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Review

A survey on non-linear optimization problems in wireless sensor networks

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ABSTRACT

Wireless Sensor Networks (WSN) pose several problems in terms of scalability and energy consumption. Many of them have been treated in the literature by means of optimization techniques. Depending on the optimization goal and constraints, a great number of these problems are solved using Linear Programming (LP) techniques whenever possible or the network model is further simplified to fit into this category, albeit accuracy may be degraded. In many other cases, simplifying the network model is not feasible and optimization requires more complex techniques. For these cases, non-linear optimization becomes an interesting alternative. Non-linear optimization is beneficial provided that more accurate results are needed or linear optimization is not achievable or yields unreliable results. This paper surveys recent and significant contributions regarding non-linear optimization problems in WSN, selecting the most relevant objectives to optimize and classify the problems under consideration into several categories. Furthermore, this work provides insights in many important problems requiring non-linear optimization and, finally, discusses current open questions in this area.

1. Introduction

The Internet of Things (IoT) paradigm and the cost reduction of wireless sensor technology have driven the popularization of Wireless Sensor Networks (WSN). Deployment of large WSN face non-trivial problems such as lifetime or capacity. In order to deal with these problems many authors have proposed different algorithms and optimization frameworks to solve routing, energy consumption or node deployment, among others. Regarding WSN optimization, there is a huge variety of problems in the literature which, can be broadly categorized into linear and non-linear optimization problems.

A WSN can be defined as a large number of connected nodes sensing some type of physical magnitude (Akyildiz et al., 2002). In these networks, nodes position does not need to be predetermined for a great number of applications. This fact, allows a random deployment of the nodes but requires network protocols to have self-organizing capabilities similar to those of wireless ad-hoc networks. However, classical protocols for wireless ad-hoc networks are not well suited for WSN. To illustrate this, some key differences between ad-hoc networks and WSN are (He et al., 2003; Bai et al., 2006):

- The number of nodes in a sensor network can be of several orders of magnitude higher than in an ad-hoc network.
- Sensor nodes are prone to failures.
- Sensor nodes are limited in power, computational capabilities, and memory.

In the related literature, almost all WSN assume that there are only a single sink per WSN (Buratti et al., 2009) (Fig. 1). This scenario suffers from lack of scalability because increasing the number of nodes has a significant impact on parameters such as throughput, network lifetime or routing. In a more general scenario, a WSN may contain more than one sink (Fig. 2). This approach provides some benefits such as less routing load because a sink will be closer with higher probability what will reduce the average number of packets that a node may have to route. As a consequence, with more sinks it is expected to obtain longer network lifetimes. However, including more sinks requires more complex protocols to be dealt with; for instance, sink selection.

Optimization techniques at different design levels assist designers in meeting application requirements. WSN optimization techniques can be generally classified into static or dynamic (Zarrabi et al., 2011). Static optimizations deal with WSN at deployment time and remain fixed for the overall WSN's lifetime. Whereas static optimizations are suitable for stable/predictable applications, they are inflexible and do not adapt to changing application requirements and environmental stimuli. Dynamic optimizations, instead, provide more flexibility by

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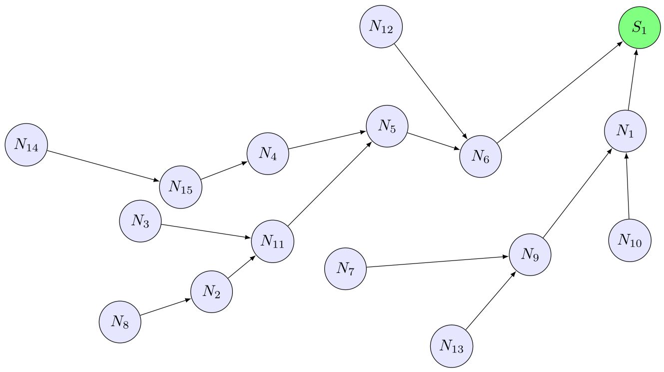


Fig. 1. Example of a WSN deployment with only one sink (S_1).

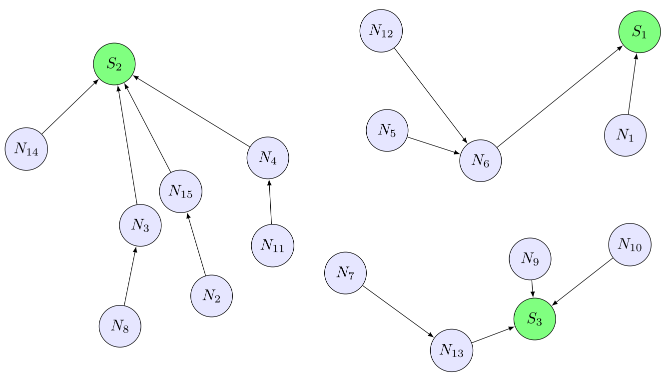


Fig. 2. Example of a WSN deployment with several sinks (S_1, S_2, and S_3).

continuously optimizing a WSN/sensor node during runtime, providing better adaptation to changing conditions and ongoing environmental stimuli (e.g. WSN using renewable energy sources) (Abu-Baker et al., 2010).

There are several works in literature dealing with different aspects of WSN, including their purpose and applications (Iqbal et al., 2016, 2015; Martinez et al., 2004; Rashid and Rehmani, 2016; Amjad et al., 2016). However, WSN pose many concerns that require the definition of different optimization problems. Many of these problems are linear and can be derived by using classical methods. However, linear optimization limits the type of problems that can be solved or the complexity of the models that can be studied. Some problems, such as node localization in WSN or node deployment are modeled by non-linear function goals or constraints. Traditional solutions consist of approaching these problems by linear versions which offer suboptimal solutions. The main reason for this simplification lays in the computation power requirements to solve these problems. The improvement on computational power and the appearance of new algorithms has allowed to formulate new WSN problems with non-linear features. So, non-linear optimization approaches are, a priori, the best fit to the WSN context. This paper surveys the recent and significant contributions on non-linear optimization problems for WSN, classifying them into four categories: (1) node localization, (2) nodes positioning accuracy, (3) energy saving, and (4) routing and achievable bitrate. These are the most usual and demanded goals to optimize in the formulation of the non-linear optimization problems, and their constraints may contain one (or some) of the remaining categories. Furthermore, this work presents some of the issues that still require further research. Another type of optimization problems that recently drawn the attention of the research community is related to cognitive radio sensor networks (CRSN), where it is studied the optimization of the radio spectrum re-use. Relevant works, in the field, including surveys, can be used as a starting point to gain knowledge in this emerging technology (Akan et al., 2009; Akyildiz et al., 2008; Bukhari et al., 2016; Ahmad et al., 2015). However, the employment of cognitive radio in WSN blurs the key idea of simple sensor nodes, since they increase their implementation complexity, energy waste, and consequently cost (magnified in addition by the extremely high number of nodes expected in those networks). This together with the specific treatment of the CRSN in the related literature as a new technology that undoubtedly has its own identity, make that we consider their study out of the scope of this paper.

The rest of this paper is organized as follows. Section 2 introduces some of the most common optimization problems and solvers in WSN. Sections 3–6 present a survey of non-linear optimization problems in WSN categorized according to their respective optimization goal and discuss the state of the art of current non-linear optimization approaches in WSN, their solvers and open questions. Finally, Section 7 concludes.

2. WSN optimization

When optimizing WSN, it is necessary to consider the problem formulation itself, and the solving method that will be applied. A type of problem and the selected solver will depend on factors such as the optimization goal or network model.

- *WSN optimization problems*. In the context of WSN, optimization can be applied at different design levels. This approach helps meet custom requirements which depend on the purpose of the application (Munir and Gordon-ross, 2010). Most approaches in the literature focus on data link-level and network-level optimizations defining MAC (Medium Access Control) and routing protocols, respectively. MAC protocols typically target load balancing, throughput and energy consumption optimizations, whereas routing protocols usually address query dissemination, data delivery delays, and network topologies.

  Apart from network protocols, a different type of problem is optimal sensor deployment (Gogu et al., 2012). The WSN deployment problem goal is to minimize the number of sensors in an area of interest while ensuring the full coverage and connectivity. As explained in Gogu et al. (2012), this problem is NP-hard (no polynomial hard). Once the sensors are deployed, coverage describes how well the sensors observe their inspection area or certain moving targets within it. In this context, nodes need to know the path that minimizes the maximum distance between every point on the path and its nearest sensor node; i.e. it represents the shortest path connecting to end points, which is also NP-hard.
- *WSN optimization solvers*. There are many general strategies to deal with optimization problems and some others explicitly designed for WSN. Among the latter, in Fischione et al. (2009) and Fischione and Jonsson (2011), authors proposed an optimization approach designed for WSN. They classified these problems into a new category denoted F-Lipschitz, which is claimed to be fast and based on a Lipschitz property of the constraints. In particular, the optimization problem requires the objective function to be increased and the constraints transformed into contractive Lipschitz functions. By casting a WSN optimization problem as an F-Lipschitz problem, authors showed that this form is robust to quantization errors and not sensitive to perturbation of the constraints, which is quite important for WSN having low computational precision. Moreover, F-Lipschitz optimization solves problems traditionally approached by Lagrangian methods more efficiently (a Lagrangian method is a problem transformation that redefines the objective function using the problem constraints and allows determining how much these the optimum is affected by each of the problem constraints). Among others, the authors proposed F-Lipschitz to distributed detection and radio power allocation.

  Other common non-linear problem solvers are genetic algorithms (GA) (Tan et al., 2011). A GA is a global search algorithm that mimics the process of the natural selection using techniques inspired by genetic mechanisms. The optimization problem can be modeled accordingly based on its biological environment. Each chromosome in a population represents a possible solution of the optimization problem. After endless rounds of evolution, the weak chromosomes are discarded, leaving only the fit and the strong chromosomes in the population. An example of GA optimization is found in Wong et al. (2012), in which authors introduced an optimization framework for distributed and collaborative beamforming in WSN. The objective of the optimization is to reduce the sidelobe level. In this case, the complexity of the objective function and the system constraints make the utilization of GA a reasonable option.

Bio-inspired intelligent optimization has also been proposed in the WSN arena (Jabbar et al., 2013). Applied to WSN, bio-inspired optimization can be understood as surviving of the best. One of these frameworks is Ant Colony Optimization, which is inspired by the behavior of ants. Thus, if ants are considered vertices then, the pheromone trails between them represent the edges forming a graph theory problem. Due to the decentralized and distributed nature of ants in foraging between the source and destination, the algorithmic complexity of their foraging behavior (routing) turns to the worst case of $O(2^n)$, i.e. an NP-hard problem.

Particle Swarm Optimization (PSO) (Kulkarni and Venayagamoorthy, 2011; Kennedy and Eberhart, 1997) is a population-based search problem where each particle is defined by a potential solution of a problem in a D-dimensional space. With the ith particle represented as $X_i = (X_{i1}, X_{i2}, \ldots, X_{iD})$, each particle adjusts its position close to the optimal according to its own experience and that of neighboring particles. There is a large family of useful PSO-based algorithms to solve non-convex non-linear optimization problems (a function is considered convex in an interval if it is differentiable within the interval and has only a local minimum).

Many solvers, as it will be indicated later in this paper, are based on ad-hoc solutions for custom WNS problems. Although this may provide good performance, the lack of generality is a major issue because a big effort is required to solve a single problem.

Non-linear optimization has been applied to WSN problems in which linear approaches obtain solutions much less precise or unreachable. Under this premise, in the following sections, we highlight those valuable papers that offer a non-linear optimization-based technique which better fits the set of proposed defined categories (localization, positioning, energy and routing). Fig. 3 shows an overview including these categories, their specific objective functions and a set of well-known nonlinear techniques to solve the optimization problem.

