

立体幻方的构造

时佑民

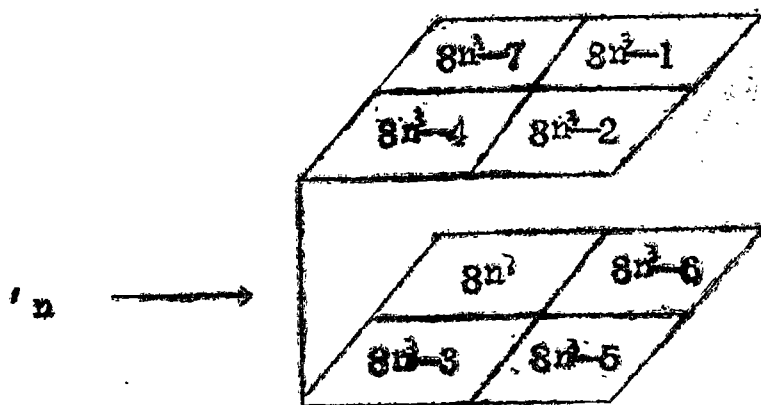
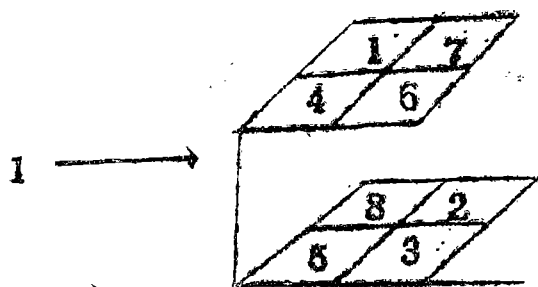
(图书馆)

提 要

本文首先提出奇数阶立体幻方及 2^K 阶($K=2, 3, \dots$)立体幻方的最简构造方法,并解决了最困难的一种立体幻方(即偶数阶中非 2^K 阶又不能通过复合方法来构造的幻方,如12阶等可通过复合方法得到)的构造方法。

构造方法简述

(一) 奇数阶幻方的造法是,按自然数大小顺序由小到大依次填写,从中间一层开始,



每写完 $2m+1$ 个换一层。直至填满空格。

2^K 阶立体幻方造法是先按自然数填写一个数字方阵,然后把其中一半数字和与它中心对称的数字进行交换即可,详细作法可通过阅读本文下面所举例子了解到。

(二) 6阶、10阶等最困难的幻方的造法:

(1) 要造一个3维 $2n=2m+2'$ 阶幻方,可先造一个3维 n 阶幻方,然后把 n 阶幻方中的每个数替换成相邻的8个自然数如下:

(2) 画出调整图 (即注明数字交换符号的参考图) 写出加法分解式:

$$\textcircled{1} \quad 4n = 6k_1 + 2k_2, \quad k_1, k_2, e \{0, 1, \dots, n\}$$

$$\textcircled{2} \quad n = 3k_3 - k_4 + 5k_5 - 3k_6$$

$$k_3, k_4, k_5, k_6 \in \{0, 1, \dots, n\}$$

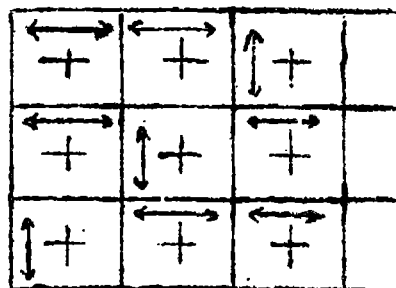
$k_i (i=1, 2, \dots, 6)$ 作为上面方程的非负整数解, 它们表示应划在调整图中的数字交换符号的个数。

(三) 3维6阶幻方造法:

(1) 先作一个3维3阶幻方: (图1)

12	23	7	一层
22	9	11	
8	10	24	
26	1	15	二层
3	14	25	
13	27	2	
4	18	20	三层
17	19	6	
21	5	16	

(图1)



(图2)

(2) 据分解式: $2n \times 2 = 6k_1 + 2k_2$ ($n=3$)

$$k_1 = 2, \quad k_2 = 0$$

$$\text{和: } n = 3k_3 - k_4 + 5k_5 - 3k_6$$

$$k_3 = 1, \quad k_4 = k_5 = k_6 = 0$$

画出调整图 (图2)

