Honors Calculus II: Chapter 15 Conics

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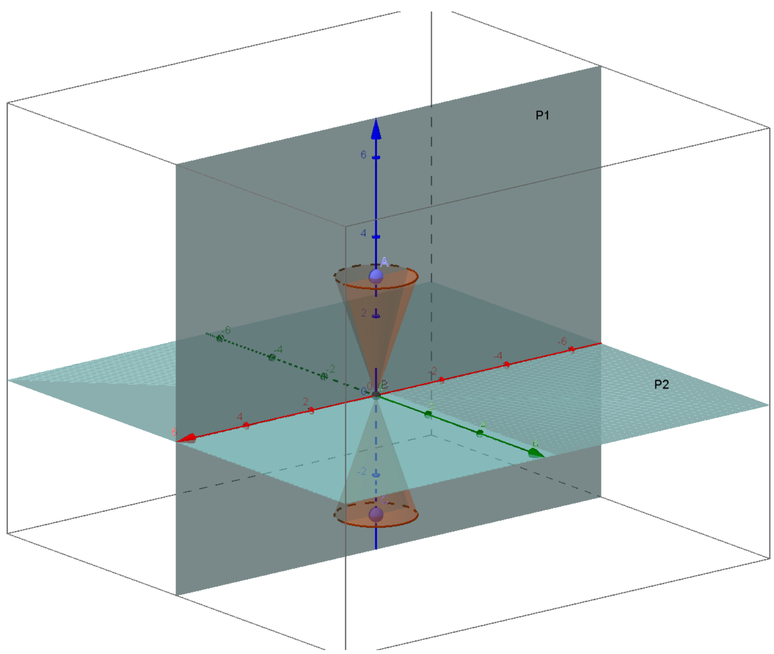
Part I
Introduction

Greeks - visual, real world oriented.
Symmetry:



1. front/back symmetry in P1
2. top/bottom symmetry in P2

- Plane parallel to P1 gives intersection "hyperbola"
- Plane parallel to P2 gives intersection "circle"

- Plane transverse to P1 gives intersection "parabola"
- Plane transverse to P2 gives intersection "ellipse"

Descartes - introduced the coordinate system

Cartesian plane or 3-space:

Cone:

$$x^2 + y^2 = z^2$$

Plane:

$$ax + by + cz = d$$

Intersection: quadratic in 2-space (found by solving intersection of cone and plane equation).

1 Point/Line

Cartesian coordinates in 2-space
Euclidian distance:

$$d(P1, P2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Describe the locus of points P, such that

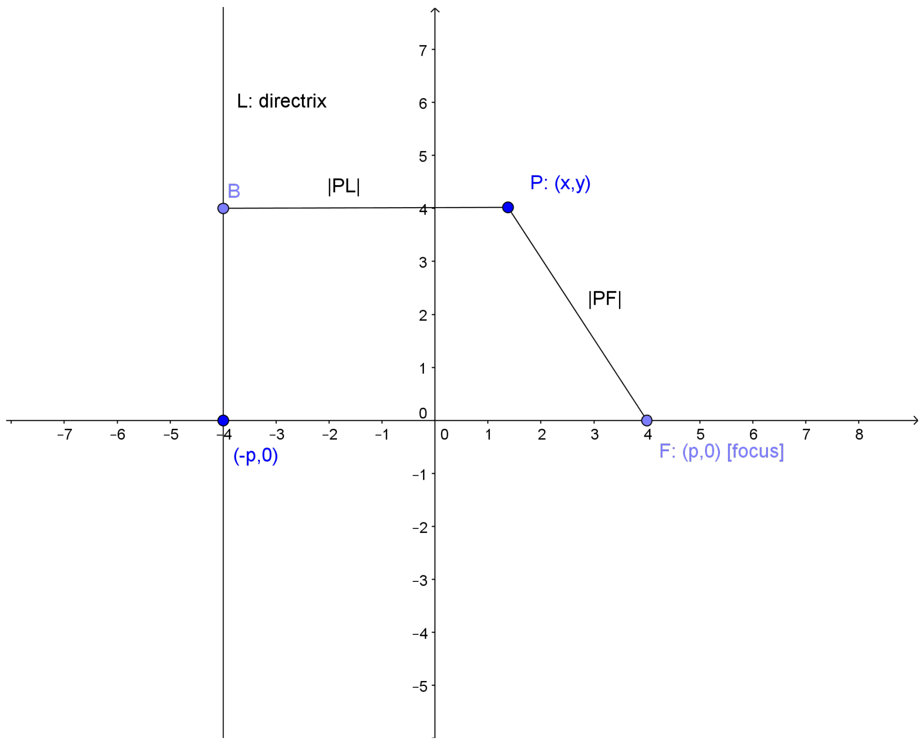
$$e|PL| = |PF|$$

Where **e** is some constant describing the eccentricity of the conic section, **L** is the directrix and **F** is the focus or foci of the conic.

