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A Study on the Impedance Calculation by using Equivalent Model in Catenary System

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Abstract

Electric railroad systems consist of rolling stock, track, signal and catenary system. In the catenary system, one of the most important factors is the impedance according to the design and characteristic. Before the catenary system is designed, the impedance should be precedently researched. The railroad catenary system is complex system which is composed by five conductors. The five conductors classify up and down feeders, up and down contact wire group, rail group. Therefore, we should compose the catenary system of the equivalent five-conductors model. In this paper, we suggest a geometrical model and a equivalent conductor model by using geometric mean radius of five conductors in the catenary system. Also, we calculate demanded parameter values in the model. By using those, line constants of five conductors are analyzed by applying the equivalent method called as the condensed joint matrix.

Keywords **:** *Impedance, Catenary system, Line constants, Conductor model, Electric railroad*

1. Introduction

A catenary system is the most important in the electric railroad. Impedance is a electrical factor of the catenary system and influenced upon the system characteristic. Therefore, impedance is estimated by preceding research. The catenary system is considered by the equivalent five-conductors model in the electrical aspect. Five-conductors consist of up and down feeders, up and down contact wire group, rail group. Contact and messenger wires are included in the contact wire group because of a conductor by using droppers. Also, rails, overhead protection wires and ground wires are included in the rail group due to common earth [1].

On account of existing five conductors, the impedance divides into the self impedance and mutual impedance. The self impedance defines a drop of electric pressure rates per unit length regarding the feedback current to the ground through the catenary. The mutual impedance defines the induced voltage rates from the current of a conductor to another conductor [2].

There is a modeling method from many conductors to one equivalent conductor by using the geometric mean radius. In case of rail group in the catenary system, distance between conductors is far. So, the method using the geometric mean radius is not able to be used.

This thesis shows the reducing process about the real system of the filed by using the proposed theory. Namely, the catenary system is showed by the matrix and applied to the condensed joint matrix. Also, the impedance of reduced system is provided.

2. Composition of Catenary System

Fig. 1 is geometric structures in the catenary system. As shown Fig. 1, up and down rails, overhead protection wires and ground wires are connected by the common earth [3]. As shown Fig. 2, the contact wire and messenger wire are connected at intervals of 4.50[m] or 6.75[m].

2.1 Self Impedance on Feeder and Wires

As shown Fig. 3, self impedance on a feeder and wires is calculated by using Carson law. The self impedance is gained by using length of imagined earth return path.

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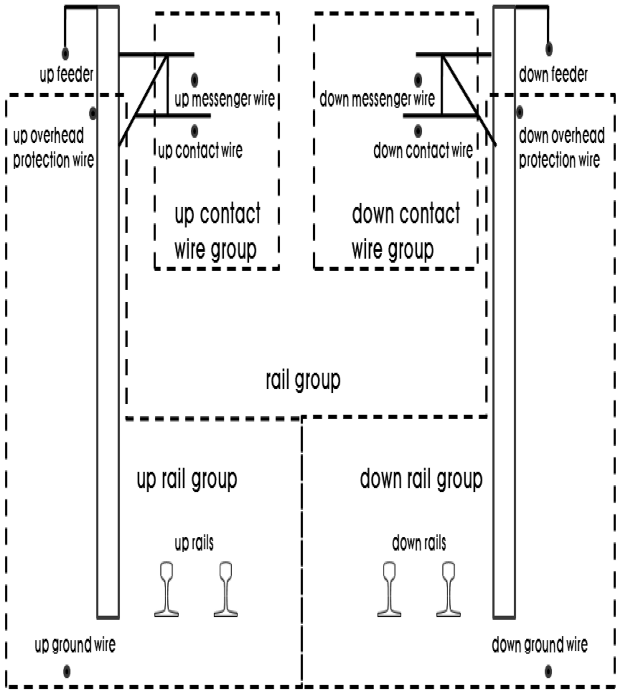