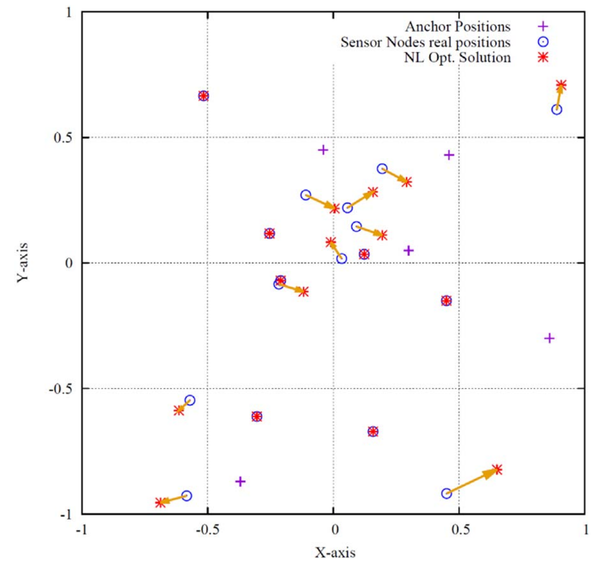


Fig. 4. Comparison of a real WSN deployment and the estimated locations using an NL Optimization technique (Wang et al., 2012). The orange arrows show the deviation between the real and theoretical results.

3. Non-linear node localization optimization

Most applications in WSN such as target tracking, geographical-based routing or data aggregation require to learn about the physical location (geometrical coordinates) of nodes that a priori not know their position in the deployment area. In this sense, Fig. 4 shows a WSN deployment example that could be improved for instance, for tracking by using NL optimization, enabling the estimation of the location of each node more precisely. Although there are linear solutions to solve these type of problems, non-linear ones have demonstrated to provide a much better accuracy at the cost of requiring more computations. Special attention is paid to the advantages and shortcomings of the different proposals reviewed, although they all share two common starting points. Firstly, irrespective of the chosen algorithm, all proposals group nodes into two categories according to whether they have or they do not have anchor nodes:

1. *Anchor nodes*. They are aware of their position in the network and, thus, can be used by other nodes as a reference for position estimation.

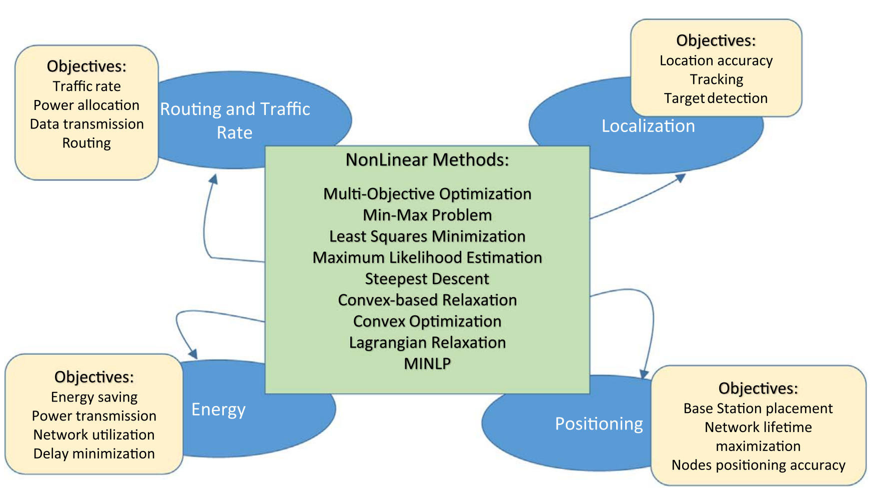


Fig. 3. Overview of categories for non-linear optimization problems: Object functions and nonlinear techniques.

2. *Non-anchor (sensor) nodes*. These nodes are not aware of their position and some method for their calculation is required.

Secondly, the standard formulation of a node localization problem is given by:

$$\min_{\hat{x}}\left\{J_N = \sum_{k=1}^{M}\sum_{j\in S_k}(\hat{d}_{kj}-\tilde{d}_{kj})^2 + \sum_{i=1}^{N}\sum_{j\in S_i}(\hat{d}_{ij}-\tilde{d}_{ij})^2\right\}, \tag{1}$$

where N is the number of non-anchor nodes, M is the number of anchor nodes, J_N is the estimated aggregated localization error, $\hat{d}_{kj}$ denotes the real distance between arbitrary nodes k and j, $\tilde{d}_{jk}$ is the estimated distance between nodes k and j, and S_k and S_i are the set of anchor and non-anchor nodes, respectively.

3.1. Classification

Studying deeply each one of these proposals, they can be classified into four main groups: (i) review of algorithms and techniques to improve the node localization precision in WSN; (ii) localization based on geometrical distances; (iii) localization based on received signal strength (RSS), and (iv) studies focused on the target detection and tracking.

Regarding the first category (i), in Niewiadomska-Szynkiewicz et al. (2011), authors presented a brief survey of non-convex heuristic optimization techniques to address the location concern in WSN. In particular, this survey describes and argues solutions, further dividing them into three types: (1) localization techniques such as the geometry-based ones, (2) location systems with heuristic optimization such as simulated annealing based systems, genetic algorithms or evolutionary strategy based systems, and additionally (3) hybrid solutions, combining schemes (1) and (2). The authors also designed and evaluated two hybrid schemes which fuse iterative multitrilateration together with some of the previous non-convex optimization techniques. Results proved that, in a large-scale WSN, the hybrid solution based on multitrilateration and simulated annealing results in an efficient and robust localization method. In this same group, we should highlight the reference Han et al. (2013) which furnishes a wide study referred to localization algorithms, reclassifying them as a function of the mobility of anchor/non-anchor nodes. Unlike our work focused on non-linear optimization solutions for nodes location accuracy, the work in Han et al. (2013) is a survey that analyzes location algorithms published until the year 2011 regardless of the mathematical technique, simulation framework or experimental test-bed employed for designing, implementing and validating each one of these algorithms.

Several studies dealt with the category numbered above as (ii). For instance, for large-scale WSN in outdoor scenarios, the work presented in Wang et al. (2012) introduces a mobile node to assist the localization of unknown nodes. To accomplish it, each unknown node receives at least three messages denoted as virtual beacons from the mobile node and at different positions. A cost function based on a hyper spherical cross mechanism achieves node self-positioning, in order to later discover the node placement through an algorithm proposed by the authors and modeled by a non-linear least squares method. The results showed a better behavior with respect to other well-known algorithms such as the Weighted-Centroid or Constraint, although authors did not justify the reason why these traditional algorithms are the most appropriate to be compared with. Authors of Chehri et al. (2008) described a non-linear optimization algorithm for node geo-location in WSN. It uses a gradient descent as well as non-linear least-squares optimization to attain a function minimum. Despite the fact that it performs better than other procedures consisting of finding intersection circles or hyperbolas, it is also true that it has a centric architecture which may lead to scalability concerns for large networks.

In Jiang et al. (2009), the non-linear least squares technique is planned to simplify a maximum likelihood estimation problem but, in this case, this optimization deteriorates further the node placement accuracy. In this study, two new algorithms were designed and implemented to reduce computation requirements while keeping the same location accuracy. The first one is a modification of the traditional Steepest Descent method and the latter is based on the well-known quasi-Newton method. Authors analyzed both algorithms by comparing them against traditional methods, verifying that (1) they enhance the location precision and (2) require much less computational power. However, even with a lower computational cost, these algorithms also suffer from scalability issues. In Gopakumar and Jacob (2009), a heuristic is implemented to solve the localization problem in a distributed WSN. In this work, authors developed an algorithm based on particle swarm optimization considering the noisy distance measurement in its formulation. Results demonstrated its localization accuracy by comparing with another study based on simulated annealing.

Currently, an interesting approach to enhance the localization precision is the use of convex relaxation. In Ji et al. (2013) a localization problem was formulated as a rank-constrained semidefinite program (SDP), where the rank corresponds to the goal dimension in which devices should be placed. To accurately derive the node positions and, therefore, to enhance the expectations of the fixed-dimensional localization problem, authors proposed a non-convex rank constraint named Schatten quasinorm, which is replaced by a convex surrogate, resulting in a convex optimization problem. This is solved by an interior-point algorithm. In Simonetto and Leus (2014), authors concentrated their effort on proposing a convex relaxation program based on maximum likelihood (ML) formulation to massage the non-convex problem. Once the problem was formulated, authors contributed, first, deriving an edge-based approach of ML convex relaxation to lessen the computational requirements. Second, by designing a distributed algorithm to solve this edge-based convex approach. In both papers (Ji et al., 2013 and Simonetto and Leus, 2014), numerical results reflected that the authors' solution improved localization accuracy, overcoming standard convex relaxations.

The third group (iii) concentrates on the work improving the estimation accuracy of node positions by exploiting the benefits of the RSS measurement. Following with convex relaxation, in Tomic et al. (2015), authors used an array of passive anchor nodes to collect the noisy RSS measurement from the remaining network nodes and, therefore, to estimate their position. To this end, first, authors calculated the maximum likelihood estimator which may have multiple local optima, making difficult to achieve a global solution. To mitigate this, they derived a new non-convex estimator that approximates the previous estimator for low noise environments. In a next step, authors relaxed this non-convex estimator even more by applying convex relaxation techniques. The resulting convex model was intensely checked and results were extensively validated by comparing with other approaches. In Lei et al. (2010), a localization algorithm based on Particle Swarm Optimization (PSO) and a mobile anchor is proposed. In detail, the region to monitor is divided into grids and the mobile anchor deploys virtual anchors on the vertex of each grid. The result is a non-linear optimization problem and PSO is the solving method. On the emerging field of the device-free localization (DFL), authors in Xiao et al. (2015) formulated a non-linear optimization problem, identifying those links that really affect the target localization and omitting links associated with large noise that are far away from the target and significantly degrade the accuracy of the localization procedure. To detect what links are dispensable, authors employed the RSS measurement estimated at the receiver. Finally, the proposed model whose constraints and objective function are formulated by geometrical aspects is solved by convex optimization. Additionally, to compare the results of the authors' proposal with other approaches, authors developed a test-bed which allowed to contrast the theoretical results with real experiments, validating their solution.

A variation of node localization is the possibility of accomplishing services as target detection or tracking, which often require non-linear optimization techniques. The fourth category (iv) comprises proposals leading to tackle this aspect. In Mansouri et al. (2009), authors proposed a quantized variational filtering (QVF) for target tracking. QVF is used to predict a node's position at a sampling instant. Based on this data, the optimization consisted of minimizing the power transmission and the path loss. Even if authors claimed QVF is simple, its more important drawback is that it is centralized and presents scalability concerns. For large-scale WSN employing data fusion, the work proposed in Yuan et al. (2008) showed, on the one hand, the relevance of placing nodes at optimal locations to enhance maximum target detection and, on the other hand, the high computational cost to model this problem under non-linear optimization. To face it, authors presented three algorithms modeled by a probabilistic data fusion method. Authors validated their approaches by comparing against a greedy algorithm and through numerical and simulation results using real data traces. In Kerse et al. (2013), authors separated the target tracking into two problems: (i) initial localization where the target locations are estimated employing a standard technique for sparse system identification called mixed-norm minimization, and (ii) pure tracking using a stochastic gradient descent to solve the non-linear optimization problem. Authors assessed their proposal in a network of few nodes, although without validating its feasibility by comparing with other well-established work. These drawbacks are also detected in Liu et al. (2014), where authors solved the nonlinearity using an interior-point algorithm.

3.2. Discussion and future research lines

In Table 1, we condense the main features of the works described in Section 3.1. For a better understanding of the concept referred to the localization accuracy included as feature within this table, Figs. 5 and 6 show two of the possible scenarios (Medium and High) in accordance with the efficiency of the NL Optimization technique. Additionally to Table 1 and Figs. 5 and 6, in the following paragraphs, we discuss those relevant aspects which may help the specialized audience to further learn about (i) the pros/cons of the most interesting non-linear optimization methods, (ii) the influence of physical features/phenomena on mathematical models, and (iii) the crucial requirement of modeling the node power consumption.

