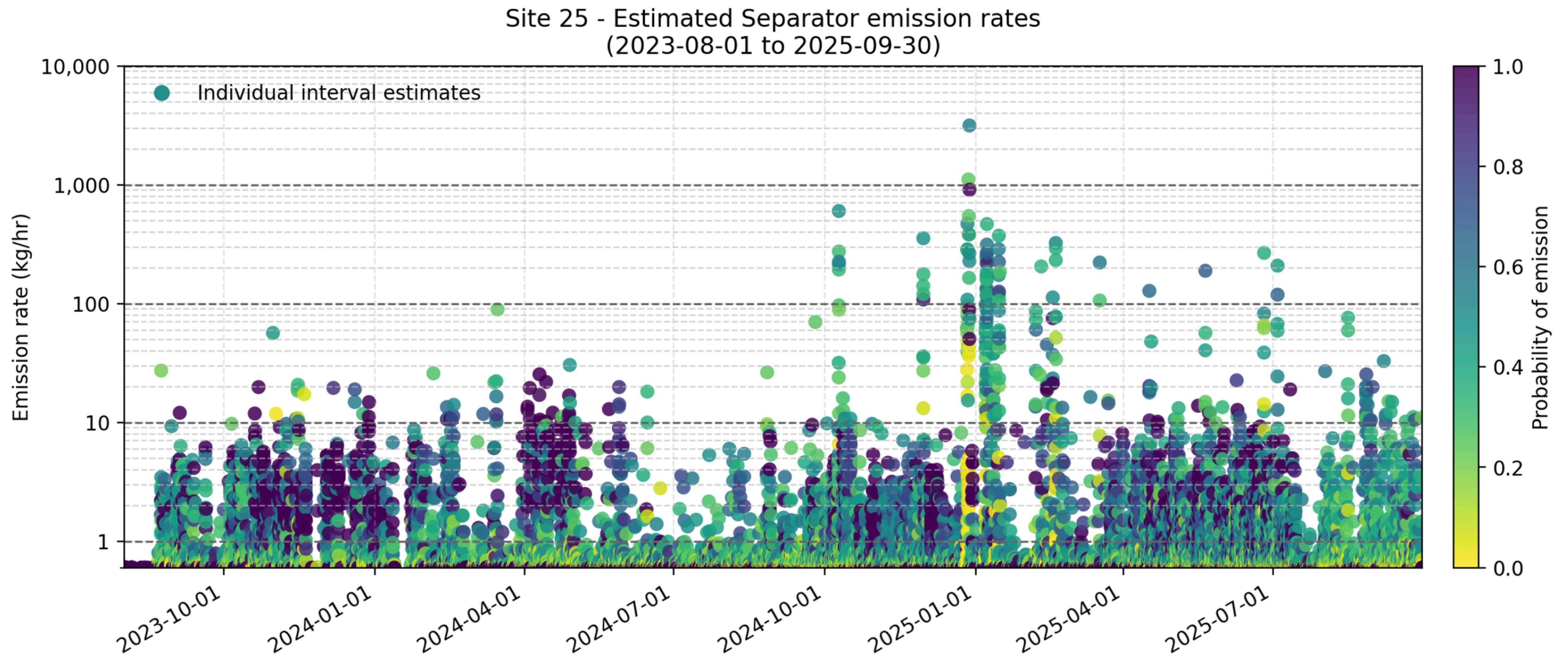
# Sales line emission rate estimates over time



