CSE 2321 Homework 3

Brendan Whitaker

AU17

1. (a) $POW(A) = \{\varnothing, \{a\}, \{c\}, \{a, c\}\}$.
   (b) $POW(B) = \{\varnothing, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}\}$.
   (c) $|POW(A) \cap POW(B)| = |POW(A \cap B)|$ since $POW(A) \cap POW(B) = \{\varnothing, \{c\}\} \Rightarrow |POW(A) \cap POW(B)| = 2$, and $POW(A \cap B) = \{\varnothing, \{c\}\} \Rightarrow |POW(A \cap B)| = 2$, since $A \cap B = \{c\}$.
   (d) $|POW(A) \cup POW(B)| \neq |POW(A \cup B)|$.
   Note $POW(A) \cup POW(B) = \{\varnothing, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}, \{a\}, \{a, c\}\}$
   $\Rightarrow |POW(A) \cup POW(B)| = 10$, and $|POW(A \cup B)| = 16$,
   since $A \cup B = \{a, b, c, d\}$, and $2^{|A \cup B|} = 2^4 = 16$.
   (e) $|POW(A - B)| \neq |POW(B - A)|$.
   Note $A - B = \{a\} \Rightarrow POW(A - B) = \{\varnothing, \{a\}\} \Rightarrow |POW(A - B)| = 2$. Also $B - A = \{b, d\} \Rightarrow POW(B - A) = \{\varnothing, \{d\}, \{b\}, \{b, d\}\} \Rightarrow |POW(B - A)| = 4$.

2. (a) True.
   (b) False. Let $x \in A \cap C$. Then $x \in A \Rightarrow x \in (A \cup (B - C))$. However, $x \notin ((A \cup B) - C)$, since $x \in C$.
   (c) False. Let $x \in A - (B \cup C)$. Then $x \notin (B \cap C) \Rightarrow x \in (A - (B \cap C))$ since $x \in A$. But $x \notin C \Rightarrow x \notin ((A - B) \cap C)$.
   (d) True. Follows from defining the membership of x in the sets A, B, C as predicates $P(x), Q(x), R(x)$. Then we may apply laws from propositional logic, and the truth of the statement follows from the commutative and distributive laws (see slide S2 of propositional logic).

3. (a) $\forall x \in D, (\neg R(x) \wedge \neg Q(x)) \Rightarrow \neg P(x)$.
   (b) $|\{x \in D | P(x)\}| = 1$.
   (c) $|\{x \in D | R(x)\}| \geq 2$.
   (d) $\forall x \in D, P(x) \Rightarrow R(x)$.
   (e) $|\{x \in D | P(x)\}| \leq 1$.
   (f) $\forall x \in D, Q(x) \Rightarrow P(x)$.
   (g) $\forall x \in D, (R(x) \wedge P(x)) \vee \neg Q(x)$.
   (h) All objects with broken windows are cars.
   (i) Everything is in the garage and has a broken window.
   (j) There is a car in the garage.
   (k) There is a car and there is an object with a broken window.

4. (a) $\mathbb{N} \cap \mathbb{R}) = \mathbb{N}$.

   Proof. Note $\mathbb{N} \subset \mathbb{R}$. □

   (b) $(\mathbb{R} - \mathbb{Q}) = \{x \in \mathbb{R} | x \text{ is irrational}\}$.

*Professor Close, MWF 12:40pm

Proof. Note $\mathbb{Q} \subset \mathbb{R}$, and that $\mathbb{Q}$ is defined as the set of all rationals. $\square$

(c) $(\mathbb{Z}^- \cup \mathbb{N}) = \mathbb{Z}$.

Proof. Note $\mathbb{Z}^- = \{x \in \mathbb{Z} | x \leq -1\}$, and $\mathbb{N} = \{x \in \mathbb{Z} | x \geq 0\}$. Observe there are no integers in the interval $-1 < x < 0$. $\square$

(d) $(\mathbb{Z} \cup \mathbb{N}) = \mathbb{Z}$.

Proof. Observe $\mathbb{N} \subset \mathbb{Z}$. $\square$

5. A sufficient condition on x for $\frac{x}{4}$ to be an even integer is to have $x \in 8\mathbb{Z}$.

   Proof. Let $x \in 8\mathbb{Z}$. Then $\exists k \in \mathbb{Z}$ s.t. $x = 8k$. Then $\frac{x}{4} = \frac{8k}{4} = 2k$ which is an even integer by definition, since $k \in \mathbb{Z}$. $\square$

6. If $n \geq 1$, $n \in \mathbb{N}$, a necessary but not sufficient condition for $n+1$ to be a prime is that $n \notin \{8k+7 | k \in \mathbb{Z}\}$.

   Proof. Let $n + 1 \in \mathbb{P}$. Then suppose $n \in \{8k + 7 | k \in \mathbb{Z}\} \Rightarrow n = 8k + 7, k \in \mathbb{Z} \Rightarrow n + 1 = 8k + 8 \Rightarrow 8 \mid (n + 1) \Rightarrow 2 \mid (n + 1) \Rightarrow n + 1$ is not prime, since the only divisor of prime numbers are 1 and the number itself, and we found two distinct divisors, both greater than 1. Thus our condition is necessary. We show it is not sufficient. Consider $6 \notin \{8k + 7 | k \in \mathbb{Z}\}$. But 6 is not prime, hence the condition is not sufficient. $\square$