- One of the current trends is to solve the sensor network localization problem through equations generating a convex-based relaxation model (Ji et al., 2013; Simonetto and Leus, 2014; Tomic et al., 2015). As argued in Simonetto and Leus (2014), the results are checked by comparing with other traditional techniques (labeled as Results Check in Table 1) and show a localization precision (localization accuracy label) of the devices much better than these techniques with a reasonable computational cost (localization time label). However, as a weakness of these studies, it should be noted that the energy waste that implementing these techniques implies has not been assessed.
- Heuristic Optimization by Simulated Annealing (Niewiadomska-Szynkiewicz et al., 2011) or Probabilistic Data Fusion (Yuan et al., 2008) techniques achieve, in their corresponding work, acceptable optimum values for the proposed objectives, employing a moderate computational cost. In particular, the model designed in Yuan et al. (2008) stands out for its robust analytical solution for target detection applications in WSN, additionally strengthened by a complete set of experiments accomplished in a real test-bed.
- Unlike aforementioned methods, solutions based on traditional techniques as Non-linear Least Squares Minimization (Wang et al., 2012; Chehri et al., 2008), Maximum Likelihood Estimation (Jiang et al., 2009) or Fitness function (Lei et al., 2010), model the location optimization problem more easily, therefore involving a shorter time in reaching the nodes positioning. However, the obtained results reveal lower accuracy than, for instance, the techniques used in papers (Simonetto and Leus, 2014) or (Niewiadomska-Szynkiewicz et al., 2011).
- As indicated in Jiang et al. (2009), to improve the location accuracy feature, constraints modeling transmission range, coverage radio or the variation of them due to, for instance, the noise, must be considered in the optimization problem. The details of this issue can be found in Chehri et al. (2008) where authors emphasized the existence of a correlation between these features/phenomena and the accuracy of localization. So, a set of constraints including the aforementioned features/phenomena are formulated again in the corresponding mathematical models of, on the one hand, the target detection and tracking studies (Lei et al., 2010; Yuan et al., 2008; Kerse et al., 2013) and, on the other hand, the RSS-based location works (Tomic et al., 2015; Lei et al., 2010; Xiao et al., 2015).

Concerning future research, RSS-based location optimization whose constraints include: (i) features such as radio propagation model, noise or transmission power, and (ii) anchors position represents a complete solution that overcomes studies only supported by geometrical or noise constraints. However, this RSS-based work lacks of a thorough energy study (only the work in Mansouri et al., 2009 optimizes together node localization and energy). The node power consumption is a crucial requirement in the design of a WSN because often the network lifetime is the ultimate figure of merit that makes possible or restricts the use of a WSN. It is therefore reasonable to think that to achieve a complete non-linear optimization solution for nodes location accuracy, target detection or tracking is essential to model the node energy waste and include it in the analytical problem formulation.

4. Non-linear nodes positioning accuracy optimization

This subsection describes the most relevant work which providing appropriate nodes positioning accuracy using non-linear optimization techniques. It updates the positioning of existing network nodes or provides an optimal location to new devices with the goal of, for instance, reaching the maximum range in tasks of sensing monitoring and collecting data within an area of interest, thus improving, if possible, metrics such as the bitrate or network lifetime. In the survey in Tennina et al. (2008), authors accomplished an interesting comparative analysis of diverse optimization techniques (including non-linear models). Namely, these techniques are: Triangulation, Steepest Descent, Non-Linear Least Squares, Conjugate Gradient, and an enhanced version of the Steepest Descent, which is the main contribution of this study. The numerical results were calculated in terms of the positioning error (mean and standard deviation) and computation time, showing that (1) Non-Linear Least Squares and Steepest Descent obtain the lowest error, (2) Conjugate Gradient is the technique that reaches the solution faster, while (3) the enhanced Steepest Descent achieves results similar to Non-Linear Least Squares and Steepest Descent, but with less computational effort. Finally, (4) the Triangulation method provides the worst performance in terms of error accuracy. As a weakness, the topology selected on the numerical study is simply composed by four nodes connected to each other.

4.1. Classification

Analyzing in detail each one of these proposals, they can be divided into two main categories: (i) optimal location of base stations with the main goal of reducing the network power consumption, and (ii) minimization of the positioning error for all network nodes.

There are also some works that, rather than focus on nodes positioning address optimal location of base stations, which we have grouped into the first category. In Zadeh (2010), a framework to optimally place a base station within a WSN is proposed. The author modeled the problem as a non-linear least squares minimization

Table 1
Node Localization Studies Comparison under non-linear optimization techniques.

Paper	Year	Objective	Non-linear parameter	Planned non-linear method	Approach	Results check	Topology	Localization accuracy	Localization time	Energy consumption
Jiang et al. (2009)	2009	Location Accuracy	Node Localization (geometry)	Maximum Likelihood Estimation/ Least Squares Solution /Steepest Descent Method	Custom Algorithm	No	All	Medium/High	Low	N/A
Wang et al. (2012)	2012	Location Accuracy	Energy	Non-linear Least Squares Minimization	Divide and Conquer Algorithm	Yes	Tree	Medium/High	N/A	N/A
Yuan et al. (2008)	2008	Target Detection	Energy/Noise	Probabilistic Data Fusion Method	Custom Algorithm	No	Cluster	High	Low	N/A
Niewiadomska-Szynkiewicz et al. (2011)	2011	Survey/ Location Accuracy	Node Localization (geometry)	N/A	Heuristic Optimization by Simulated Annealing	Yes	All	High	Medium	N/A
Simonetto and Leus (2014)	2015	Location Accuracy	Node Localization (geometry)	Maximum Likelihood Estimation/ Convex-based Relaxation	Custom Algorithm based on Alternating Direction Method of Multipliers	Yes	All	Very High	Low	N/A
Ji et al. (2013)	2013	Location Accuracy	Node Localization (geometry)	Convex-based Relaxation	Interior Point Algorithm	Yes	All	High	N/A	N/A
Chehri et al. (2008)	2008	Location Accuracy	Node Localization (geometry)	Non-linear Least Squares Minimization	Custom Algorithm based on Differential Evolution	Yes	All	Medium/High	Medium	N/A
Mansouri et al. (2009)	2009	Tracking	Node Localization (geometry)/Energy	N/A	Custom Algorithm	Yes	Cluster	N/A	N/A	Low
Lei et al. (2010)	2010	Location Accuracy	Node Localization/RSSI Measurements	Path Loss Model for Wireless Channel Unknown Environment	Custom Algorithm based on PSO	No	All	Medium	N/A	N/A
Tomic et al. (2015)	2015	Location Accuracy	Node Localization and RSS Measurements	Maximum Likelihood Estimator/ Convex-based Relaxation	Custom Algorithm	Yes	All	High	Low	N/A
Xiao et al. (2015)	2015	Location Accuracy	Node Localization	Convex Optimization	Optimal Point Calculation	Yes	Cluster	High	N/A	N/A
Gopakumar and Jacob (2009)	2009	Location Accuracy	Node Localization (geometry)	N/A	Metaheuristic Algorithm based on Tabu Search and PSO	Yes	All	Medium	Low	N/A
Liu et al. (2014)	2013	Tracking	Node Localization and Tracking Model	Non-myopic Quantizer Design	Interior Point Algorithm	No	All	Medium	N/A	N/A
Kerse et al. (2013)	2013	Tracking	Node Localization and Tracking Model	Mixed-norm Minimization	Gradient Algorithm	Yes	All	Medium	N/A	N/A

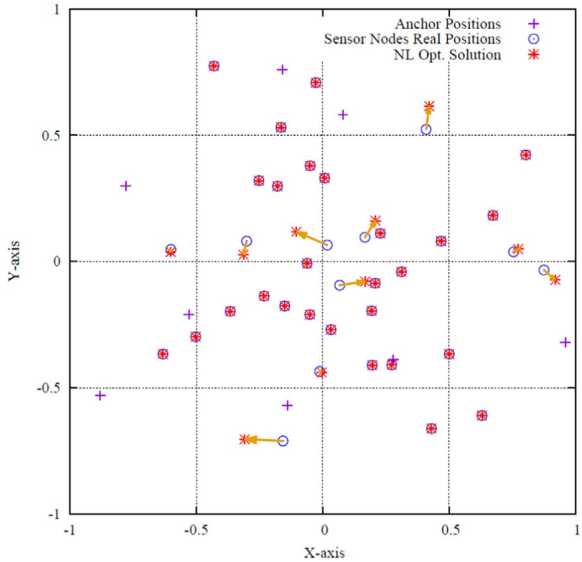


Fig. 5. A *medium* localization accuracy scenario. Comparison between real positions of sensor nodes respect to the theoretical results derived by an NL Optimization.

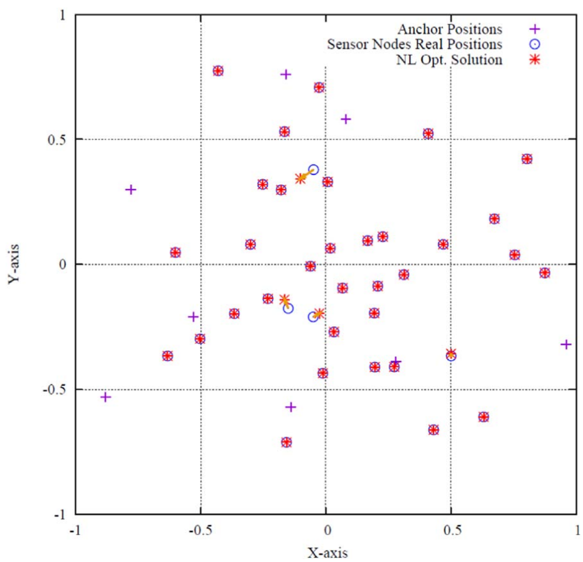


Fig. 6. A *high* localization accuracy scenario. Comparison between real positions of sensor nodes with respect to the theoretical results derived by an NL Optimization.

according to equation (2):

$$\min_{x} \frac{1}{2} \sum_{i=1}^{n} (f_i(x))^2, \tag{2}$$

where the goal is to determine the vector of parameter x in order to minimize the sum of squared residuals, and $f_i(x)$ are given functions that best fit their curves with respect to the known set of points. The main outcomes of Zadeh (2010) are summarized as follows:

- The optimization framework finds the best location for the base station by providing the least total consumed communication energy.
- Optimization can be performed in a distributed fashion through the local interaction between the nodes without the need of a global knowledge.

The work in Zadeh et al. (2012) contributed to enhance the performance of WSN by a metric-aware optimal technique. BS positioning was derived from the available resources, the amount of traffic dispatched by sensor devices and, their operation in a dynamic environment. As an outstanding feature, the WSN is modeled by a non-linear communication channel represented by a path loss exponent greater than two. Under these premises, authors proposed a non-linear least squares curve fitting problem, solved, as a last resort, by a distributed algorithm which also minimizes the WSN energy consumption. Even though authors performed a complete set of tests and simulations offering an exhaustive detail in their results, they are not contrasted against any related work of the scientific literature.