# Sales line emission rate estimates over time

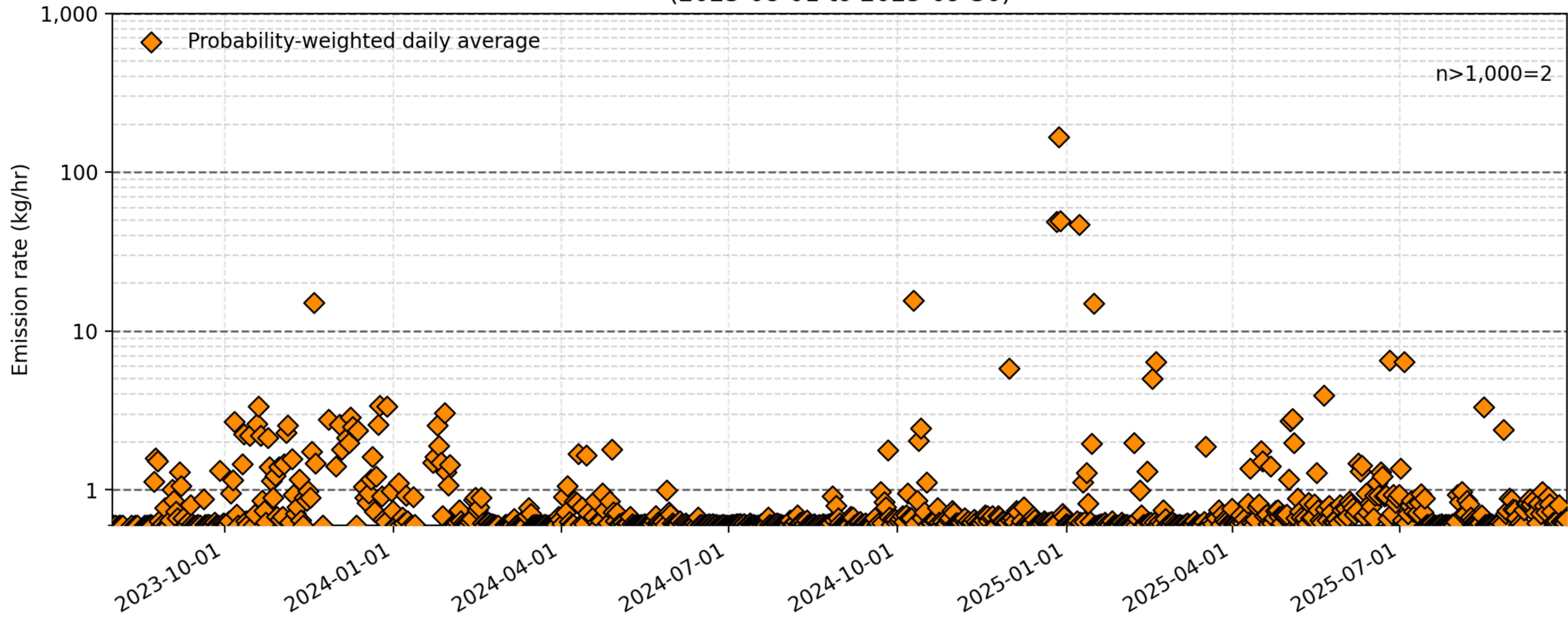


# Separator emission rate estimates over time

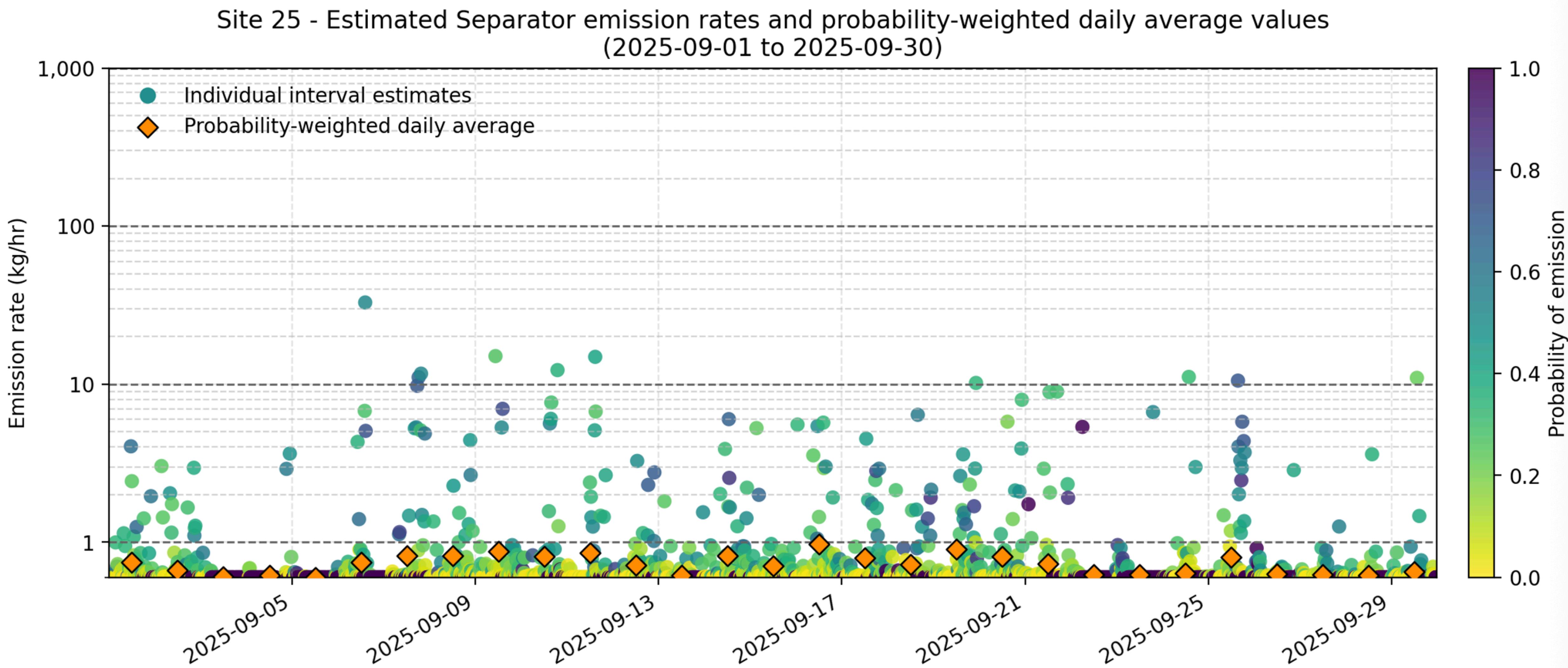


# Separator emission rate estimates over time

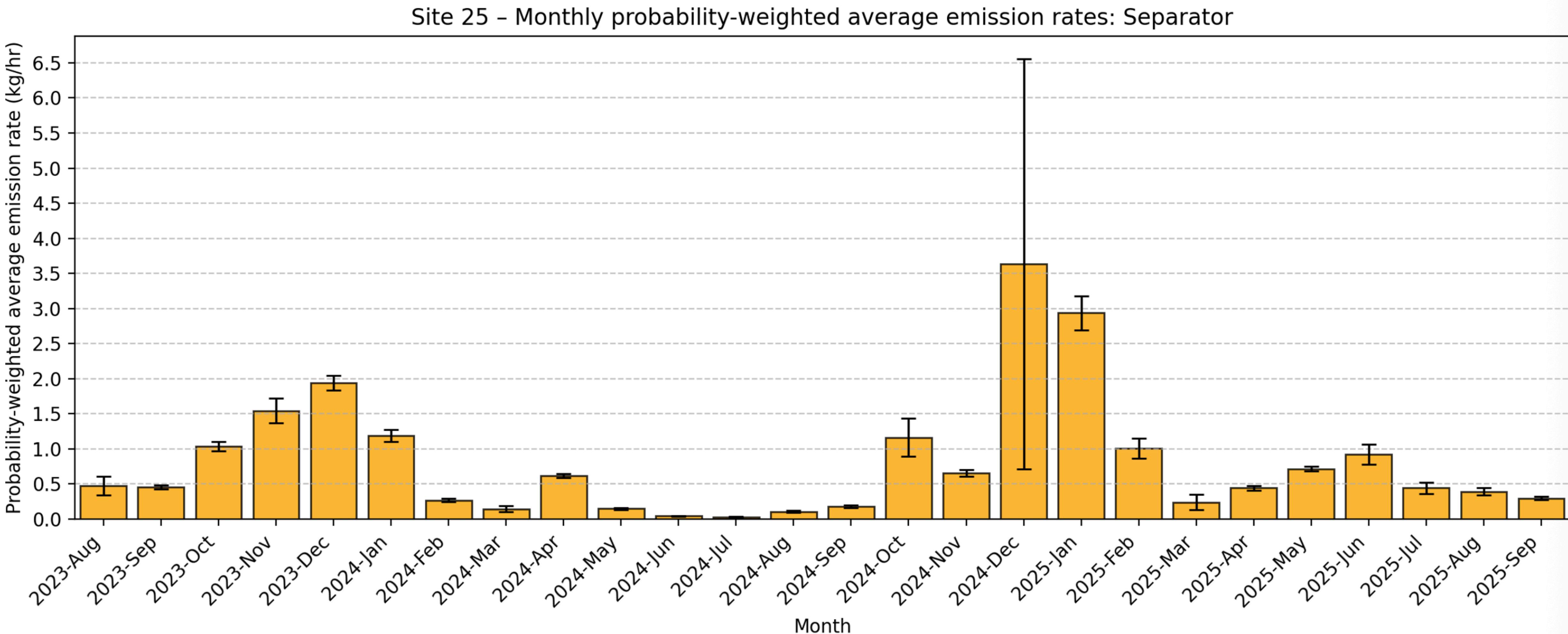
Site 25 - Estimated Separator probability-weighted daily average values  
(2023-08-01 to 2025-09-30)



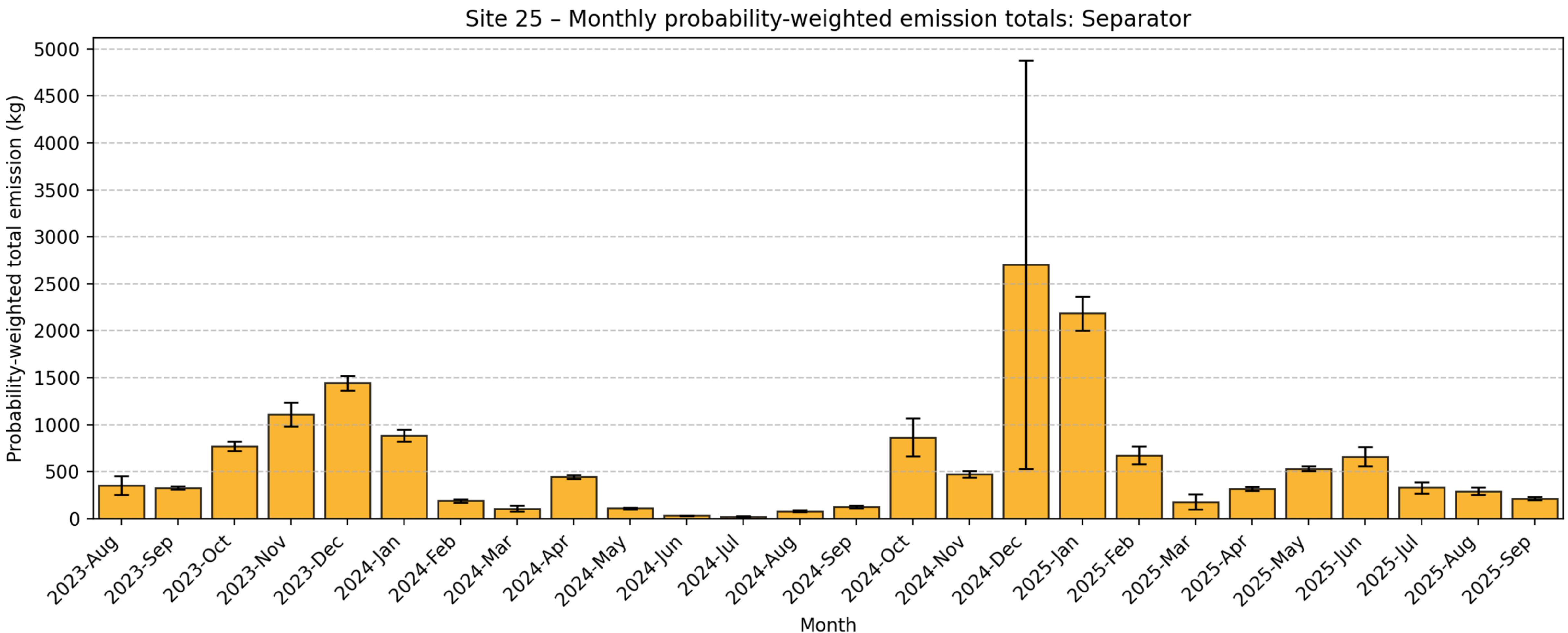
# Separator emission rate estimates over time



