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CSE 2221 Homework 17

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STAT 4202 Final Cheat Sheet

A statistic $\hat{\theta}$ is **unbiased estimator** of θ if and only if $E(\hat{\theta}) = \theta$.
$\hat{\theta}$ is an unbias estim of θ and $Var(\hat{\theta}) = \left[nE\left[\left(\frac{\partial \ln f(X)}{\partial \theta}\right)^2\right]\right]^{-1}$ then $\hat{\theta}$ is **min var. unbias estim**.
$\hat{\theta}$ **consistent estimator** if and only if $\lim_{n\to\infty} \mathbb{P}(|\hat{\theta} - \theta| < c) = 1$.
$\hat{\theta}$ unbiased (asymptotically) estimator and $Var(\hat{\theta}) \to 0$ as $n \to \infty$ then $\hat{\theta}$ is **consistent**.
$\hat{\theta}$ consistent but $Var(\hat{\theta})$ doesn't $\to 0$ is
$\hat{\theta}$ is **sufficient** iff the joint probability can be factored to $f(x_i, \theta) = g(\hat{\theta}, \theta)h(x_i)$.
If $U_1 = h(U_2)$ and both sufficient, then its false that $Var(\hat{\theta}_1) \geq Var(\hat{\theta}_2)$.
k^{th} **sample moment** $m'_k = \frac{1}{n}\sum_i x_i^k$.
Method of Moments $\mu'_1 = m'_1,\ \mu'_2 = m'_2 \implies \mu'_1 = E(X) = \bar{x},\ \mu'_2 = E(X^2) = \frac{1}{n}\sum x_i^2$. Solve for $m'_i s$.
MLE $L(\theta) = f(x_i)$, $\partial_\theta \ln L(\theta) = 0$ and solve for θ. Check $\partial_\theta^2 < 0$. Binom MLE $\hat{\theta} = X/n$. Unif is nth order stat. lots are $\bar{X}$. If two variables, check $J > 0$.
Estimation of Mean known σ, then $\bar{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ is $1-\alpha$ confidence estimate for μ.
If from a normal pop., then $\bar{X} \pm t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}$ is $1-\alpha$ confidence.
Difference of Means Known σ_1, σ_2, $(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ is $1-\alpha$ confidence.
If from normal,small samps assume $\sigma_1 = \sigma_2 = \sigma$, then $(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2,n_1+n_2-2}\sqrt{\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$.
Estimation of Props $X \sim Binom$ and n large, $\frac{X}{n} \pm z_{\alpha/2}\sqrt{\frac{\frac{X}{n}(1-\frac{X}{n})}{n}}$ is $1-\alpha$.
Difference of Props $X_1, X_2 Binom$ and n_1, n_2 large, $\frac{x_1}{n_1} - \frac{x_2}{n_2} \pm z_{\alpha/2}\sqrt{\frac{\frac{x_1}{n_1}(1-\frac{x_1}{n_1})}{n_1} + \frac{\frac{x_2}{n_2}(1-\frac{x_2}{n_2})}{n_2}}$ is $1-\alpha$.
Estimation of Variance s^2 from normal then $\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}$ is $1-\alpha$.
Ratio of Var s_1^2, s_2^2 from normal then $\frac{s_1^2}{s_2^2}\frac{1}{f_{\alpha/2,n_1-1,n_2-1}} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{s_1^2}{s_2^2}f_{\alpha/2,n_2-1,n_1-1}$ is $1-\alpha$.
$\alpha =$ **Type I Error** $= \mathbb{P}(\text{reject } H_0 | H_0 \text{ true})$. $\beta =$ **Type II Error** $= \mathbb{P}(\text{accept } H_0 | H_0 \text{ false})$.
Neyman Pearson Lemma Testing $H_0 : \theta = \theta_0$ vs. $H_1 : \theta = \theta_1$, this test provides the most powerful critical region. $L_0 = \prod f(x_i, \theta_0),\ L_1 = \prod f(x_i, \theta_1)$ Reject H_0 if $L_0/L_1 \leq k$ inside crit.
LRT Test $H_0 : \theta = \theta_0$ vs. $H_1 : \theta \neq \theta_0$. max $L_0 = \prod f(x_i, MLE\ \theta_0)$ and max $L_1 = \prod f(x_i, \theta_{MLE})$. $\Lambda =$max $L_0/$ max L_1, $\Lambda \leq k$ crit region. Large n, $-2\ln\Lambda \sim \chi_1^2$.
p-value $p = \mathbb{P}$(as extreme or more extreme observation under H_0). Reject H_0 if $p < \alpha$.
Remember, larger samples make it easier to reject H_0.
Tests for Mean $H_0 : \mu = \mu_0$ vs. $H_1 : \mu \neq \mu_0, \mu > \mu_0, \mu < \mu_0$. Reject H_0 if $|Z| > z_{\alpha/2}, Z > z_\alpha, z < -z_\alpha$ with $Z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$. **t test** If σ unknown and normal pop, use t_{n-1} and s.
Difference of Means Test $H_0 : \mu_1 - \mu_2 = \delta$ vs. $H_1 : \mu_1 - \mu_2 \neq \delta, > \delta, < \delta$. Reject H_0 if $|Z| > z_{\alpha/2}, Z > z_\alpha, Z < -z_\alpha$ with $Z = \frac{\bar{x}_1 - \bar{x}_2 - \delta}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$. Can use $t_{n_1+n_2-2}$ and s^2 as well.
If we have paired data, subtract one from the other for "new" data set and do one-sample test.
Tests for Variance Test $H_0 : \sigma^2 = \sigma_0^2$ vs. $H_1 : \sigma \neq, >, < \sigma_0^2$. Reject H_0 if $\chi^2 > \chi^2_{\alpha/2,n-1}$ or $\chi^2 < \chi^2_{1-\alpha/2,n-1}, \chi^2 > \chi^2_{\alpha,n-1}, \chi^2 < \chi^2_{1-\alpha,n-1}$ where $\chi^2 = (n-1)s^2/\sigma_0^2$.
Two samples Test $H_0 : \sigma_1^2 = \sigma_2^2$ vs $H_1 : \sigma_1^2 \neq, >, < \sigma_2^2$. Reject H_0 if $F > f_{\alpha/2,n_1-1,n_2-1}$ or $F < f_{\alpha/2,n_1-2,n_1-1}, F > f_{\alpha,n_1-1,n_2-1}, F < f_{\alpha/2,n_2-1,n_1-1}$ where $F = s_1^2/s_2^2$.
Test for Props Using Binom under H_0, find p-vlaue. If n is large $Z = \frac{x - n\theta_0}{\sqrt{n\theta_0(1-\theta_0)}}$ is standard normal.