The work in Gu et al. (2013) underlines the design of a mechanism to prolong the large-scale WSN lifetime by optimal positioning of multiple base stations. Additionally to wireless transmission features and routing concerns and, as a strong requirement, authors considered in their analytical model, the fact of the high cost of BS and the difficulty of their placement in geographically constrained fields. To solve this and other aspects, they formulated a mixed integer non-linear programing problem, which is NP-hard. In large-scale WSN, feasible solutions to the base stations positioning problem are reached by means of a heuristic algorithm. In the study developed in Arkin et al. (2014), authors imposed four restrictions to the optimization problem in order to localize the base station and find a transmission scheme, so that the network lifetime is maximized. These four principal restrictions are: (1) low-duty cycle of the WSN nodes to reduce the power consumption, (2) low end-to-end delay of data delivery to achieve time-critical data, (3) clock synchronization to maintain a low duty-cycle, and (4) a tree topology. Authors conducted a thorough study and concluded that concerning tree topologies consisting of three (or more) levels, the solution to the optimization problem is NP-hard, designing in turn an algorithm to solve it. As main shortcoming, the results of this algorithm are not compared with any other study, thus jeopardizing the scope of their contribution.

The second category copes with the objective of increasing the positioning precision (or minimizing the positioning error) of new or existing network nodes. As aforementioned, non-linear least squares is a popular optimization technique employed in the positioning of BS or in facilitating the resolution of optimization problems focused on localization. However, this technique can be also applied to enhancing the nodes positioning accuracy. In this sense, the work in Feng and Zhang (2012) deployed a WSN employing the DV-Hop algorithm, a typical range-free positioning solution, but it is not able to guarantee an appropriate node placement accuracy. To overcome this, authors proposed a non-linear least squares model, which is solved by implementing a modified shuttled frog leaping algorithm. This algorithm combines techniques based on the chaos mapping and Cauchy mutation to optimize the position of WSN devices. So, authors performed different simulations and demonstrated that the average location error is reduced around a 15% in comparison with the original DV-hop algorithm. Nevertheless, the non-linear least squares technique is not always the optimal solution for the nodes positioning in WSN. The calculation of maximum likelihood estimator is also required in Gholami et al. (2011) where the turn-around time is an objective in the optimization problem as well as the nodes positioning. Authors formulated a non-linear and non-convex problem which is simplified by a linear model. Although simulations result in a lower bound value for this estimator, they are not validated by means of other alternative studies or algorithms.

The authors of Ferentinos and Tsiligiridis (2006) introduced an optimization framework to determine where nodes must be placed. Moreover, they proposed a multiobjective optimization in which three different parameters are considered, namely, node density, connectivity, and energy consumption. The complexity of this multiobjective optimization relied on the fact that improving one of the parameters worsens the others. To overcome this, each parameter has an associated weight that indicates its importance. As a consequence, the formulated problem is NP-hard and grows quadratically with the number of nodes. The authors in Bhuiyan et al. (2012) appropriately placed devices in a structural health monitoring system (SHM) by means of optimizing multiple objectives (network lifetime, low communication cost, and fault tolerance). To achieve this goal, authors defined diverse non-lineal contraints such as connectivity, transmission load, and data delivery and designed their own algorithms. In a tree topology, these algorithms distinguish between two types of nodes according to their capabilities: high-end node or base station and low-end nodes. The result was tested in a real environment, i.e. a building using Imote2 devices. Concerning SHM, authors in Li et al. (2010) studied where placing the nodes to best capture the structure properties along with maximizing the durability of the WSN. As contraints, authors modeled the connectivity and the data delivery. Finally, to solve the non-linear optimization problem, authors designed their own algorithms which were validated for any type of topology and were implemented in a well-known communication tower in China using Imote2 nodes.

In Dashtestani et al. (2014), authors derived the optimal nodes positioning in an underwater WSN under a multi-hop hierarchical tree topology. As their main feature, the non-linear optimization problem was converted into a linear programming problem thanks to the fact that it satisfies the Karush-Kuhn-Tucker conditions. Numerical results only showed the performance for a network composed by a few nodes, which raises scalability concerns. Authors of Ngo et al. (2014) explored the feasibility of collaborative beamforming from an airbone wireless sensor network consisting of multirotor aerial vehicles. An effective collaboration of these vehicles must allow a successful communication at a given altitude, thus avoiding the problems due to the noisy surface environment. To this end, two proposals were presented to reduce sidelobes caused by beam pattern fluctuations of the unsteady aerial vehicles in the sky. As the problem posed is non-convex and do not guarantee a global solution, these proposals were further formulated through a set of non-linear constraint functions to minimize a cost function by the reconfiguration of each vehicle position in the 3-D space, thus assuring an upper bounded response for each interfering signal direction. However, although both proposals lead to results in the same range with low computational cost, authors did not compare them with any other study or non-linear optimization technique. Finally, following with 3-D positioning, the work in Hashim et al. (2016) provides an approach with a twofold contribution: (i) to optimize the positioning of relay nodes in a grid topology, and (ii) to extend the network lifetime. The nonlinear optimization problem is formulated through a Laplacian matrix with initial relay nodes positioning and solved by an artificial bee colony algorithm. Authors implemented another well-known technique (the Shortest Path 3-D grid Deployment) to validate their proposal. Results demonstrate the feasibility of the algorithm in dense networks; however the model lacks constraints related to the noise or radio propagation parameters which facilitates its fast convergence.

4.2. Discussion and future research lines

Once the different papers referred in Section 4 and summarized in Table 2 have been thoroughly analyzed, we should highlight and argue about the most remarkable aspects which must be taken into account to place the devices of a scalable WSN optimally. Following these guidelines, it is possible, for instance, to monitor a given area of interest with the optimal number and positioning of devices, thus saving costs and effort. Therefore, the aspects to consider are as follows.

- A Multi-objective Optimization (Ferentinos and Tsiligiridis, 2006) technique approaches a pareto-optimal solution for a set of three objectives. To enhance the optimization process, authors implemented a Genetic Algorithm-based solution which gives sub-optimum values for the proposed objectives. The result is a powerful tool for nodes positioning although the time required to find a solution (labeled as positioning time in Table 2), being reasonable, is higher than other proposals. To solve the Multi-objective problem, authors in Bhuiyan et al. (2012) proposed their own algorithms, highlighting their validation by comparing them with other proposals (as the label Results Check indicated) and testing in a real scenario. Results show a long-term WSN (low energy consumption) where nodes are suitably placed for the expected functionality.
- As would occur in the previous section, well-known techniques as Non-linear Least Squares Minimization (Zadeh et al., 2012), (Feng and Zhang, 2012), Maximum Likelihood Estimation (Gholami et al., 2011), or Steepest Descent (Tennina et al., 2008) reach the nodes positioning faster than, for instance, the mathematical model presented in Ferentinos and Tsiligiridis (2006) at the expense of a lower positioning accuracy.
- Reference Zadeh et al. (2012) is the only work found in the recent scientific literature which satisfies a complete positioning optimization study referred to the set localization precision, computational cost, and energy consumption. However, the results offered in Zadeh et al. (2012) are not contrasted against any other related work or non-linear optimization method. In this sense, the validation of the proposed models through, for instance, a simulation framework is essential for the credibility of the achieved results.
- There are few studies which analyze the energy consumption in their positioning-based proposals motivated by (1) the increase of the complexity both, for the formulation and the non-linear model, and (2) finding a feasible solution including the tradeoff between the network power consumption and nodes positioning accuracy. In WSN, the network energy waste is a fundamental concern to study and evaluate. Furthermore, tree-hierarchical or cluster topologies are very sensitive to the energy consumption because neighbors close to the base station or the cluster heads handle more traffic than the remaining network nodes, and, therefore, their power consumption is clearly higher. This is the reason why studies such as Tennina et al. (2008), Feng and Zhang (2012), Gholami et al. (2011) or Ngo et al. (2014), could accomplish more consistency in their results if an energy study/optimization was furnished.
- Concerning work in Zadeh et al. (2012), Arkin et al. (2014), they reach the best placement for a single Base Station. However, none of them considered the particular case of more than one Base Station. This design feature is satisfied in Gu et al. (2013), contributing to improve, for instance, the scalability and robustness of the WSN.

The Multi-objective Optimization seems to be an excellent solution to achieve a good performance in positioning accuracy. However, the topologies analyzed (cluster and tree) are not conclusive to determine if this type of optimization is appropriate for other complex topologies, such as random or mesh. A future research line must be addressed in this direction to find solutions computationally acceptable for such mesh/random topologies. Furthermore, the energy consumed must be included in the analysis and be part of both, the objective function (minimization) and the constraints.

5. Non-linear energy saving optimization

Energy consumption is one of the most common optimization goals in WSN, since they are typically deployed in large areas where battery replacement is possible only at extreme cost. Furthermore, a long network lifetime is a desirable design goal, and energy optimization can be used as an indirect means for optimizing other parameters such as routing or network traffic. As well it is possible to optimize some network parameter such as packet size to reduce energy consumption (Sankarasubramaniam et al., 2003).

The optimization of WSN traditionally involves the extension of the network lifetime and it is usually derived by a non-linear formulation. A first group of approaches is formed by complex models applied to batteries or energy depletion that introduce additional complexity (in fact, the non-linearity effect) on the problem formulation and there-

Table 2
Node Positioning Studies Comparison under non-linear optimization techniques.

Paper	Year	Objective	Non-linear parameter	Planned non-linear method	Approach	Results check	Topology	Positioning accuracy	Positioning time	Energy consumption
Zadeh et al. (2012)	2012	BS Placement-Network Lifetime Maximization	BS Positioning (geometry)	Non-linear Least Squares Minimization	Custom Algorithm	No	All	Medium/High	Low	Low
Gu et al. (2013)	2013	Network Lifetime Maximization	N/A	Mixed Integer Non-linear Programing	Custom Algorithm based on Heuristic Optimization	Yes	All	Medium/High	Medium	Low
Feng and Zhang (2012)	2012	Nodes Positioning Accuracy	Node Positioning (geometry)	Non-linear Least Squares Minimization	Custom Algorithm based on GA and PSO	No	All	Medium/High	Low	N/A
Arkin et al. (2014)	2014	BS Placement/Network Lifetime Maximization	BS Positioning (geometry)/Energy	NP-Hard	Linear Programming	Yes	Tree	High	N/A	Low
Dashtestani et al. (2014)	2014	Nodes Positioning Accuracy	Energy	Karush-Kuhn-Tucker Conditions	Linear Programming	No	Tree (few nodes)	Medium	N/A	Low/Medium
Gholami et al. (2011)	2011	Nodes Positioning Accuracy	Nodes Positioning (geometry)	Maximum Likelihood Estimation	Linear Programming	No	All	Medium	Low	N/A
Tennina et al. (2008)	2008	Survey/Node Positioning Accuracy	Node Positioning (geometry)	Steepest Descent	Custom Algorithm	Yes	Few nodes	Medium/High	Low	N/A
Ngo et al. (2014)	2014	Nodes Positioning Accuracy in aerial WSN	Beam Pattern	N/A	Interior Point Algorithm	No	N/A	Low/Medium	Low	N/A
Ferentinos and Tsiligiridis (2006)	2006	Cluster Size	Energy	Multi-objective Optimization	Custom Algorithm based on GA	No	Cluster	High	Medium	Low
Bhuiyan et al. (2012)	2012	Nodes Positioning/Lifetime	Energy	Multi-objective Optimization	Authors' Algorithm	Yes	Tree	High	High	Low
Li et al. (2010)	2010	Nodes Positioning/Lifetime	Energy	N/A	Custom Algorithm	Yes	Any	High	Medium	Low
Hashim et al. (2016)	2016	Relay Nodes Positioning/Lifetime	Relay Node Positioning (geometry)	Laplacian Matrix	Artificial Bee Colony	Yes	Grid	High	Low	Low

fore, requiring more computation and processing tasks. In other cases, the energy model is not highly complex and the non-linearity is not given by the energy model. This second group is characterized by the introduction in the problem of multi-objectives (e.g. network utilization) or more precise physical phenomena such as channel capacity or propagation models, which is also a cause of non-linearity.

