CSE 2321 HOMEWORK 7

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2. (a)

$$\begin{aligned} t &= \sum_{i=1}^{6n^2}\sum_{j=3}^{n}\sum_{k=1}^{n} c = c\sum_{i=1}^{6n^2}\sum_{j=3}^{n}(n-1+1) = c\sum_{i=1}^{6n^2}\sum_{j=3}^{n} n = c\sum_{i=1}^{6n^2}(n-3+1)n \\ &= (6n^2-1+1)cn(n-2) = 6cn^3(n-2) = 6cn^4 - 12cn^3 \approx cn^4. \end{aligned} \tag{1}$$

(b)

$$\begin{aligned} t &= \sum_{i=1}^{10}\sum_{j=3}^{n}\sum_{k=1}^{n\lfloor log_2 n\rfloor} c = c\sum_{i=1}^{10}\sum_{j=3}^{n} nlog_2 n = \sum_{i=1}^{10} cn(n-2)log_2 n \\ &= 10cn(n-2)log_2 n = 10cn^2 log_2 n - 20cnlog_2 n \approx cn^2 log_2 n. \end{aligned} \tag{2}$$

(c)

$$t = \sum_{i=n}^{2n}\sum_{10i+7}^{10i+21} c = c\sum_{i=n}^{2n}(10i-10i+21-7+1) = c\sum_{i=n}^{2n} 15 = (n+1)15c \approx cn.$$

(d)

$$t = \sum_{i=4}^{n^2}\sum_{j=6}^{3i\lfloor log_2 i\rfloor} c = \sum_{i=4}^{n^2} c(3ilog_2 i - 5).$$

We find the upper bound. Substituting n^2 for i, the inside of the summation becomes $3cn^2log_2(n^2)$. Plugging in, we have

$$\begin{aligned} t &\le \sum_{i=4}^{n^2} c(3n^2 log_2(n^2) - 5) = (n^2-3)c(3n^2log_2(n^2)-5) \\ &= 3cn^4log_2(n^2) - 5cn^2 - 9cn^2log_2(n^2) + 15c \le 3cn^4log_2(n^2) + 15c = 6cn^4log_2 n + 15c. \end{aligned} \tag{3}$$

Now we compute the lower bound. We split the summation

$$t = \sum_{i=4}^{n^2} c(3ilog_2 i - 5) = \sum_{i=4}^{\frac{n^2-2}{2}} c(3ilog_2 i - 5) + \sum_{i=\frac{n^2}{2}}^{n^2} c(3ilog_2 i - 5).$$

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We substitute $n^2/2$ for i, and the inside of the summation becomes $c(3(\frac{n^2}{2})log_2(\frac{n^2}{2})-5)$. Then we have

$$\begin{aligned}
t &= \sum_{i=4}^{\frac{n^2-2}{2}} c(3ilog_2 i-5) + \sum_{i=\frac{n^2}{2}}^{n^2} c(3ilog_2 i-5) \geq \sum_{i=\frac{n^2}{2}}^{n^2} c(3ilog_2 i-5) \\
&\geq c\sum_{i=\frac{n^2}{2}}^{n^2} \frac{3}{2}n^2 log_2(n^2/2)-5 = c(n^2/2+1)\left(\frac{3}{2}n^2 log_2(n^2/2)-5\right) \\
&= c\left(\frac{3}{4}n^4 log_2(n^2/2)+\frac{3}{2}n^2 log_2(n^2/2)-5n^2/2-5\right) \\
&\geq c\left(\frac{3}{4}n^4 log_2(n^2/2)-5n^2/2-5\right) = c\left(\frac{3}{4}n^4(log_2(n^2)-1)-5n^2/2-5\right) \\
&= \frac{3}{2}cn^4 log_2 n - \frac{3}{4}cn^4 - \frac{5}{2}cn^2 - 5c.
\end{aligned} \tag{4}$$

Now note since the dominating term in both the upper bound and lower bound is of the form $kcn^4log_2(n)$ for some constant $k \in \mathbb{R}^{>0}$, we have that the upper bound and lower bound give us a running time of $t \approx cn^4log_2(n)$.