Diff of k props $\chi^2 = \sum_1^k \frac{(x_i - n_i\theta_i)^2}{n_i\theta_i(1-\theta_i)}$ is test stat. Reject H_0 that all props are equal in favor of H_1 that they aren't all equal if $\chi^2 > \chi^2_{\alpha,k}$. Using $\hat{\theta} = \sum x_i / \sum n_i$ and reduce df by 1.

r x c table RIP

	Outcome 1	Outcome 2	Outcome 3	
Sample 1	f_{11}	f_{12}	f_{13}	$f_{1\cdot}$
Sample 2	f_{21}	f_{22}	f_{23}	$f_{2\cdot}$
Sample 3	f_{31}	f_{32}	f_{33}	$f_{3\cdot}$
	$f_{\cdot 1}$	$f_{\cdot 2}$	$f_{\cdot 3}$	f

Test H_0 Samples are independent vs. H_1 not independent. Reject H_0 if $\chi^2 = \sum_1^3 \sum_1^3 \frac{(f_{ij} - e_{ij})^2}{e_{ij}} > \chi^2_{(r-1)(c-1)}$ where $e_{ij} = f_{i\cdot} f_{\cdot j} / f$.

Goodness of Fit Test whether data comes from dist. Reject if $\chi^2 > \sum \frac{(f_i - e_i)^2}{e_i} > \chi^2_{\alpha,m-t-1}$ where f_i observed and e_i expected. Combine columns until expected value > 5 and use $\hat{\theta}$ MLE if not given.

Sign Test Test $\mu = \mu_0$ vs something else. In observed data, values $< \mu_0$ get a $+$ and values $< \mu_0$ get a $-$. Under H_0, $X =$ number of $+$ is $Binom(\theta = .5)$ so use p-value.