5.1. Classification

The first group considers that the battery depletion does not occur in a strict instant and the power is continuous and slowly consumed. Therefore, these authors presented complex models for the energy consumption (or the remaining power) which is adjusted more clearly to the real battery behavior. Usually, these models are logarithmic-based formulations, thus resulting in a non-linearity of the problem. Under these conditions, each study exemplifies a reduction of the network power consumption in a different way, which makes their comparison difficult.

The work in Zhang et al. (2009) proposed a node deployment scheme having into account the *current effect*. Based on the battery model, the authors assigned a different communication radio to nodes in a given network. Thus, the energy consumption of each node across the entire network is balanced, and network lifetime maximized. Meanwhile, the battery capacity is also fully utilized by balancing the traffic load among nodes. Lifetime maximization is performed by reducing the transmission power for a specific network topology, a sequence of concentric rings of nodes, being the sink at the center of these rings. This problem is resolved by Lagrangian functions where the Lagrange coefficients are calculated as in Gandelli et al. (2007). Results indicate that the number of rings is critical in order to save energy, establishing the least number of them as the best solution.

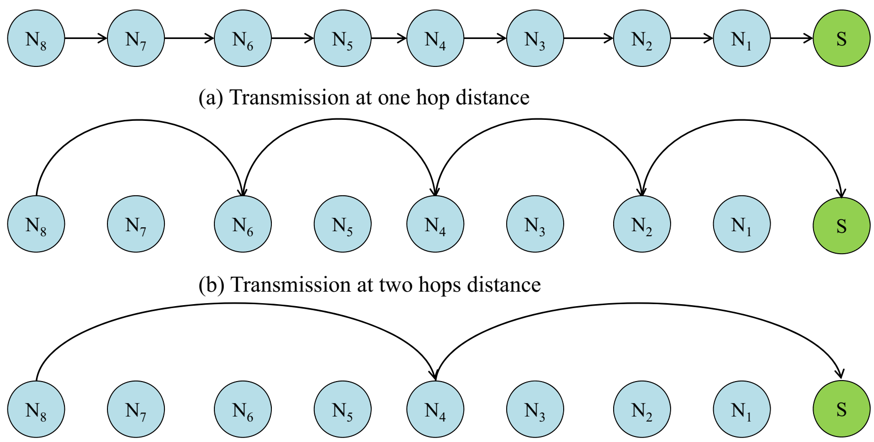
Within this first group, we also include papers with linear modeling for the power supplies, but a modification of the time variable transforms the energy consumption function in a nonlinear expression. In this case, paper (Naeem et al., 2015) presented an interesting result, since, in order to optimize the energy consumption, authors modeled a transmission power control for the transmitter systems as a min-max problem with high computation cost. The problem is resolved by two different methods, an iterative algorithm and another based on their own estimators. The first solution clearly offers more accurate values than using estimators but the results of both techniques converge when the number of nodes increases. The same research group presented an energy consumption optimization using cognitive radio (CR) (Naeem et al., 2013), proposing the scheduled transmission in time slots using free radio spectrum. The channel capacity of CR problems is modeled as an upper bounded logarithmic function, formulating a Non-Coherent-Mixed Integer Non Linear Problem (NC-MINLP). This problem is considered to be NP-hard which sharply increases the computational cost. To solve it, authors proposed a hybrid low-complexity estimation-of-distribution algorithm (EDA) based on a solution population, which is iteratively reduced until finding the minimal solution. This solution provides high flexibility to configure this type of problem. The work in Deng et al. (2012) is also reducing energy consumption and scheduling the CR spectrum use. The problem turns in NL by introducing scheduling in the management and utilization of spectrum through a decision making tool. Furthermore, by introducing CR, the utilization/non-utilization of spectrum is also a square root equation, and therefore nonlinear. The solution is obtained intuitively by a general framework which depending on the function shape indicates the optimal scheduling. Then, simulations to determine the exact optimal results are required. The curves obtained for the theoretical and simulation results are consistently compared.

An optimization problem including renewal power supply is presented in Shadmehr et al. (2012), in which authors minimized the energy consumed by coupling with a wired network. The goal is to recover energy and, therefore, to reduce the cost of energy transmission thanks to the coupling with wired networks that may be around. The model of energy recovery is complex and poses a square root equation. To solve this nonlinear problem, authors proposed to use a Genetic Swarm Optimization (GSO), which is deeply discussed in Gandelli et al. (2007). They verified that energy results indeed are renewed enough to achieve an acceptable network operation performance.

Finally, regarding this first group, the energy consumption minimization is also combined with the optimization of other design variables such as the communication channel utilization. Authors of Krishnamachari and Ordóñez (2003) introduced two non-linear optimization models that can be used to analyze the fundamental limits on the performance of information routing. They explored the influence of different fairness requirements of the network on the overall performance of the WSN by imposing constraints on the maximum percentage of the total information that each source node can send to the sink. Thus, in a completely fair WSN every node contributes with the same percentage of the total information to the sink. On the contrary, in a completely unfair WSN all the information to the sink could be from a single source node. The non-linearity of these two formulations comes from using the Shannon Theorem as one of the optimization constraints. Both flow-based formulations involve maximum information extraction, but one of them comprises the minimization of the total energy utilization. Results showed that higher fairness constraints result in a significant decrease in the information extraction and a higher energy consumption. In Haider and Yusuf (2009), a similar problem is presented but authors applied fuzzy logic to find optimal solutions. Results are similar to other papers analyzed in Section 6.

Although the non-linear models for the battery discharge commented above are specially relevant, the number of papers increases when additional optimization parameters are included in simple energy consumption models such as the ON/OFF one, resulting in a nonlinear problem. This second group of approaches assumes errors on the battery models, but they are non-critical since authors are not interested in instantaneous power consumption. Instead, in long term estimations, the energy values are not constrained by punctual energy consumptions and the average behavior is similar on both groups. This factor gives flexibility to introduce new design variables to convert a linear problem into an NL optimization. Concerning this group, different cases may be distinguished; (i) non-linearity by modeling physical conditions such as the propagation model, (ii) non-linearity caused by the traffic model, (iii) non-linearity by restricted transmission delays or real-time conditions, and (iv) non-linearity by network deployment conditions such as mobility or other topological requirements. The following paragraphs detail these cases and their main contributions to the improvement of the WSN operation.

In case (i), the characterization of new physical aspects such as the radio propagation issues (including the path-loss model), fading, modulation techniques or packet transmission losses is very usual. These aspects translate into an NL problem since they are well-known non-linear functions. Under these premises, we should highlight two representative papers, Farjow et al. (2012) and Egbogah and Fapojuwo (2013). The work in Farjow et al. (2012) analyzes routing optimization together with energy consumption minimization including optimization constraints in the problem such as fading, specific propagation model and modulation technique (NC-MFSK, non-coherent M-ary FSK). An evaluation of the transmitted power and energy consumption is conducted, based on the average distance of the nodes and the number of hops for a particular deployment. Due to its complexity, the problem is not directly resolved, but it is evaluated by means of an adaptive algorithm. Fig. 7 presents the advantage of modifying the power transmission, reaching to nodes far away from source (at several hops distance) and allowing the unrequested nodes to sleep thus saving energy, but increasing the computational optimization cost. In Fig. 7(a) all intermediate nodes between the source and the sink are involved in the packet retransmission, while in Fig. 7(c), only node 4 relays the packets to the sink, remaining the rest of nodes asleep. Paper (Egbogah



(a) Transmission at one hop distance

(b) Transmission at two hops distance

(c) Transmission at four hops distance

Fig. 7. Operational scheme of adaptive power transmission approaches.

and Fapojuwo, 2013) is different from the previous studies because it considers a WBAN (Wireless Body Area Network) intended for soldiers, in which the optimization of the set energy consumption, network utilization and payload size is derived. Then, packet losses are minimized. Authors presented a combined and well-known model, the Karush-Kuhn-Tucker (KKT), which models all these parameters at once. The result is obtained by the Lagrange operators, whose values are configured, on the one hand, by specific device settings, and on the other hand, by the study in Gandelli et al. (2007). According to their authors, this work reflects the high flexibility of these models for evaluating any variable figure of merit (such as transmission rate, delay, packet losses, etc).

For case (ii), non-linearity caused by the generated traffic model, multimedia traffic is usually the main constraint for the optimization problem. Under this condition, reference Li et al. (2015b) is a paper including multimedia traffic and mobile nodes. This complicates substantially the problem because it considers QoS (Quality of Service) requirements and communication channel propagation models (in particular, path-loss and fading). The problem proposed is NP-hard (hard nonlinear-polynomial) incurring in a high computational cost. To simplify the original problem, its equations are linearized and authors presented an iterative 'branch-and-bound' algorithm where, to accelerate its solving process, diverse problem-specific features are exploited (e.g., to analyze isolated pairs of nodes). It is worth mentioning the work in Wani and Rahnavard (2011), which includes WCSN (Wireless Camera Sensor Network) using the Imote2 device (Nachman et al., 2008). Its authors minimized the energy consumption and the images transmitted losses by reducing the images sampling (processed image). Different variables such as mutual coherence matrix are defined to evaluate the similarity between the acquired and processed images. In particular, this parameter imposes a non linear constraint. This image sampling reduction, with 30% fewer samples, makes the images look good enough.

Included in the same case (ii), a new non-linearity source appears when transmitted data is compressed. This technique is very useful to reduce the energy consumption in medium/high traffic transfers and thus extending the network lifetime. In this field, the work in Incebacak et al. (2015) implements a model for data compression (the optimal compression -OC- one) with three compression levels, which reduces the number of transmitted packets at the expense of charging the processing operation. The mathematical problem derives in a MIP (Mixed Integer problem) problem, which is resolved analytically with a freeware framework, the GAMS (General Algebraic Modeling System), compatible with 35 different solvers (specifications about them is not provided). The results obtained in terms of network lifetime reveals that the compression extends it in the order of 20%, but always in the case that the compression is performed when it is required. For instance, very small packets transmission does not require this task.

Regarding case (iii), non-linearity by restricted transmission delays or real-time conditions, two subcases may be further analyzed separately, (1) when delays are restricted by TDMA (Time Division Multiple Access) schemes, and (2) when delays or QoS requirements are imposed by the operation of the own application. Concerning the first subcase, we should highlight two papers, namely, Shi and Fapojuwo (2010) and Gu et al. (2011). Paper (Shi and Fapojuwo, 2010) deals with the minimization of the set energy and delay in clustered TDMA networks. In the formulation of the problem, FSK modulation and packet losses are introduced. The solution is obtained from heuristics that determine when the nodes can or cannot transmit. Results showed that the energy drops as the distance between nodes increases. The work in Gu et al. (2011) copes with minimizing the energy consumption and delay subject, as constraints, to routing techniques and TDMA medium access control. The resulting problem is again an MINLP. Then, carrying out different simplifications, the MINLP becomes a straightforward linear problem, but with a binary variable, which authors solved by a heuristic value called Relax and Sequential Fixing (RSF). The result benefits both the scheduling delay and energy consumed. Other relevant paper (Dedeoglu et al., 2012), conducts delay optimization combined with the minimization of packet loss in TDMA systems. The problem assumes that the data correlations in space and time are redundant, and can be reasonably minimized to improve energy consumption. As the energy model an ON/OFF scheme is chosen. The Slepian-Wolf method for correlated data is used leading to a convex problem. The results demonstrated that at the highest correlation, the most energy is consumed.

The second subcase is represented by different interesting studies. The work in Saifullah et al. (2014) is a draft from the University of Washington applied to the WirelessHart standard, where the energy optimization also takes into account the minimization of delays. This work exploits the control theory, which implies integral equations for the transmission rate calculation. To solve the problem, authors contributed with a method called subgradient based on the Lagrangian approach and the dual Lagrangian function. This method is refined with new mechanisms, the first is the use of Greedy heuristics, the second adds penalties for the bitrates that work best, and the third uses derivatives of convex optimization functions. Authors conducted simulations regardless of the topology, which is more expensive in computing but gives the best result. The convex method is slightly better than the rest according to those results. Also, we should mention papers (Rao et al., 2009; Li et al., 2015a), which present similar cases of application for real-time services (where it is assumed that there are restrictive requirements of QoS), combining

Table 3
Competitive comparison related to NL energy-saving optimization studies.