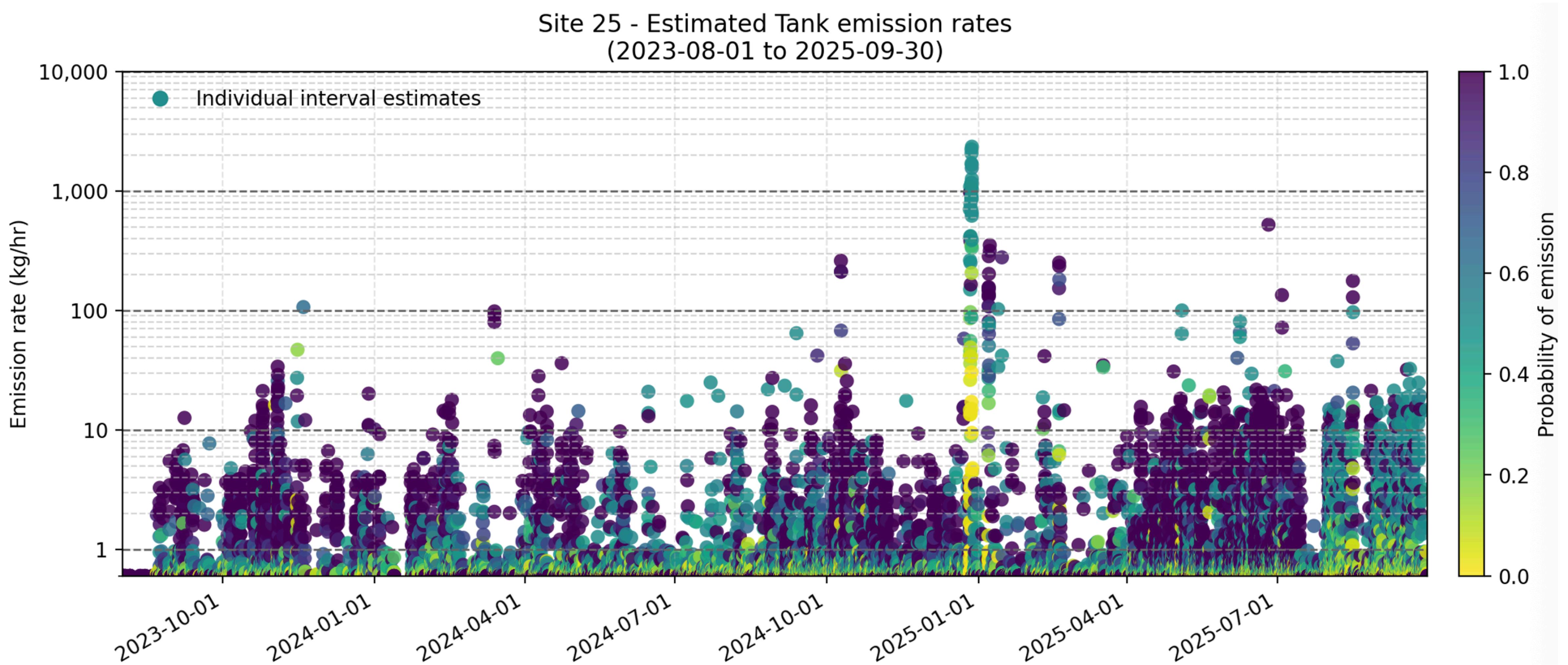
# Separator emission rate estimates over time



# Separator emission rate estimates over time

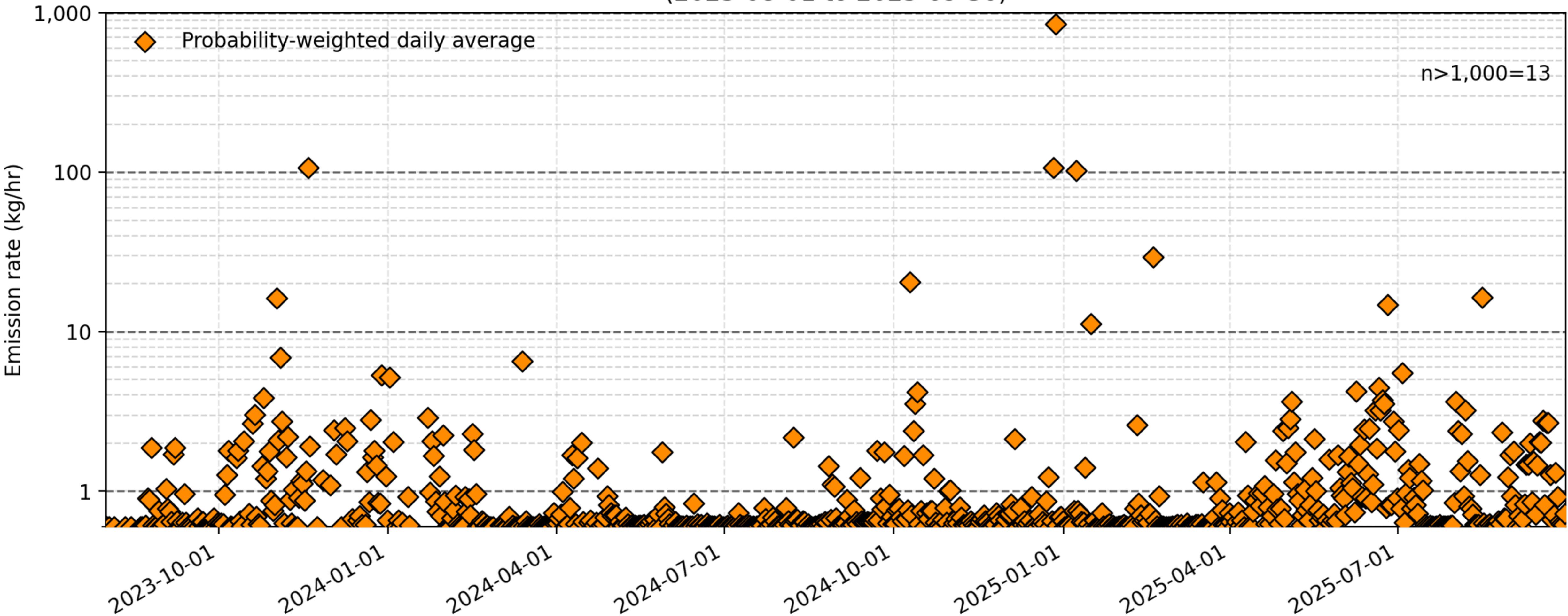


# Tank emission rate estimates over time



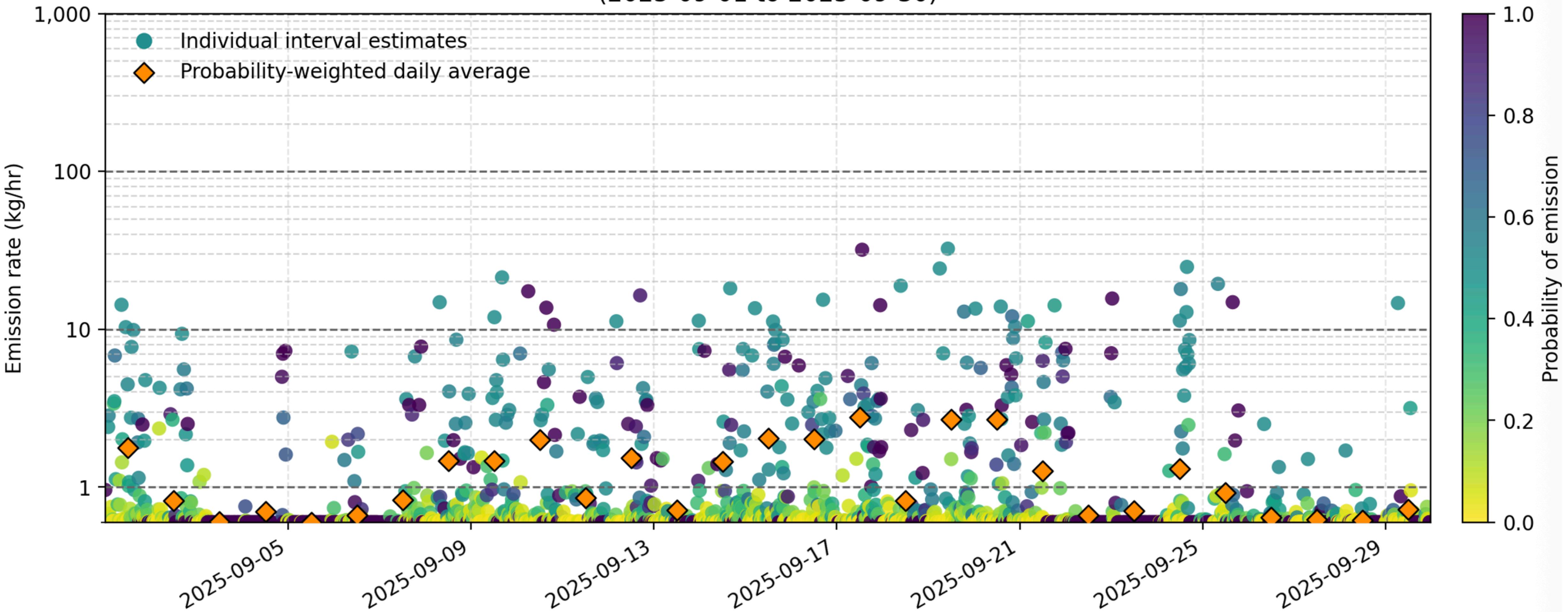
# Tank emission rate estimates over time