(3) 据3维3阶幻方和调整图画出6阶幻方 (图3)

(四) 3维10阶幻方造法: ($10 = 2n = 2 \times 5, n=5$)

(1) 写出分解式 $4n = 6k_1 + 2k_2$, 可取: $k_1 = 3, k_2 = 1$,

$$n = 5k_5, \quad k_5 = 1$$

(2) 先作一个3维5阶幻方 (图4):

(3) 用类似的方法作出3维10阶幻方调整图 (图5):

95	89	183	177	52	55
92	94	180	182	49	54
175	169	68	71	87	81
172	174	65	70	84	86
60	63	79	73	191	185
57	62	76	78	188	190

I

90	96	178	184	53	50
93	91	181	179	56	51
170	176	69	66	82	88
173	171	72	67	85	83
61	58	74	80	186	192
64	59	77	75	189	187

II

207	201	7	1	116	119
204	206	4	6	113	118
23	17	108	111	199	193
20	22	105	110	196	198
100	103	215	209	15	9
97	102	212	214	12	14

III

202	208	2	8	117	114
205	203	5	3	120	115
18	24	109	106	194	200
21	19	112	107	197	195
101	98	210	216	10	16
104	99	213	211	13	11

IV

31	25	143	137	156	159
28	30	140	142	153	158
135	129	148	151	47	41
132	134	145	150	44	46
164	167	39	33	127	121
161	166	36	38	124	126

V

26	32	138	144	157	154
29	27	141	139	160	155
130	136	149	146	42	48
133	131	152	147	45	43
165	162	34	40	122	128
168	163	37	35	125	123

VI

图3 3维6阶幻方

48	55	82	114	16
54	81	113	20	47
85	112	19	46	53
111	18	50	52	84
17	49	51	83	115

I

60	87	119	21	28
86	118	25	27	59
117	24	26	58	90
23	30	57	89	116
29	56	88	120	22

II

92	124	1	33	65
123	5	32	64	91
4	31	63	95	122
35	62	94	121	3
61	93	125	2	34

III

104	6	38	70	97
10	37	69	96	103
36	68	100	102	9
67	99	101	8	40
98	105	7	39	66

IV

11	43	75	77	109
42	74	76	108	15
73	80	107	14	41
79	106	13	45	72
110	12	44	71	78

V

+	+	+	+	+
+	+	+	+	+
+	+	+	+	+
+	+	+	+	+
+	+	+	+	+

图4 (3维5阶幻方)

图5 (3维10阶幻方调整图)

(4) 作出3维10阶幻方(图6):

377 383 380 382	433 439 436 438	655 654 652 649	911 905 908 910	127 121 126 124
425 431 428 430	647 646 644 641	903 897 900 902	159 153 158 156	369 375 372 374
679 678 676 673	895 889 892 894	151 145 150 148	361 367 364 366	417 423 420 422
887 881 884 886	143 137 142 140	393 399 396 398	409 415 412 414	671 670 668 665
135 129 134 132	385 391 388 390	401 407 404 406	663 662 660 657	919 913 916 918

I

384 378 381 379	440 434 437 435	650 651 653 656	906 912 909 907	122 128 123 125
432 426 429 427	642 643 645 648	898 904 901 899	154 160 155 157	376 370 373 371
674 675 677 680	890 896 893 891	146 152 147 149	368 362 365 363	424 418 421 419
882 888 885 883	138 144 139 141	400 394 397 395	416 410 413 411	666 667 669 672
130 136 131 133	392 386 389 387	408 402 405 403	658 659 661 664	914 920 917 915

II

473 479 476 478	689 695 692 694	951 950 948 945	167 161 164 166	223 217 222 220
681 687 684 686	943 942 940 937	199 193 196 198	215 209 214 212	465 471 468 470
935 934 932 929	191 185 188 190	207 201 206 204	457 463 460 462	713 719 716 718
183 177 180 182	239 233 238 236	449 455 452 454	705 711 708 710	927 926 924 921
231 225 230 228	441 447 444 446	697 703 700 702	959 958 956 953	175 169 172 174