Paper	Year	Objective/Improvement	Non-linear parameter	Non-linear method	Approach	Results check	Topology	Computational complexity	Energy accuracy	Other parameter accuracy
Zhang et al. (2009)	2009	Energy Saving via an NL Energy Consumption model	Energy Consumption	NL Problem solved via Lagrange Operators	Analytical	No	Random	High	Low	Sensing (High)
Naeem et al. (2013)	2013	Energy Saving using CR Models	Energy Consumption	NC-MINLP (NP-hard)	Custom Algorithm	No	Random	High	High	N/A
Naeem et al. (2015)	2015	Energy Saving/Power Transmission	Energy Consumption	Min-Max Problem	Analytical/ Heuristic Estimator	Yes	Concentric Rings	High/Medium	High	Power Transmission (Medium)
Deng et al. (2012)	2011	Energy Saving using CR Models/Spectrum Use	Spectrum Use	NL (integer) Problem without Theoretical Solution	Graphical Overview	No	Random	Very high	Low	Spectrum use (Medium)
Shadmehr et al. (2012)	2012	Energy Saving/Power Coupling	Energy consumption	Proposed GSO	Analytical	No	Any	High	Medium	Power coupling (High)
Krishnamachari and Ordóñez (2003)	2003	Energy saving/Network Utilization	Information Delivered	NL problem Derived of the Energy Model	Custom Algorithm	No	Cluster	Low	Low	Network Utilization (Medium)
Haider and Yusuf (2009)	2009	Energy Saving/Network Utilization	Energy Consumption	Fuzzy Logic	Custom Algorithm	No	Cluster	Low	Unknown	Network Utilization (Low)
Farjow et al. (2012)	2012	Energy Saving using Routing and Transmission Power	Routing under fading conditions	NL Problem caused by Modulation/Noise Models	Simulation	Different Network Sizes	Random	Depends on the Network Size	High	Power Transmission (High)
Egbogah and Fapojuwo (2013)	2013	Energy Saving/Network Utilization/Payload Size	Energy Consumption/ Network Utilization/ Packet Size	NL non convex Problem solved via Karush-Kuhn-Tucker	Analytical	Yes	Cluster	High	High	Network Utilization (High)/Payload size (Medium)
Li et al. (2015b)	2015	Energy Saving/ Mobility/ Multimedia Traffic	Traffic model	Channel and Fading Models, NP-hard Problem	Custom Algorithm using Linear Bounds	Yes	Node-to-node	Medium	Low	Mobility (Low)/Traffic (Medium)
Wani and Rahnavard (2011)	2011	Energy Saving/Path Losses for Image tTaffic	Traffic Model	NL Problem solved via Coherence Matrix between Images	No solution, improvement via Sampling	No	Node-to-node	Low	Low	Path Losses Reduction (High)
Incebacak et al. (2015)	2016	Energy Saving	Traffic Model (compressed)	MIP Problem	Yes, analytically solved with Software GAMS	No	Random	High	High	Compression Level (low)
Shi and Fapojuwo (2010)	2010	Energy Saving/Delay Minimization	Transmission Delay	Multi-Objective Problem solved via Heuristic Estimators	Custom Algorithm	No	Linear, grid, random	Depend on the network size	High	Delay Minimization (Medium)
Gu et al. (2011)	2011	Energy Saving/Delay Minimization	Routing Schema	MINLP, own Heuristic (relax and sequential fixing)	Analytical	Yes	Cluster	Medium	High	Delay Minimization (High)
Dedeoglu et al. (2012)	2012	Energy Saving/Losses Reduction for Correlated Traffic	Transmission Delay/ Packet Losses	Slepian-Wolf Model/ Convex Problem	Analytical	Yes	Cluster	Low	Medium	Packet Losses Reduction (Medium)
Saifullah et al. (2014)	2011	Energy Saving/Delay Minimization	Transmission Rate	Control Scheduling/ Subgradient Method and Lagrangian Operators	Analytical/ Simulation	Yes	Not Defined	High	High	Delay Minimization (High)
Rao et al. (2009)	2009	Energy Saving/Network Utilization	Transmission Losses	Distributed Problem solved via Lagrange Operators	Analytical	No	Cells	High	Medium	Network Utilization (Low)
Li et al. (2015a)	2015	Energy Saving/Network Utilization	Transmission Losses	Joint Restricted Master Problem and Pricing Problem	Custom Algorithm	No	Cells	Low	High	Network Utilization (High)
Maleki et al. (2014)	2014	Energy Saving/ Transmission Power	Topology	MINLP	Simulation	No	Random	High	Low	Transmission Power (Low)
Yun and Xia (2010)	2010	Energy Saving	Transmission Delay	MINLP	Analytical	Two Models	Random	Medium	Medium	N/A

energy saving with the maximum utilization of the network (bitrate). The main difference is that the work in Li et al. (2015a) introduces in its model the bit error rate. However, the study in Rao et al. (2009) poses a quadratic problem solved by derivation obtaining the envelope of the bitrate function and later the coefficients of the Lagrange variables. The problem formulated in Li et al. (2015a) is iteratively solved by the execution of what is known as a ‘Master Restricted Problem’ algorithm (a restrictive problem that is solved with linear approximations executed many times), along with a ‘Pricing Problem’, which is a simpler and reproducible cost problem. The outcomes are compared with linear results, obtaining a more accurate WSN performance. The main drawback of both papers is the topology under consideration (nodes located in the vertexes of a hexagonal cell), which implies a very particular case (where even the traffic pattern is predefined). The particular node distribution facilitates the energy saving tasks, since it guarantees each node has always six neighbor nodes, which make easier to find a node which may receive the packets from the original node, thus alleviating the operations to search, find and wait for an available one.

Finally, case (iv) non-linearity by network deployment conditions is well represented by paper (Maleki et al., 2014). It includes a mixed wired/wireless topology, in the context of green communications, thereby necessarily optimizing energy consumption. The transmission power and the battery model are also modeled as a linear function, but authors included Euclidean distances because they analyzed a 2D deployment. An MINLP problem is posed and solved by simulation. Apart from the linear dependence on the number of nodes, results revealed that the wireless network wastes more energy than the wired one. In the same scenario as Gu et al. (2011), the work in Yun and Xia (2010) proposed another modeling technique. The different premise with Gu et al. (2011) is that time delays are disregarded. This offers again an MINLP problem, which is simplified using queuing models, though achieving suboptimal solutions. The results indicated that when delays are not included in the optimization process the lifetime increases in the same proportion as the number of nodes.

Some authors have addressed the optimization of the physical layer in WSN. Although similar to regular wireless networks, in WSN, as aforementioned, energy consumption is a primary design goal and, thus, optimizing the physical layer can lead to more efficient higher layers. In Holland et al. (2011), the authors proposed a method for finding the optimal physical layer parameters to minimize energy consumption in a multi-hop WSN. To conduct the optimization, authors defined an objective function consisting of a metric that specifies the energy per successfully received bit. This metric is a function of three physical layer parameters: hop distance, transmit energy, and modulation scheme. In addition, this latter metric also depends on the channel model. Thus, given a specific channel model and a constraint on any two of the three physical layer parameters, the optimization allows to determine the remaining physical layer parameters that will minimize energy consumption. Results showed some important facts such as:

- Using optimal transmitted energy and optimal relay distance is crucial in achieving energy efficiency.
- Optimizing the transmitted energy without optimizing the relay distance is not enough to achieve the best performance.
- It is preferable to overestimate the transmitted energy than to underestimate it.
- If the system operates at an optimum distance, then the transmitted and received energy become independent from the channel noise. This implies that if channel noise varies, it is only necessary to modify the operating distance without requiring to modify the transmitted energy.

5.2. Discussion and future research lines

Once the different works referred to this parameter are summarized in Table 3, we want to remark and discuss the most relevant aspects which must be considered when a WSN is being designed for optimal energy consumption. Under these premises, the readers/researchers should be able to (1) select the best parameter to be optimized, (2) choose the most convenient models for the optimization purpose, and (3) evaluate the employed technique/-s in order to achieve the required accuracy for the results. Therefore, the main aspects to be considered are described on the subsequent paragraphs as a guideline.

Currently, energy saving is demanded in almost every WSN deployment. This is due to, firstly, the objective itself, namely to extend the network lifetime and, secondly, the technological horizon based on green communications (Naeem et al., 2015). Indeed, the future employment of renewal power supplies is not enough to obviate this factor on the WSN design in order to separate the network operation and the energy supply behavior in all circumstances (Maleki et al., 2014). Therefore, designers should consider an appropriate model for the energy consumption which must be, in general cases, simplified as much as possible. This is recommended since complex energy consumption models do not guarantee the perfect supply behavior (Zhang et al., 2009), and, moreover, the long-term accuracy is usually similar on all of them (i.e., comparing results in Deng et al., 2012; Shi and Fapojuwo, 2010).

In general, a simplified energy consumption model may be combined with any other parameter to get the optimal network operation. The network utilization is the most common one (Krishnamachari and Ordóñez, 2003; Haider and Yusuf, 2009; Egbogah and Fapojuwo, 2013; Rao et al., 2009; Li et al., 2015b), but other parameters are also optimized such as the power transmission (Naeem et al., 2015; Farjow et al., 2012; Maleki et al., 2014), the transmission delay (Shi and Fapojuwo, 2010; Gu et al., 2013; Saifullah et al., 2014) or the packet losses reduction (Dedeoglu et al., 2012; Li et al., 2015b; Wani and Rahnavard, 2011). The network utilization maximization is currently used as the main parameter which improves the global network operation. Moreover, it is easy to model it by the sum/difference among input and output traffics for each link, however increasing the complexity of any other parameter or physical phenomena as the radio propagation noise (Li et al., 2015b). Transmission power optimization is a powerful tool since designers may find different transmission control power algorithms in the literature (Vales-Alonso et al., 2007) which may have been adapted to the optimization goals, thus achieving accurate results. These algorithms also help to reduce the interference inter nodes, thus improving globally the network performance. Network delay minimization is only required when deployment is designed for time-dependent applications, such as TDMA (Time Division Multiplexing Access) systems. However, the reduction of packet transmission losses is applied when traffic is especially sensitive to path losses, such as, multimedia traffic. In these cases, a modification of the transmission bitrate should be demanded in order to achieve acceptable results. Finally, many different parameters or physical phenomena can be modeled by different non-linear equations, depending on the specific application. Therefore, the simplified energy consumption model is required for all of them to achieve reasonable results with low-medium computational effort.

