CM summary - chapter 2

Terminology

H_n	harmonic number $H_n = \sum_{k=1}^{n} \frac{1}{k} = 1 + \frac{1}{2} + \cdots + \frac{1}{n}$ $H_0 = 0$
Δ	difference operator $\Delta f(x) = f(x+1) - f(x)$
E	shift operator $Ef(x) = f(x+1)$
D	derivative operator
$\sum f(x)\delta x$	indefinite sum
$\sum_a^b f(x)\delta x$	definite sum
$\sum$	antidifference operator $\Delta f(n) = g(n) \iff \sum g(n)\delta n = f(n)$ $\Delta f(n) = g(n) \implies \sum_a^b g(n) = f(b) - f(a)$ $\Delta f(n) = g(n) \implies \sum_a^b g(x)\delta x = -\sum_b^a g(x)\delta x$

Finite calculus analogies with integral calculus

$$\begin{tikzcd}
\Delta f(x) \arrow[r, leftrightarrow] \arrow[d, leftrightarrow] & \sum f(x)\delta x \arrow[d, leftrightarrow] \\
Df(x) \arrow[r, leftrightarrow] & \int f(x)dx
\end{tikzcd}$$

In the above $\delta x = 1$ (reference)

Falling/Rising factorial identities

$$x^{\underline{-m}} = \frac{1}{(x+1)\cdots(x+m)} \quad \text{for } m > 0$$
$$x^{\underline{m}} = \frac{x!}{(x-n)!} = (x-n+1)(x-n+2)\cdots(x-1)x \quad \text{for } m > 0$$
$$x^{\underline{+2}} = x(x-1)$$
$$x^{\underline{+1}} = x$$
$$x^{\underline{0}} = 1$$
$$x^{\underline{-1}} = \frac{1}{x+1}$$
$$x^{\underline{-2}} = \frac{1}{(x+1)(x+2)}$$

Summation by parts

Difference of a product	$\Delta(u \cdot v) = u\Delta v + E\ v\Delta u$
Summation by parts	$\sum u \cdot \Delta v = uv - \sum E\ v\Delta u$ E is applied only to the term immediately after it

Finite difference table

(table 55 in the book)

$\mathbf{f(x)}$	$\mathbf{\Delta f(x)}$	**note**
$x^{\underline{m}}$	$m \cdot x^{\underline{m-1}}$	
H_x	$\frac{1}{x+1} = x^{\underline{-1}}$	
c^x	$(c-1)c^x$	
$\frac{x^{\underline{m+1}}}{m+1}$	$x^{\underline{m}}$	$m \neq -1$
$c \cdot u$	$c \cdot \Delta u$	
$u + v$	$\Delta u + \Delta v$	
$\frac{c^x}{c-1}$	c^x	
2^x	2^x	

Sum/Product properties

sum property	product property
$\sum_{k \in K} ca_k = c \sum_{k \in K} a_k$	$\prod_{k \in K} a_k^c = \left(\prod_{k \in K} a_k\right)^c$
$\sum_{k \in K} a_k + b_k = \sum_{k \in K} a_k + \sum_{k \in K} b_k$	$\prod_{k \in K} a_k b_k = \prod_{k \in K} a_k \cdot \prod_{k \in K} b_k$
$\sum_{\substack{k \in K \\ j \in K}} a_{j,k} = \sum_{k \in K} \sum_{j \in J} a_{j,k}$	$\prod_{\substack{k \in K \\ j \in K}} a_{j,k} = \prod_{k \in K} \prod_{j \in J} a_{j,k}$
$\sum_{k \in K} a_k = \sum_{k \in K} a_{p(k)}$	$\prod_{k \in K} a_k = \prod_{k \in K} a_{p(k)}$
$\sum_{k \in K} a_k = \sum_{k} a_k \cdot [k \in K]$	$\prod_{k \in K} a_k = \prod_{k} a_k^{[k \in K]}$
$\sum_{k \in K} 1 = \|K\|$	$\prod_{k \in K} c = c^{\|K\|}$