CSE 2331 HOMEWORK 1

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1. Write the asymptotic time complexity of the given functions.
   (a) $\Theta(6^n)$.
   (b) $\Theta(n^{0.3})$.
   (c) $\Theta(log_4(n))$.
   (d) $\Theta(n^{1.1})$.
   (e) $\Theta(7^{2n})$.
   (f) $\Theta(n^{0.5})$.
   (g) $\Theta(n)$.
   (h) $\Theta(n)$.
   (i) $\Theta(n^{0.5})$.
   (j) $\Theta(n^{0.5}log_2(n))$.
   (k) $\Theta(n^{0.6})$.
   (l) $\Theta(n^6)$.
   (m) $\Theta(1)$.
   (n) $\Theta(n^{1.5})$.
   (o) $\Theta(n)$.
   (p) $\Theta((log_5(n))^3)$.
   (q) $\Theta(log_3(n))$.
   (r) $\Theta(5^n)$.
   (s) $\Theta(((log_2(n))^2)$.
   (t) $\Theta(nlog_7(n))$.
   (u) $\Theta(n^2)$.
   (v) $\Theta(8^n)$.
   (w) $\Theta(log_5(n))$.
   (x) $\Theta(5^{2n})$.
   (y) $\Theta(log_5(n))$.

2. Let $f(n) = n^2(log_2(n))^2$. Then $f(n) \in O(n^3/log_2(n))$, since $n^2(log_2(n))^2 = \frac{n^3(log_2(n))^2}{nlog_2(n)}$. So we have

$$f(n) = \frac{n^3}{log_2(n)} \cdot \frac{(log_2(n))^2}{n}, \tag{1}$$

and since $(log_2(n))^2 \in O(n)$, we know $\frac{(log_2(n))^2}{n} \in O(1)$. Thus $f(n) = \frac{n^3}{log_2(n)}O(1) \in O(\frac{n^3}{log_2(n)})$. Also, we have $f(n) \in \Omega(n^2log_2(n))$, since $log_2(n) \in \Omega(1)$. Now $f(n) \notin \Theta(\frac{n^3}{log_2(n)})$, since $\frac{(log_2(n))^2}{n} \notin \Theta(1)$, and $f(n) \notin \Theta(n^2log_2(n))$ since $\log_2(n) \notin \Theta(1)$. Hence $f(n)$ is a function with the desired properties.

3. Let $f(n) = n^{0.55}$.

4. *Prove that $3\sqrt{2n^5 - 2n^3 + 23} \in \Theta(n^{2.5})$ using the definition of $\Theta(n^{2.5})$ as functions $f(n)$ such that $c_1n^{2.5} \leq f(n) \leq c_2n^{2.5}$ for constants $c_1, c_2 > 0$ for all large n.*

   Proof. Note

$$3\sqrt{2n^5 - 2n^3 + 23} \leq 3\sqrt{2n^5 - 2n^5 + 23n^5} = 3\sqrt{23n^5} = 3\sqrt{23}n^{2.5}. \tag{2}$$

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And also

$$3\sqrt{2n^5 - 2n^3 + 23} \geq 3\sqrt{2n^5} = 3\sqrt{2}n^{2.5}. \tag{3}$$

So we have

$$3\sqrt{2}n^{2.5} \leq 3\sqrt{2n^5 - 2n^3 + 23} \leq 3\sqrt{23}n^{2.5}, \tag{4}$$

where $3\sqrt{2} < 3\sqrt{23}$, so we must have that $3\sqrt{2n^5 - 2n^3 + 23} \in \Theta(n^{2.5})$. □

5. Observe:

$$\begin{aligned}
\lim_{n\to\infty} \frac{7\sqrt{7n^2+8n}(\log_4(3n+2))^3}{6n\log_5(6n^3+n^2)\cdot\log_9(6n+13)} &= \lim_{n\to\infty} \frac{7\sqrt{7n^2+8n}(k_1\log_2(3n+2))^3}{6nk_2\log_2(6n^3+n^2)\cdot k_3\log_2(6n+13)} \\
&= \lim_{n\to\infty} \frac{7\sqrt{7n^2}(k_1\log_2(3n))^3}{6nk_2\log_2(6n^3)\cdot k_3\log_2(6n)} \\
&= \lim_{n\to\infty} \frac{7\sqrt{7n^2}(k_1(\log_2(n)+k_4))^3}{6nk_2(\log_2(n^3)+k_5)\cdot k_3(\log_2(n)+k_6)} \\
&= \lim_{n\to\infty} \frac{7\sqrt{7}n(k_1(\log_2(n)))^3}{18nk_2k_3(\log_2(n))^2} \\
&= \lim_{n\to\infty} \frac{7\sqrt{7}nk_7(\log_2(n))^3}{18nk_2k_3(\log_2(n))^2} \\
&= \lim_{n\to\infty} \frac{7\sqrt{7}k_7\log_2(n)}{18k_2k_3} \\
&= \lim_{n\to\infty} k_8\log_2(n) = \infty.
\end{aligned} \tag{5}$$

Thus $f(n) \in \Omega(g(n))$.

6. *Prove that if $f(n) \in O(g(n))$, and $f(n) \in O(h(n))$, then $f(n) \in O(g(n)+h(n))$, where $f, g, h : \mathbb{N} \to \mathbb{R}^{\geq 0}$.*

Proof. Since $f(n) \in O(g(n))$ we know that $\exists k \in \mathbb{R}^+$, and $N_k \in \mathbb{N}$ s.t. $f(n) \leq kg(n)$ $\forall n \in \mathbb{N}$ s.t. $n \geq N_k$. Similarly, since $f(n) \in O(h(n))$ we know that $\exists l \in \mathbb{R}^+$ and $N_l \in \mathbb{N}$ s.t. $f(n) \leq lh(n)$ $\forall n \in \mathbb{N}$ s.t. $n \geq N_l$. So let $m = kl$, and let $N_m = \max(N_k, N_l)$. Then $m(g(n)+h(n)) \geq kg(n)$ and $m(g(n)+h(n)) \geq lh(n)$, and $N_m \geq N_k, N_l$, thus $f(n) \leq g(n)+h(n)$ $\forall n \in \mathbb{N}$ s.t. $n \geq N_m$. □