Concerning the results relevance, we have evaluated and compared four different techniques for the energy saving parameter. Indeed, we should note the work in Naeem et al. (2013) for its direct analytical results, reference Farjow et al. (2012) for relevant simulation results, Shi and Fapojuwo (2010) for using heuristic estimators, and finally (Li et al., 2015b) for an interesting customized algorithm proposal. The study related to the energy consumption per node can be found in Fig. 8, where the values are presented as a function of the number of hops to reach the sink node. To provide a realistic comparison, we have calculated the values which authors did not offer in their respective

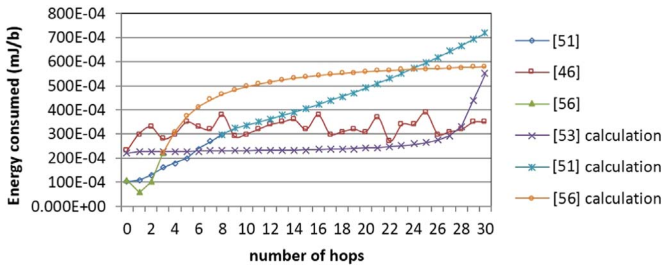


Fig. 8. Energy consumption per node (millijoules per bit) for four different optimization techniques.

work (Farjow et al., 2012; Shi and Fapojuwo, 2010; Li et al., 2015b) by running their provided formulas. All of them present good optimization results for the energy saving parameter as indicated in Table 3. We notice that some studies such as Naeem et al. (2013) and Shi and Fapojuwo (2010) reveal a more constant energy consumption regardless of the distance to the sink. The work in Shi and Fapojuwo (2010) explains this effect as a consequence of the linear topology employed, which seems an improbable case. However, the results in Naeem et al. (2013) are clearly more convincing but no other parameter has been studied. Even the energy consumption tendency showed in papers (Farjow et al., 2012) and (Li et al., 2015b) is more predictable. Reference Farjow et al. (2012) offers a clearly linear behavior, as a consequence of the simulation process employed. On the contrary, work (Li et al., 2015b) exhibits an asymptotic trend given by the estimator applied.

Considering the computational complexity, as a general paradigm, increasing the computational cost improves the quality and accuracy of results. The procedure to achieve the results (i.e. analytical, algorithm or simulation) does not modify this intuitive premise. Here, we notice that the high computational effort such as in Li et al. (2015b), Incebacak et al. (2015), is questionable for many local deployments. For instance, the computational (time and resources) cost to achieve the optimal solutions is not justified for many particular deployments on precision agriculture or industrial applications, where owners only demand results for a limited period of time. In this sense, high or very high computational cost is only required for long-term monitoring or multi-replicated applications. Under other circumstances, designers may opt for low-medium computational complexity techniques as specified in Table 3, which also provide feasible suboptimal solutions.

Analyzing Table 3, pure analytical approaches offer more accurate solutions. In particular, genetic algorithms (i.e., GSO) are a promising technique to obtain the optimal solution. Additionally, Lagrangian expressions using the Lagrange operators are also highly employed, thanks to the wide knowledge of them in the traditional mathematical literature. However, the complexity of these techniques usually fosters their substitution or complementary evaluation using other methods, such as specific authors algorithms or simulation framework proposals. The employment of heuristic estimators is also a remarkable tool to get close to the optimal results (suboptimal) with medium computational effort. It should be noted that we exclude the studies which employ linear simplifications or bounds (i.e. Rao et al., 2009; Li et al., 2015b; Yun and Xia, 2010) because they affect negatively the results accuracy.

As regards the solution technique, we highlight a general lack on the NL problems found in the literature. Besides, simulation frameworks are not commonly employed for results validation. Simulations are very useful tools since they not only validate analytical values, but also they may offer additional information about other essential parameters. Therefore, simulators help to analyze full parameters set. We should note studies (Saifullah et al., 2014; Naeem et al., 2015) which offer two different calculation techniques in order to validate results.

A final and optional aspect to consider in order to achieve an appropriate design is information redundancies. They are unavoidable in a WSN deployment. This is due to the limited computational resources of sensor nodes, which implies low information processing. This task is thus performed by the final service, which discards the redundant information, but at that moment, redundant transmitted packets have unnecessarily reduced the energy disposable on the network. In this regard, data fusion techniques (Liu et al., 2014) may be useful for minimizing this information redundancy before the NL problem is considered.

Finally, at this point, we want to propose different research lines to the reader which are still open for the improvement of the solution of the non-linear problems centered on the energy consumption minimization. The first one is obviously the generation or application of more efficient calculation techniques, which reduce computational complexity without degrading the results accuracy. The exploitation of new genetic algorithms (different from the typical GSO) or simplified version of them may facilitate the computational work. The second one is the integration of simulators in the plain optimization process. They can be introduced just to validate results, or even, to mix analytical and simulated operations. In this second case, they can provide faster results which may be further combined with numerical values obtained from the analysis. A third one is to simplify the physical models to facilitate their integration into the overall problem without affecting the parameters accuracy. In this sense, propagation, fading, path losses or noise models are susceptible of being simplified in order to achieve for instance, a convex expression, and a simpler computational case.

6. Non-linear routing and traffic rate optimization

Routing optimization in WSN is commonly associated to energy constraints or throughput maximization (Asorey-Cacheda et al., 2013). In this type of problems, the optimization goal may be the maximization of the transmission rate for every node or, given some other restriction, such as energy consumption or a fixed transmission rate, the minimization of the total amount of transmitted flows, which optimizes routing.

6.1. Classification

Typically, routing optimization involves only linear constraints but, depending on the problem, there may be cases where the optimization process uses a non-linear formulation. One of these cases can be found in Mohajerzadeh et al. (2011) in which its authors discussed an optimized routing protocol based on fuzzy variables. In this work, authors aimed to provide fairness in energy consumption, i.e. all nodes have the same lifetime, while trying to achieve the best possible routing. Fuzzy variables are used to implement environmental contention in the problem formulation.

Routing is also affected by sensor traffic rate generation. This topic is studied in Park (2015), in which the author developed a modeling framework for wireless sensor and actuator networks (WSAN) to optimize the traffic generation rate. The optimization problem for the proposed model minimizes the maximum outage probability while guaranteeing the schedulability constraint of the networks to achieve the maximum traffic rate. In this model, it is the probability of at least successfully transmitting a sensing measurement which causes this problem to be non-linear. This constraint is expressed as follows:

$$\gamma = 1 - \Omega_i \geq \left(p_{s,i} + (1 - p_{s,i}) p_{a,i}^{\frac{h_i - d - c}{t_i}} \right)^{\frac{\delta_i - d}{t_i}}, \tag{3}$$

where γ is the outage probability, $p_{s,i}$ and $p_{a,i}$ are the packet error probabilities of sensors and actuators, respectively, h_i is the sensor sampling interval, t_i is the controller update interval, c is the computation time, δ_i is the maximum allowable state update interval, d is the transmission delay, and Ω_i is the minimum probability with which the maximum allowable state update interval should be achieved. To solve this problem, the author substituted the constraint above by a linear approximation:

$$u \geq \frac{\delta_i - d}{t_i}\left(\log p_{s,i} + \frac{(1-p_{s,i})p_{a,i}^{b}}{p_{s,i}}\left(1 + \left(\frac{h_i - d_i - c}{t_i} - b\right)\log p_{a,i}\right)\right), \quad (4)$$

where $u = \ln\gamma$ and the exponentially increasing function $p_{a,i}^{(h_i-d-c)/t_i}$ is approximated by the power series at constant b.

By doing so, the non-convex non-linear optimization problem is approximated by a linear one. Results showed that optimum requires assigning more network resources to actuating links in order to guarantee system stability.

Video quality in sensor networks is affected by rate allocation. In Arar et al. (2015), authors proposed the optimization of both power and rate allocation for wireless video sensor networks. The optimization problem allocates the required power for compression and video transmission tasks as well as the channel resources (time slots and frequency bands) at each network node. The problem is similar to the one in Park (2015), in which the objective function is linear and only one of the constraints is non-linear, making this optimization problem a non-convex, non-linear one. To solve it, authors proposed to approach the non-linear constraint by a linear one and to solve the problem using a custom algorithm. As expected, results showed that allocating power to the encoding or transmission processes depends on the channel conditions.

It should be also mention paper (Shokrzadeh et al., 2012), in which the traffic rate is predicted by means of estimations done for each node. Thereby, the traffic rate becomes a constraint in the routing optimization.

In Guo et al. (2014), the authors described the impact of the energy and lifetime in WSN on the establishment of a reliable network by using data aggregation. Data aggregation is a process by which data from multiple child nodes along the routing tree can be aggregated and compressed in order to reduce the communication load of the network. The compression process and the compression ratio is related to data correlation and redundancy. An example of how data aggregation works is depicted in Figs. 9 and 10. Fig. 9 shows a regular WSN in which the arrow width is proportional to the aggregated data flow (throughput) being transmitted. Fig. 10 represents the same example but using data aggregation. It can be observed that a significant reduction of the traffic load can be achieved when several flows are aggregated (if sensor data is correlated). However, due to the uncertainty of the compression ratio in different application scenarios, it is necessary to use an abstract parameter to denote the data reduction due to aggregation. Regarding the energy model, three consumption factors are considered: energy required to transmit data, energy required to receive data, and energy devoted to aggregate data. To achieve the optimization goals, authors proposed an NP-hard multi-objective optimization problem to maximize network lifetime and minimize energy consumption. Because this problem cannot be easily approached by a linearized version, authors used Discrete PSO (DPSO) optimization (Kennedy and Eberhart, 1997) to obtain the optimal solutions. Simulations showed that it is possible to determine optimal aggregation routes with lower energy consumption while considering aggregation cost, which extends network lifetime.

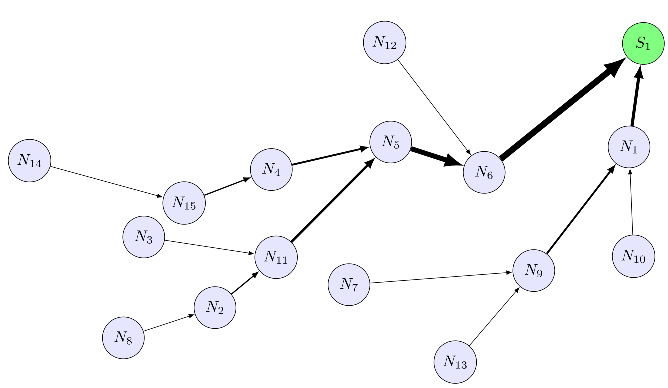


Fig. 9. Example of a WSN deployment without data aggregation (arrow width is proportional to data flow).

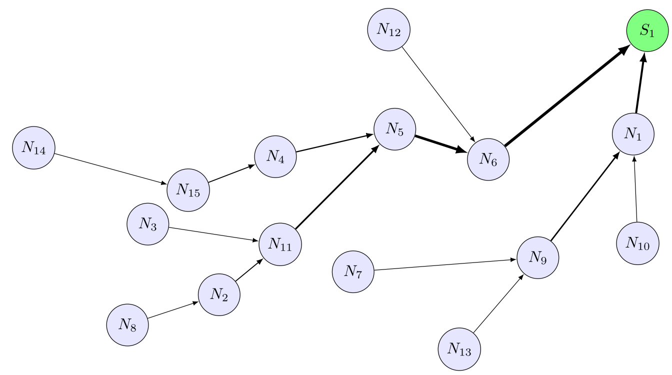


Fig. 10. Example of a WSN deployment using data aggregation (arrow width is proportional to data flow).

Regarding data aggregation trees, the authors of Lin et al. (2009) presented a non-linear mathematical formulation, whose goal is to minimize the total energy consumption of data transmission subject to data aggregation trees and data retransmissions. The proposed solving approach is based on the Lagrangian relaxation to construct a MAC aware energy-efficient data aggregation tree that jointly considers the tradeoff between data aggregation and data retransmission by using the Lagrangian multipliers.

Similar to the work presented in Guo et al. (2014), the authors of Rao et al. (2011) studied an integrated scheme to optimize the total system performance of real-time flows over a real time WSN by exploiting multi-path routing and dynamic sampling rate assignment. In this case, optimization consisted of a holistic optimization problem that derives the nonlinear objective function and linear constraints. In order to solve the problem, a distributed algorithm was developed based on the primal-dual method and dual decomposition technique. However, this method may produce potential oscillations in the assignments of path bitrates, which was solved by adding a quadratic term to the objective function of the optimization problem.