Site 25 - Estimated Tank probability-weighted daily average values  
(2023-08-01 to 2025-09-30)

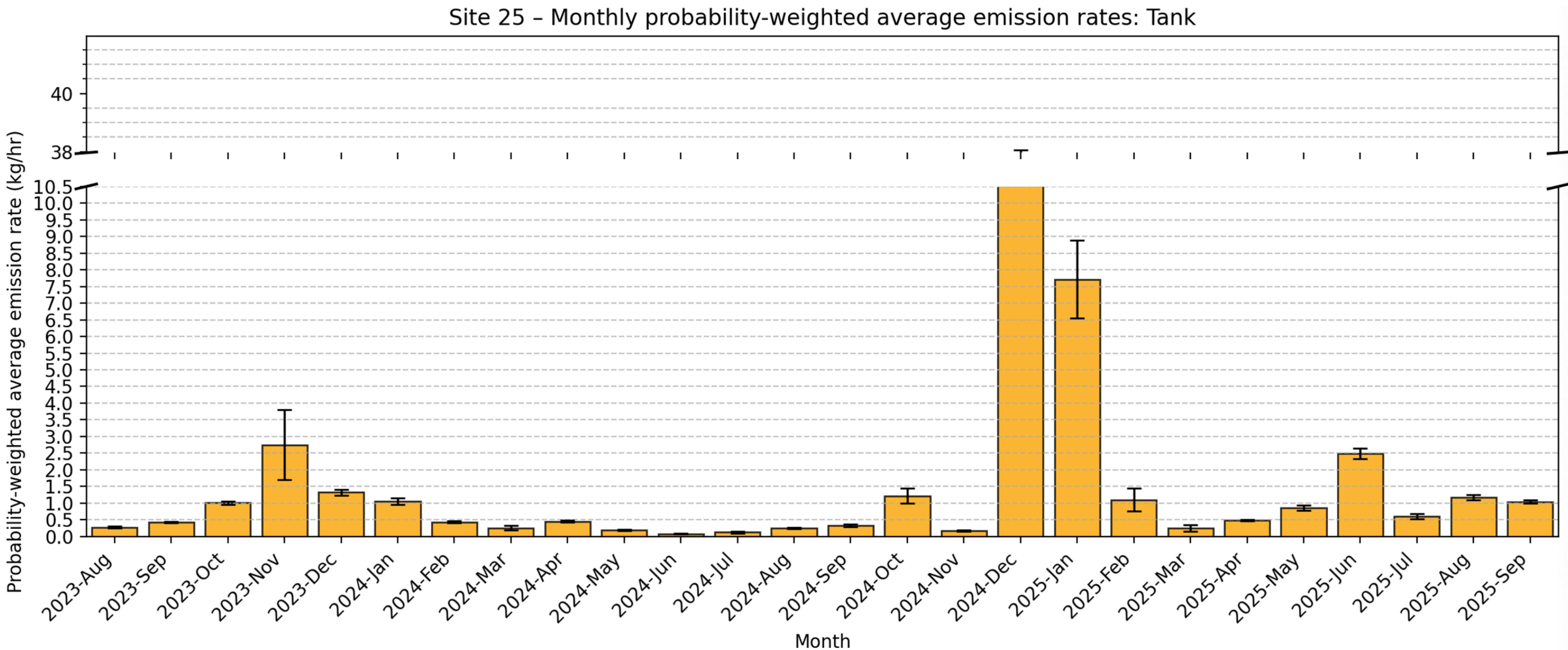


# Tank emission rate estimates over time

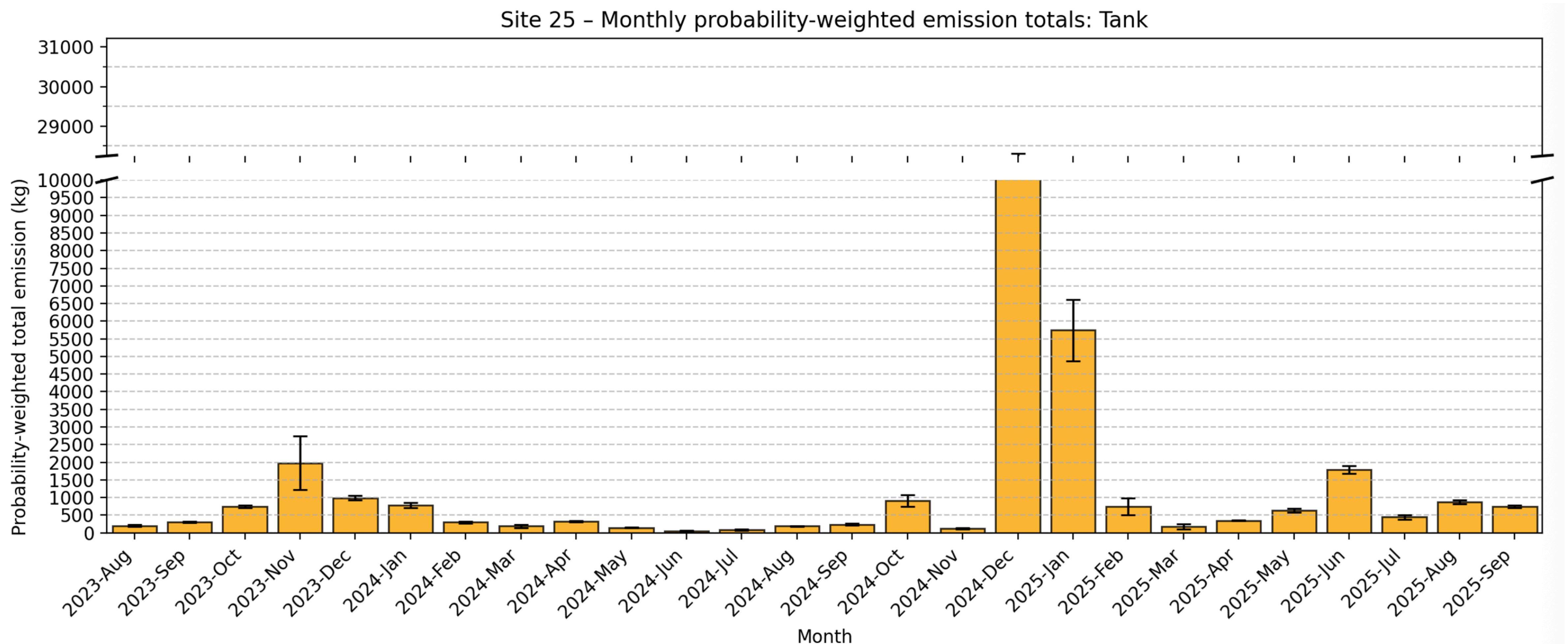
Site 25 - Estimated Tank emission rates and probability-weighted daily average values  
(2025-09-01 to 2025-09-30)