III

$\begin{array}{r} 480 \\ 477 \end{array}$	$\begin{array}{r} 474 \\ 475 \end{array}$	$\begin{array}{r} 696 \\ 693 \end{array}$	$\begin{array}{r} 690 \\ 691 \end{array}$	$\begin{array}{r} 946 \\ 949 \end{array}$	$\begin{array}{r} 947 \\ 952 \end{array}$	$\begin{array}{r} 162 \\ 165 \end{array}$	$\begin{array}{r} 168 \\ 163 \end{array}$	$\begin{array}{r} 218 \\ 219 \end{array}$	$\begin{array}{r} 224 \\ 221 \end{array}$
$\begin{array}{r} 688 \\ 685 \end{array}$	$\begin{array}{r} 682 \\ 683 \end{array}$	$\begin{array}{r} 938 \\ 941 \end{array}$	$\begin{array}{r} 939 \\ 944 \end{array}$	$\begin{array}{r} 194 \\ 197 \end{array}$	$\begin{array}{r} 200 \\ 195 \end{array}$	$\begin{array}{r} 210 \\ 211 \end{array}$	$\begin{array}{r} 216 \\ 213 \end{array}$	$\begin{array}{r} 472 \\ 469 \end{array}$	$\begin{array}{r} 466 \\ 467 \end{array}$
$\begin{array}{r} 930 \\ 933 \end{array}$	$\begin{array}{r} 931 \\ 936 \end{array}$	$\begin{array}{r} 186 \\ 189 \end{array}$	$\begin{array}{r} 192 \\ 187 \end{array}$	$\begin{array}{r} 202 \\ 203 \end{array}$	$\begin{array}{r} 208 \\ 205 \end{array}$	$\begin{array}{r} 464 \\ 461 \end{array}$	$\begin{array}{r} 458 \\ 459 \end{array}$	$\begin{array}{r} 720 \\ 717 \end{array}$	$\begin{array}{r} 714 \\ 715 \end{array}$
$\begin{array}{r} 178 \\ 181 \end{array}$	$\begin{array}{r} 184 \\ 179 \end{array}$	$\begin{array}{r} 234 \\ 235 \end{array}$	$\begin{array}{r} 240 \\ 237 \end{array}$	$\begin{array}{r} 456 \\ 453 \end{array}$	$\begin{array}{r} 450 \\ 451 \end{array}$	$\begin{array}{r} 712 \\ 709 \end{array}$	$\begin{array}{r} 706 \\ 707 \end{array}$	$\begin{array}{r} 922 \\ 925 \end{array}$	$\begin{array}{r} 923 \\ 928 \end{array}$
$\begin{array}{r} 226 \\ 227 \end{array}$	$\begin{array}{r} 232 \\ 229 \end{array}$	$\begin{array}{r} 448 \\ 445 \end{array}$	$\begin{array}{r} 442 \\ 443 \end{array}$	$\begin{array}{r} 704 \\ 701 \end{array}$	$\begin{array}{r} 698 \\ 699 \end{array}$	$\begin{array}{r} 954 \\ 957 \end{array}$	$\begin{array}{r} 955 \\ 960 \end{array}$	$\begin{array}{r} 170 \\ 173 \end{array}$	$\begin{array}{r} 176 \\ 171 \end{array}$