Fig. 1 Geometric structure in catenary system

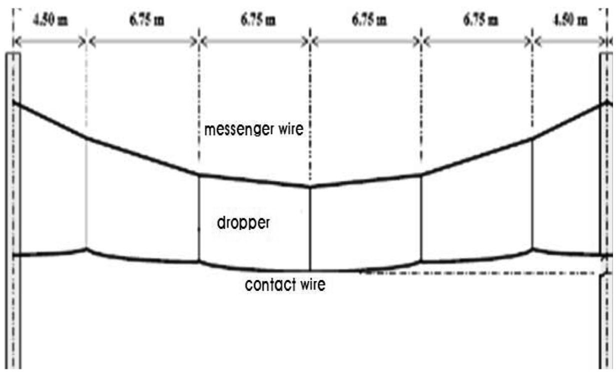

Fig. 2 Internals of droppers

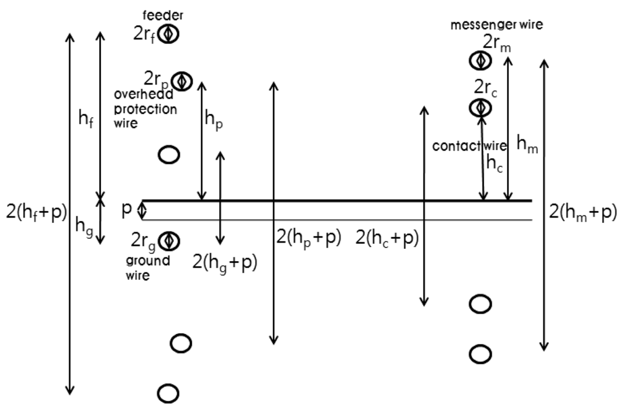

Fig. 3 Relation for self impedance among feeder, wires and earth

The complex depth of earth return method assumes that the current in conductor returns through an imagined earth path located directly under the original conductor at a depth of (h+2p) as shown Fig. 3, in which prime(') refers to the imagined earth return conductor of conductor and p to the skin depth of the ground [4].

The length of imagined earth return path is the sum of twice height from the earth to a conductor and the skin depth of ground. As shown (11), unit on the skin depth of ground is complex. So, the length of imagined earth return path is complex. Equation (1) ~ (5) are the self impedance on a feeder and wires [5].

$$Z_f = R_f + j\omega \frac{\mu_0}{2\pi} \ln \frac{2(h_f+\rho)}{r_f} \tag{1}$$

$$Z_c = R_c + j\omega \frac{\mu_0}{2\pi} \ln \frac{2(h_c+\rho)}{r_c} \tag{2}$$

$$Z_m = R_m + j\omega \frac{\mu_0}{2\pi} \ln \frac{2(h_m+\rho)}{r_m} \tag{3}$$

$$Z_p = R_p + j\omega \frac{\mu_0}{2\pi} \ln \frac{2(h_p+\rho)}{r_p} \tag{4}$$

$$Z_g = R_g + j\omega \frac{\mu_0}{2\pi} \ln \frac{2(h_g+\rho)}{r_g} \tag{5}$$

$$R_f = \frac{l_f}{\sigma_f S_f} \tag{6}$$

$$R_c = \frac{l_c}{\sigma_c S_c} \tag{7}$$

$$R_m = \frac{l_m}{\sigma_m S_m} \tag{8}$$

$$R_p = \frac{l_p}{\sigma_p S_p} \tag{9}$$

$$R_g = \frac{l_g}{\sigma_g S_g} \tag{10}$$

$$\rho = \sqrt{\frac{p}{j\omega\mu_0}} \tag{11}$$

R_f, R_c, R_m, R_p and R_g are the resistance on a feeder, contact, messenger, overhead protection and ground wire. Second term means the self inductance in (1). ω is the angular frequency and σ_f, σ_c, σ_m, σ_p and σ_g are the conductivity of a feeder, contact, messenger, overhead protection and ground wire. l_f, l_c, l_m, l_p and l_g are the length of a feeder, contact,