# Tank emission rate estimates over time

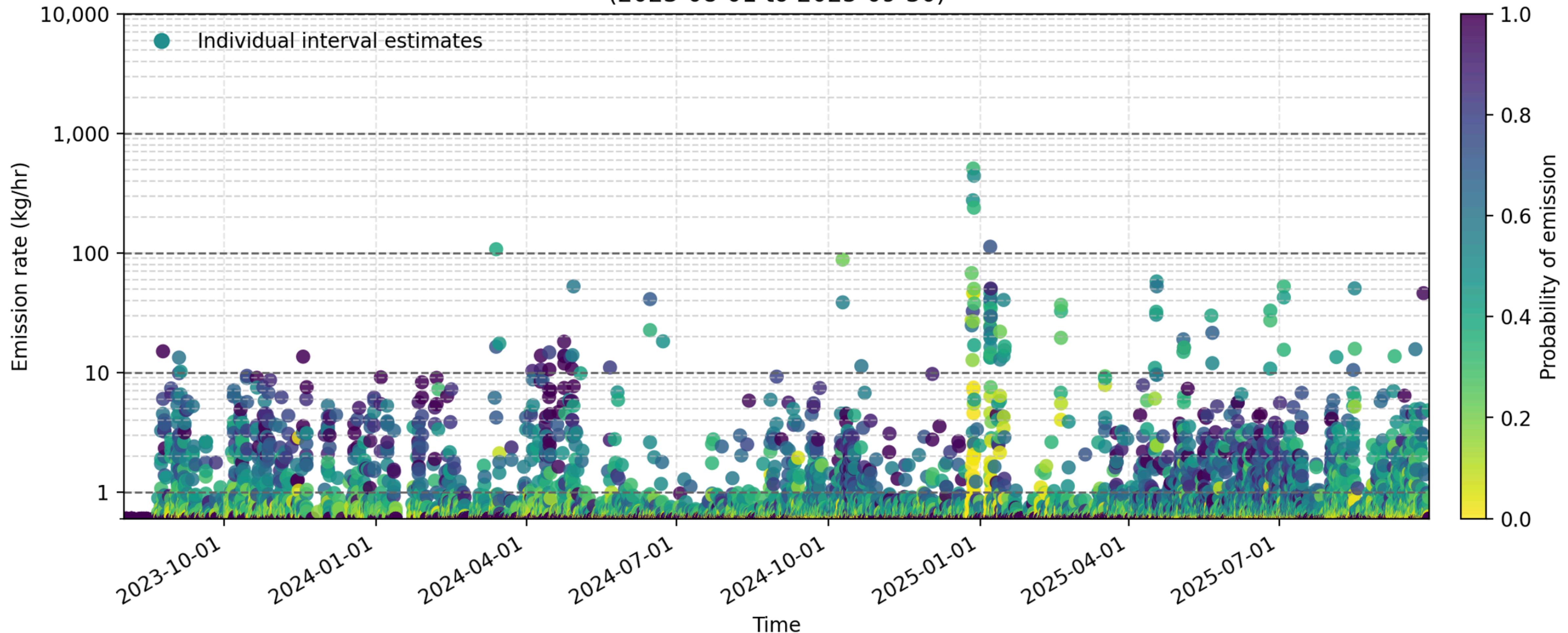


# Tank emission rate estimates over time



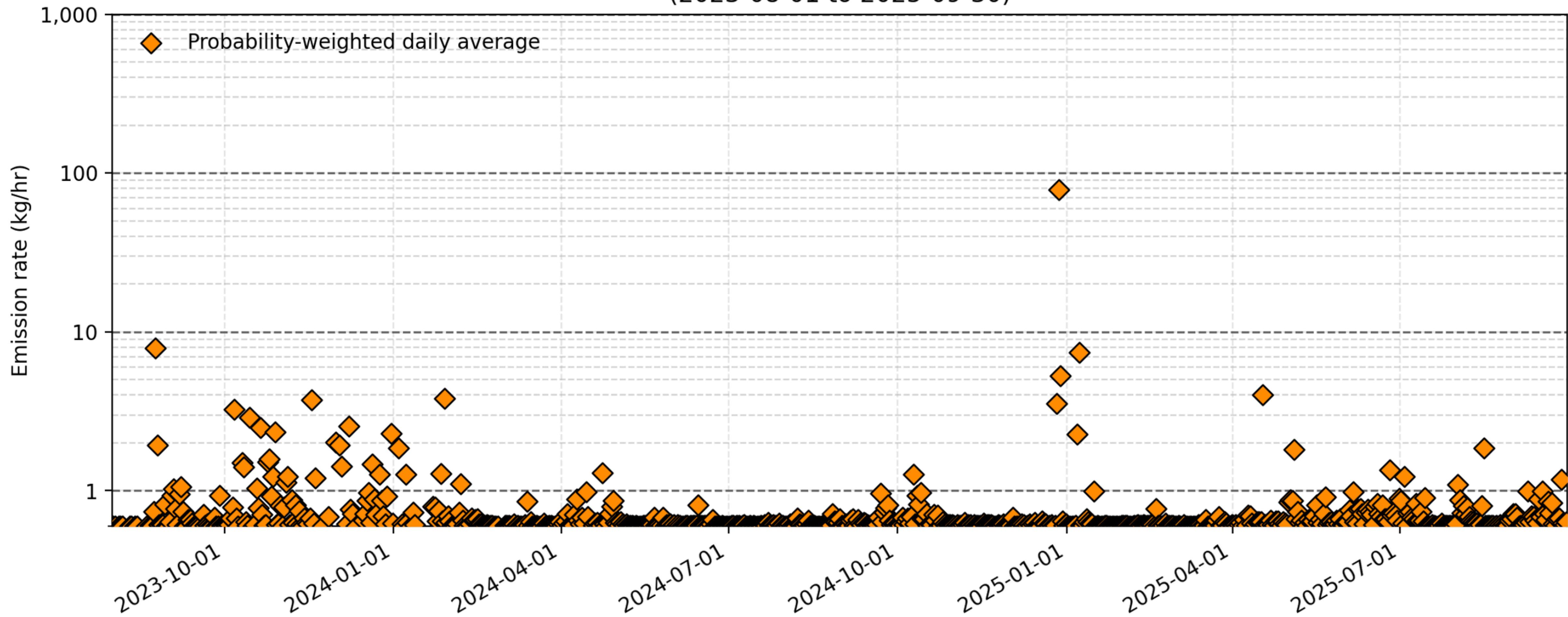
# Wellhead emission rate estimates over time

Site 25 - Estimated Wellhead emission rates  
(2023-08-01 to 2025-09-30)

