MATH 5590H BONUS

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Exercise. *Find a group $G = \langle S \rangle$ s.t. $G' \neq [S, S]$.*

Proof. Consider $A_4 = \langle (1,2,3), (1,2,4) \rangle$. Then
$$[S, S] = \langle (1,2,3)(1,2,4)(1,3,2)(1,4,2), (1,2,4)(1,2,3)(1,4,2)(1,3,2) \rangle.$$
Now note
$$(1,2,3)(1,2,4)(1,3,2)(1,4,2) = (1,3)(2,4)(1,4)(2,3) = (1,2)(3,4).$$
And
$$(1,2,4)(1,2,3)(1,4,2)(1,3,2) = (1,2)(3,4).$$
Thus $[S, S] = \langle (1,2)(3,4) \rangle$. Now note that $[(1,2,3),(2,3,4)] \in [G, G]$, and
$$[(1,2,3),(2,3,4)] = (1,2,3)(2,3,4)(1,3,2)(2,4,3) = (1,4)(2,3).$$
Then since $(1,4)(2,3) \notin \langle (1,2)(3,4) \rangle$, we know $[S, S] \neq G'$. $\qquad\square$