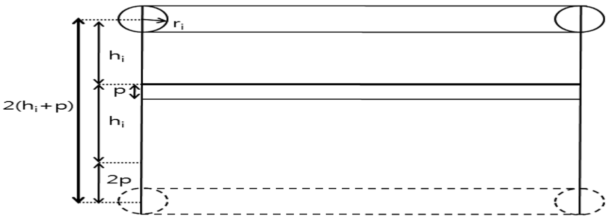

Fig. 4 Earth return path

messenger, overhead protection and ground wire. S_f, S_c, S_m, S_p and S_g are the cross section on a feeder, contact, messenger, overhead protection and ground wire. μ_0 is the permeability in free space.ρ is the skin depth of ground and r_f, r_c, r_m, r_p and r_g are the radius of a feeder, contact, messenger, overhead protection and ground wire. p is the resistance rate on ground. Fig. 4 is the imagined earth return path.

2.2 Self Impedance on Rail

Equation (12) is the resistance of a rail [6].

$$R_r = \frac{l_r}{\sigma_r S_r} \tag{12}$$

where l_r is the length of a rail and σ_r is the conductivity of a rail. S_r is the cross section on a rail. Equation (13) is self inductance on a rail by using the Ampere law [6].

$$L_r = \frac{l_r \mu}{8\pi} + \frac{l_r \mu_0}{2\pi}\ln\left(\frac{b_r - r_r}{r_r}\right) \tag{13}$$

where b_r is the distance between rails and r_r is the radius of cross section on a rail. μ is the permeability of a rail. Equation (14) is the self impedance on a rail.

$$Z_r = R_r + j\omega L_r \tag{14}$$

2.3 Mutual Impedance Between Feeder and Wires

Equation (15) ~ (24) are the mutual impedance between a feeder and wires with reference to Fig. 5 [7].

$$Z_{fc} = j\omega\frac{\mu_0}{2\pi}\ln\frac{\sqrt{(h_f+h_c+2\rho)^2+x_{fc}^2}}{\sqrt{(h_f-h_c)^2+x_{fc}^2}} = j\omega\frac{\mu_0}{2\pi}\ln\frac{D'_{fe}}{D_{fc}} \tag{15}$$

$$Z_{fm} = j\omega\frac{\mu_0}{2\pi}\ln\frac{\sqrt{(h_f+h_m+2\rho)^2+x_{fm}^2}}{\sqrt{(h_f-h_m)^2+x_{fm}^2}} = j\omega\frac{\mu_0}{2\pi}\ln\frac{D'_{fm}}{D_{fm}} \tag{16}$$

$$Z_{fp} = j\omega\frac{\mu_0}{2\pi}\ln\frac{\sqrt{(h_f+h_p+2\rho)^2+x_{fp}^2}}{\sqrt{(h_f-h_c)^2+x_{fp}^2}} = j\omega\frac{\mu_0}{2\pi}\ln\frac{D'_{fp}}{D_{fp}} \tag{17}$$

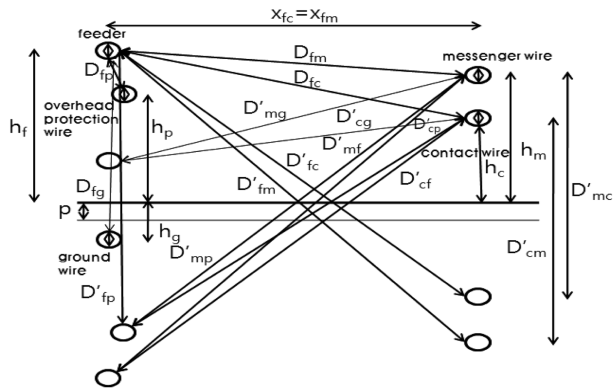

Fig. 5 Relation for mutual impedance among feeder, wires and earth

$$Z_{fg} = j\omega\frac{\mu_0}{2\pi}\ln\frac{\sqrt{(h_f+h_g+2\rho)^2+x_{fg}^2}}{\sqrt{(h_f-h_g)^2+x_{fg}^2}} = j\omega\frac{\mu_0}{2\pi}\ln\frac{D'_{fg}}{D_{fg}} \tag{18}$$

