CSE 2321 NOTES OCT 30

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2. (a)

$$t = \sum_{i=1}^{n} \sum_{j=1}^{\sqrt{i}} \sum_{k=2}^{\lfloor nlog_2 n \rfloor} c = c \sum_{i=1}^{n} \sum_{j=1}^{\sqrt{i}} (nlog_2 n - 1) = c \sum_{i=1}^{n} \sqrt{i}(nlog_2 n - 1). \tag{1}$$

We find the upper bound, plugging in n for i

$$t \leq c \sum_{i=1}^{n} \sqrt{n}(nlog_2 n - 1) \leq c \sum_{i=1}^{n} \sqrt{n}(nlog_2 n) = cn\sqrt{n}(nlog_2 n) = \boxed{cn^{5/2} log_2 n}.$$

Then, we split the summation for the lower bound

$$t = c \sum_{i=1}^{\frac{n}{2}-1} \sqrt{i}(nlog_2 n - 1) + c \sum_{i=\frac{n}{2}}^{n} \sqrt{i}(nlog_2 n - 1) \geq c \sum_{i=\frac{n}{2}}^{n} \sqrt{i}(nlog_2 n - 1).$$

And plugging in $\frac{n}{2}$ for i, we get

$$\begin{aligned} t &\geq c \sum_{i=\frac{n}{2}}^{n} \sqrt{\frac{n}{2}}(nlog_2 n - 1) = c(n/2+1)\sqrt{\frac{n}{2}}(nlog_2 n - 1) \geq c(n/2)\sqrt{\frac{n}{2}}(nlog_2 n - 1) \\ &= \boxed{c\frac{n^{5/2}}{2\sqrt{2}} log_2 n} - c\frac{n^{3/2}}{2\sqrt{2}} \end{aligned} \tag{2}$$

Now since $\frac{1}{2\sqrt{2}} < 1$, we have $c\frac{n^{5/2}}{2\sqrt{2}} log_2 n \leq t \leq cn^{5/2} log_2 n$.

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