Wilcoxon Signed Ranked Test One sample. ASSUME symmetric about μ. Rank differences $x_i - \mu_0$ irregardless of sign. T^+ is sum of ranks for positive diff, T^- similar. T is min of two. Test $H_0 : \mu = \mu_0$ vs. $H_1 : \mu \neq, >, < \mu_0$ and reject H_0 if $T < T_\alpha, T^- < T_{2\alpha}, T^+ < T_{2\alpha}$.

$\mu_{T^+} = \frac{1}{4}(n)(n+1)$ and $Var(T^+) = \frac{1}{24}(n(n+1)(2n+1))$. Can see that $Z = \frac{T^+ - \mu}{\sigma} \approx N(0,1)$.

Ranked Sum Test, U, Wilcoxon Test Two Sample. ASSUME cont. pop. Rank all observations from 1 to $n_1 + n_2$. Compute $W_1 =$ sum of ranks from sample 1, W_2 from sample 2. $U_1 = W_1 - \frac{1}{2}(n_1)(n_1+1)$ and $U_2 = W_2 - \frac{1}{2}(n_2)(n_2+1)$ and U is min. $U_1 + U_2 = n_1 n_2$.

Test $H_0 : \mu_1 = \mu_2$ vs $H_1 : \mu_1 \neq, >, < \mu_2$. Reject H_0 if $U < U_\alpha, U_2 < U_{2\alpha}, U_1 < U_{2\alpha}$.

$\mu_{U_1} = \frac{1}{2} n_1 n_2$ and $Var(U_1) = \frac{1}{12} n_1 n_2 (n_1 + n_2 + 1)$. Then for large n, $Z = \frac{U_1 - \mu}{\sigma} \approx N(0,1)$.

H Test We have k samples. Rank all data. Test H_0 all μ_i equal vs. H_1 not all equal. Reject H_0 if $H > \chi^2_{k-1}$ where $H = \frac{12}{n(n+1)} \sum_1^k \frac{R_i^2}{n_i} - 3(n+1)$ where R_i is the sum of ranks.

$\mu_{Y|X=x} = E(Y|X=x) = \int y \cdot w(y|X=x)\, dy$ where $w(y|X=x)$ is conditional distribution of Y given $X = x$. Integrate out x for marginal then divide $f(x,y)/h(x)$.

If $\mu_{Y|X=x}$ is linear in x then $\mu_{Y|X=x} = \mu_2 + \rho\frac{\sigma_2}{\sigma_1}(x - \mu_1)$ where $\mu_1 = E(X), \mu_2 = E(Y), \sigma_1^2 = Var(X), \sigma_2^2 = Var(Y), \rho = \frac{Cov(X,Y)}{\sigma_1\sigma_2}$.

Least Squares Estimate $\hat{y} = \hat{\alpha} + \hat{\beta}x$ are $\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$ and $\hat{\beta} = S_{xy}/S_{xx}$ were $S_{xy} = \sum x_i y_i - \frac{1}{n}\sum x_i \sum y_i$ and $s_{xx} = \sum x_i^2 - \frac{1}{n}(\sum x_i)^2$.

Normal Regression Analysis $n\hat{\sigma}^2/\sigma^2 \sim \chi^2_{n-2}$. $t = \frac{\hat{\beta} - \beta}{\hat{\sigma}}\sqrt{\frac{(n-2)S_{xx}}{n}} \sim t_{n-2}$ with $\hat{\beta}$ as above. $\hat{\sigma}^2 = \frac{1}{n}(S_{yy} - \hat{\beta}S_{xy})$ and we want to estimate β. $\hat{\beta} \pm t_{\alpha/2,n-2}\hat{\sigma}\sqrt{\frac{n}{(n-2)S_{xx}}}$ is $1 - \alpha$.

Normal Correlation Analysis If X, Y bivariate normal, $\hat{\mu}_1 = \bar{x}, \hat{\mu}_2 = \bar{y}, \hat{\sigma}_1^2 = \frac{1}{n}\sum(x_i - \bar{x})^2, \hat{\rho} = \frac{s_{xy}}{\sqrt{s_{xx}s_{yy}}}$. ρ measures strength of linear relationship. $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$.

$$Z = \frac{\frac{1}{2}\ln\left(\frac{1+R}{1-R}\right) - \frac{1}{2}\ln\left(\frac{1+\rho}{1-\rho}\right)}{\sqrt{\frac{1}{n-3}}} \sim N(0,1)$$

Under H_0, we will have a value of ρ. $r = \hat{\rho}$.