1.1 Algebraic Expression of Conics

Rotate and Translate to Standard Form:

(Rotations will not be covered in this course)



Note: Uppercase P denotes a point, while lowercase p denotes a length value.

$$
\begin{aligned}
|PF| &= \\
\sqrt{(x-p)^2+(y-0)^2} &= e|PL| \\
\sqrt{(x-p)^2+(y-0)^2} &= e|x+p| \\
(\sqrt{(x-p)^2+(y)^2})^2 &= (e|x+p|)^2 \\
(x-p)^2+y^2 &= e^2(x^2+2xp+p^2) \\
x^2-2xp+p^2+y^2 &= e^2(x^2+2xp+p^2)
\end{aligned} \tag{1}
$$

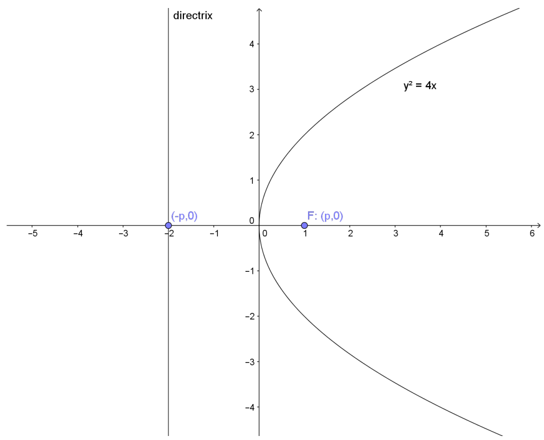
1.1.1 Case 1: Parabola (e=1)

$$
\begin{aligned}
x^2-2xp+p^2+y^2 &= (1)^2(x^2+2xp+p^2) \\
x^2-2xp+p^2+y^2 &= (x^2+2xp+p^2) \\
\cancel{x^2}-2xp+\cancel{p^2}+y^2 &= \cancel{x^2}+2xp+\cancel{p^2}
\end{aligned} \tag{2}
$$

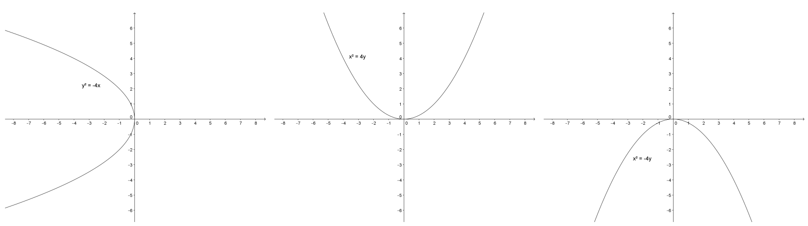
Equation of a Parabola:

$$\boxed{y^2 = 4px}$$

Standard Graph:

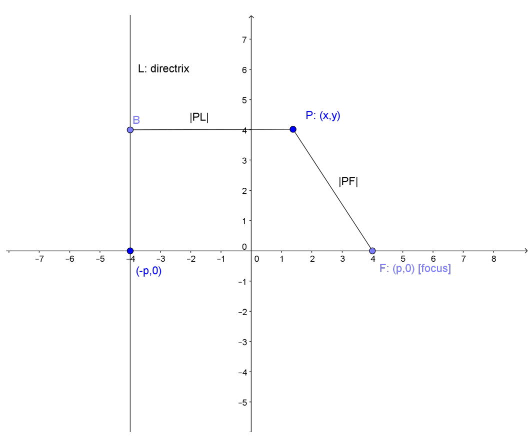


Variations:



1.1.2 Case 2: Ellipse (0<1<e)

Same Picture:



$$x^2 - 2xp + p^2 + y^2 = e^2(x^2 + 2xp + p^2)$$