The work presented in Farjow et al. (2012) (also commented in Section 5) introduced a framework to optimize routing and reduce energy consumption. The main difference to other related works is that the algorithm contemplates the consumed energy per bit instead of the energy per node. The procedure used to minimize the energy per bit consist in assigning an appropriate modulation according to the transmission distance and the amount of data. This constraint is what makes this problem non-linear. To solve it, authors developed a custom algorithm which provides close-to-optimal solutions.

Traffic routing in WSN also presents security issues when it is required to hide the location of data sources and sinks. In Tapiador et al. (2011), the authors explored the problem of how to derive routing policies which minimize the path predictability whilst observing certain QoS restrictions. An example of the optimization goal can be observed in Fig. 11 in which data is relayed to multiple nodes in order to make it difficult for an external observer to determine the location of both transmitter and sink nodes. To achieve this, authors developed a non-linear optimization problem with linear constraints to derive efficient routing strategies. Thus, the optimization goal was to build aggregated flows yielding to maximum uncertainty (Shannon entropy) to an external observer while limiting end-to-end communication costs. Maximum entropy is achieved with a uniform distribution, which is equivalent to minimizing the standard deviation while minimizing packet transmissions. This optimization goal can be formulated as a classical least squares error optimization problem which can be solved by any of the well known available numerical techniques. Albeit using

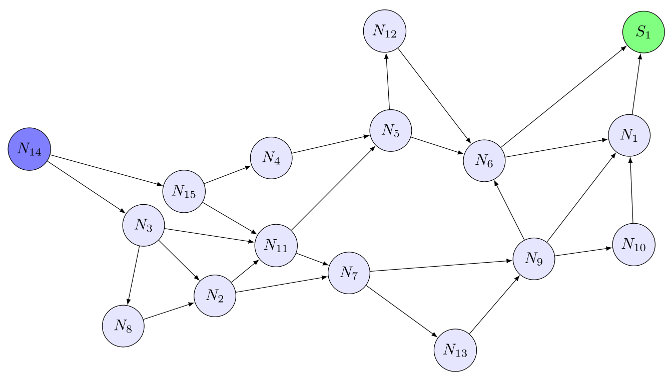


Fig. 11. Data transmission from N_{14} to S_1 in which the goal is to hide both transmitter and sink nodes.

aggregated flows may contribute to hide a node location it might produce other security issues that should be considered as well but are out of the scope of Tapiador et al. (2011).

The work in Chakraborty (2009) studied the problem of battery drain of those nodes closer to a sink. The best way to alleviate this issue is to use cluster-based routing. Despite this, clusters close to the sink still suffer from the same battery drain problems. To solve it, the author proposed an optimization framework that allowed to reconfigure clusters in order to avoid the battery drain of the closest nodes to the sink. The resulting problem is non-linear but the author derived a linear approximation that can be solved with classical methods. The main outcomes of this contribution was that there is only a solution when the cluster diameter is 6 (in terms of hop count) or less.

Routing in environments with node mobility is addressed in Gatzianas and Georgiadis (2008). The authors posed an optimization framework for a WSN with a mobile sink node. The idea behind it was that a mobile sink can extend network lifetime because mobility permits avoiding traffic saturation (and consequently battery depletion) of those nodes close to the sink because the network load is uniformly distributed among all nodes. The problem formulation produced a non-convex, non-linear problem that is cast into a simpler problem by using the Lagrange multipliers and removing some constraints in the original dual problem. Finally, the optimization problem was solved using a subgradient algorithm. Authors studied the efficiency of the proposed algorithm by means of simulations and demonstrated the accuracy of the method.

Energy consumption optimization has been studied by many authors. The work presented in Shi et al. (2014) explored a new paradigm. This paper presented a non-linear optimization framework for a wireless charging vehicle to extend lifetime of WSN using power transfer. The goal of this problem is to optimize the vehicle charging route cycle which leads to several integration and differentiation terms and makes it a non-polynomial optimization problem. To solve it, authors proposed an algorithm that provides an optimal solution in $N+1$ phases (where N is the number of network nodes).

6.2. Discussion and future research lines

As discussed along this section, most work dealing with routing or traffic rate optimization in WSN uses these objectives as a means of optimizing parameters that are not considered in the model such as the network lifetime, but being the primary goal in practice. Among others, there are some relevant points that should be mentioned here and summarized in Table 4:

- Most of the problems related to routing and traffic rate optimization fall into the non-convex, non-linear category. In some cases, these problems require the utilization of custom algorithms (Farjow et al., 2012; Mohajerzadeh et al., 2011; Rao et al., 2011; Shi et al., 2014). Other authors prefer to produce linearized versions of their optimization problems which can be solved using classical methods such as simplex (Park, 2015; Arar et al., 2015; Chakraborty, 2009). The authors in Lin et al. (2009) used Lagrangian relaxation to simplify the non-linear optimization problem and produce a linearized version. An exception is (Tapiador et al., 2011), this work describes a convex problem which is suitable for classical solving methods.
- All the work analyzed lead to optimal solutions (or suboptimal in case of using linear approaches). However, in Rao et al. (2011), under some conditions, optimization may produce oscillations in routing, not converging to an optimal solution. To overcome this issue, the authors introduced a quadratic term in the optimization problem producing less accurate results. Another exception can be found in Chakraborty (2009). In this case, the optimization problem can only be solved if the cluster sizes are limited to a maximum of 6 nodes.
- Most of the papers reviewed focus on network lifetime. One of the paradigms, which is specific of WSN, is data aggregation (Guo et al., 2014; Lin et al., 2009; Rao et al., 2011). Data aggregation may drastically reduce transmissions but at the expense of adding network load to those nodes at the end of the transmission tree. This extra load may have a significant impact on node energy consumption and make data aggregation useless. Moreover, data aggregation depends on the type of data being transmitted. Thus, the data aggregation level cannot be predicted beforehand. In some cases, if data correlation is low, data aggregation benefits may be negligible. Data aggregation optimization poses some interesting questions that, depending on other issues, such as, among others, network topology, energy constraints or other complementary WSN optimization goals, may lead to significantly complex non-linear optimization problems.
- The authors in Tapiador et al. (2011) focused their work on security issues. There are situations in which it is important to hide a node localization to external attackers. In this scenario, an attacker seeks to locate the source or the sink nodes. Attacking a relay node may be successful but WSN are usually redundant and, in a general case, a new route can be found. Solution to these problems consists in distributing traffic among all nodes in the WSN so that link utilization is uniform and makes it difficult for an external observer to determine the origin or destination of the data traffic. However, this approach presents some drawbacks such as larger energy consumption and more bandwidth utilization, and it is contrary to the data aggregation paradigm in which transmissions are based on network trees. This question is analyzed in Tapiador et al. (2011), in which authors developed an optimization framework having into consideration security concerns when using data aggregation. Although results are promising, security is gaining momentum in modern WSN what may have a significant impact when developing systems to operate these networks.
- The development of routing schemes for WSN with mobile sinks as proposed by the authors of paper (Gatzianas and Georgiadis, 2008) leads to a new type of problems. Due to the complexity of the original problem, authors used a dual relaxation consisting of adding Lagrange multipliers and removing some constraints. Thus, only suboptimal solutions can be found. This new problem is solved by using a subgradient algorithm whose efficiency is analyzed by computer simulation.
- An uncommon optimization problem is presented in work (Shi et al., 2014). This paper described WSN using charging vehicles to extend network lifetime. It does not seem that in the short term there will be network devices implementing wireless charging technology but it is possible that in the mid and long term a considerable amount of devices will use it. The authors of Shi et al. (2014) assumed a terrestrial vehicle in their scenario. In the future, charging might be performed by other type of vehicles such as drones of any type.

Table 4
Routing Studies Comparison under non-linear optimization techniques.

Paper	Year	Objective	Non-linear parameter	Non-linear method	Approach	Results check	Topology	Node coordination	Energy consumption
Farjow et al. (2012)	2012	Bit Energy	Transmission power	Custom Algorithm	Yes	No	All	Centralized	Low
Mohajerzadeh et al. (2011)	2011	Routing Protocol	Fairness in network energy	Custom Algorithm	Yes	Yes	All	Distributed	Low
Park (2015)	2015	Traffic Rate	Outage probability	Approximation of a non-convex non-linear Problem to a linear one	Yes	No	All	N/A	Medium
Arar et al. (2015)	2015	Power Allocation	Power and traffic rate	Approximation of a non-convex non-linear Problem to a linear one	Yes	No	All	Centralized	Low
Shokrzadeh et al. (2012)	2012	Optimum network central point	Agent movement	Custom algorithm	Yes	Yes	All	Centralized	Low
Guo et al. (2014)	2014	Minimize Data Transmission using Data Aggregation	Energy	Non-linear Multi-Objective Problem approximated to a linearized version and solved using discrete Particle Swarm Optimization	Yes	Yes	Tree	Centralized	Low
Lin et al. (2009)	2009	Minimize Data Transmission using Data Aggregation	Energy	Lagrangian Relaxation	Yes	No	Tree	Distributed	Low
Rao et al. (2011)	2011	Multipath Routing and Dynamic Rate Assignment	Source rate	Custom Algorithm	Some Solutions may produce Oscillations	No	All	Distributed	Low
Tapiador et al. (2011)	2011	Minimize Path Predictability	Shannon entropy	Least Squares Error	Yes	No	All	Centralized	High
Chakraborty (2009)	2009	Cluster-based Routing	Transmission cost	Linear Approximation	Yes (for limited cluster sizes)	No	All	Centralized	Low
Gatzianas and Georgiadis (2008)	2008	Routing optimization with Mobile Sinks	Survivability time	Dual Relaxation using Lagrange Multipliers	Yes	Yes	All	Centralized	Low
Shi et al. (2014)	2014	Vehicle Charging Route	Renewable cycle	Custom Algorithm	Yes	No	All	Centralized	N/A

Routing and traffic rate optimization pose many challenges that require further research. Routing and traffic rate suffer mostly from scalability issues as the network size increases because more bandwidth has to be allocated for retransmissions and finding optimal routes becomes more difficult. This problem becomes more important if the WSN must optimize other parameters such as energy consumption as routing algorithms should distribute traffic uniformly across all nodes. In his context, there are many proposals in the literature than may contribute to solve these problems such as wireless cooperative diversity (Gomez-Cuba et al., 2012), but with a complexity that makes very difficult the development of custom algorithms.

Other relevant issues are related to security as it is generally not considered in optimization but is important for questions such as secure data relaying, node hiding, energy draining or jamming (Caviglione and Merlo, 2012). All these issues should be integrated along with other optimization goals what might invalidate some previous results or require different routing and traffic allocation algorithms.

7. Conclusions

There are many works in the open literature dealing with WSN optimization. Large scale WSN pose many problems which make it very difficult to accomplish efficient and reliable network deployments. Commonly, these problems affect energy consumption, traffic routing, node deployment or node localization. In order to solve them, many authors have proposed different optimization frameworks and algorithms. Moreover, these solutions rely on linear models of WSN. Albeit this is enough in many cases, in others it is a limitation to achieve better results. Reasons behind this can be found in the inherent simplicity of linear optimization to provide a solution. However, in some cases these results are poor or the network model is not suitable for a linear model.

To face this concern, non-linear optimization provides better accuracy and, in some occasions, it is simply the only option that can be considered. This paper offers a survey on non-linear optimization problems for WSN. We analyze the most relevant contributions in recent years. The interest for this analysis relies on (i) identifying and categorizing the non-linear optimization problems in WSN following the premise that these categories are the most usual goals to be optimized (node positioning, node localization, energy saving, and traffic routing), and (ii) solving these problems by means of particular non-linear optimization techniques. Finally, this work shows readers/researchers open issues and future trends regarding WSN and non-linear optimization.

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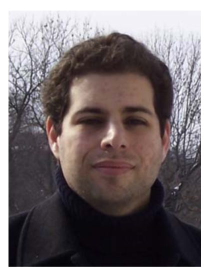

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