CSE 2321 HOMEWORK 9

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1. (a) We find the bounds of the summation for the first while loop.

$$\begin{array}{c|c} \text{iterations} & i = n \\ \hline 1 & 6n \\ 2 & 6^2 n \\ \vdots & \vdots \\ k & 6^k n \end{array}.$$

So we have $5n^3 = 6^k n \Rightarrow 6^k = 5n^2 \Rightarrow k = \log_6(5n^2) = \boxed{2\log_6(n)} + \log_6(5)$. So $1 \leq k \leq \lfloor 2\log_6(n) \rfloor$. Now we find the bounds for the second while loop

$$\begin{array}{c|c} \text{iterations} & j = 6n^2 \\ \hline 1 & 6n^2/4 \\ 2 & 6n^2/4^2 \\ \vdots & \vdots \\ l & 6n^2/4^l \end{array}.$$

So we have $3 = 6n^2/4^l \Rightarrow 4^l = 2n^2 \Rightarrow l = \log_4(2n^2) = \boxed{2\log_4(n)} + \log_4(2)$. Thus $1 \leq l \leq \lfloor 2\log_4(n) \rfloor$. Hence the running time is given by

$$\begin{aligned} t = \sum_{k=1}^{\lfloor 2\log_6(n) \rfloor} \sum_{l=1}^{\lfloor 2\log_4(n) \rfloor} c = \sum_{k=1}^{\lfloor 2\log_6(n) \rfloor} 2\log_4(n) c = \frac{4}{\log_2(4)\log_2(6)} (\log_2(n))^2 c \\ = \boxed{\Theta((\log_2(n))^2).} \end{aligned} \tag{1}$$

(b) We find the bounds for the while loop

$$\begin{array}{c|c} \text{iterations} & j = 2i \\ \hline 1 & 3 \cdot 2i \\ 2 & 3^2 \cdot 2i \\ \vdots & \vdots \\ k & 3^k \cdot 2i \end{array}.$$

So $3^k \cdot 2i = i^4 \Rightarrow k =$
$log3(i^3/2) = 3\log_3(i) + \log_3(2)$. So $1 \leq k \leq \lfloor 3$
$log3(i) \rfloor$. Thus the running time is given by

$$t = \sum_{i=1}^{3n^2} \sum_{k=1}^{\lfloor 3\log_3(i) \rfloor} c = \sum_{i=1}^{3n^2} 3\log_3(i) c = \sum_{i=1}^{n^2-1} 3\log_3(i) c + \sum_{i=n^2}^{3n^2} 3\log_3(i) c. \tag{2}$$

We find an upper bound, plugging in $3n^2$ for i

$$t \leq \sum_{i=1}^{3n^2} 3\log_3(3n^2) c = 9n^2(2(\log_3(n)) + 1) c = \boxed{18cn^2\log_3(n)} + 9cn^2. \tag{3}$$

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Now we find a lower bound, splitting the sum and plugging in n^2 for i

$$t \geq \sum_{i=n^2}^{3n^2} 3\log_3(n^2)c = \boxed{12cn^2\log_3(n).} \tag{4}$$

So $t = c_1 n^2 \log_3(n)$, where $12c \leq c_1 \leq 18c$, so $t = \boxed{\Theta(n^2\log_3(n)).}$

(c) We find the bounds of the summation for the first while loop.

iterations	$i = n$
1	$n+5$
2	$n+2\cdot 5$
$\vdots$	$\vdots$
k	$n+5k$

.

So $2n^3 = n + 5k \Rightarrow 5k = 2n^3 - n \Rightarrow k = \boxed{\lfloor \frac{2}{5}n^3 \rfloor} - \frac{n}{5}$. So $1 \leq k \leq \lfloor \frac{2}{5}n^3 \rfloor$. Now we find the bounds of the summation for the second while loop

iterations	$j = i^2$
1	$i^2/4$
2	$i^2/4^2$
$\vdots$	$\vdots$
l	$i^2/4^l$

.

So we have $i^2/4^l = i \Rightarrow 4^l = i \Rightarrow l = \boxed{\lfloor \log_4(i) \rfloor.}$ Thus $1 \leq l \leq \lfloor \log_4(i) \rfloor$. Then

$$t = \sum_{k=1}^{\lfloor \frac{2}{5}n^3 \rfloor} \sum_{l=1}^{\lfloor \log_4(i) \rfloor} c = \sum_{k=1}^{\lfloor \frac{2}{5}n^3 \rfloor} \log_4(i)c = \sum_{k=1}^{\lfloor \frac{1}{5}n^3 \rfloor - 1} \log_4(i)c + \sum_{k=\lfloor \frac{1}{5}n^3 \rfloor}^{\lfloor \frac{2}{5}n^3 \rfloor} \log_4(i)c. \tag{5}$$

We find an upper bound, plugging in $\frac{2}{5}n^3$ for k. Then $i = n + 2n^3$, so since we are taking an upper bound, we let $i = 3n^3$. Thus we have

$$\begin{aligned} t &\leq \sum_{k=1}^{\lfloor \frac{2}{5}n^3 \rfloor} \log_4(3n^3)c = \frac{2}{5}cn^3\log_4(3n^3) = \frac{2}{5}cn^3(3\log_4(n) + \log_4(3)) \\ &= \boxed{\frac{6}{5}cn^3\log_4(n)} + \frac{2}{5}cn^3\log_4(3). \end{aligned} \tag{6}$$

Now we find a lower bound, splitting the summation and plugging in $\frac{1}{5}n^3$ for k. Then $i = n + n^3$. So since we are finding a lower bound, we take $i = n^3$. Then we have

$$t \geq \sum_{k=\lfloor \frac{1}{5}n^3 \rfloor}^{\lfloor \frac{2}{5}n^3 \rfloor} \log_4(n^3)c = \frac{1}{5}n^3\log_4(n^3)c = \boxed{\frac{3}{5}cn^3\log_4(n).} \tag{7}$$

Hence $t = c_2 n^3 \log_4(n)$, where $\frac{3}{5}c \leq c_2 \leq \frac{6}{5}c$. So $t = \boxed{\Theta(n^3\log_4(n)).}$

2. Note

$$\begin{aligned} f_a(n) &= \Theta(n^4 \log(n)), \\ f_b(n) &= \Theta(4^n), \\ f_c(n) &= \Theta(n^{0.8}), \\ f_d(n) &= \Theta(1), \\ f_e(n) &= \Theta(n^{0.7}), \\ f_f(n) &= \Theta(n^5), \\ f_g(n) &= \Theta(n^2), \\ f_h(n) &= \Theta(n^{1.5}), \\ f_i(n) &= \Theta(n^{2.5}), \\ f_j(n) &= \Theta(n^2 \log(n)), \\ f_k(n) &= \Theta(n \log(n)), \\ f_l(n) &= \Theta(n^4), \\ f_m(n) &= \Theta(1), \\ f_n(n) &= \Theta(n^n), \end{aligned} \tag{8}$$

Let $\nu \in \{a, b, ..., n\}^{14}$ s.t.

$$f_{\nu_i} = O(f_{\nu_{i+1}}) \tag{9}$$

for $1 \leq i \leq 14$. Then

$$\nu = (d, m, e, c, k, h, g, j, i, l, a, f, b, n). \tag{10}$$

Also note $f_m(n) = \Theta(f_d(n))$.