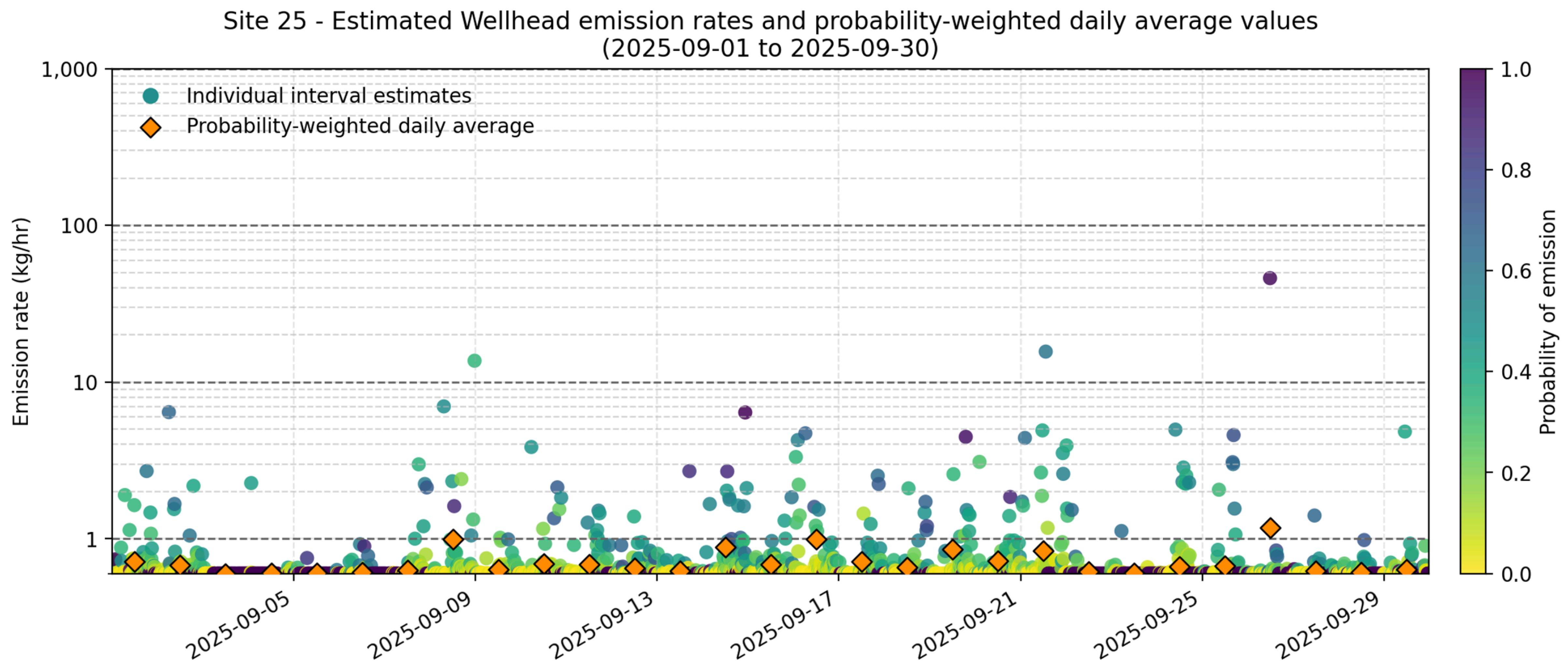


# Wellhead emission rate estimates over time

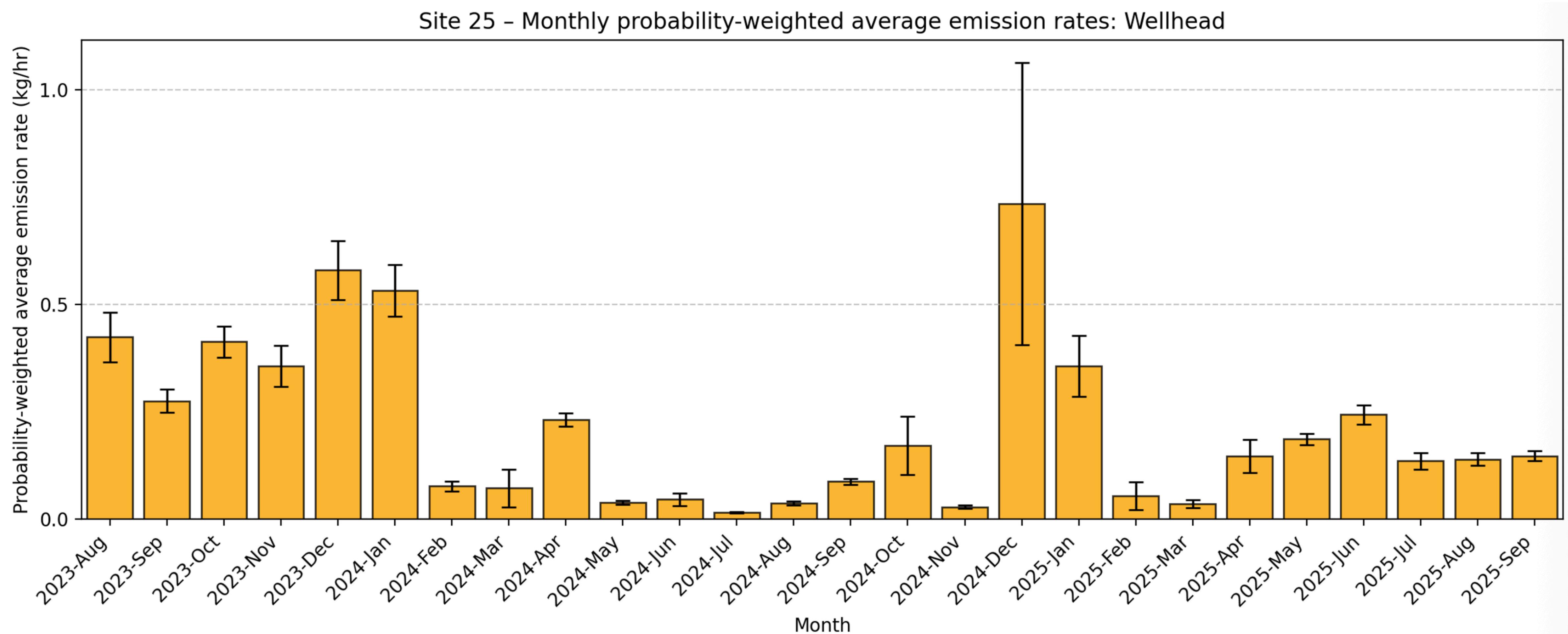
Site 25 - Estimated Wellhead probability-weighted daily average values  
(2023-08-01 to 2025-09-30)