$$Z_{cm} = j\omega\frac{\mu_0}{2\pi}\ln\frac{\sqrt{(h_f+h_m+2\rho)^2+x_{cm}^2}}{\sqrt{(h_c-h_m)^2+x_{cm}^2}} = j\omega\frac{\mu_0}{2\pi}\ln\frac{D'_{cm}}{D_{cm}} \tag{19}$$

$$Z_{cp} = j\omega\frac{\mu_0}{2\pi}\ln\frac{\sqrt{(h_c+h_p+2\rho)^2+x_{cp}^2}}{\sqrt{(h_c-h_p)^2+x_{cp}^2}} = j\omega\frac{\mu_0}{2\pi}\ln\frac{D'_{cp}}{D_{cp}} \tag{20}$$

$$Z_{cg} = j\omega\frac{\mu_0}{2\pi}\ln\frac{\sqrt{(h_c+h_g+2\rho)^2+x_{cg}^2}}{\sqrt{(h_c-h_g)^2+x_{cg}^2}} = j\omega\frac{\mu_0}{2\pi}\ln\frac{D'_{cg}}{D_{cg}} \tag{21}$$

$$Z_{mp} = j\omega\frac{\mu_0}{2\pi}\ln\frac{\sqrt{(h_m+h_p+2\rho)^2+x_{mp}^2}}{\sqrt{(h_m-h_p)^2+x_{mp}^2}} = j\omega\frac{\mu_0}{2\pi}\ln\frac{D'_{mp}}{D_{mp}} \tag{22}$$

$$Z_{mg} = j\omega\frac{\mu_0}{2\pi}\ln\frac{\sqrt{(h_m+h_g+2\rho)^2+x_{mg}^2}}{\sqrt{(h_m-h_g)^2+x_{mg}^2}} = j\omega\frac{\mu_0}{2\pi}\ln\frac{D'_{mg}}{D_{mg}}$$

$$Z_{pg} = j\omega\frac{\mu_0}{2\pi}\ln\frac{\sqrt{(h_p+h_g+2\rho)^2+x_{pg}^2}}{\sqrt{(h_p-h_g)^2+x_{pg}^2}} = j\omega\frac{\mu_0}{2\pi}\ln\frac{D'_{pg}}{D_{pg}} \tag{24}$$

2.4 Mutual Impedance Among Feeder, Wires and Rails

As shown Fig. 6, the model for calculating mutual impedance is as below.

Return current which flows to the opposite direction of the feeders and wires is considered to calculate mutual impedance among feeder, wires and rails [8]. Equation (25)~(30) are the mutual impedance among a feeder,

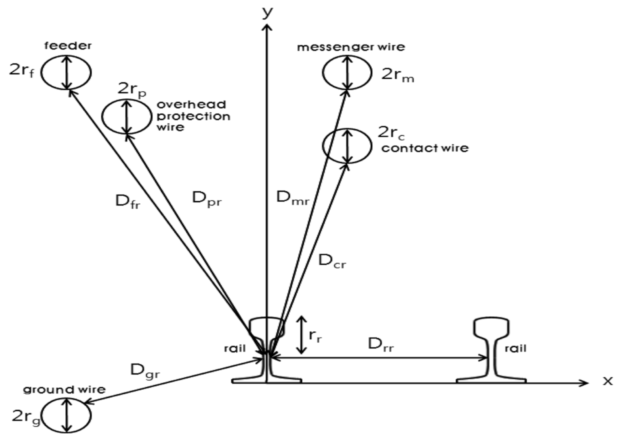

Fig. 6 Relation among feeder, wires and rails

wires and a rail with reference to Fig. 6.