IV

$\begin{array}{r} 729 \\ 732 \end{array}$	$\begin{array}{r} 735 \\ 734 \end{array}$	$\begin{array}{r} 985 \\ 988 \end{array}$	$\begin{array}{r} 991 \\ 990 \end{array}$	$\begin{array}{r} 7 \\ 4 \end{array}$	$\begin{array}{r} 6 \\ 1 \end{array}$	$\begin{array}{r} 263 \\ 260 \end{array}$	$\begin{array}{r} 257 \\ 262 \end{array}$	$\begin{array}{r} 519 \\ 518 \end{array}$	$\begin{array}{r} 513 \\ 516 \end{array}$
$\begin{array}{r} 977 \\ 980 \end{array}$	$\begin{array}{r} 983 \\ 982 \end{array}$	$\begin{array}{r} 39 \\ 36 \end{array}$	$\begin{array}{r} 38 \\ 33 \end{array}$	$\begin{array}{r} 255 \\ 252 \end{array}$	$\begin{array}{r} 249 \\ 254 \end{array}$	$\begin{array}{r} 511 \\ 510 \end{array}$	$\begin{array}{r} 505 \\ 508 \end{array}$	$\begin{array}{r} 721 \\ 724 \end{array}$	$\begin{array}{r} 727 \\ 726 \end{array}$
$\begin{array}{r} 31 \\ 28 \end{array}$	$\begin{array}{r} 30 \\ 25 \end{array}$	$\begin{array}{r} 247 \\ 244 \end{array}$	$\begin{array}{r} 241 \\ 246 \end{array}$	$\begin{array}{r} 503 \\ 502 \end{array}$	$\begin{array}{r} 497 \\ 500 \end{array}$	$\begin{array}{r} 753 \\ 756 \end{array}$	$\begin{array}{r} 759 \\ 758 \end{array}$	$\begin{array}{r} 969 \\ 972 \end{array}$	$\begin{array}{r} 975 \\ 974 \end{array}$
$\begin{array}{r} 279 \\ 276 \end{array}$	$\begin{array}{r} 273 \\ 278 \end{array}$	$\begin{array}{r} 495 \\ 494 \end{array}$	$\begin{array}{r} 489 \\ 492 \end{array}$	$\begin{array}{r} 745 \\ 748 \end{array}$	$\begin{array}{r} 751 \\ 750 \end{array}$	$\begin{array}{r} 961 \\ 964 \end{array}$	$\begin{array}{r} 967 \\ 966 \end{array}$	$\begin{array}{r} 23 \\ 20 \end{array}$	$\begin{array}{r} 22 \\ 17 \end{array}$
$\begin{array}{r} 487 \\ 486 \end{array}$	$\begin{array}{r} 481 \\ 484 \end{array}$	$\begin{array}{r} 737 \\ 740 \end{array}$	$\begin{array}{r} 743 \\ 742 \end{array}$	$\begin{array}{r} 993 \\ 996 \end{array}$	$\begin{array}{r} 999 \\ 998 \end{array}$	$\begin{array}{r} 15 \\ 12 \end{array}$	$\begin{array}{r} 14 \\ 9 \end{array}$	$\begin{array}{r} 271 \\ 268 \end{array}$	$\begin{array}{r} 265 \\ 270 \end{array}$

V

$\begin{array}{r} 736 \\ 733 \end{array}$	$\begin{array}{r} 730 \\ 731 \end{array}$	$\begin{array}{r} 992 \\ 989 \end{array}$	$\begin{array}{r} 986 \\ 987 \end{array}$	$\begin{array}{r} 2 \\ 5 \end{array}$	$\begin{array}{r} 3 \\ 8 \end{array}$	$\begin{array}{r} 258 \\ 261 \end{array}$	$\begin{array}{r} 264 \\ 259 \end{array}$	$\begin{array}{r} 514 \\ 515 \end{array}$	$\begin{array}{r} 520 \\ 517 \end{array}$
$\begin{array}{r} 984 \\ 981 \end{array}$	$\begin{array}{r} 978 \\ 979 \end{array}$	$\begin{array}{r} 34 \\ 37 \end{array}$	$\begin{array}{r} 35 \\ 40 \end{array}$	$\begin{array}{r} 250 \\ 253 \end{array}$	$\begin{array}{r} 256 \\ 251 \end{array}$	$\begin{array}{r} 506 \\ 507 \end{array}$	$\begin{array}{r} 512 \\ 509 \end{array}$	$\begin{array}{r} 728 \\ 725 \end{array}$	$\begin{array}{r} 722 \\ 723 \end{array}$
$\begin{array}{r} 26 \\ 29 \end{array}$	$\begin{array}{r} 27 \\ 32 \end{array}$	$\begin{array}{r} 242 \\ 245 \end{array}$	$\begin{array}{r} 248 \\ 243 \end{array}$	$\begin{array}{r} 498 \\ 499 \end{array}$	$\begin{array}{r} 504 \\ 501 \end{array}$	$\begin{array}{r} 760 \\ 757 \end{array}$	$\begin{array}{r} 754 \\ 755 \end{array}$	$\begin{array}{r} 976 \\ 973 \end{array}$	$\begin{array}{r} 970 \\ 971 \end{array}$
$\begin{array}{r} 274 \\ 277 \end{array}$	$\begin{array}{r} 280 \\ 275 \end{array}$	$\begin{array}{r} 490 \\ 491 \end{array}$	$\begin{array}{r} 496 \\ 493 \end{array}$	$\begin{array}{r} 752 \\ 749 \end{array}$	$\begin{array}{r} 746 \\ 747 \end{array}$	$\begin{array}{r} 968 \\ 965 \end{array}$	$\begin{array}{r} 962 \\ 963 \end{array}$	$\begin{array}{r} 18 \\ 21 \end{array}$	$\begin{array}{r} 19 \\ 24 \end{array}$
$\begin{array}{r} 482 \\ 483 \end{array}$	$\begin{array}{r} 488 \\ 485 \end{array}$	$\begin{array}{r} 744 \\ 741 \end{array}$	$\begin{array}{r} 738 \\ 739 \end{array}$	$\begin{array}{r} 1000 \\ 997 \end{array}$	$\begin{array}{r} 994 \\ 995 \end{array}$	$\begin{array}{r} 10 \\ 13 \end{array}$	$\begin{array}{r} 11 \\ 16 \end{array}$	$\begin{array}{r} 266 \\ 269 \end{array}$	$\begin{array}{r} 272 \\ 267 \end{array}$

