CSE 2321 HOMEWORK 8

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(a)

$$\begin{aligned} t &= \sum_{i=n}^{2n} \sum_{j=10i+5}^{11i+5} c = \sum_{i=n}^{2n} (11i + 5 - (10i+5) + 1)c = \sum_{i=n}^{2n} (i+1)c = \sum_{n+1}^{2n+1} ic \\ &= \sum_{i=1}^{2n+1} ic - \sum_{i=1}^{n} ic. \end{aligned} \tag{1}$$

Now, making use of arithmetic series, we have

$$\begin{aligned} t &= \frac{(2n+1)(2n+2)}{2} c - \frac{n(n+1)}{2} c = (2n+1)(n+1)c - (1/2)n(n+1)c \\ &= c((2n^2 + 3n + 1) - (\frac{1}{2}n^2 + \frac{1}{2}n)) = \boxed{\frac{3}{2}cn^2} + \frac{5}{2}cn + c \end{aligned} \tag{2}$$

(b)

$$t = \sum_{i=1}^{n^2} \sum_{j=i}^{n^2} c = \sum_{i=1}^{n^2} (n^2 - i + 1)c = c\sum_{i=1}^{n^2} n^2 - c\sum_{i=1}^{n^2} i + c\sum_{i=1}^{n^2} 1. \tag{3}$$

Again, making use of arithmetic series, we have

$$t = c(n^2n^2) - c\frac{n^2(n^2+1)}{2} + cn^2 = cn^4 - \frac{c}{2}(n^4 + n^2) + cn^2 = \boxed{\frac{1}{2}cn^4} + \frac{1}{2}cn^2. \tag{4}$$

(c)

$$t = \sum_{i=4n}^{6n^3} \sum_{j=4}^{i} \sum_{k=j}^{i} c = \sum_{i=4n}^{6n^3} \sum_{j=4}^{i} (i - j + 1)c = c\sum_{i=4n}^{6n^3} \left(\sum_{j=4}^{i} i - \sum_{j=4}^{i} j + \sum_{j=4}^{i} 1 \right). \tag{5}$$

And by arithmetic series we have

$$\begin{aligned} t &= c\sum_{i=4n}^{6n^3} \left((i-3)i - \left(\sum_{j=1}^{i} j - \sum_{j=1}^{3} j \right) + (i-3) \right) \\ &= c\sum_{i=4n}^{6n^3} \left((i-3)i - \left(\frac{i(i+1)}{2} - 6 \right) + (i-3) \right) \\ &= c\sum_{i=4n}^{6n^3} \left(i^2 - 2i - 3 - \frac{1}{2}i^2 - \frac{1}{2}i + 6 \right) = c\sum_{i=4n}^{6n^3} \left(\frac{1}{2}i^2 - \frac{5}{2}i + 3 \right). \end{aligned} \tag{6}$$

We find an upper bound by plugging in $6n^3$ for i

$$\begin{aligned} t &\le c\sum_{i=4n}^{6n^3} \left(\frac{1}{2}(6n^3)^2 - \frac{5}{2}(6n^3) + 3 \right) \le c\sum_{i=4n}^{6n^3} \left(18n^6 + 3 \right) = (6n^3 - 4n + 1)(18n^6 + 3) \\ &\le (6n^3 + 1)(18n^6 + 3) = \boxed{108n^9} + 18n^3 + 18n^6 + 3. \end{aligned} \tag{7}$$

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Similarly, we find a lower bound by splitting the summation

$$\begin{aligned} t &= c\sum_{i=4n}^{n^3-1}\left(\frac{1}{2}i^2-\frac{5}{2}i+3\right)+c\sum_{i=n^3}^{6n^3}\left(\frac{1}{2}i^2-\frac{5}{2}i+3\right) \\ &\geq c\sum_{i=n^3}^{6n^3}\left(\frac{1}{2}i^2-\frac{5}{2}i+3\right)\geq c\sum_{i=n^3}^{6n^3}\left(\frac{1}{2}i^2-\frac{5}{2}i\right). \end{aligned} \tag{8}$$

Plugging in n^3 for i we get

$$\begin{aligned} t &\geq c\sum_{i=n^3}^{6n^3}\left(\frac{1}{2}(n^3)^2-\frac{5}{2}n^3\right)=c(6n^3-n^3+1)\left(\frac{1}{2}n^6-\frac{5}{2}n^3\right) \\ &\geq c(5n^3)\left(\frac{1}{2}n^6-\frac{5}{2}n^3\right)=\boxed{\frac{5}{2}cn^9}-\frac{25}{2}cn^6. \end{aligned} \tag{9}$$

Thus $\boxed{t=c'n^9}$ where $\frac{5}{2}c\leq c'\leq 108$.

(d)

$$\begin{aligned} t &= \sum_{i=\lfloor n/5\rfloor}^{n}\sum_{j=i}^{n}c=\sum_{i=\lfloor n/5\rfloor}^{n}(n-i+1)c \\ &= c\sum_{i=\lfloor n/5\rfloor}^{n}n+c\sum_{i=\lfloor n/5\rfloor}^{n}1-c\sum_{i=\lfloor n/5\rfloor}^{n}i \\ &= c(n-\frac{n}{5}+1)n+c(n-\frac{n}{5}+1)-c\sum_{i=\lfloor n/5\rfloor}^{n}i \\ &= c(\frac{4}{5}n+1)n+c(\frac{4}{5}n+1)-c\sum_{i=\lfloor n/5\rfloor}^{n}i \\ &= c\left(\frac{4}{5}n^2+\frac{9}{5}n+1\right)-c\sum_{i=\lfloor n/5\rfloor}^{n}i \\ &= c\left(\frac{4}{5}n^2+\frac{9}{5}n+1\right)-c\left(\sum_{i=1}^{n}i-\sum_{i=1}^{\lfloor n/5\rfloor-1}i\right). \end{aligned} \tag{10}$$

Again making use of arithmetic series

$$\begin{aligned} t &= c\left(\frac{4}{5}n^2+\frac{9}{5}n+1\right)-c\left(\frac{n(n+1)}{2}-\frac{(n/5-1)(n/5)}{2}\right) \\ &= c\left(\frac{4}{5}n^2+\frac{9}{5}n+1\right)-c\left(\frac{12}{25}n^2+\frac{3}{5}n\right) \\ &= \boxed{\frac{8}{25}cn^2}+\frac{6}{5}cn+c. \end{aligned} \tag{11}$$