$$Z_{fr} = j\omega\frac{\mu_0 l_r}{\pi}\ln\left(\frac{D_{fr}-r_f}{r_r}\right) \tag{25}$$

$$Z_{cr} = j\omega\frac{\mu_0 l_r}{\pi}\ln\left(\frac{D_{cr}-r_c}{r_r}\right) \tag{26}$$

$$Z_{mr} = j\omega\frac{\mu_0 l_r}{\pi}\ln\left(\frac{D_{mr}-r_m}{r_r}\right) \tag{27}$$

$$Z_{pr} = j\omega\frac{\mu_0 l_r}{\pi}\ln\left(\frac{D_{pr}-r_p}{r_r}\right) \tag{28}$$

$$Z_{gr} = j\omega\frac{\mu_0 l_r}{\pi}\ln\left(\frac{D_{gr}-r_g}{r_r}\right) \tag{29}$$

$$Z_{rr} = j\omega\frac{\mu_0 l_r}{\pi}\ln\left(\frac{D_{rr}-r_r}{r_r}\right) \tag{30}$$

3. Equivalent Circuit for Catenary System

There are three equivalent circuit models. One is the self impedance in equivalent conductor model. Another is the mutual impedance between conductor group and a conductor. The other is the mutual impedance between conductor group and conductor group [9].

3.1 Self Impedance in Equivalent Conductor Model

As shown Fig. 7, two conductors are replaced with a conductor. In other words, the self impedance and mutual impedance in two conductors are substituted for self impedance [9,10].

Equation (31) is the self impedance in case that the catenary system consists of the contact wire and messenger wire ($Z_1 \neq Z_2$).

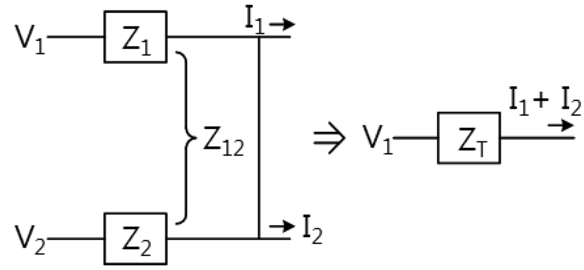

Fig. 7 Equivalent model for two conductors

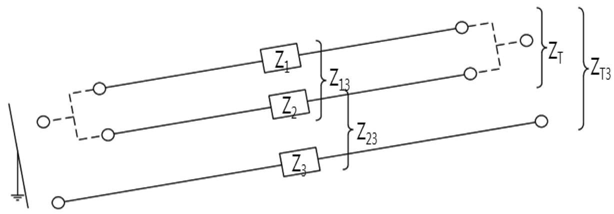

Fig. 8 Equivalent model between conductor group and conductor

$$Z_T = \frac{Z_c \cdot Z_m \cdot Z_{cm}^2}{Z_c + Z_m - 2Z_{cm}} \tag{31}$$

Equation (32) is the self impedance in case of rails ($Z_1 = Z_2$).

$$Z_T = \frac{Z_r + Z_{rr}}{2} \tag{32}$$

3.2 Mutual Impedance between Conductor Group and Conductor

Fig.8 is the equivalent model between conductor group and a conductor [9,11].

Equation (33) is the mutual impedance between the contact wire group which consists of a contact and messenger wire and a feeder.

$$Z_{T3} = \frac{Z_{cf}(Z_m - Z_{cm}) + (Z_{mf}(Z_c - Z_{cf}))}{Z_c + Z_m - 2Z_{cm}} \tag{33}$$

Equation (34) is the mutual impedance for the rails and feeder.

$$Z_{T3} = \frac{Z_{rf\_left} + Z_{rf\_right}}{2} \tag{34}$$

Z_{rf_left} is the mutual impedance between a rail located on left and a feeder. Z_{rf_right} is the mutual impedance between a rail located on right and a feeder.

