CSE 2321 Homework 2

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AU17

1. (a) $P \Rightarrow R$
   i. $P \Rightarrow \neg R$
   ii. If your number if prime, then it is not evenly divisible by an integer other than 1 and itself.
   iii. Claim: $P \Leftrightarrow \neg R$.

   Proof. Note that $\mathbb{P}$, the set of all primes is defined as the set of all integers of the form n divisible only by 1 and n. But this is precisely the same set as that defined by R. Let A be the set defined in $\neg R$. Formally, $x \in A \Rightarrow x \in \mathbb{P} \Rightarrow A \subset \mathbb{P}$. Similarly, $x \in \mathbb{P} \Rightarrow x \in A \Rightarrow \mathbb{P} \subset A$. □

   (b) $S \Rightarrow Q$.
   i. $S \Rightarrow \neg Q$.
   ii. If your number is odd, then it is not even.
   iii. Claim: $S \Leftrightarrow \neg Q$.

   Proof. Observe that $\mathbb{Z} = \{2k : k \in \mathbb{Z}\} \dot{\bigcup} \{2k+1 : k \in \mathbb{Z}\}$. □

   (c) $(R \quad S) \Rightarrow P$
   i. $(\neg R \wedge S) \Rightarrow P$
   ii. If your number is not evenly divisible by any integer other than 1 and itself, and it is odd, then it is prime.
   iii. Claim: $(\neg R \wedge S) \Leftrightarrow P$.

   Proof. By part (a) we know that $\neg R \Leftrightarrow P$. So we must only show that $P \Rightarrow S$. But all the primes other than 2 are odd, and all numbers between 10 and 1,000,000 are greater than 2. □

   (d) $S \Rightarrow P$.
   i. $\neg S \Rightarrow \neg P$.
   ii. If your number is not odd, then it is not prime.
   iii. The implication cannot be replaced by a biconditional, since 15 is not prime, but it is odd.

2. (a) Some houses don't have electricity.
   i. $D = \{\text{the set of all houses}\}$.
   ii. Let $P(x)$ denote the truth value of the statement "house x has electricity". Then we write $\exists x \in D \ \neg P(x)$.
   iii. $\forall x \in D \ P(x)$.
   iv. All houses have electricity.

   (b) None of the houses have plumbing.
   i. $D = \{\text{the set of all houses}\}$.

[*]Professor Close, MWF 12:40pm

ii. Let $P(x)$ denote the truth value of the statement "house x has plumbing". Then we write $\forall x \in D\ \neg P(x)$.

iii. $\exists x \in D\ P(x)$.

iv. Some houses have plumbing.

(c) Some cabins have heat but do not have A/C.

i. $D = \{\text{the set of all cabins}\}$.

ii. Let $P(x)$ denote the truth value of the statement "cabin x has heat". Let $Q(x)$ denote the truth value of the statement "cabin x has A/C". Then we write $\exists x \in D\ (P(x) \wedge \neg Q(x))$.

iii. $(\neg P(x) \vee Q(x))\ \forall x \in D$.

iv. All cabins either do not have heat, or have A/C.

(d) At least one cabin does not have heat, and some cabins do not have A/C.

i. $D = \{\text{the set of all cabins}\}$.

ii. Let $P(x)$ denote the truth value of the statement "cabin x has heat". Let $Q(x)$ denote the truth value of the statement "cabin x has A/C". Then we write $\exists x, y \in D\ \neg P(x) \wedge \neg Q(y)$.

iii. $(P(x)\ \forall x \in D) \vee (Q(x)\ \forall x \in D)$.

iv. All cabins have heat or all cabins have A/C.

(e) For every x and y there exists a z such that $x + y = z$.

i. $D = \mathbb{Z}$.

ii. Let $P(x, y, z)$ denote the truth value of the equality $x + y = z$. Then we write $\forall x, y \in D$, $\exists z \in D\ P(x, y, z)$.

iii. $\exists x, y \in D\ \neg P(x, y, z)\ \forall z \in D$.

iv. There exist x and y such that $x + y \neq z$ for all z.

(f) For every x and y, $\frac{x}{y} = \frac{y}{x}$.

i. $D = \mathbb{Z}$.

ii. Let $P(x, y)$ denote the truth value of the equality $\frac{x}{y} = \frac{y}{x}$. Then we write $\forall x, y \in D$, $P(x, y)$.

iii. $\exists x, y \in D$, $\neg P(x, y)$.

iv. There exist x,y such that $\frac{x}{y} \neq \frac{y}{x}$.

3. (a) Having a driver's license and car insurance are sufficient conditions for owning a car in Ohio.

i. Let $P =$ you have a driver's license, let $Q =$ you have car insurance, and let $S =$ you have a car in Ohio.

ii. $(P \wedge Q) \Rightarrow S$.

iii. $S \Rightarrow (P \wedge Q)$. Owning a car in Ohio is a sufficient condition for having a driver's license and car insurance.

iv. $\neg S \Rightarrow \neg(P \wedge Q)$. Not owning a car in Ohio is a sufficient condition for not having a driver's license and not having car insurance.

(b) It is necessary for a student to complete assignments and attend class to do well in CSE 2321.

i. Let P = the student completes assignments, let Q = the student attends class, and let S = the student does well in CSE 2321.

ii. $S \Rightarrow (P \wedge Q)$.

iii. $(P \wedge Q) \Rightarrow S$. It is necessary for a student to do well in CSE 2321 in order to complete assignments and attend class.

iv. $(\neg P \vee \neg Q) \Rightarrow \neg S$. Not completing assignments or not attending class are sufficient conditions for not doing well in CSE 2321.