VI

$\frac{825}{828} \mid \frac{831}{830}$	$\frac{41}{44} \mid \frac{47}{46}$	$\frac{303}{300} \mid \frac{302}{297}$	$\frac{559}{556} \mid \frac{553}{558}$	$\frac{775}{774} \mid \frac{769}{772}$
$\frac{73}{76} \mid \frac{79}{78}$	$\frac{295}{292} \mid \frac{294}{289}$	$\frac{551}{548} \mid \frac{545}{550}$	$\frac{767}{766} \mid \frac{761}{764}$	$\frac{817}{820} \mid \frac{823}{822}$
$\frac{287}{284} \mid \frac{286}{281}$	$\frac{543}{540} \mid \frac{537}{542}$	$\frac{799}{798} \mid \frac{793}{796}$	$\frac{809}{812} \mid \frac{815}{814}$	$\frac{65}{68} \mid \frac{71}{70}$
$\frac{535}{532} \mid \frac{529}{534}$	$\frac{791}{790} \mid \frac{785}{788}$	$\frac{801}{804} \mid \frac{807}{806}$	$\frac{57}{60} \mid \frac{63}{62}$	$\frac{319}{316} \mid \frac{318}{313}$
$\frac{783}{782} \mid \frac{777}{780}$	$\frac{833}{836} \mid \frac{839}{838}$	$\frac{49}{52} \mid \frac{55}{54}$	$\frac{311}{308} \mid \frac{310}{305}$	$\frac{527}{524} \mid \frac{521}{526}$

VII

$\frac{832}{829} \mid \frac{826}{827}$	$\frac{48}{45} \mid \frac{42}{43}$	$\frac{298}{301} \mid \frac{299}{304}$	$\frac{554}{557} \mid \frac{560}{555}$	$\frac{770}{771} \mid \frac{776}{773}$
$\frac{80}{77} \mid \frac{74}{75}$	$\frac{290}{293} \mid \frac{291}{296}$	$\frac{546}{549} \mid \frac{552}{547}$	$\frac{762}{763} \mid \frac{768}{765}$	$\frac{824}{821} \mid \frac{818}{819}$
$\frac{282}{285} \mid \frac{283}{288}$	$\frac{538}{541} \mid \frac{544}{539}$	$\frac{794}{795} \mid \frac{800}{797}$	$\frac{816}{813} \mid \frac{810}{811}$	$\frac{72}{69} \mid \frac{66}{67}$
$\frac{530}{533} \mid \frac{536}{531}$	$\frac{786}{787} \mid \frac{792}{789}$	$\frac{808}{805} \mid \frac{802}{803}$	$\frac{64}{61} \mid \frac{58}{59}$	$\frac{314}{317} \mid \frac{315}{320}$
$\frac{778}{779} \mid \frac{784}{781}$	$\frac{840}{837} \mid \frac{834}{835}$	$\frac{56}{53} \mid \frac{50}{51}$	$\frac{306}{309} \mid \frac{307}{312}$	$\frac{522}{525} \mid \frac{528}{523}$