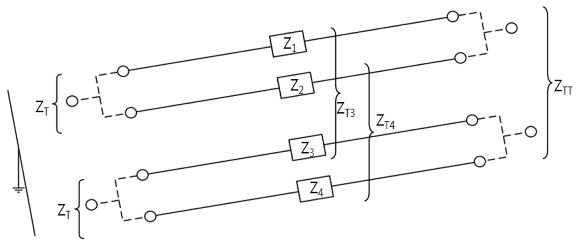

Fig. 9 Equivalent model between conductor groups

3.3 Mutual Impedance between Conductor Groups

Fig. 9 is the model for analyzing mutual impedance between conductor groups [9,12].

When there are the contact wire group and rails, (35) is the mutual impedance.

$$Z_{TT} = \frac{Z_{lr\_left}(Z_m - Z_{cm}) + Z_{lr\_right}(Z_c - Z_{cm})}{Z_c + Z_m - 2Z_{cm}} \tag{35}$$

Z_{lr_left} is the mutual impedance between the contact wire group and a rail located on left. Z_{lr_right} is the mutual impedance between the contact wire group and a rail located on right. Equation (36) is the mutual impedance between up rails and down rails.

$$Z_{TT} = \frac{Z_{rr\_1} + Z_{rr\_2} + Z_{rr\_3} + Z_{rr\_4}}{4} \tag{36}$$

Z_{rr_1} is the mutual impedance between a up rail located on left and a down rail located on left. Z_{rr_2} is the mutual impedance between a up rail located on left and a down rail located on right. Z_{rr_3} is the mutual impedance between a up rail located on right and a down rail located on left. Z_{rr_4} is the mutual impedance between a up rail located on right and a down rail located on right.

Fig. 10 shows the self and mutual impedance on the feeder, contact wire and messenger wire. As the contact wire and messenger wire is connected by the dropper, the electric potential is same.

Equation (37) is the relation between the voltage and current.

$$\begin{bmatrix} V_f \\ V_c \\ 0 \end{bmatrix} = \begin{bmatrix} Z_f & Z_{fc} & Z_{fc} - Z_{fm} \\ Z_{cf} & Z_c & Z_c - Z_{cm} \\ Z_{cf} - Z_{mf} & Z_c - Z_{mc} & Z_c - Z_{mc} \end{bmatrix} \begin{bmatrix} I_f \\ I_c + I_m \\ -I_m \end{bmatrix} \tag{37}$$

Kron law called as the condensed joint matrix is applied to remove factors in three by three. So, the impedance on the messenger wire is included on the self impedance on the equivalent contact wire group and mutual impedance between the feeder and contact wire. Equation (38) is the self impedance and mutual impedance on the equivalent conductor group [9,13].

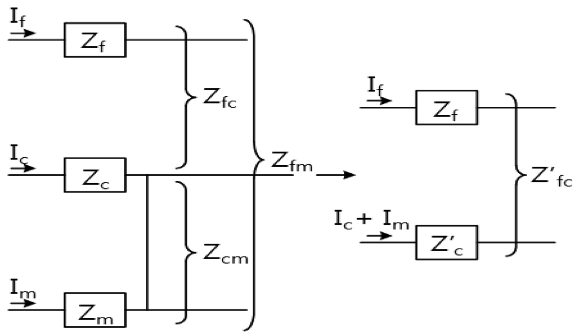

Fig. 10 Equivalent model between feeder and contact wire group

Table 1 Geometric Structure in Catenary System

Up line	X[m]	Y[m]
rail$_{_left}$	-2.87	0.6
rail$_{_right}$	-1.43	0.6
feeder	-3.75	8.06
contact wire	-2.15	5.8
messenger wire	-3.75	6.76
overhead protection wire	-5.28	6.1
ground wire	-6.15	-0.75
Down line	X[m]	Y[m]
rail$_{_left}$	1.43	0.6
rail$_{_right}$	2.87	0.6
feeder	3.75	8.06
contact wire	2.15	5.8
messenger wire	2.15	6.76
overhead protection wire	5.28	6.1
ground wire	6.15	0.75

