## CSE 6331 HOMEWORK 5 (REDUX)

## BRENDAN WHITAKER

1. Consider the first approach to the longest common subsequence problem, where we solved the problem using the forward approach. Now, solve it using the backward approach. Your answer must include: the definition of L(i,j), the definition of  $\varphi(k,j)$ , the recurrence, boundary conditions, and the goal.

We view  $(x_1, x_2, ..., x_k)$ , the sequence of subproblems as a subsequence of A. Using the backward approach, our options for  $x_k$  are  $a_1, ..., a_n$ . Let L(i, j) denote the length of the longest common subsequence of  $A_i = (a_1, ..., a_i)$  and  $B_j = (b_1, ..., b_j)$ . Let  $\varphi(k, j)$  be the index of the last character in  $B_j$  that is equal to  $a_k$ , or 0 if no such character. Our recurrence is:

$$L(i,j) = \begin{cases} 1 + \max_{1 \le k \le i, \varphi(k,j) \ge 1} \left\{ L(k-1,\varphi(k,j)-1) \right\} \\ 0 & \text{if the set for the max is empty} \end{cases} . \tag{1}$$

Boundary condition: L(0,j) = L(i,0) = 0 for all i,j. Running time:  $\Theta(n^3)$ .