VIII

$\frac{81}{84} \mid \frac{87}{86}$	$\frac{337}{340} \mid \frac{343}{342}$	$\frac{599}{596} \mid \frac{598}{593}$	$\frac{615}{612} \mid \frac{609}{614}$	$\frac{871}{870} \mid \frac{865}{868}$
$\frac{329}{332} \mid \frac{335}{334}$	$\frac{591}{588} \mid \frac{590}{585}$	$\frac{607}{604} \mid \frac{601}{606}$	$\frac{863}{862} \mid \frac{857}{860}$	$\frac{113}{116} \mid \frac{119}{118}$
$\frac{583}{580} \mid \frac{582}{577}$	$\frac{639}{636} \mid \frac{633}{638}$	$\frac{855}{854} \mid \frac{849}{852}$	$\frac{105}{108} \mid \frac{111}{110}$	$\frac{321}{324} \mid \frac{327}{326}$
$\frac{631}{628} \mid \frac{625}{630}$	$\frac{847}{846} \mid \frac{841}{844}$	$\frac{97}{100} \mid \frac{103}{102}$	$\frac{353}{356} \mid \frac{359}{358}$	$\frac{575}{572} \mid \frac{574}{569}$
$\frac{879}{878} \mid \frac{873}{876}$	$\frac{89}{92} \mid \frac{95}{94}$	$\frac{345}{348} \mid \frac{351}{350}$	$\frac{567}{564} \mid \frac{566}{561}$	$\frac{623}{620} \mid \frac{617}{622}$

IX

$\begin{array}{r l} 88 & 82 \\ \hline 85 & 83 \end{array}$	$\begin{array}{r l} 344 & 338 \\ \hline 341 & 339 \end{array}$	$\begin{array}{r l} 594 & 595 \\ \hline 597 & 600 \end{array}$	$\begin{array}{r l} 610 & 616 \\ \hline 613 & 611 \end{array}$	$\begin{array}{r l} 866 & 872 \\ \hline 867 & 869 \end{array}$
$\begin{array}{r l} 336 & 330 \\ \hline 333 & 331 \end{array}$	$\begin{array}{r l} 586 & 587 \\ \hline 589 & 592 \end{array}$	$\begin{array}{r l} 602 & 608 \\ \hline 605 & 603 \end{array}$	$\begin{array}{r l} 858 & 864 \\ \hline 859 & 861 \end{array}$	$\begin{array}{r l} 120 & 114 \\ \hline 117 & 115 \end{array}$
$\begin{array}{r l} 578 & 579 \\ \hline 581 & 584 \end{array}$	$\begin{array}{r l} 634 & 640 \\ \hline 637 & 635 \end{array}$	$\begin{array}{r l} 850 & 856 \\ \hline 851 & 853 \end{array}$	$\begin{array}{r l} 112 & 106 \\ \hline 109 & 107 \end{array}$	$\begin{array}{r l} 328 & 322 \\ \hline 325 & 323 \end{array}$
$\begin{array}{r l} 626 & 632 \\ \hline 629 & 627 \end{array}$	$\begin{array}{r l} 842 & 848 \\ \hline 843 & 845 \end{array}$	$\begin{array}{r l} 104 & 98 \\ \hline 101 & 99 \end{array}$	$\begin{array}{r l} 360 & 354 \\ \hline 357 & 355 \end{array}$	$\begin{array}{r l} 570 & 571 \\ \hline 573 & 576 \end{array}$
$\begin{array}{r l} 874 & 880 \\ \hline 875 & 877 \end{array}$	$\begin{array}{r l} 96 & 90 \\ \hline 93 & 91 \end{array}$	$\begin{array}{r l} 352 & 346 \\ \hline 349 & 347 \end{array}$	$\begin{array}{r l} 562 & 563 \\ \hline 565 & 568 \end{array}$	$\begin{array}{r l} 618 & 624 \\ \hline 621 & 619 \end{array}$

X

(五) 3维10阶幻方性质:

1. 全体数为自然数截段:

$$\{1, 2, \dots, 10^3\}$$

2. 特征数为: 5005, 即每行、每列、每竖的各数之和及每条对角线上各数之和都等于5005。

3. 其中两条大对角线上各数为:

$$\textcircled{1} 377 + 379 + 943 + 944 + 503 + 501 + 57 + 59 + 623 + 619 = 5005$$

$$\textcircled{2} 918 + 914 + 710 + 712 + 500 + 498 + 289 + 290 + 86 + 88 = 5005$$

参 考 资 料

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