$$\begin{bmatrix} V_f \\ V_c \end{bmatrix} = \begin{bmatrix} Z'_f & Z'_{fc} \\ Z'_{cf} & Z'_c \end{bmatrix} \begin{bmatrix} I_f \\ I_c + I_m \end{bmatrix} \tag{38}$$

$$Z'_f = Z_f - \frac{(Z_{cf} - Z_{mf}) + (Z_{fc} - Z_{fm})}{Z_c + Z_m - 2Z_{cm}} \tag{39}$$

$$Z'_c = Z_f - \frac{(Z_c - Z_{mc}) + (Z_c - Z_{cm})}{Z_c + Z_m - 2Z_{cm}} \tag{40}$$

$$Z'_{fc} = Z'_c - \frac{(Z_c - Z_{mc}) + (Z_{fc} - Z_{fm})}{Z_c + Z_m - 2Z_{cm}} \tag{41}$$

$$Z'_{cf} = Z'_{fc} \tag{42}$$

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feeder	3.75	8.06
contact wire	2.15	5.8
messenger wire	2.15	6.76
overhead protection wire	5.28	6.1
ground wire	6.15	0.75

Table 2 Conductor Characteristic

Parameter	Resistance (mΩ/m)	Material	Cross section (mm^2)	Radius (cm)
rail	0.126	60 kg/m	76.9	3.85
feeder	0.1173	Cu150 mm^2	150	0.68
contact wire	0.1173	Cu150 mm^2	150	0.68
messenger wire	0.4474	Bz65 mm^2	65.49	0.525
overhead protection wire	0.239	Cu75 mm^2	75.25	0.555
ground wire	0.484	Cu38 mm^2	37.16	0.39

4. Simulation

The impedance on the equivalent circuit model in the catenary system is estimated. The frequency is 60[Hz] and resistance rate on ground is 100[Ω·m]. As shown Fig. 1, the geometric structure on catenary system is Table 1 in the Cartesian coordinates [14].

Table 2 is the conductor characteristic.

Table 3 Conductor Characteristic

Para meter	Up feeder	Down feeder	Up messen ger wire	Up contact wire	Down messen ger wire	Down contact wire	Up rail_left	Up rail_right	Down rail_left	Down rail_right	Up overhead protection wire	Down overhead protection wire	Up ground wire	Down ground wire
Up feeder	0.570	0.563	0.463	0.453	0.573	0.563	0.563	0.553	0.543	0.543	0.444	0.543	0.523	0.553
Down feeder	0.563	0.570	0.573	0.573	0.483	0.453	0.543	0.543	0.563	0.553	0.543	0.444	0.553	0.523
Up messen ger wire	0.463	0.573	1.051	0.513	0.404	0.404	0.573	0.573	0.563	0.563	0.424	0.563	0.553	0.553
Up contact wire	0.453	0.573	0.513	0.912	0.404	0.404	0.553	0.553	0.563	0.573	0.424	0.563	0.553	0.563
Down messen ger wire	0.573	0.483	0.404	0.404	1.051	0.513	0.553	0.563	0.573	0.573	0.563	0.424	0.553	0.553
Down contact wire	0.563	0.453	0.404	0.404	0.513	0.912	0.563	0.573	0.553	0.553	0.563	0.424	0.563	0.553
Up rail_left	0.563	0.543	0.573	0.553	0.553	0.563	2.640	0.454	0.553	0.404	0.573	0.543	0.543	0.424
Up rail_right	0.553	0.543	0.573	0.553	0.563	0.573	0.454	2.640	0.404	0.454	0.573	0.563	0.563	0.593
Down rail_left	0.543	0.563	0.563	0.563	0.573	0.553	0.553	0.404	2.640	0.454	0.543	0.573	0.424	0.543
Down rail_right	0.543	0.553	0.563	0.573	0.573	0.553	0.404	0.454	0.454	2.640	0.553	0.573	0.593	0.563