# Wellhead emission rate estimates over time

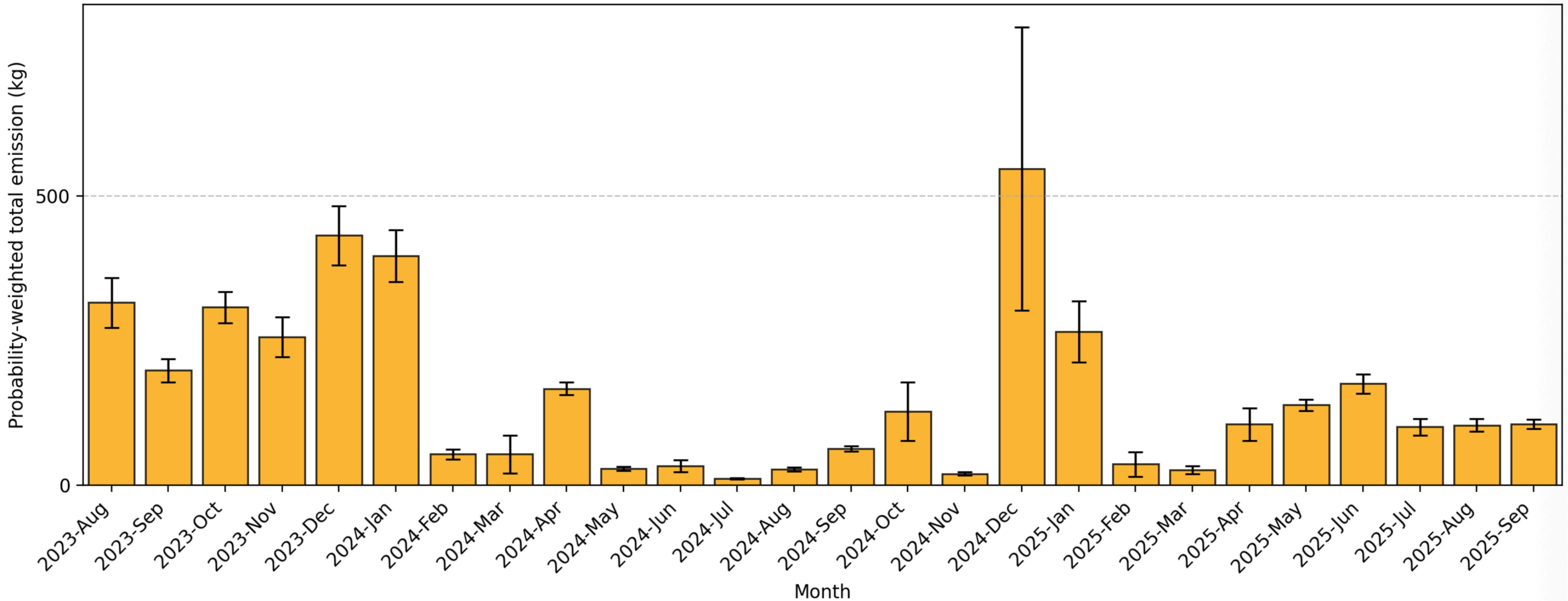


# Wellhead emission rate estimates over time



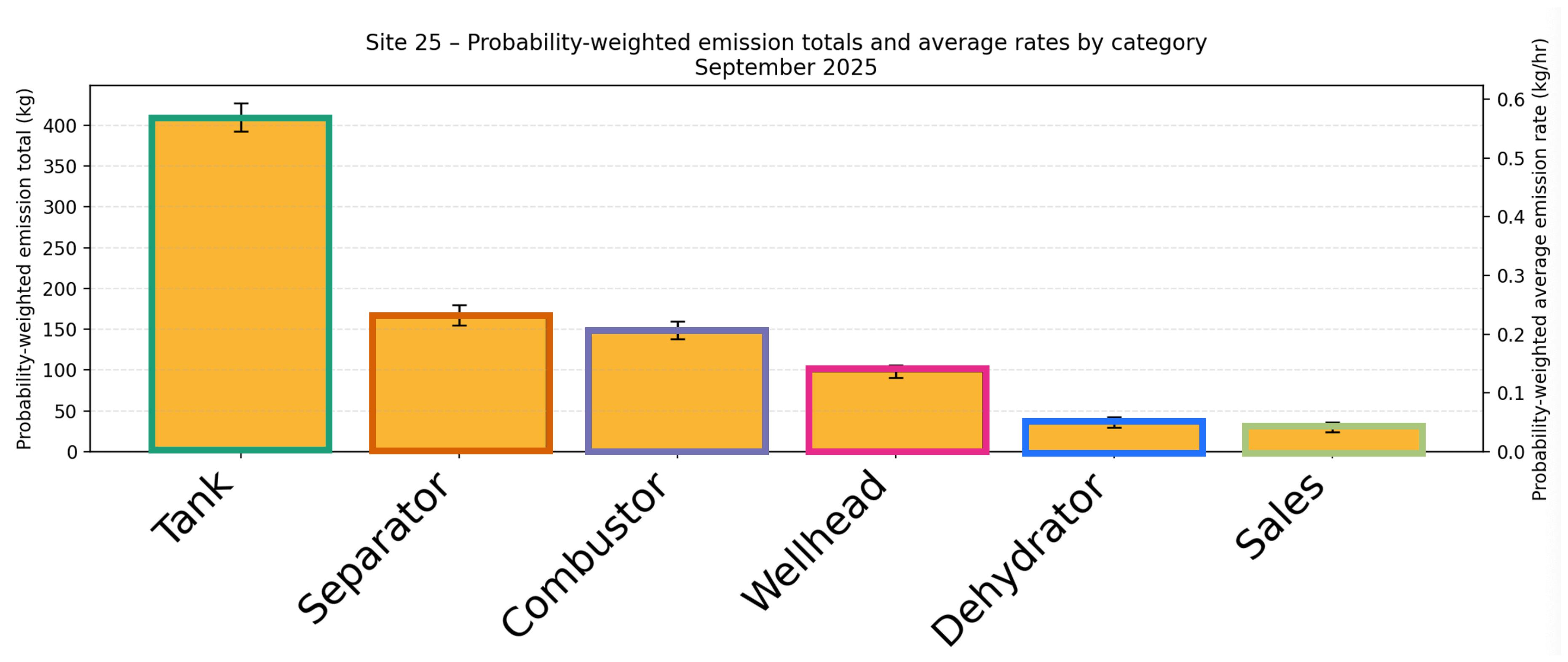
# Wellhead emission rate estimates over time