Table 3 Continued

Para meter	Up feeder	Down feeder	Up messen ger wire	Up contact wire	Down messen ger wire	Down contact wire	Up rail_left	Up rail_right	Down rail_left	Down rail_right	Up overhead protection wire	Down overhead protection wire	Up ground wire	Down ground wire
Up overhead protection wire	0.444	0.543	0.424	0.424	0.563	0.563	0.573	0.573	0.543	0.553	1.047	0.553	0.523	0.573
Down overhead protection wire	0.543	0.444	0.563	0.563	0.424	0.424	0.543	0.563	0.573	0.573	0.553	1.047	0.573	0.523
Up ground wire	0.523	0.553	0.553	0.553	0.553	0.563	0.543	0.563	0.424	0.593	0.523	0.573	0.993	0.323
Down ground wire	0.553	0.523	0.553	0.563	0.553	0.553	0.424	0.593	0.543	0.563	0.573	0.523	0.323	0.993

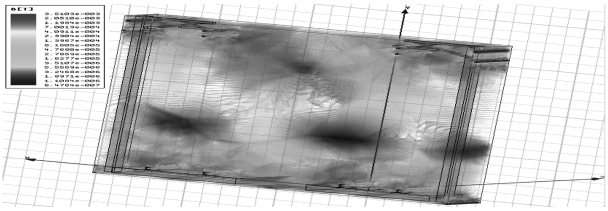

Fig. 11 Catenary system model in Maxwell program

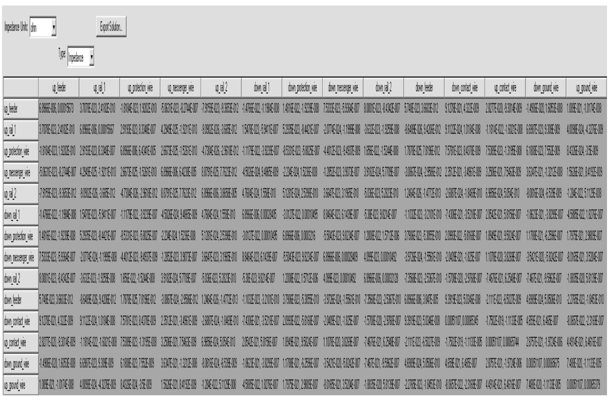

Fig. 12 Simulation results

Table 3 is the results of the amplitude of impedance by using impedance equations in reference to Table 1 and Table 2.

Fig.11 is the catenary system model in Maxwell program with reference to Table 1 and Table 2. Fig.12 is the results of Maxwell program [15].

As compared between Table 3 and simulation results, error rate is about 7~10[%]. Table 4 is the results of self and mutual impedance on five-conductor in the equivalent circuit model.

Table 4 Impedance in Equivalent Model

Para meter	Up contact wire group	Up feeder group	Rail group	Down contact wire group	Down feeder group
Up contact wire group	0.751	0.444	0.374	0.404	0.374
Up feeder group	0.444	0.927	0.365	0.374	0.365
Rail group	0.374	0.365	0.482	0.374	0.365
Down contact wire group	0.404	0.374	0.374	0.751	0.444
Down feeder group	0.374	0.365	0.365	0.444	0.927

5. Conclusion

In this paper, the geometric structure in the catenary system is presented and a equivalent conductor is suggested by using matrix including five-conductor in the catenary system.

Error rates between numerical analysis and simulation analysis is about 7~10[%] due to the earth return method. The impedance on rails is the highest and the impedance on a feeder is the lowest in the conductor. While the impedance on feeder group is the highest and the impedance on a rail group is the lowest in the conductor group. As feeder group is connected by a series circuit, the impedance on feeder group is high. As rail group is connected by a parallel circuit, the impedance on rail group is low. This thesis is applied to the line constant calculation in the catenary system, current distribution, a drop of electric pressure and harmonic analysis.

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