Site 25 – Monthly probability-weighted emission totals: Wellhead

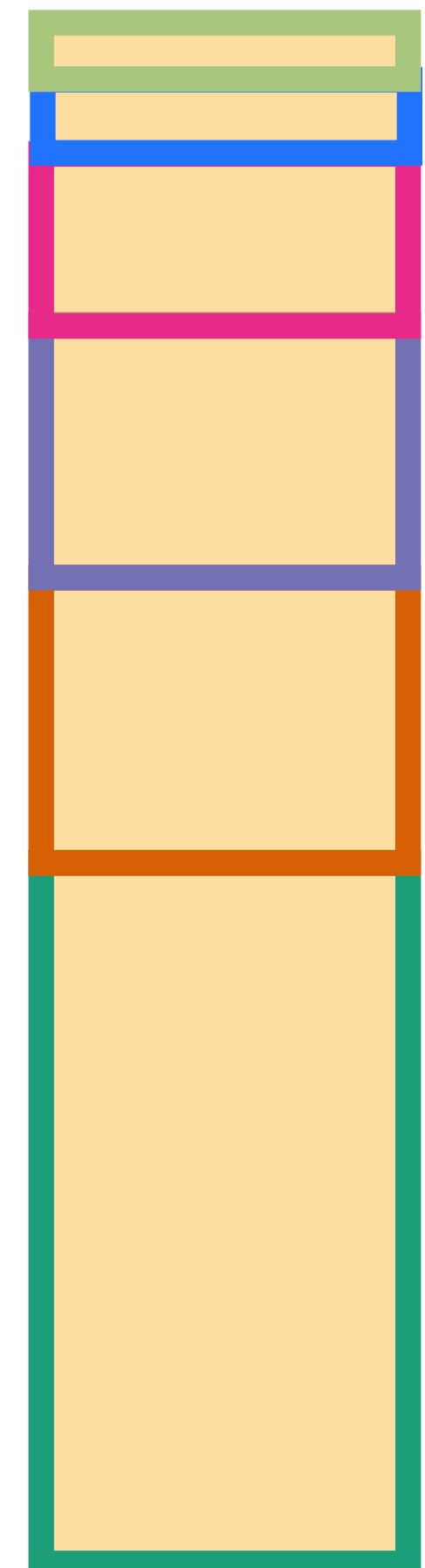


# Site 25: September 2025 inventory

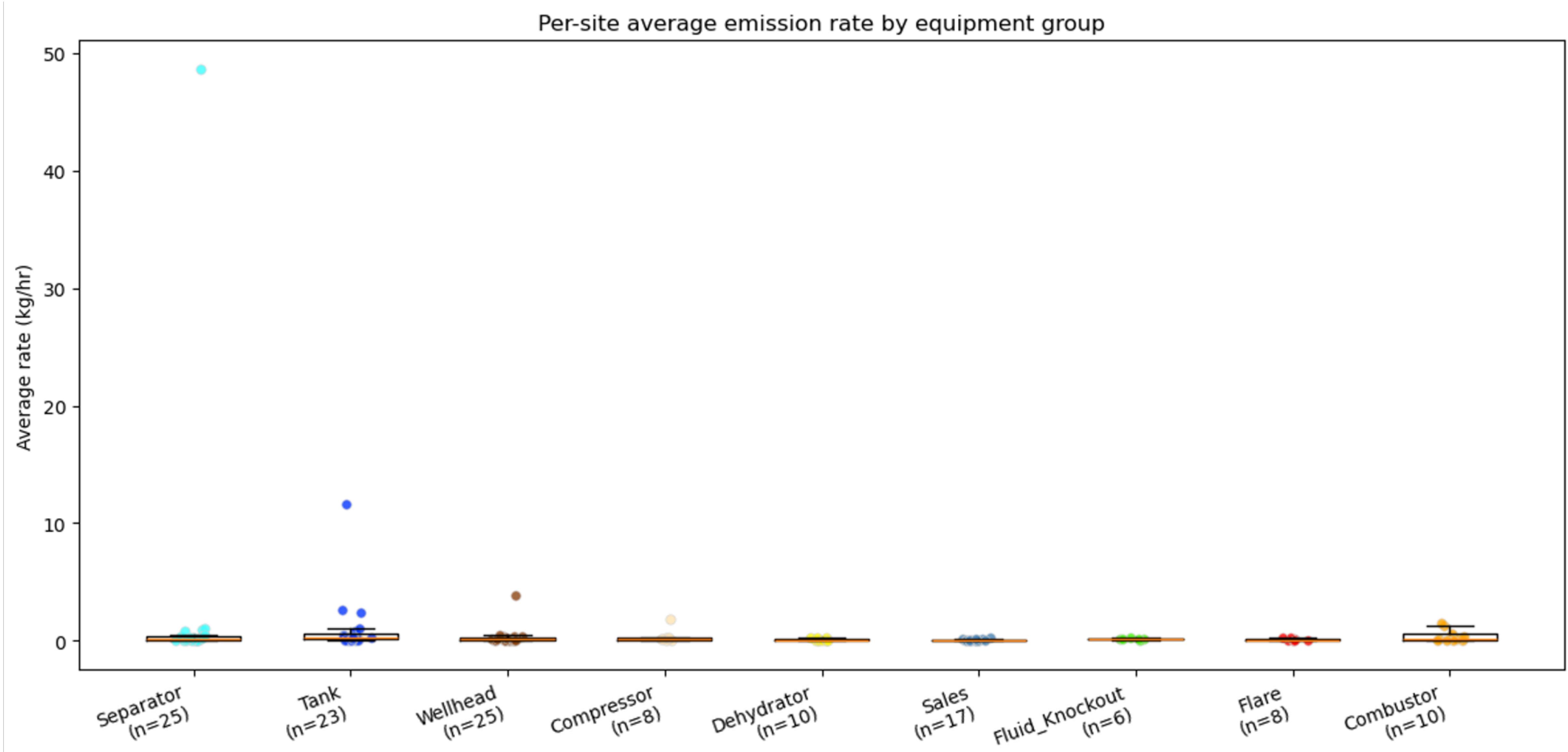
890 [830, 951] kg



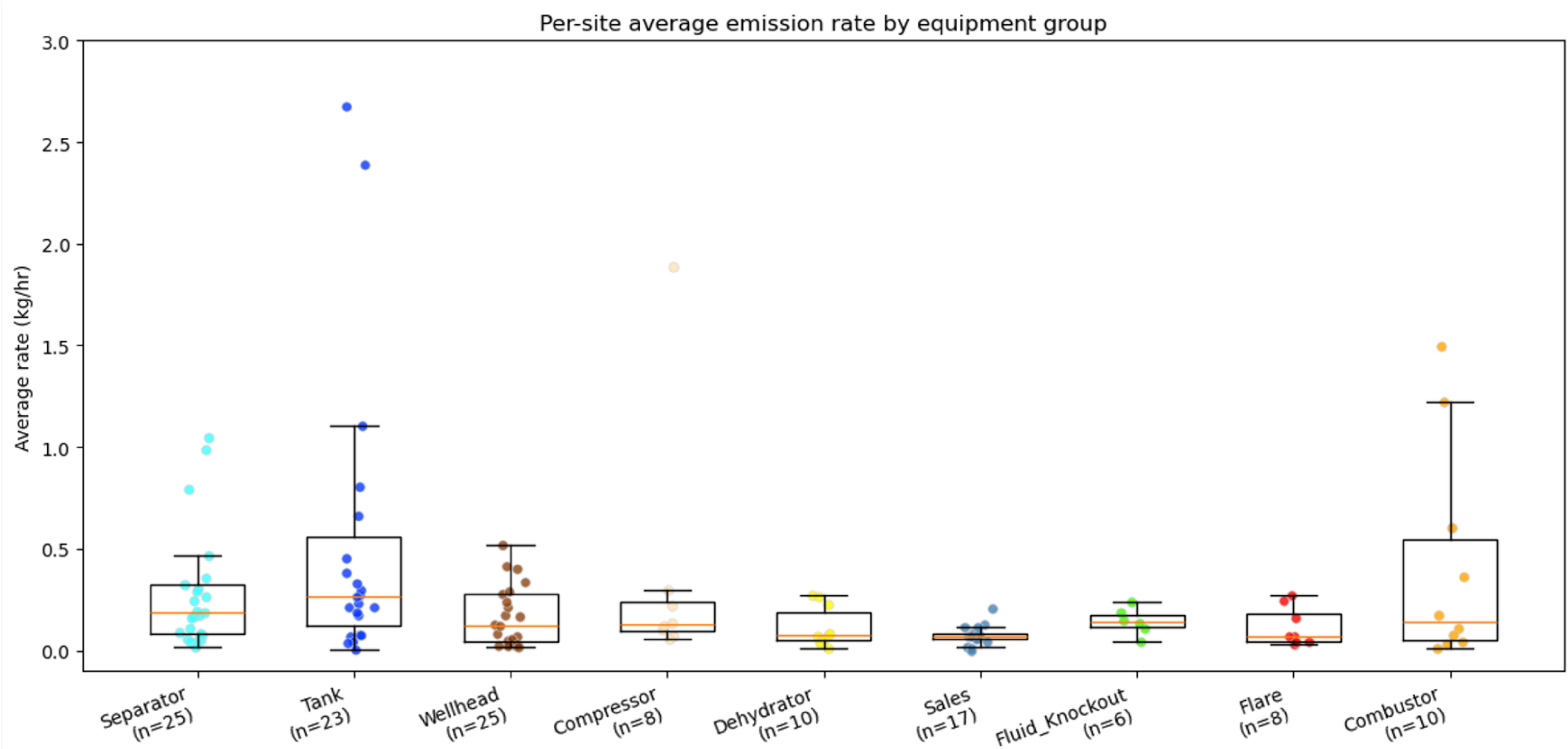
Site-level



# Summary across all 26 sites

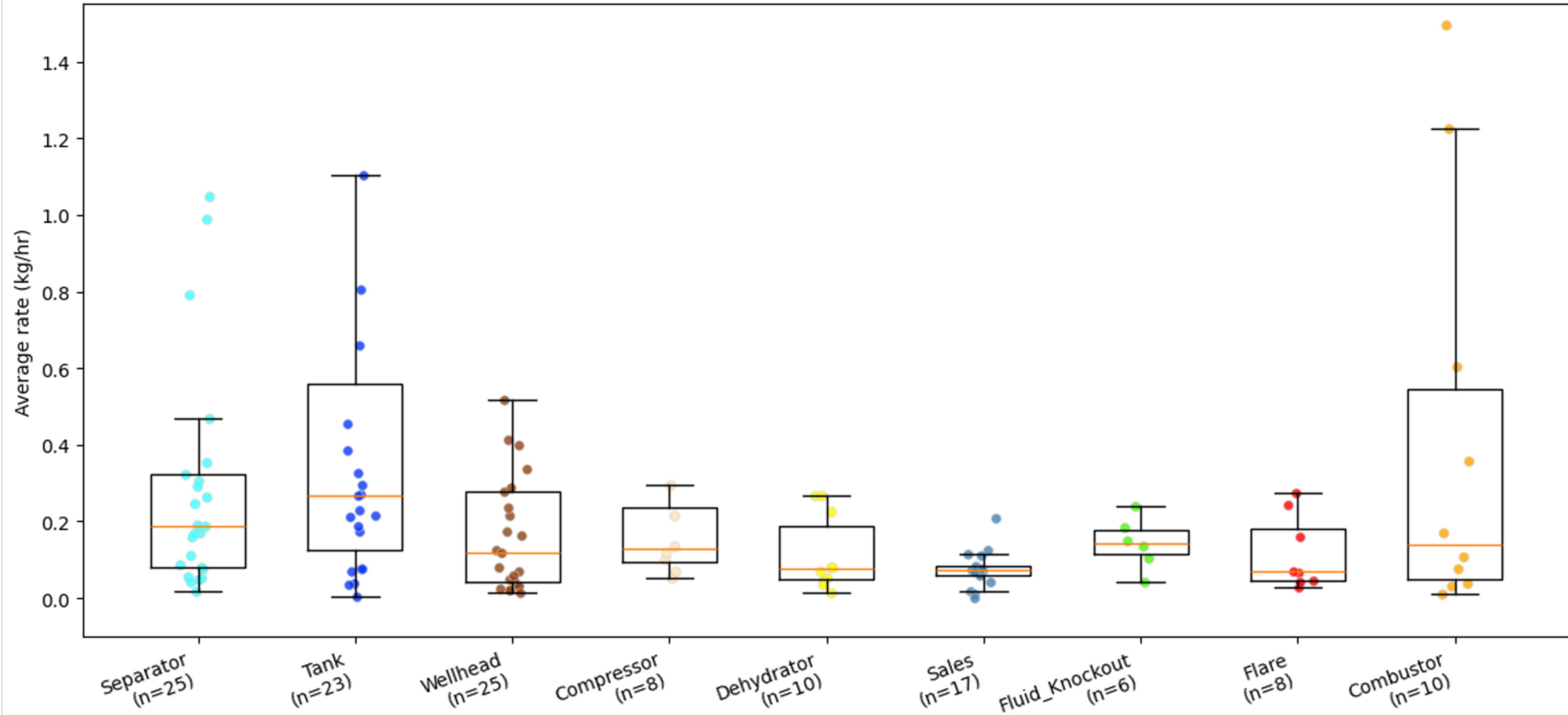


# Summary across all 26 sites



# Summary across all 26 sites

Per-site average emission rate by equipment group



## Concluding thought:

- CMS provide enough measurements to create fully measurement-based inventories at the site-level... IF
  - You account for periods of no information
  - You have an unbiased inverse model (or know how to correct for the bias)
- There's a lot of information in the CMS-based emission rates, and we are just getting started analyzing it

## Next steps:

- Compare to other inventory methods (UT MII, CSU MAES, GHGRP)
  - We have already done this on 5 sites for the COBE project in Colorado
- Use CMS-based emission rate estimates to inform “prototypical sites” or subsets of sites where distributions are similar
  - E.g. conventional wells

# Thank you! Questions?

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COLORADO SCHOOL OF  
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**ENERGY**